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PHYSICS

for IIT-JEE 2012-13

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Exercises

MECHANICS I

B.M. Sharma



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CENGAGE
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**Physics for IIT-JEE 2012-13:
Mechanics I**

B.M. Sharma

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Preface

Since the time the IIT-JEE (Indian Institute of Technology Joint Entrance Examination) started, the examination scheme and the methodology have witnessed many a change. From the lengthy subjective problems of 1950s to the matching column type questions of the present day, the paper-setting pattern and the approach have changed. A variety of questions have been framed to test an aspirant's calibre, aptitude, and attitude for engineering field and profession. Across all these years, however, there is one thing that has not changed about the IIT-JEE, i.e., its objective of testing an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grass-root level.

No subject can be mastered overnight; nor can a subject be mastered just by formulae-based practice. Mastering a subject is an expedition that starts with the basics, goes through the illustrations that go on the lines of a concept, leads finally to the application domain (which aims at using the learnt concept(s) in problem-solving with accuracy) in a highly structured manner.

This series of books is an attempt at coming face-to-face with the latest IIT-JEE pattern in its own format, which is going to be highly advantageous to an aspirant for securing a good rank. A thorough knowledge of the contemporary pattern of the IIT-JEE is a must. This series of books features all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion reason type, matrix match type, or paragraph-based, thought-type questions. Not discounting to need for skilled and guided practice, the material in the book has been enriched with a large number of fully solved concept-application exercises so that every step in learning is ensured for the understanding and application of the subject.

This whole series of books adopts a multi-faceted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant. Each book in the series has a sizeable portion devoted to questions and problems from previous years' IIT-JEE papers, which will help students get a feel and pattern of the questions asked in the examination. The best part about this series of books is that almost all the exercises and problem have been provided with not just answers but also solutions.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which an aspirant must follow to accomplish success in IIT-JEE.

B. M. SHARMA

Basic Mathematics

- Elementary Algebra
- Elementary Trigonometry
- Basic Coordinate Geometry
- Differentiation
- Integral of a Function
- Application in Physics

Mathematics is the supporting tool of physics. The elementary knowledge of basic maths is useful in problem solving in physics. Basic knowledge of elementary algebra, trigonometry, coordinate geometry and basic calculus is must before going into the depth of physics.

ELEMENTARY ALGEBRA

Common Formulae

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
4. $(a + b)(a - b) = a^2 - b^2$
5. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
6. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
7. $(a + b)^2 - (a - b)^2 = 4ab$
8. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

Polynomial, Linear, and Quadratic Equations

Real Polynomial

Let $a_0, a_1, a_2, \dots, a_n$ be real numbers and x a real variable, then $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ is called a *real polynomial*.

Degree or Index of a Polynomial

The highest power appearing in a polynomial is called its degree. For example, $f(x) = x^3 + 8x + 3$ is a polynomial of degree 3.

Students must note here that it is not necessary that the highest power must be of a single variable only. For example, $f(x) = 3x^2y + y^2 + 2$ is a polynomial of degree 3 because of variable y in the term x^2y . We add powers of the variables in a term to find degree of a polynomial irrespective of the nature of variables. Thus, in the present case, x^2y is having power of $2 + 1 = 3$ and hence the degree of the given polynomial is 3.

Linear Equations

Equations having polynomials of unit degree are called linear equations, e.g., $x + y = 2$ or $2x + 3 = 5$. Such equations always represent a straight line on a graph.

Quadratic Equations

Equations of second degree are called quadratic equations. The general form of a quadratic equation is as given below:

$$ax^2 + bx + c = 0, \text{ where } a \neq 0.$$

Roots of a Quadratic Equation

Solutions of a quadratic equation are called its roots. Roots are those values of a variable such as x for which the given quadratic equation collapses to zero. As a rule, a quadratic equation always has two roots which may or may not be equal.

Roots of a quadratic equation are generally represented by α and β .

Let $ax^2 + bx + c = 0$ be a quadratic equation. Then:

1. Its roots are $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$; $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.
2. Hence, its solution is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
3. Sum of its roots is given by $\alpha + \beta = \frac{-b}{a}$.
4. Product of its roots is given by $\alpha\beta = \frac{c}{a}$.
5. Difference of its roots is given by $\alpha - \beta = \frac{\sqrt{b^2 - 4ac}}{a}$.

Binomial Expression and Theorem

An algebraic expression containing two terms is called a *binomial expression*.

For example, $(a + b)$, $(2x - 3y)$, $\left(x + \frac{1}{y}\right)$, $\left(x + \frac{3}{x}\right)$, etc., are binomial expressions.

Binomial Theorem for Positive Integral Index

The general form of a binomial expression is $(x + a)^n$, where n is any positive integer (called index) and x and a are real numbers.

Binomial theorem states:

$$(x + a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} \cdot a^1 + {}^nC_2 x^{n-2} \cdot a^2 + \dots + {}^nC_r x^{n-r} \cdot a^r + \dots + {}^nC_n \cdot a^n$$

where ${}^nC_r = \frac{n!}{r!(n-r)!}$ and $n! = n(n-1)(n-2) \dots 3 \times 2 \times 1$, the product of first n natural numbers [e.g., $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$].

Important Points

- Total number of terms in the expansion = $(n + 1)$.
- In every successive term in the expansion, the power of x goes on decreasing by 1 and that of a increasing by 1, so that the sum of the powers of x and a in each term is always equal to n .
- ${}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n$ are called binomial coefficients.
- The expression $n!$ is read as "factorial n ". So,

$${}^nC_1 = \frac{n!}{1!(n-1)!} = \frac{n(n-1)(n-2) \dots 3 \times 2 \times 1}{1(n-1)(n-2) \dots 3 \times 2 \times 1} = n.$$
 Similarly, ${}^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2 \times 1}, \dots, {}^nC_n = 1.$

Binomial Theorem for Any Index

If n is positive, negative or fraction and x is any real number such that $-1 < x < 1$, i.e., x lies between -1 and $+1$, then according to the binomial theorem

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \infty \text{ terms.}$$

Note:

- If n is a positive integer, then the expansion will have $(n + 1)$ terms.
- If n is a negative integer or a fraction, then the number of terms in the expansion will be infinite, i.e., there will be no last term.
- If $|x| \ll 1$, then only first two terms of the expansion are significant. It is so because the values of the second and higher order terms become very small and can be neglected. Thus, in this case, the binomial expansion reduces to the following simplified forms when $|x| \ll 1$:

$$(1+x)^n = 1+nx, \quad (1+x)^{-n} = 1-nx, \\ (1-x)^n = 1-nx, \quad \text{and} \quad (1-x)^{-n} = 1+nx.$$

Illustration 1.1 Calculate $(1001)^{1/3}$.

Sol. We can write 1001 as: $1001 = 1000 \left(1 + \frac{1}{1000}\right)$, so that we have

$$\begin{aligned} (1001)^{1/3} &= \left[1000 \left(1 + \frac{1}{1000}\right)\right]^{1/3} = 10 \left[1 + \frac{1}{1000}\right]^{1/3} \\ &= 10(1 + 0.001)^{1/3} = 10\left(1 + \frac{1}{3} \times 0.001\right) \\ &= 10.003333 \end{aligned}$$

Illustration 1.2 Expand $(1+x)^{-3}$.

$$\begin{aligned} \text{Sol. } (1+x)^{-3} &= 1 + (-3)x + \frac{(-3)(-3-1)x^2}{2!} \\ &\quad + \frac{(-3)(-3-1)(-3-2)x^3}{3!} + \dots \\ &= 1 - 3x + \frac{12}{2}x^2 - \frac{60}{3 \times 2}x^3 + \dots \\ &= 1 - 3x + 6x^2 - 10x^3 + \dots \end{aligned}$$

Concept Application Exercise 1.1

1. Expand $(1+x)^{-2}$.
2. Using binomial expansion, simplify the following expression: $Q \left[\left(1 + \frac{\Delta x}{x}\right)^3 - 1 \right]$, assuming Δx to be small in comparison to x .

ELEMENTARY TRIGONOMETRY**System of Measurement of an Angle**

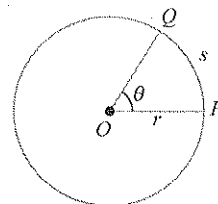
There are different types of measurement of an angle.

1. **Sexagesimal system:** In this system,
1 right angle = 90° (degree); 1 degree = $60'$ (minutes)
1 minute = $60''$ (seconds)

2. **Centesimal system:** In this system,
1 right angle = 100 grades (100 g)
1 grade = 100 minutes ($100'$)
1 minute = 100 seconds ($100''$)
3. **Circular system:** In this system, angle is measured in radian.
 π radians = 180°

Consider a particle moves from a position P to position Q along a circle of radius r with centre at O (see Fig. 1.1). Then,

$$\text{Angle } \theta = \frac{\text{Arc length } PQ}{\text{Radius of circle}} = \frac{s}{r} \Rightarrow s = r\theta$$

**Fig. 1.1**

If the length of arc PQ = radius of circle r , then $\theta = 1$ radian.

Radian

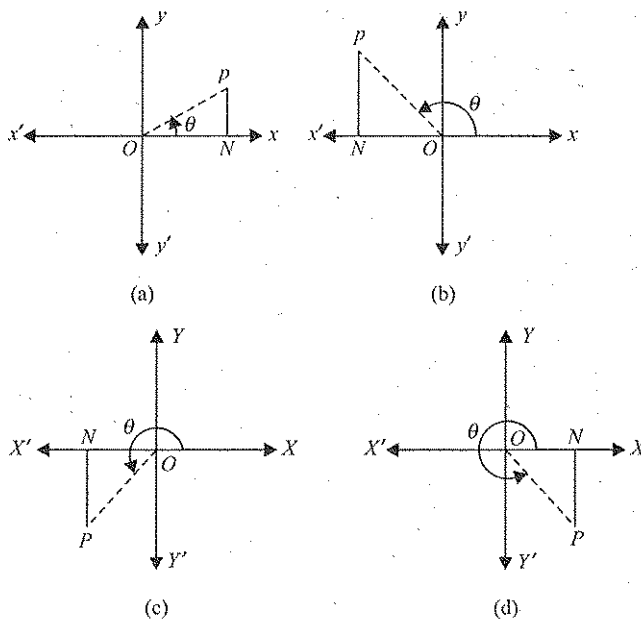
When a body completes one revolution, then $\theta = 2\pi$ rad.

$$\therefore 2\pi \text{ rad} = 360^\circ \text{ or } 2 \times 3.14 \text{ rad} = 360^\circ$$

$$\Rightarrow 1 \text{ rad} = \frac{360^\circ}{2 \times 3.14} = 57.3^\circ$$

Four Quadrants and Sign Conventions

Consider two mutually perpendicular lines intersecting at O . These two mutually perpendicular lines divide the plane into four parts called quadrants (see Fig. 1.2).

**Fig. 1.2**

Points to Remember

1. To determine the sign of a trigonometrical ratio in any quadrant, OP will be taken as positive in all four quadrants.
2. In first quadrant, all trigonometrical ratios are positive (see Fig. 1.3).

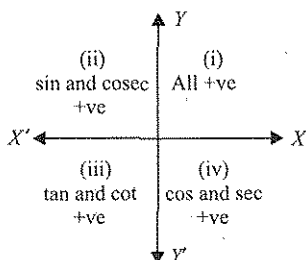


Fig. 1.3

3. In second quadrant, only $\sin \theta$ and $\operatorname{cosec} \theta$ are positive.
4. In third quadrant, only $\tan \theta$ and $\cot \theta$ are positive.
5. In fourth quadrant, only $\cos \theta$ and $\sec \theta$ are positive.
6. The value of $\sin \theta$ and $\cos \theta$ are such that $-1 \leq \sin \theta \leq 1$ and $-1 \leq \cos \theta \leq 1$.
7. But $\tan \theta$ and $\cot \theta$ can take any real value.

The Graphs of sin and cos Functions

The function $y = \sin x$, where x is any dimensionless quantity, is called a sine function. The argument x is usually measured in radian. The function $y = \sin x$ is plotted in Fig. 1.4(a). The maximum positive and negative values of a sine function are +1 and -1, respectively. Between $x = 0$ and $x = \pi$, the function is positive, the peak value of +1 occurs at $x = \frac{\pi}{2}$. Similarly, for the interval $x = \pi$ to $x = 2\pi$, the function is negative and the negative peak occurs at $x = \frac{3\pi}{2}$. The sine function is a periodic function, with a period of 2π . That is, the pattern of the graph repeats itself after an interval of 2π . Mathematically, it may be stated as $y = \sin x = \sin(2\pi + x) = \sin(4\pi + x) = \dots$ and so on.

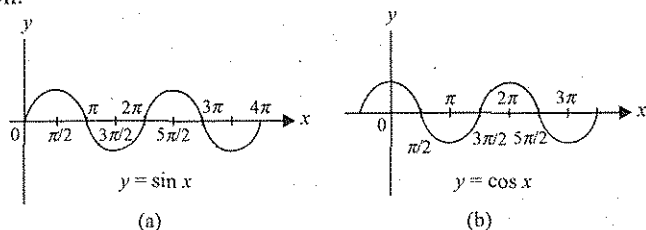


Fig. 1.4

If the graph of the sine function is displaced to the left through $\frac{\pi}{2}$, we get the graph of cosine function: $y = \cos x$ as shown in Fig. 1.4(b). The cosine function is also a periodic function with a period of 2π .

Some Important Trigonometric Formulae

1. a. $\sin^2 \theta + \cos^2 \theta = 1$
b. $1 + \tan^2 \theta = \sec^2 \theta$

2. Addition and subtraction formulae:

- a. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- b. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- c. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

3. Multiple formulae:

- a. $\sin 2A = 2 \sin A \cos A$
- b. $\cos 2A = \cos^2 A - \sin^2 A$
- c. $\cos 2A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$
- d. $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Trigonometrical Ratios of Allied Angles

The angles whose sum or difference with angle θ is zero or a multiple of 90° are called angles allied to θ . Commonly used T -ratios of some of the allied angles are given below.

1. $\sin(-\theta) = -\sin \theta$
 $\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta$
 $\cos(-\theta) = \cos \theta$
 $\sec(-\theta) = \sec \theta$
 $\tan(-\theta) = -\tan \theta$
 $\cot(-\theta) = -\cot \theta$

Note:

- As angle $-\theta$ lies in the fourth quadrant, only $\cos \theta$ and $\sec \theta$ are positive, e.g., $\sin(-30^\circ) = -\sin 30^\circ$ and $\cos(-30^\circ) = +\cos 30^\circ$.

2. $\sin(90^\circ - \theta) = \cos \theta$
 $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$
 $\cos(90^\circ - \theta) = \sin \theta$
 $\sec(90^\circ - \theta) = \operatorname{cosec} \theta$
 $\tan(90^\circ - \theta) = \cot \theta$
 $\cot(90^\circ - \theta) = \tan \theta$

Note:

- As angle $(90^\circ - \theta)$ lies in first quadrant, all T -ratios are positive, e.g., $\sin 30^\circ = \sin(90^\circ - 60^\circ) = \cos 60^\circ$.

3. $\sin(90^\circ + \theta) = \cos \theta$
 $\operatorname{cosec}(90^\circ + \theta) = \sec \theta$
 $\cos(90^\circ + \theta) = -\sin \theta$
 $\sec(90^\circ + \theta) = -\operatorname{cosec} \theta$
 $\tan(90^\circ + \theta) = -\cot \theta$
 $\cot(90^\circ + \theta) = -\tan \theta$

Note:

- As angle $(90^\circ + \theta)$ lies in 2nd quadrant, therefore only $\sin \theta$ and $\operatorname{cosec} \theta$ are positive, e.g., $\sin 120^\circ = \sin(90^\circ + 30^\circ) = \cos 30^\circ$.

4. $\sin(180^\circ - \theta) = \sin \theta$
 $\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec} \theta$
 $\cos(180^\circ - \theta) = -\cos \theta$
 $\sec(180^\circ - \theta) = -\sec \theta$
 $\tan(180^\circ - \theta) = -\tan \theta$
 $\cot(180^\circ - \theta) = -\cot \theta$

Note:

- As angle $(180^\circ - \theta)$ lies in 2nd quadrant, therefore, only $\sin \theta$ and $\operatorname{cosec} \theta$ are positive, e.g.,
 $\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ$.

$$\begin{aligned} 5. \quad \sin(180^\circ + \theta) &= -\sin \theta \\ \cos(180^\circ + \theta) &= -\cos \theta \\ \tan(180^\circ + \theta) &= \tan \theta \end{aligned}$$

Note:

- As angle $(180^\circ + \theta)$ lies in 3rd quadrant, therefore only $\tan \theta$ is positive, e.g., $\tan 210^\circ = \tan(180^\circ + 30^\circ) = \tan 30^\circ$.

$$\begin{aligned} 6. \quad \sin(270^\circ + \theta) &= -\cos \theta \\ \cos(270^\circ + \theta) &= \sin \theta \\ \tan(270^\circ + \theta) &= -\cot \theta \end{aligned}$$

Note:

- As angle $(270^\circ + \theta)$ lies in the 4th quadrant, therefore only $\cos \theta$ is positive, e.g., $\sin 300^\circ = \sin(270^\circ + 30^\circ) = -\cos 30^\circ$.

$$\begin{aligned} 7. \quad \sin(360^\circ - \theta) &= -\sin \theta \\ \cos(360^\circ - \theta) &= \cos \theta \\ \tan(360^\circ - \theta) &= -\tan \theta \end{aligned}$$

Note:

- As angle $(360^\circ - \theta)$ lies in the 4th quadrant, therefore only $\cos \theta$ is positive.

$$\begin{aligned} 8. \quad \sin(360^\circ + \theta) &= \sin \theta \\ \cos(360^\circ + \theta) &= \cos \theta \\ \tan(360^\circ + \theta) &= \tan \theta \end{aligned}$$

Note:

- As angle $(360^\circ + \theta)$ lies in the first quadrant, therefore all the T-ratios are positive, e.g., $\sin 400^\circ = \sin(360^\circ + 40^\circ) = \sin 40^\circ$.

Illustration 1.3

Given that $\sin 30^\circ = 1/2$ and $\cos 30^\circ = \sqrt{3}/2$, determine the values of $\sin 60^\circ$, $\sin 120^\circ$, $\sin 240^\circ$, $\sin 300^\circ$, and $\sin(-30^\circ)$.

Sol.

$$1. \quad \sin 60^\circ = ?$$

First, we should determine the quadrant in which 60° lies. It is obviously first quadrant. Then, we should recall whether $\sin$ in first quadrant is positive or negative. "All Silver Tea Cups" tells us that all the TRs are positive in the first quadrant, therefore, $\sin 60^\circ$ must be positive.

Now, we should write 60° in such a way that it is $\pm 30^\circ$ with any of the two axes (the horizontal XOX' and the vertical YOY'). So, we can write $\sin 60^\circ = \sin(90^\circ - 30^\circ)$.

Now, we can recall from the TRs on the previous page that $\sin(90^\circ - \theta) = \cos \theta$

$$\Rightarrow \sin 60^\circ = \sin(90^\circ - 30^\circ) = \cos 30^\circ = \sqrt{3}/2.$$

- Similarly, we can find out the value of $\sin 120^\circ$. This angle lies in second quadrant. In second quadrant, $\sin$ is positive. Therefore, $\sin 120^\circ = \text{some positive value}$.

$$\Rightarrow \sin 120^\circ = \sin(90^\circ + 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

- $\sin 240^\circ$ lies in third quadrant. So, it should be negative.

$$\Rightarrow \sin 240^\circ = \sin(270^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

- $\sin 300^\circ$ lies in fourth quadrant where $\sin$ is negative.

$$\Rightarrow \sin 300^\circ = \sin(270^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

- $\sin(-30^\circ)$ lies in fourth quadrant, so it should be negative.

$$\Rightarrow \sin(-30^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

Inverse Trigonometric Functions

Inverse trigonometric functions are also called anti-trigonometric functions. These are represented by putting a superscript ' -1 ' on the corresponding trigonometric function whose inverse are to be obtained, e.g.,

inverse of $\sin \theta = x$, means $\theta = \sin^{-1} x$

It is read as "sine inverse x ". Just as trigonometric operation on any angle gives a particular value, inverse trigonometric operation on any value (or number) will return its corresponding angle.

Properties of Inverse Trigonometric Functions:

- $\sin^{-1}(\sin \theta) = \theta$ and $\sin(\sin^{-1} x) = x$; provided $-\pi/2 \leq \theta \leq \pi/2$ and $-1 \leq x \leq 1$.
- $\cos^{-1}(\cos \theta) = \theta$ and $\cos(\cos^{-1} x) = x$; provided $0 \leq \theta \leq \pi$ and $-1 \leq x \leq 1$.
- $\tan^{-1}(\tan \theta) = \theta$ and $\tan(\tan^{-1} x) = x$; provided $-\pi/2 \leq \theta \leq \pi/2$ and $-\infty \leq x \leq \infty$.

Illustration 1.4

Find the value of $\sin^{-1} 1$.

Sol. Let $y = \sin^{-1} 1 = \sin^{-1}(\sin \pi/2) = \pi/2$

[$\because \sin \pi/2 = 1$ and $\sin^{-1}(\sin \theta) = \theta$ for $-\pi/2 \leq \theta \leq \pi/2$]

Illustration 1.5

Find the value of $\cos^{-1}(-1/2)$.

Sol. Let $y = \cos^{-1}(-1/2) = \cos^{-1}\left(\cos \frac{2\pi}{3}\right) = \frac{2\pi}{3}$

[$\because \cos(2\pi/3) = -1/2$ and $\cos^{-1}(\cos \theta) = \theta$ for $0 \leq \theta \leq \pi$]

BASIC COORDINATE GEOMETRY

If you have to specify the position of a point in space, how will you do it? This is the easiest application of coordinate geometry. We can give, assign or find out exact numerical values of the position of points, lines, curves, slopes, etc. All this is done with the help of coordinate systems. There are many types of coordinate systems such as rectangular, polar, spherical, cylindrical, etc. It is generally the right handed rectangular axes coordinate system which you will be using in physics. This system consists of:

1. Origin**2. Axis or Axes**

If the point is known to be on a given line or in a particular direction, only one coordinate is necessary to specify its position; if it is in a plane, two coordinates are required; if it is in space, three coordinates are needed.

Origin

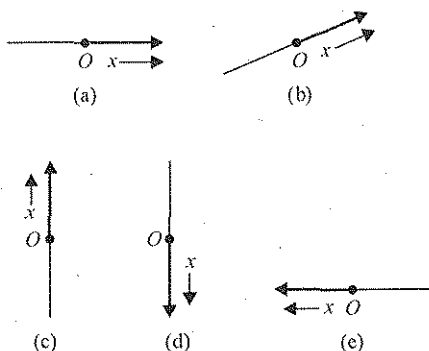
This is any fixed point which is convenient to you. Say, in a room, you can consider any corner of the room as the origin. On a sheet of paper, you can mark any point on it and consider it as the origin. All measurements are taken basically with respect to this point called origin.

Axis or Axes

Any fixed direction passing through the origin and convenient to you can be taken as an axis. If the position of a point or positions of all the points under consideration always happen to be in a particular direction, then only one axis is required. This is generally called the x -axis. If the positions of all the points under consideration are always in a plane, two axes are needed. These are generally called x and y axes. If the points are distributed in a space, three axes are taken which are called x , y , and z -axes. If x , y , and z -axes are mutually perpendicular, the system is called rectangular axes coordinate system.

Important Points

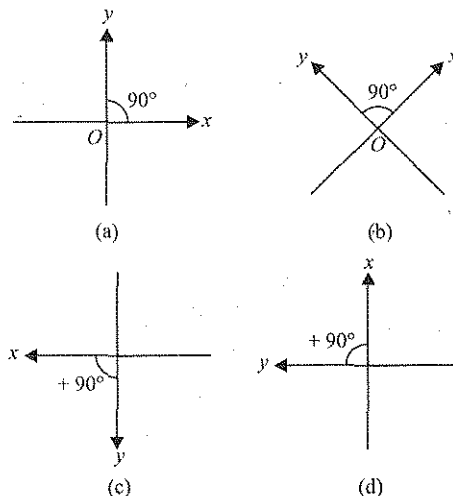
1. Origin can be any fixed point convenient to you. It is denoted by O .
2. x -axis is any fixed direction passing through the origin and convenient to you (Fig. 1.5). Thus, it is not at all necessary that the (so called) horizontal line passing through the origin is x -axis.

**Fig. 1.5**

3. Unless otherwise explicitly mentioned, all angles are always measured from the direction of x -axis (called the positive direction of x -axis). Positive angles are measured in anticlockwise direction and negative in clockwise direction.

4. y -axis is any fixed direction passing through the origin perpendicular to the x -axis, convenient to you. Perpendicular means making an angle of $+90^\circ$ with the positive direction of x -axis. Students may feel that once the origin and x -axis have been fixed, the position of y -axis also gets fixed accordingly. But, it is not the case. y -axis can be any fixed direction which is in the plane passing through the origin and the x -axis and perpendicular to x -axis.

Thus, x and y -axes can be any direction as shown in Fig. 1.6.

**Fig. 1.6**

5. Once origin, x - and y -axes are fixed, z -axis becomes automatically fixed. Convenience of the observer goes away. z -axis is the fixed direction passing through the origin and perpendicular to both x - and y -axes.

Position of a Point

As you already know it well, in case of plane coordinate geometry, i.e., when the position of a point always remains contained in a plane (called x - y plane), the position of a point is specified by its distances from the origin along (or parallel to) x and y -axes, as shown in Fig. 1.7.

You can easily observe that the coordinates (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , and (x_4, y_4) in Fig. 1.7 are $(4, 2)$, $(-4, 3)$, $(-5, -4)$, and $(2, -2)$, respectively.

Distance Formulae

1. The distance between two points (x_1, y_1) and (x_2, y_2) $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
2. The distance of the point (x_1, y_1) from the origin $= \sqrt{(x_1^2 + y_1^2)}$.

The coordinates of the mid-point of the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

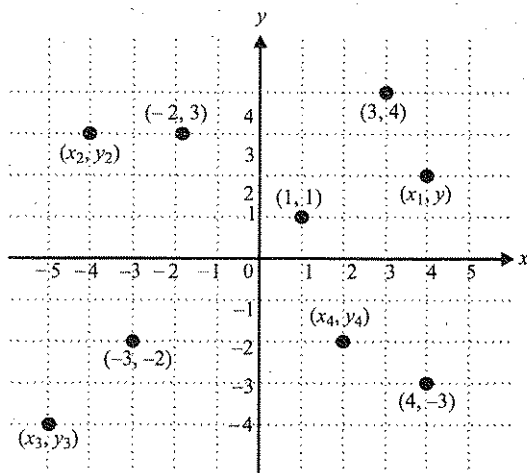


Fig. 1.7

Slope of a Line

The slope of a line joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is denoted by m and is given by $m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$, where θ is the angle which the line makes with the positive direction of x -axis (Fig. 1.8).

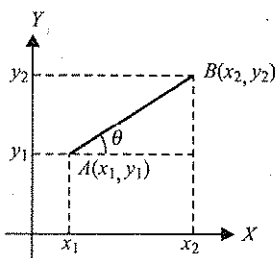


Fig. 1.8

Straight Line Equations

1. $Ax + By + C = 0$, is the general form of the equation of a straight line.
2. Equation of x -axis is $y = 0$.
3. Equation of y -axis is $x = 0$.
4. Equation of a straight line parallel to y -axis and at a distance a from it is given by $x = a$.
5. Equation of a straight line parallel to x -axis and at a distance b from it is given by $y = b$.
 - a. Constant function, $x = a$ (Fig. 1.9).

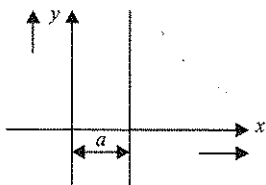


Fig. 1.9

- b. Constant function, $y = b$ (Fig. 1.10).
6. $y = mx + c$ is a line which cuts off an intercept c on y -axis and makes an angle θ with the +ve direction of x -axis in

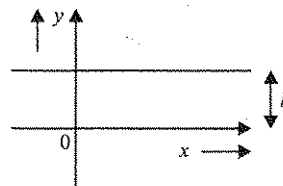


Fig. 1.10

anticlockwise direction; and $m = \tan \theta$ is called its slope or gradient.

7. $y = mx$, is any line through the origin and having slope m .

- a. When $c = 0$, $y = mx$

The graph between x and y will be a straight line as x bears the direct dependence on y (Fig. 1.11).

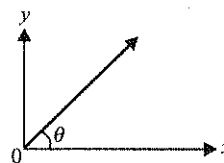


Fig. 1.11

Here, m represents the slope of line.

$$\therefore \frac{dy}{dx} = m = \tan \theta$$

- b. When $c \neq 0$, $y = mx + c$

Graph for this equation is also a straight line but with a +ve intercept on y -axis. As when x goes to zero, y accordingly takes value c . So, the straight line will start from $y = c$ instead of origin (Fig. 1.12).

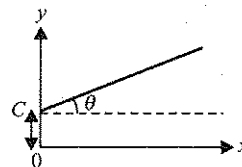


Fig. 1.12

$m = \tan \theta$ is the slope of the straight line here also.

- c. When $c = 0$, $m < 0$, $y = mx$

For $m < 0$, $\theta > 90^\circ$ (Fig. 1.13).

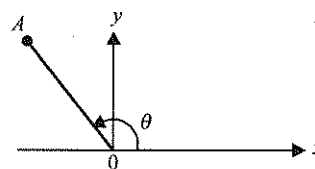


Fig. 1.13

- d. When $c \neq 0$, $m < 0$, $y = mx + c$

For $m < 0$, $\theta > 90^\circ$ (Fig. 1.14).

8. $\frac{x}{a} + \frac{y}{b} = 1$, is a line in intercept form where a and b are the intercepts on the axes of x and y , respectively (Fig. 1.15).
9. $y - y_1 = m(x - x_1)$, is the equation of a line through a given point (x_1, y_1) and having slope m .

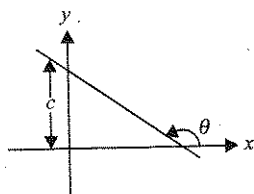


Fig. 1.14

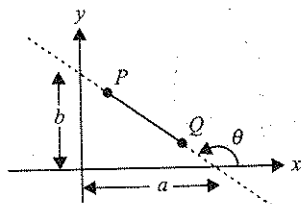


Fig. 1.15

Slope

The slope m of the line $Ax + By + C = 0$ is given by

$$m = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y} = \frac{-A}{B}$$

Illustration 1.6 Consider two points $P_1(2, 7)$ and $P_2(6, 15)$. Write the equation and draw the straight line through these points.

Sol.

Step 1. Obtain the gradient which is m .

Step 2. c can be found by using the (x, y) values of any given point.

$$\text{Step 1. Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 7}{6 - 2} = \frac{\text{Height}}{\text{Distance}} = \frac{8}{4} = 2.$$

$$\text{So, } y = 2x + c. \quad (1)$$

Step 2. To find c , put $(x = 2, y = 7)$ or $(x = 6, y = 15)$ in equation (1).

$$7 = 2 \times 2 + c \Rightarrow c = 3$$

So, $m = 2, c = 3$.

$\therefore$ the equation becomes $y = 2x + 3$.

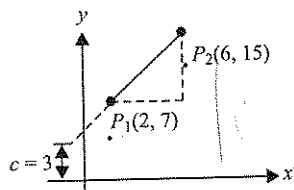


Fig. 1.16

Illustration 1.7 Plot the line $2x - 3y = 12$.

Sol. Method 1:

$$\text{Given } 3y = 2x - 12 \Rightarrow y = \frac{2}{3}x - 4$$

$$\therefore \text{ using } y = mx + c, m = \frac{2}{3} \text{ and } c = -4$$

Here, the positive slope (m) means that the angle made by the line with x -axis should be less than 90° and negative c means the line will intercept with negative y -axis (Fig. 1.17).

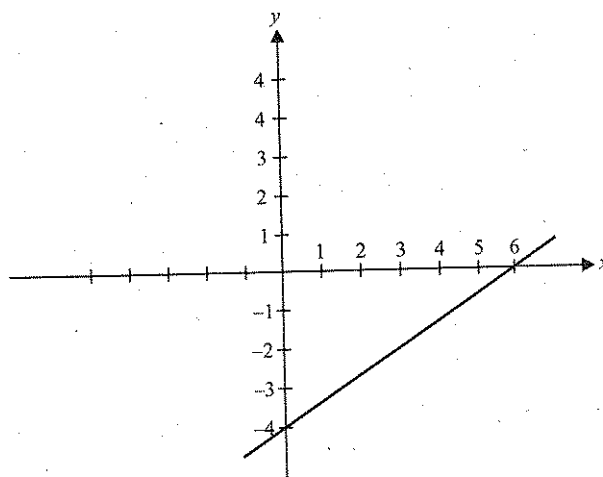


Fig. 1.17

$$\text{If } y = 0, \text{ then } \frac{2}{3}x = 4 \Rightarrow x = \frac{4 \times 3}{2} = 6$$

$$\text{Method 2: Using } \frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{2x}{12} - \frac{3y}{12} = 1$$

$$\Rightarrow \frac{x}{6} + \frac{y}{-4} = 1$$

Illustration 1.8 Plot the line $-3x - 5y = 15$.

Sol. Method 1:

$$\text{Given } 5y = -3x - 15$$

$$\therefore \text{ using } y = mx + c, y = -\frac{3}{5}x - 3$$

Here, the slope is negative, i.e., the line makes an angle greater than 90° with x -axis. As intercept is negative, it indicates that the line will cut y -axis at negative side of it (Fig. 1.18).

$$\text{When } x = 0, y = -3.$$

$$\text{When } y = 0, x = -5.$$

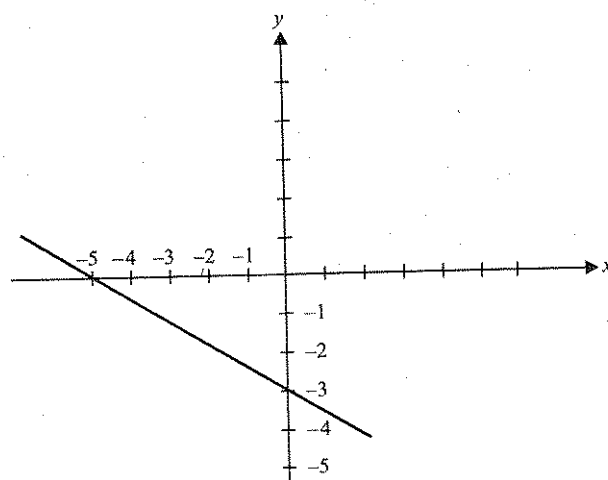


Fig. 1.18

$$\text{Method 2: Using } \frac{x}{a} + \frac{y}{b} = 1; \frac{-3x}{15} - \frac{5y}{15} = 1$$

$$\Rightarrow \frac{x}{(-5)} + \frac{y}{(-3)} = 1$$

Concept Application Exercise 1.2

- Plot the lines: (i) $3x + 2y = 0$, (ii) $x - 3y + 6 = 0$.
- If a particle starts moving with initial velocity $u = 1$ m/s and with acceleration $a = 2$ m/s², the velocity of the particle at any time is given by $v = u + at = 1 + 2t = 1 + 2t$; plot the velocity-time graph of the particle.
- A particle starts moving with initial velocity $u = 25$ m/s with retardation $a = -2$ m/s². Draw the velocity-time graph.

Parabola: The Quadratic Equations

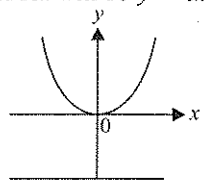
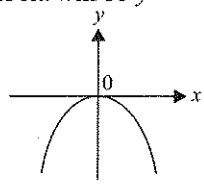
Let us now discuss graphs of quadratic equations.

For equation $y = ax^2 + bx + c$ (where a , b , and c are constants), the graph between x and y will be an asymmetric parabola. As long as $a \neq 0$, this equation represents a quadratic function. So, what is the simplest quadratic equation? It is $y = ax^2$ (obtained by putting $b = 0$, $c = 0$), which is the equation of parabola.

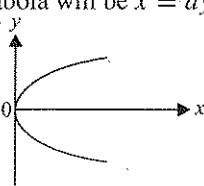
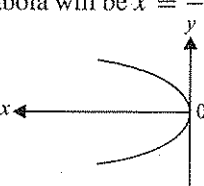
Conclusion

Equation of parabola is a quadratic equation in its simplest form. This parabola has its vertex at origin $(0, 0)$ because when we put $x = 0$, it gives $y = 0$.

- The graph for $y = ax^2$ will be a symmetric parabola about y -axis. The orientation of parabola will be decided by the sign of a .

When a is Positive	When a is Negative
The equation of the parabola will be $y = ax^2$.	The equation of the parabola will be $y = -ax^2$.
	

- If we exchange x and y in this equation, i.e., $x = ay^2$, then the axis of symmetry changes and becomes x -axis. As we know this orientation changes as per the sign of a , so the orientation will be opposite when a is negative.

When a is Positive	When a is Negative
The equation of the parabola will be $x = ay^2$.	The equation of the parabola will be $x = -ay^2$.
	

- For equation with $c = 0$, $y = ax^2 + bx$. The graph between x and y is an asymmetric parabola, but the orientation of the graph varies with the signs of a and b . Let us take the special case when both a and b are positive.

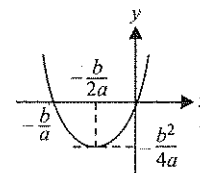


Fig. 1.19

When $y = 0$, then $x = 0$ or $x = -b/a$. At $x = -b/2a$, y is min and $y_{\min} = -b^2/4a$. It is known as vertex (see Fig. 1.19).

Plotting Quadratic Equations

- General quadratic equation is $y = ax^2 + bx + c$.
- The graph of a quadratic equation is always a parabola.
- Orientation of graph depends upon sign of a .
 - When a is +ve, the graph will open up.
 - When a is -ve, the graph will open down.
- The x -coordinate of the vertex is equal to $-b/2a$, i.e., $x = -b/2a$.
- Put this value back in given equation and find y . Point (x, y) so obtained represents the vertex.
- Choose two values of x which are to the right or left of the x -coordinate of the vertex.
- Substitution of these values will give values of y .

Using these values of (x, y) , the graph can be plotted successfully.

Note: Since a parabola is symmetric about the line passing through its vertex, the mirror image of points taken with the same value will give other side of parabola.

Illustration 1.9

Plot the graph for the equation

$$y = -x^2 + 4x - 1.$$

Sol. $a = -1$, $b = 4$, $c = -1$. As a is negative, so parabola should open down.

$$\text{Vertex: } x = \frac{-b}{2a} = 2. \text{ Put this value of } x \text{ to get } y = 3.$$

Hence, the vertex of the parabola is $(2, 3)$.

Assume two values of x as follows and find corresponding values of y .

x	1	-1
y	2	-6

Points obtained: $(1, 2)$, $(-1, -6)$

All points obtained: $(2, 3)$ (Vertex), $[1, 2]$, $[-1, -6]$

Symmetry of parabola: Mirror image points of $(1, 2)$ and $(-1, -6)$ are $(3, 2)$ and $(5, -6)$.

Now, sketch the parabola as shown in Fig. 1.20.

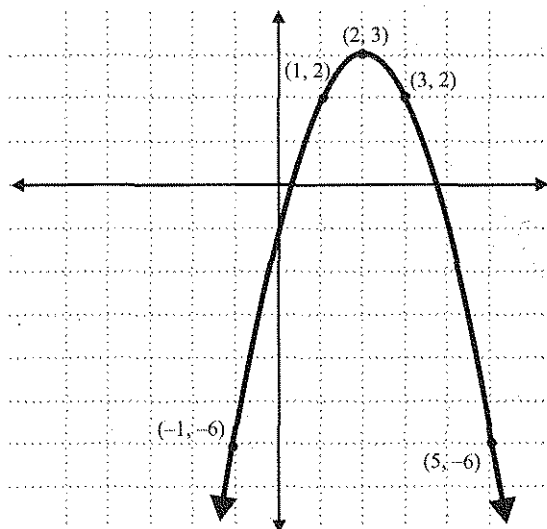


Fig. 1.20

Illustration 1.10**Plot the graph for the equation**

$$y = x^2 - 4x.$$

Sol. $a = 1$, $b = -4$. As a is positive, so parabola should open up. As $c = 0$, so parabola will pass through origin.

Vertex: $x = \frac{-b}{2a} = 2$, so $y = -4 \Rightarrow \text{Vertex} = (2, -4)$

Assuming two values of x ,

x	1	0
y	-3	0

All points obtained: $(2, -4)$, $(1, -3)$, $(0, 0)$

Symmetry of parabola: Mirror image points of $(1, -3)$ and $(0, 0)$ are $(3, -3)$ and $(4, 0)$.

Now, sketch the parabola as shown in Fig. 1.21.

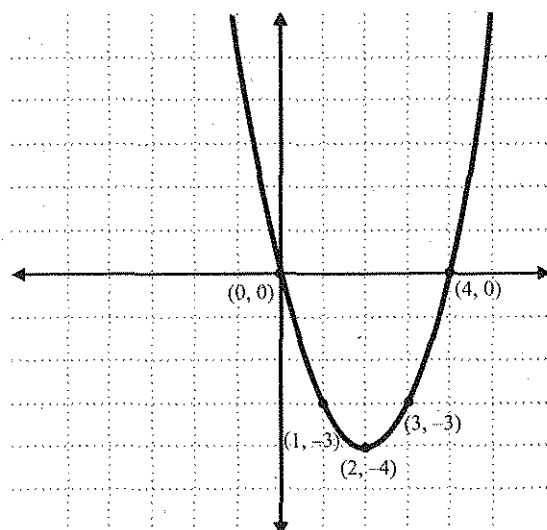


Fig. 1.21

Concept Application Exercise 1.3

- Find the vertex of the following quadratic equations and plot the graph:
 - $y = x^2 - 8x$
 - $y = -2x^2 + 3$
 - $y = x^2 - 6x + 4$
- If a particle starts moving along x -axis from origin with initial velocity $u = 1$ m/s and acceleration $a = 2$ m/s², the relationship between displacement and time will be

$$x = ut + \frac{1}{2}at^2 = 1 \times t + \frac{1}{2} \times 2 \times t^2 = t + t^2.$$
 Draw the displacement (x)-time (t) graph.

DIFFERENTIATION

The purpose of differential calculus is to study the nature (i.e., increase or decrease) and the amount of variation in a quantity when another quantity (on which first quantity depends) varies independently. In our day-to-day life, we often face such types of situations, e.g., growth of plants, expansion of solids on heating, variation in the velocity of a uniformly accelerated object, growth in the population of a country.

Quantity: Anything which can be measured is called a quantity.

Constants and Variables: A quantity whose value remains constant throughout the mathematical operation is called a constant, e.g., integers, fractions, π , e , etc. On the other hand, a quantity which can have any numerical value within certain specific limits is called a variable. A variable is usually represented by u , v , w , x , y , z , etc.

Dependent and Independent Variables: A variable which can have any arbitrary value within specific limits is called as independent variable whereas one whose value depends upon the numerical values assigned to the independent variable is defined as dependant variable.

Differential Coefficient or Derivative of a Function

Suppose y be a function of x , i.e., $y = f(x)$. (i)

The value of the function or the dependent variable y depends on the value of independent variable x . If we change the value of independent variable x to $x + \Delta x$, then value of the function will also change. Let it becomes $y + \Delta y$. Hence

$$y + \Delta y = f(x + \Delta x) \quad \text{(ii)}$$

Subtracting equation (i) from (ii), we get

$$y + \Delta y - y = f(x + \Delta x) - f(x)$$

$$\text{or } \Delta y = f(x + \Delta x) - f(x) \quad \text{(iii)}$$

Above equation provides the change in the value of function y , when the value of variable x is changed from x to $x + \Delta x$.

Dividing both sides of the equation (iii) by Δx , we get

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad \text{(iv)}$$

$$\begin{aligned}\text{Maximum value of } v &= 6 \times \frac{4}{9} - 6 \times \frac{8}{27} = \frac{8}{3} - \frac{16}{9} \\ &= \frac{8}{9} \text{ ms}^{-1} \text{ (by putting } t = \frac{2}{3} \text{ in } v)\end{aligned}$$

Example 1.25 On a certain planet, the instantaneous velocity of a ball thrown up is given by $v = 2t - 6$ (v is in ms^{-1} and t is in sec).

1. Find the expression for the displacement of the particle, given that the particle started its journey at $x = 1$ m.
2. What is the value of g on the surface of this planet?

Sol. 1. To find the displacements, we need to integrate the velocity, i.e.,

$$x = \int v dt = \int (2t - 6) dt = \frac{2t^2}{2} - 6t + c = t^2 - 6t + c$$

where c is the constant of integration. Its value can be calculated from the given boundary condition that $x = 1$, $t = 0$.

$$\Rightarrow c = 1 \text{ m} \Rightarrow x = t^2 - 6t + 1$$

2. g is acceleration due to gravity. So, to find acceleration we need to differentiate velocity.

$$\text{As } v = 2t - 6 \Rightarrow a = \frac{dv}{dt} = 2 \text{ ms}^{-2}$$

So, the acceleration due to gravity on the planet under consideration is 2 m/s^{-2} .

Example 1.26 Let the instantaneous velocity of a rocket, just after landing, is given by the expression, $v = 2t + 3t^2$ (where v is in ms^{-1} and t is in seconds). Find out the distance traveled by the rocket from $t = 2$ s to $t = 3$ s.

Sol. To find distance traveled we need to integrate v . [The limits of integration will be from 2 to 3 as we have to find the distance traveled between $t = 2$ s and $t = 3$ s.]

$$\begin{aligned}x &= \int_2^3 v dt = \int_2^3 (2t + 3t^2) dt = \left[\frac{2t^2}{2} + \frac{3t^3}{3} \right]_2^3 \\ &= \left[t^2 + t^3 \right]_2^3 = 24 \text{ m}\end{aligned}$$

Example 1.27 A particle moves with a constant acceleration $a = 2 \text{ ms}^{-2}$ along a straight line. If it moves with an initial velocity of 5 ms^{-1} , then obtain an expression for its instantaneous velocity.

Sol. We know that acceleration is time rate of change of velocity, i.e., $a = \frac{dv}{dt}$ and differentiation is the inverse operation of integration. So, by integrating acceleration we can obtain the expression of velocity.

$$\text{So, } v = \int a dt = 2 \int dt = 2t + c$$

where c is the constant of integration and its value can be obtained from the initial conditions.

$$\text{At } t = 0, v = 5 \text{ ms}^{-1}. \text{ We have } 5 = 2 \times 0 + c$$

$$\Rightarrow c = 5 \text{ ms}^{-1}$$

Therefore, $v = 2t + 5$ is the required expression for the instantaneous velocity.

Example 1.28 In the previous problem, if the particle occupies a position $x = 7$ m at $t = 1$ s, then obtain an expression for the instantaneous displacement of the particle.

Sol. Again, we can use the idea that displacement is the integration of velocity w.r.t. time.

$$\text{So, } x = \int v dt = \int (2t + 5) dt = \frac{2t^2}{2} + 5t + c = t^2 + 5t + c$$

where c is the constant of integration. Its value can be determined by using the given condition. (As particular details have been given about the particle.)

$$\text{At } t = 1 \text{ s, } x = 7 \text{ m} \Rightarrow 7 = 1^2 + 5 \times 1 + c \Rightarrow c = 1 \text{ m}$$

$$\text{Hence, the expression becomes } x = t^2 + 5t + 1$$

Concept Application Exercise 1.6

1. Displacement of a particle is given by $y = (6t^2 + 3t + 4)$ m, where t is in second. Calculate the instantaneous speed of the particle.
2. The velocity of a particle is given by $v = 12 + 3(t + 7t^2)$. What is the acceleration of the particle?
3. A particle starts from origin with uniform acceleration. Its displacement after t seconds is given in meter by the relation $x = 2 + 5t + 7t^2$. Calculate the magnitude of its
 - a. initial velocity,
 - b. velocity at $t = 4$ s,
 - c. uniform acceleration, and
 - d. displacement at $t = 5$ s.
4. The acceleration of a particle is given by $a = t^3 - 3t^2 + 5$, where a is in ms^{-2} and t in sec. At $t = 1$ s, the displacement and velocity are 8.30 m and 6.25 ms^{-1} , respectively. Calculate the displacement and velocity at $t = 2$ sec.
5. A particle starts moving along x -axis from $t = 0$, its position varying with time as $x = 2t^3 - 3t^2 + 1$.
 - a. At which time instants is its velocity zero?
 - b. What is the velocity when it passes through origin?
6. A particle moves along x -axis obeying the equation $x = t(t - 1)(t - 2)$, where x is in meter and t is in second
 - a. Find the initial velocity of the particle.
 - b. Find the initial acceleration of the particle.
 - c. Find the time when the displacement of the particle is zero.
 - d. Find the displacement when the velocity of the particle is zero.
 - e. Find the acceleration of the particle when its velocity is zero.
7. The speed of a car increases uniformly from zero to 10 ms^{-1} in 2 s and then remains constant (Fig. 1.29).
 - a. Find the distance traveled by the car in the first two seconds.

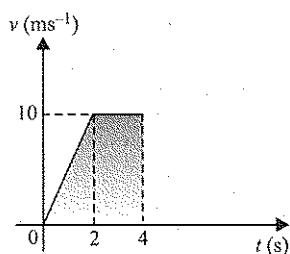


Fig. 1.29

- b. Find the distance traveled by the car in the next two seconds.
 - c. Find the total distance traveled in 4 s.
8. A car accelerates from rest with 2 ms^{-2} for 2 s and then decelerates constantly with 4 ms^{-2} for t_0 second to come to rest. The graph for the motion is shown in Fig. 1.30.

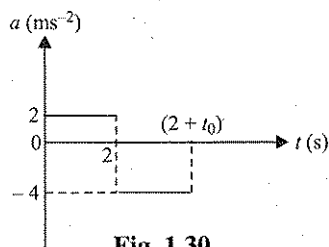


Fig. 1.30

- a. Find the maximum speed attained by the car.
 - b. Find the value of t_0 .
9. A stationary particle of mass $m = 1.5 \text{ kg}$ is acted upon by a variable force. The variation of force with respect to displacement is plotted in the following Fig. 1.31.

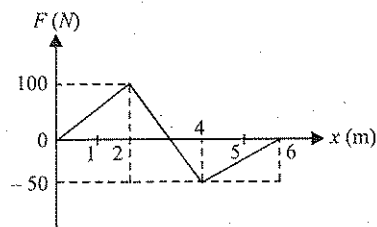


Fig. 1.31

- a. Calculate the velocity acquired by the particle after getting displaced through 6 m.
 - b. What is the maximum speed attained by the particle and at what time is it attained?
10. A body moving along x -axis has, at any instant, its x -coordinate to be $x = a + bt + ct^2$. What is the acceleration of the particle?
11. The displacement of a body at any time t after starting is given by $s = 15t - 0.4t^2$. Find the time when the velocity of the body will be 7 ms^{-1} .
12. A particle moves along a straight line such that its displacement at any time t is given by $s = t^3 - 6t^2 + 3t + 4 \text{ m}$. Find the velocity when the acceleration is 0.
13. The coordinates of a moving particle at time t are given by $x = ct^2$ and $y = bt^2$. Find the speed of the particle at any time t .
14. The displacement x of a particle moving in one dimension under the action of a constant force is related to time t by the equation $t = \sqrt{x} + 3$, where x is in meter and t is in seconds. Find the displacement of the particle when its velocity is zero.

Vectors

- Scalars
- Vectors
- Representation of Vector
- Notation of Vector
- Introduction of Different Types of Vectors
- Addition of Vectors
- Triangle Law of Vector Addition
- Parallelogram Law of Vector Addition
- Addition of More than Two Vectors
- Vectors Addition by Analytical Method
- Rectangular Components of a Vector in Two Dimensions
- Rectangular Components of a Vector in Three Dimensions
- Product of Two Vectors
- Cross Product Method 1: Using Component Form
- Cross Product Method 2: Determinant Method

In physics, various quantities are broadly classified into two categories:

1. Scalars, and
2. Vectors.

SCALARS

Scalars are those physical quantities which have magnitude only but no direction. For example: mass, length, time, work, etc.

VECTORS

Vectors are those physical quantities which have both magnitude and direction. For example: velocity, acceleration, momentum, force, etc.

Further, vectors can be of two types:

1. **Polar vectors:** These are vectors which have a starting point or a point of application.
For example: velocity, force, linear momentum, etc.
2. **Axial vectors:** These are vectors whose directions are along the axis of rotation.
For example: angular velocity, angular momentum, torque, etc.

Note: A vector is meaningful only if we know both magnitude and direction of a vector. Without knowing direction, the description of a vector quantity is incomplete.

Use of vectors: Many laws of physics can be expressed in compact form by the use of vectors. In this way, the complicated laws are greatly simplified and then they become easier to apply. For example: $F = m\vec{a}$, $\vec{\tau} = I\vec{\alpha}$, $\vec{F} = q\vec{v} \times \vec{B}$, etc.

REPRESENTATION OF VECTOR

A vector is represented by a directed line segment, with an arrow head. For example, a vector $\vec{F}$ is represented by a directed line PQ (see Fig. 2.1).

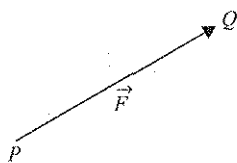


Fig. 2.1

Point P is called *tail* or *origin* of the vector. Point Q , the end point of vector, is called *tip*, *head* or *terminal point* of the vector.

1. The length of the line represents the magnitude of the vector.
2. The arrow head represents the direction of the vector.

NOTATION OF VECTOR

1. By single letter with an arrow overhead.
For example: Force can be represented by $\vec{F}$ and its magnitude is represented as $|\vec{F}|$.

2. By bold letters.

For example: Force can be represented by $\mathbf{F}$. In this case, magnitude is represented in the same way as above such as $|\vec{F}|$ or by same symbol, but *Italic F*.

INTRODUCTION TO DIFFERENT TYPES OF VECTORS

Collinear Vectors

The vectors which either act along the same line or along the parallel lines are called collinear vectors. These vectors may act either in the same direction or in the opposite direction.

Parallel Vectors

Two collinear vectors having the same direction are called parallel or like vectors (see Fig. 2.2). Angle between them is 0° .

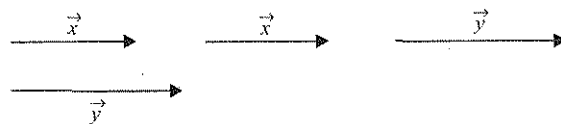


Fig. 2.2

Antiparallel Vectors

Two collinear vectors acting in opposite directions are called antiparallel or opposite vectors (see Fig. 2.3). The angle between them is 180° or π -radian.

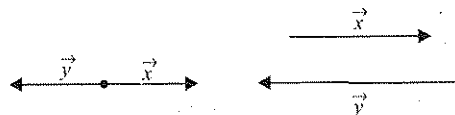


Fig. 2.3

Equal Vectors

Two vectors are said to be equal vectors if they have equal magnitude and same direction. Two equal vectors $\vec{A}$ and $\vec{B}$ are represented by two equal and parallel lines having arrow heads in the same direction (see Fig. 2.4).



Fig. 2.4

Negative of a Vector

A negative vector of a given vector is a vector having same magnitude with the direction opposite to that of given vector (see Fig. 2.5). The negative vector of $\vec{A}$ is represented by $-\vec{A}$.

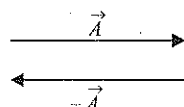


Fig. 2.5

Coplanar Vectors

These are vectors which lie in the same plane. In Fig. 2.6, $\vec{A}$, $\vec{B}$, and $\vec{C}$ are acting in the same plane, i.e., XY plane, so they are coplanar vectors.

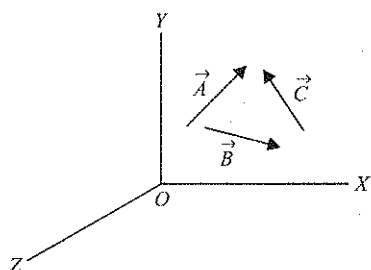


Fig. 2.6

Unit Vector

It is a vector whose magnitude is equal to unity (one). A unit vector in a given direction can be obtained by dividing a vector in that direction by its magnitude. Unit vector of $\vec{A}$ is written as $\hat{A}$ and is read as A cap.

So, $\hat{A} = \frac{\vec{A}}{A}$, where $\hat{A}$ is the unit vector along the direction of $\vec{A}$.

The direction of unit vector will be same as that of the vector from which it is obtained. It means $\hat{A}$ is parallel to $\vec{A}$.

Also, from above: $\vec{A} = A\hat{A}$ or
vector = magnitude $\times$ direction

In every direction, we can obtain a unit vector. In x , y and z directions, unit vectors are predefined (Fig. 2.7).

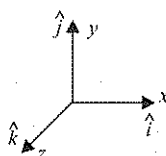


Fig. 2.7

$\hat{i}$ is the unit vector along x -axis,
 $\hat{j}$ is the unit vector along y -axis, and
 $\hat{k}$ is the unit vector along z -axis.

Now, a unit vector in any other direction can be obtained in terms of $\hat{i}$, $\hat{j}$, and $\hat{k}$ as shown in the next section.

Note: A unit vector is a dimensionless quantity, so it has no units. It represents only direction.

Expression of a Vector in terms of $\hat{i}$, $\hat{j}$, and $\hat{k}$ and Magnitude of a Vector

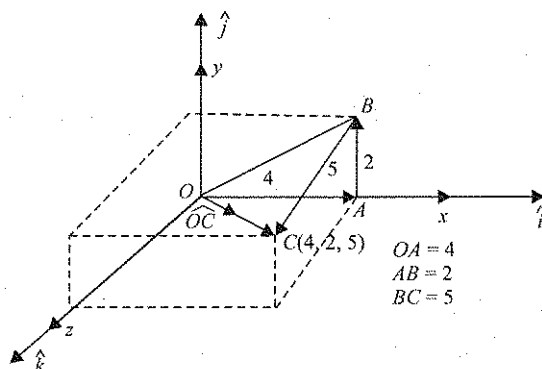


Fig. 2.8

Suppose a particle goes from O to A , 4 m along x -axis; A to B , 2 m along y -axis; and then B to C , 5 m along z -axis (Fig. 2.8). Finally, the particle reaches at C .

Coordinates of C : (4, 2, 5) m

We can write: $\vec{OA} = 4\hat{i}$, $\vec{AB} = 2\hat{j}$, and $\vec{BC} = 5\hat{k}$.

O is the initial position and C is the final position. $\vec{OC}$ is the net displacement, which is the resultant of $\vec{OA}$, $\vec{AB}$, and $\vec{BC}$.

So, we can write: $\vec{OC} = \vec{OA} + \vec{AB} + \vec{BC}$

i.e., $\vec{OC} = 4\hat{i} + 2\hat{j} + 5\hat{k}$

(Hence, here we represent $\vec{OC}$ in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$.)

Magnitude (modulus) of $\vec{OC}$: It is written as $|\vec{OC}|$ or OC .

In $\triangle OAB$: $OB^2 = OA^2 + AB^2$.

In $\triangle OBC$: $OC^2 = OB^2 + BC^2$ [$\because \angle OBC = 90^\circ$]

$\Rightarrow OC^2 = OA^2 + AB^2 + BC^2$

$\Rightarrow OC = \sqrt{OA^2 + AB^2 + BC^2}$
 $= \sqrt{4^2 + 2^2 + 5^2} = \sqrt{45}$

$\Rightarrow OC = |\vec{OC}| = \sqrt{45}$. This is the magnitude of $\vec{OC}$.

Note: Magnitude of a vector or modulus of a vector is equal to length of the vector.

Unit vector: $\hat{OC} = \frac{\vec{OC}}{OC} = \frac{4\hat{i} + 2\hat{j} + 5\hat{k}}{\sqrt{45}}$
 $= \frac{4}{\sqrt{45}}\hat{i} + \frac{2}{\sqrt{45}}\hat{j} + \frac{5}{\sqrt{45}}\hat{k}$

(Hence, we express a unit vector in the direction of $\vec{OC}$ in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$.)

Important: Direction of $\hat{OC}$ and $\vec{OC}$ will be same.

Now, in general, if we have a point P in space whose coordinates are x , y and z , then a vector from origin O to P is known as position vector of P w.r.t. origin O (Fig. 2.9). We can write:

$\vec{r} = \vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

Magnitude: $r = OP = \sqrt{x^2 + y^2 + z^2}$,

Unit vector: $\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$

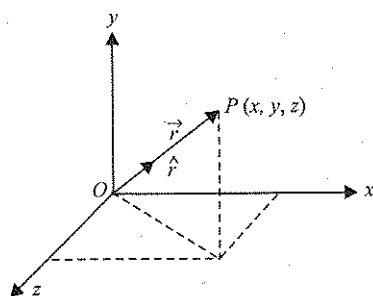


Fig. 2.9

Position Vector and Displacement Vector

Consider two points A and B in space whose coordinates are (x_1, y_1, z_1) and (x_2, y_2, z_2) , respectively (Fig. 2.10).

Position vector of A: $\vec{OA} = \vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$,

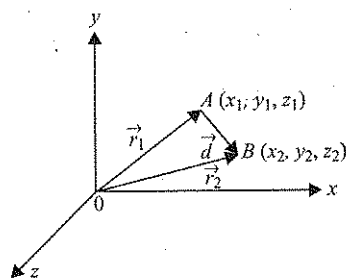


Fig. 2.10

Position vector of B: $\vec{OB} = \vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Position vector provides us some important information:

It gives an idea about the direction and the distance of the point from origin in space.

Now consider a particle which goes from point A to B. Then, $\vec{AB} = \vec{d}$ is displacement vector of the particle. Displacement vector is that vector which tells us how much and in which direction an object has changed its position in a given interval of time.

Here, $\vec{r}_1$ is initial position vector of the particle and $\vec{r}_2$ is final position vector of the particle.

$\therefore$ displacement vector = final position vector – initial position vector

$$\Rightarrow \vec{d} = \vec{r}_2 - \vec{r}_1$$

$$\Rightarrow \vec{d} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Note: Position vector gives us information about position of a point in space, displacement vector about displacement. Similarly, a velocity vector or a force vector will tell us about velocity or force and their magnitudes and directions. So, a vector gives us information about the same physical quantity by which it is known.

- Position vector is also known as radius vector.
- If A and B are two points whose coordinates are (x_1, y_1, z_1) and (x_2, y_2, z_2) , respectively, then a vector from A to B is given by (Fig. 2.11):

$$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

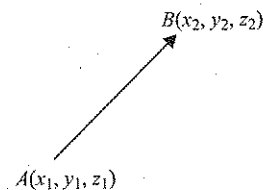


Fig. 2.11

- A vector can be completely zero, if all of its individual components are zero. For example: let a vector $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is given. If $\vec{A} = 0$, then it is possible only if $a_1 = 0$, $a_2 = 0$ and $a_3 = 0$.
- If two vectors are equal, then their individual components are also equal separately.
For example: Let two vectors $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are given. If $\vec{A} = \vec{B}$, then $a_1 = b_1$, $a_2 = b_2$ and $a_3 = b_3$ (but if $A = B$, then it means $\sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{b_1^2 + b_2^2 + b_3^2}$).

Illustration 2.1 A particle initially at point A(2, 4, 6) m moves finally to the point B(3, 2, -3) m. Write the initial position vector, final position vector and displacement vector of the particle.

Sol. Initial position vector: $\vec{r}_1 = 2\hat{i} + 4\hat{j} + 6\hat{k}$

Final position vector: $\vec{r}_2 = 3\hat{i} + 2\hat{j} - 3\hat{k}$

Displacement: $\vec{d} = \vec{r}_2 - \vec{r}_1 = (3 - 2)\hat{i} + (2 - 4)\hat{j} + (-3 - 6)\hat{k} = \hat{i} - 2\hat{j} - 9\hat{k}$

Illustration 2.2 A particle has the following displacements in succession: (i) 12 m towards east, (ii) 5 m towards north, and (iii) 6 m vertically upwards. Find the magnitude of the resultant displacement.

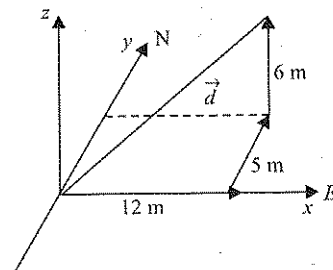


Fig. 2.12

Sol. From Fig. 2.12: $\vec{d} = 12\hat{i} + 5\hat{j} + 6\hat{k}$,

Magnitude: $d = \sqrt{12^2 + 5^2 + 6^2} = \sqrt{205}$ m

Illustration 2.3 Determine a vector which when added to the resultant of $\vec{A} = 2\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{B} = 3\hat{i} - 5\hat{j} - \hat{k}$ gives unit vector along negative y-direction.

Sol. Let $\vec{C}$ be the vector which we have to find.
Then, given: $(\vec{A} + \vec{B}) + \vec{C} = -\hat{j}$

$$\Rightarrow (2\hat{i} + 5\hat{j} - \hat{k}) + (3\hat{i} - 5\hat{j} - \hat{k}) + \vec{C} = -\hat{j}$$

$$\Rightarrow \vec{C} = -5\hat{j} - \hat{j} + 2\hat{k}$$

Resultant Vector

The resultant vector of two or more vectors is defined as that single vector which produces the same effect both in magnitude and direction as produced by individual vectors taken together.

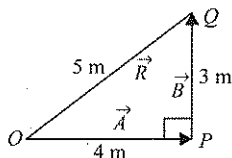


Fig. 2.13

Example: Let a person goes from O to P traveling 4 m and then P to Q traveling 3 m perpendicular to OP . The person could also go directly from O to Q , traveling 5 m (Fig. 2.13). In both the cases, displacement of the person would be same. Here, $\vec{OQ}$ produces same effect as produced by $\vec{OP}$ and $\vec{PQ}$ taken together.

So, $\vec{OQ}$ is called the resultant of $\vec{OP}$ and $\vec{PQ}$. We can also write $\vec{OQ} = \vec{OP} + \vec{PQ}$.

Note: Resultant means addition.

We cannot write $OQ = OP + PQ$, because $OQ = 5$ m and $OP + PQ = 4 + 3 = 7$ m.

Points to Remember

1. If two vectors $\vec{A}$ and $\vec{B}$ are parallel, then we can write $\vec{B} = m\vec{A}$, where m is a number. If vectors are parallel, then m is +ve and if vectors are antiparallel, then m is -ve (Fig. 2.14).

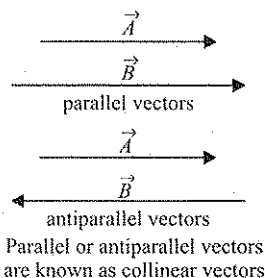


Fig. 2.14

2. If two vectors are parallel, then their unit vectors are equal, i.e., $\hat{A} = \hat{B}$ and if two vectors are antiparallel, then we have $\hat{A} = -\hat{B}$.
3. If three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are coplanar, then we can write: $\vec{A} = m\vec{B} + n\vec{C}$, where m and n are some numbers. Minimum number of unequal vectors whose sum can be zero is three and these three vectors must be coplanar.
4. If four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ are in any arbitrary directions or in different planes, then we can write: $\vec{A} = m\vec{B} + n\vec{C} + p\vec{D}$, where m , n and p are some numbers. The resultant of three non-coplanar vectors can never be zero. The minimum number of non-coplanar vectors whose sum can be zero is four.

Illustration 2.4 Two vectors $\vec{P} = 2\hat{i} - b\hat{j} + 2\hat{k}$ and $\vec{Q} = \hat{i} + \hat{j} + \hat{k}$ are parallel. Find the value of b .

Sol. If $\vec{P}$ and $\vec{Q}$ are parallel, then we have $\vec{P} = m\vec{Q}$, where m is some number.

$$\Rightarrow 2\hat{i} - b\hat{j} + 2\hat{k} = m(\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow 2\hat{i} - b\hat{j} + 2\hat{k} = m\hat{i} + m\hat{j} + m\hat{k}$$

Equating the components of $\hat{i}$ and $\hat{j}$ from both sides: $m = 2$ and $-b = m \Rightarrow b = -2$

Illustration 2.5 If vectors $2\hat{i} + 2\hat{j} - 2\hat{k}$, $5\hat{i} + y\hat{j} + \hat{k}$, and $\hat{i} + 2\hat{j} + 2\hat{k}$ are coplanar, then find the value of y .

Sol. If $\vec{A}$, $\vec{B}$, and $\vec{C}$ are coplanar, then we have: $\vec{A} = m\vec{B} + n\vec{C}$

$$\Rightarrow 2\hat{i} + 2\hat{j} - 2\hat{k} = m(5\hat{i} + y\hat{j} + \hat{k}) + n(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\Rightarrow 2\hat{i} + 2\hat{j} - 2\hat{k} = (5m + n)\hat{i} + (my + 2n)\hat{j} + (m + 2n)\hat{k}$$

Equating components of $\hat{i}$, $\hat{j}$, and $\hat{k}$ from both sides:

$$5m + n = 2, my + 2n = 2, m + 2n = -2.$$

Solving them, we get: $m = 2/3$, $n = -4/3$ and $y = 7$.

ADDITION OF VECTORS

How to Add Two Vectors Graphically (Tip to Tail Method)

1. Draw the two vectors by arrow head lines using the same suitable scale.
2. Put the second vector such that its tail coincides with the head or tip of the first vector.
3. Now, draw a single vector from the tail of the first vector to the head of the second vector. This single vector represents the resultant of the two vectors.

Let us discuss some cases:

1. When two vectors are acting in the same direction (Fig. 2.15). Let the two vectors $\vec{x}$ and $\vec{y}$ be acting in the same direction.

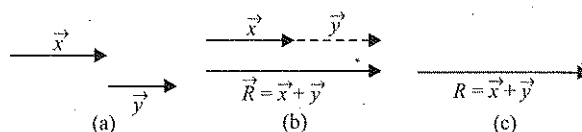


Fig. 2.15

2. When two vectors are acting in opposite directions: To find the resultant, in this case, coincide the head of $\vec{x}$ on the tail of $\vec{y}$ and then draw a single vectors $\vec{R}$ from the tail of $\vec{x}$ to the head of $\vec{y}$. The vector $\vec{R}$ gives the resultant of vectors $\vec{x}$ and $\vec{y}$ (see Fig. 2.16).

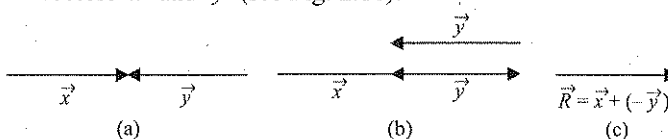


Fig. 2.16

The direction of the resultant vector is the same as that of bigger vector.

3. When two vectors are acting at some angle:

First join the tail of $\vec{y}$ with the head of $\vec{x}$ and then, to find the resultant in this case, draw a vector $\vec{R}$ from the tail of $\vec{x}$ to the head of $\vec{y}$. This single vector $\vec{R}$ drawn is the resultant vector (Fig. 2.17).

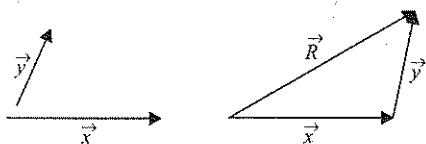


Fig. 2.17

$\vec{R}$ represents the resultant of $\vec{x}$ and $\vec{y}$ both in magnitude and direction. So, $\vec{R} = \vec{x} + \vec{y}$.

TRIANGLE LAW OF VECTOR ADDITION

If two vectors can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented by the third side of the triangle (both in magnitude and direction) taken in reverse direction.

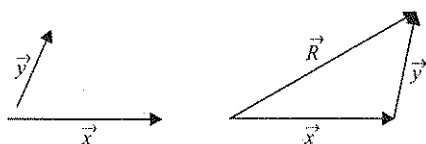


Fig. 2.18

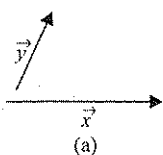
Suppose $\vec{x}$ and $\vec{y}$ are two vectors acting on a particle at the same time. They are represented as two sides of a triangle and $\vec{R}$ represents the third side (Fig. 2.18).

Thus, $\vec{R}$ represents the resultant of $\vec{x}$ and $\vec{y}$ both in magnitude and direction. So, we can write: $\vec{R} = \vec{x} + \vec{y}$.

PARALLELOGRAM LAW OF VECTOR ADDITION

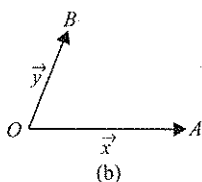
It states that if two vectors can be represented both in magnitude and direction by two adjacent sides of a parallelogram, then the resultant is represented completely both in magnitude and direction by the corresponding diagonal of the parallelogram.

(i) Consider two vectors $\vec{x}$ and $\vec{y}$ as shown.



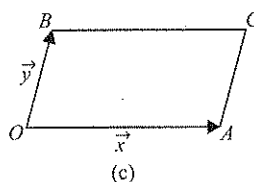
(a)

(ii) Draw these two vectors $\vec{x}$ and $\vec{y}$ from a common point O .



(b)

(iii) Now complete the parallelogram as shown.

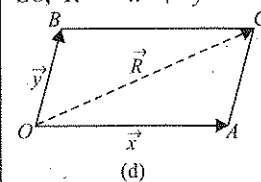


(c)

(iv) Draw the diagonal $\vec{OC}$ which represents resultant of $\vec{x}$ and $\vec{y}$.

$$\vec{OC} = \vec{OA} + \vec{OB}$$

So, $\vec{R} = \vec{x} + \vec{y}$



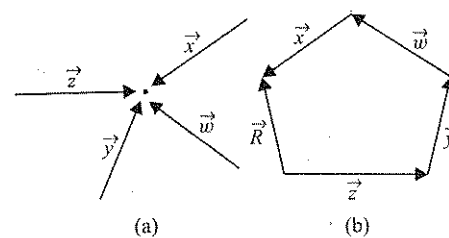
(d)

ADDITION OF MORE THAN TWO VECTORS

For this, we can use following steps:

1. Represent these vectors by arrow head lines using the same suitable scale.
2. Put these vectors in such a way that the head of one coincides with the tail of second and so on to the last vector.
3. Then, draw a single vector from the tail of first vector to the head of the last vector.
4. This single vector represents the resultant of all the vectors.

The above mentioned process may be known as *polygon law of vector addition*. It states that if any number of vectors acting on a particle at the same time are represented in magnitude and direction by the sides of an open polygon taken in order, then their resultant is represented both in magnitude and direction by the closing side of the polygon. Consider four vectors $\vec{w}$, $\vec{x}$, $\vec{y}$, $\vec{z}$ as shown in Fig. 2.19(a).



(a)

(b)

Fig. 2.19

Then, $\vec{R} = \vec{x} + \vec{w} + \vec{y} + \vec{z}$, as shown in Fig. 2.19(b), represents the resultant of $\vec{w}$, $\vec{x}$, $\vec{y}$, and $\vec{z}$.

VECTOR ADDITION BY ANALYTICAL METHOD

Here, we will treat both triangle law and the parallelogram law of vector addition analytically to find the resultant of two vectors.

1. Analytical treatment of triangle law of vector addition:

Let us consider two vectors $\vec{P}$ and $\vec{Q}$ acting simultaneously on a particle and inclined at an angle θ . Let these vectors be represented both in magnitude and direction by the two sides $\vec{OA}$ and $\vec{AC}$ of $\triangle OAC$ taken in same order. Then, the third side

$\vec{OC}$ represents the resultant (taken in opposite order) (Fig. 2.20).

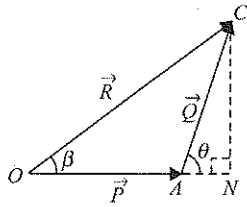


Fig. 2.20

Draw $CN \perp ON$ (extended).

From the right angled $\triangle CNO$,

$$OC^2 = R^2 = ON^2 + NC^2 = (OA + AN)^2 + NC^2$$

$$\Rightarrow R^2 = (P + AN)^2 + NC^2 \quad (i)$$

In right-angled $\triangle ANC$,

$$\sin \theta = \frac{NC}{Q} \Rightarrow NC = Q \sin \theta \text{ and } \cos \theta = \frac{AN}{Q}$$

$$\Rightarrow AN = Q \cos \theta$$

$$\text{From equation (i): } R^2 = (P + Q \cos \theta)^2 + Q^2 \sin^2 \theta$$

$$\Rightarrow R^2 = P^2 + Q^2 \cos^2 \theta + 2PQ \cos \theta + Q^2 \sin^2 \theta$$

$$\Rightarrow R^2 = P^2 + Q^2 (\cos^2 \theta + \sin^2 \theta) + 2PQ \cos \theta$$

$$\Rightarrow R^2 = P^2 + Q^2 + 2PQ \cos \theta \quad [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \quad (ii)$$

The equation of the magnitude of the resultant vector can be written in either of the following two ways.

$$1. |\vec{R}| = \sqrt{|\vec{P}|^2 + |\vec{Q}|^2 + 2|\vec{P}||\vec{Q}|\cos \theta}$$

$$2. |\vec{P} + \vec{Q}| = \sqrt{|\vec{P}|^2 + |\vec{Q}|^2 + 2|\vec{P}||\vec{Q}|\cos \theta}$$

Let β be the angle which $\vec{R}$ makes with $\vec{P}$. Then,

$$\tan \beta = \frac{NC}{ON} = \frac{NC}{OA + AN} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\therefore \beta = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right) = \tan^{-1} \left(\frac{|\vec{Q}| \sin \theta}{|\vec{P}| + |\vec{Q}| \cos \theta} \right)$$

which gives the direction of the resultant vector.

2. Analytical treatment of parallelogram law of vector addition:

Let us consider two vectors $\vec{P}$ and $\vec{Q}$ acting simultaneously at a point and let us further assume that they can be represented both in magnitude and direction by the two adjacent sides of a parallelogram $\vec{OA}$ and $\vec{OB}$ inclined at an angle θ w.r.t. each other. Draw $CN \perp ON$ (extended) (Fig. 2.21).

Remaining procedure is same as above in triangle law.

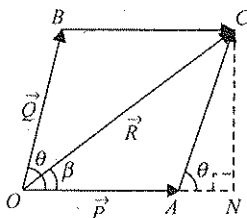


Fig. 2.21

Special Cases

Case 1. When the given vectors act in the same direction ($\theta = 0^\circ$).

$$\text{So, } R = \sqrt{P^2 + Q^2 + 2PQ \cos 0^\circ} \quad [\text{From equation (ii)}]$$

$$= \sqrt{P^2 + Q^2 + 2PQ} = \sqrt{(P + Q)^2} = P + Q$$

$$[\because \cos 0^\circ = 1]$$

or $|\vec{R}| = |\vec{P}| + |\vec{Q}|$ which represents the magnitude of the resultant.

Case 2. When the given vectors act in opposite directions ($\theta = 180^\circ$).

$$\text{So, } R = \sqrt{P^2 + Q^2 + 2PQ \cos 180^\circ} = \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P - Q)^2} \quad [\because \cos 180^\circ = -1]$$

$$R = \pm(P - Q) = P - Q \text{ or } Q - P$$

or $|\vec{R}| = ||\vec{P}| - |\vec{Q}||$ which represents the magnitude of the resultant.

Case 3. When the given vectors $\vec{P}$ and $\vec{Q}$ act at right angle to each other ($\theta = 90^\circ$).

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos 90^\circ} = \sqrt{P^2 + Q^2}$$

$$[\because \cos 90^\circ = 0]$$

$$\text{or } |\vec{R}| = \sqrt{|\vec{P}|^2 + |\vec{Q}|^2} \text{ and } \tan \beta = \frac{Q \sin 90^\circ}{P + Q \cos 90^\circ}$$

$$\text{or } \tan \beta = \frac{Q}{P} \quad [\because \sin 90^\circ = 1]$$

Note: Subtraction of two vectors can be followed from addition

Suppose we have to subtract a vector $\vec{B}$ from $\vec{A}$. So, we have to find $\vec{A} - \vec{B}$. It can also be written as $\vec{A} + (-\vec{B})$. Hence, subtraction of a vector $\vec{B}$ from $\vec{A}$ becomes as the addition of vectors $\vec{A}$ and $-\vec{B}$ (Fig. 2.22).

$$S = \sqrt{A^2 + B^2 + 2AB \cos(180 - \theta)}$$

$$\Rightarrow S = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

$$\tan \beta = \frac{B \sin(180 - \theta)}{A + B \cos(180 - \theta)} \Rightarrow \tan \beta = \frac{B \sin \theta}{A - B \cos \theta}$$

Vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$, **Vector subtraction is anti-commutative:** $\vec{A} - \vec{B} = -(\vec{B} - \vec{A})$ (Fig. 2.23).

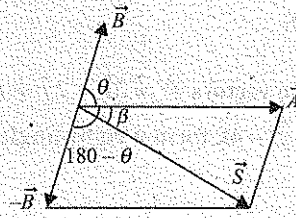


Fig. 2.22

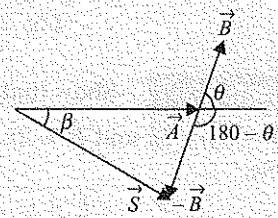


Fig. 2.23

Illustration 2.6 Two forces of 10 N and 15 N are acting at a point at an angle of 45° with each other. Find out the magnitude and the direction of their resultant force.

Sol. Given $A = 10$ N, $B = 15$ N, $\theta = 45^\circ$

$$\text{Magnitude of resultant force: } R = \sqrt{A^2 + B^2 + 2AB \cos \theta} \\ = \sqrt{10^2 + 15^2 + 2 \times 10 \times 15 \times \cos 45^\circ} = 23.76 \text{ N}$$

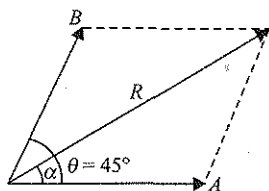


Fig. 2.24

$$\text{Direction: } \tan \alpha = \frac{B \sin \theta}{A + B \cos \theta} = \frac{15 \sin 45^\circ}{10 + 15 \cos 45^\circ} = 0.5147 \\ \Rightarrow \alpha = \tan^{-1}(0.5147) = 27^\circ$$

Illustration 2.7 Two forces of equal magnitudes are acting at a point. The magnitude of their resultant is equal to magnitude of the either. Find the angle between the force vectors.

Sol. Given $R = A = B$. Using $R^2 = A^2 + B^2 + 2AB \cos \theta$.

$$A^2 = A^2 + A^2 + 2AA \cos \theta \\ \Rightarrow \cos \theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$$

Condition for Zero Resultant Vectors

1. The resultant of two vectors can only be zero if they are equal in magnitude and opposite in direction (Fig. 2.25).



Fig. 2.25

2. The resultant of three or more vectors can be zero if they constitute a close figure when taken in same order (Fig. 2.26).

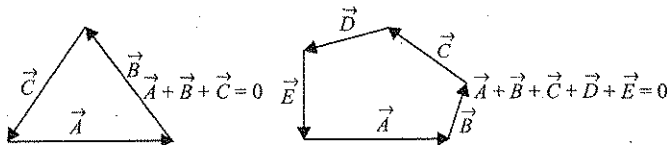


Fig. 2.26

Illustration 2.8 Show that the vectors $\vec{P} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{Q} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{R} = 2\hat{i} + \hat{j} - 4\hat{k}$ form a right-angled triangle.

Sol.

$$P = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{14}, \\ Q = \sqrt{1^2 + (-3)^2 + 5^2} = \sqrt{35},$$

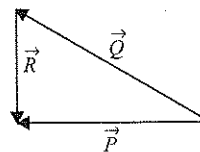


Fig. 2.27

and

$$R = \sqrt{2^2 + 1^2 + (-4)^2} = \sqrt{21}$$

We see that $\vec{P} = \vec{Q} + \vec{R}$ and $Q^2 = P^2 + R^2$ is satisfied. Hence, they will form a right-angled triangle, with $\vec{Q}$ as hypotenuse and $\vec{P}$ and $\vec{R}$ the other two sides as shown in Fig. 2.27.

Lami's Theorem

It states that if the resultant of three vectors is zero, then magnitude of a vector is directly proportional to the sine of angle between the other two vectors (see Fig. 2.28). Or it can be stated as if the resultant of three vectors is zero, then the ratio of magnitude of a vector to the sine of angle between the other two vectors is constant, i.e.,

$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$

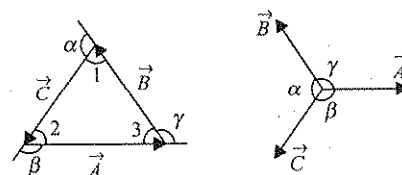


Fig. 2.28

Illustration 2.9 Given that $\vec{A} + \vec{B} + \vec{C} = \vec{0}$. Out of three vectors two are equal in magnitude and the magnitude of the third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Find the angles between the vectors.

Sol. Given $A = B$, $C = \sqrt{2}A = \sqrt{2}B = B$. From Fig. 2.29: $\alpha = \beta$ and $\alpha + \beta + \gamma = 180^\circ \Rightarrow \gamma = 180^\circ - 2\alpha$

$$\text{Apply Lami's theorem: } \frac{A}{\sin \alpha} = \frac{C}{\sin \gamma}$$

$$\Rightarrow \frac{A}{\sin \alpha} = \frac{\sqrt{2}A}{\sin(180 - 2\alpha)}$$

$$\Rightarrow \frac{1}{\sin \alpha} = \frac{\sqrt{2}}{\sin 2\alpha}$$

$$\Rightarrow \frac{1}{\sin \alpha} = \frac{\sqrt{2}}{2 \sin \alpha \cos \alpha} \Rightarrow \cos \alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$$

$$\Rightarrow \beta = 45^\circ \text{ and } \gamma = 180^\circ - 2\alpha = 90^\circ$$

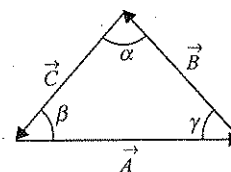


Fig. 2.29

Angle between $\vec{A}$ and $\vec{B} = 180^\circ - \gamma = 90^\circ$, angle between $\vec{B}$ and $\vec{C} = 180^\circ - \alpha = 135^\circ$, angle between $\vec{C}$ and $\vec{A} = 180^\circ - \beta = 135^\circ$.

Concept Application Exercise 2.1

1. A force of $(2\hat{i} + 3\hat{j} + \hat{k})$ N and another force of $(\hat{i} + \hat{j} + \hat{k})$ N are acting on a body. What is the magnitude of total force acting on the body?
2. If $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = 7\hat{i} + 24\hat{j}$, then find the vector having the same magnitude as that of $\vec{b}$ and parallel to $\vec{a}$.
3. In the vector diagram given below (Fig. 2.30), what is the angle between $\vec{A}$ and $\vec{B}$? (Given: $C = \frac{B}{2}$)

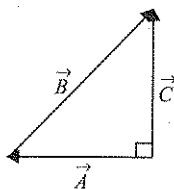


Fig. 2.30

4. What is the angle made by $3\hat{i} + 4\hat{j}$ with x-axis?
5. Three forces $\vec{A} = (\hat{i} + \hat{j} + \hat{k})$, $\vec{B} = (2\hat{i} - \hat{j} + 3\hat{k})$, and $\vec{C}$ are acting on a body which is kept at equilibrium. Find $\vec{C}$.
6. At what angle should the two force vectors $2F$ and $\sqrt{2}F$ act so that the resultant force is $\sqrt{10}F$?
7. Two forces while acting on a particle in opposite directions, have the resultant of 10 N. If they act at right angles to each other, the resultant is found to be 50 N. Find the two forces.
8. Two forces each equal to $F/2$ act at right angle. Their effect may be neutralized by a third force acting along their bisector in the opposite direction. What is the magnitude of that third force?
9. The resultant of two forces has magnitude 20 N. One of the forces is of magnitude $20\sqrt{3}$ N and makes an angle of 30° with the resultant. What is the magnitude of the other force?
10. The sum of the magnitudes of two vectors is 18. The magnitude of their resultant is 12. If the resultant is perpendicular to one of the vectors, then find the magnitudes of the two vectors.

RECTANGULAR COMPONENTS OF A VECTOR IN TWO DIMENSIONS

When a vector is split into two mutually perpendicular directions in a plane, the component vectors are called rectangular components of the given vector in a plane.

Fig. 2.31 shows vector $\vec{A}$ represented by $\vec{OP}$.

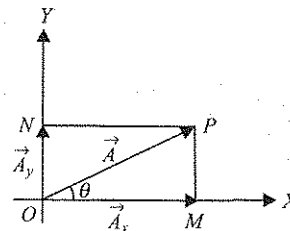


Fig. 2.31

Component along x-axis $A_x = A \cos \theta$

(Horizontal component) (1)

Component along y-axis $A_y = A \sin \theta$

(Vertical component) (2)

So, $\vec{A} = \vec{A}_x + \vec{A}_y = A_x \hat{i} + A_y \hat{j}$

Squaring and adding equations (1) and (2), we get

$$A_x^2 + A_y^2 = A^2(\sin^2 \theta + \cos^2 \theta) = A^2$$

$$\Rightarrow A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \tan \theta = \frac{PM}{OM} = \frac{A_y}{A_x}$$

So, knowledge of components of a vector gives information about angle which the vector makes with different axes.

Illustration 2.10 A force of 10 N is inclined at an angle of 30° to the horizontal. Find the horizontal and vertical components of the force.

Sol. Let $R = 10$ N. Horizontal component:

$$R_x = R \cos 30^\circ = 10\sqrt{3}/2 = 5\sqrt{3} \text{ N}$$

$$\text{Vertical component: } R_y = R \sin 30^\circ = 10/2 = 5 \text{ N}$$

Illustration 2.11 The x and y components of vector $\vec{A}$ are 4 and 6 m, respectively. The x and y components of vector $\vec{A} + \vec{B}$ are 10 and 9 m, respectively. Calculate for the vector $\vec{B}$ the followings: (i) its x and y components; (ii) its length, assuming that $\vec{A}$ and $\vec{B}$ lie in x - y plane; and (iii) the angle it makes with the x-axis.

Sol. Given $\vec{A} = 4\hat{i} + 6\hat{j}$ (1)

and $\vec{A} + \vec{B} = 10\hat{i} + 9\hat{j}$ (2)

Subtract equation (1) from (2), we get $\vec{B} = 6\hat{i} + 3\hat{j}$

1. Hence, x and y components of $\vec{B}$ are 6 and 3 m, respectively.

2. Length of $\vec{B}$ = magnitude of $\vec{B} = \sqrt{6^2 + 3^2} = 3\sqrt{5}$ m.

3. Let $\vec{B}$ makes an angle α with x-axis, then $\tan \alpha = 3/6$.

$$\Rightarrow \alpha = \tan^{-1}(1/2) = 26.6^\circ$$

RECTANGULAR COMPONENTS OF A VECTOR IN THREE DIMENSIONS

When a vector is split into mutually perpendicular directions in 3-D space, the component vectors obtained are called rectangular components of the given vector in 3-D space.

Fig. 2.32 shows vector $\vec{A}$ represented by $\vec{OP}$.

$$\text{Here, } \vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

The magnitude of $\vec{A}$ is given by, $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

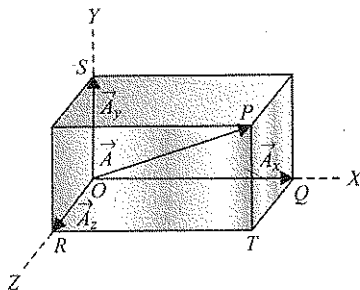


Fig. 2.32

Direction Cosines

Let A is a point in space whose coordinates are (x, y, z) , then its position vector w.r.t. the origin of coordinate system is given by: $\vec{r} = \overrightarrow{OA} = x\hat{i} + y\hat{j} + z\hat{k}$ (see Fig. 2.33).

$$\text{And } r = OA = \sqrt{x^2 + y^2 + z^2}$$

Angles of $\vec{r}$ with x -, y - and z -axis, respectively, are given by:

$$\cos \alpha = \frac{x}{r} = l, \cos \beta = \frac{y}{r} = m, \cos \gamma = \frac{z}{r} = n$$

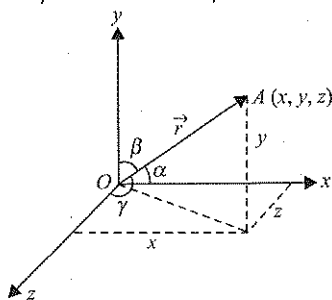


Fig. 2.33

The direction cosines l , m , and n of a vector are the cosines of the angles α , β and γ which a given vector makes with x -, y -, and z -axis, respectively.

Now, squaring and adding l , m , and n

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{x^2 + y^2 + z^2}{r^2}$$

$$\text{or } l^2 + m^2 + n^2 = \frac{r^2}{r^2} = 1$$

It means the sum of squares of the direction cosines of a vector is always unity.

Illustration 2.12 Given $\vec{A} = 5\hat{i} + 2\hat{j} + 4\hat{k}$. Find: (i) $|\vec{A}|$ and (ii) the direction cosines of vector $\vec{A}$.

Sol.

$$(i) \text{ As } \vec{A} = 5\hat{i} + 2\hat{j} + 4\hat{k} \Rightarrow |\vec{A}| = \sqrt{25 + 4 + 16} = \sqrt{45}$$

$$(ii) \cos \alpha = l = \frac{x}{r} = \frac{5}{\sqrt{45}}, \cos \beta = m = \frac{y}{r} = \frac{2}{\sqrt{45}},$$

$$\cos \gamma = n = \frac{z}{r} = \frac{4}{\sqrt{45}}$$

PRODUCT OF TWO VECTORS

There are two ways of vector multiplication.

1. Scalar or dot product
2. Vector or cross product.

Scalar or Dot Product

The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitudes of two vectors and the cosine of the smaller angle between them (Fig. 2.34). It is given by $\vec{A} \cdot \vec{B} = AB \cos \theta$.

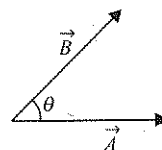


Fig. 2.34

Special Cases

1. If $\theta = 0^\circ$, $\vec{A} \cdot \vec{B} = AB$ (maximum value) [$\because \cos 0^\circ = 1$]
 2. If $\theta = 180^\circ$, $\vec{A} \cdot \vec{B} = -AB$ (negative maximum value) [$\because \cos 180^\circ = -1$]
 3. If $\theta = 90^\circ$, $\vec{A} \cdot \vec{B} = 0$ (minimum value) [$\because \cos 90^\circ = 0$]
- So, if two vectors are perpendicular, then their dot product is zero.
4. If θ is acute, then $\vec{A} \cdot \vec{B}$ is +ve. [$\because \cos \theta$ is +ve when θ is acute]
 5. If θ is obtuse, then $\vec{A} \cdot \vec{B}$ is -ve. [$\because \cos \theta$ is -ve when θ is obtuse]

Note: The dot product of two vectors is always a scalar quantity.

Dot Product of Unit Vectors Along x -, y -, and z -directions

Dot product of a unit vector with itself is unity and with other perpendicular unit vectors is zero (Fig. 2.35).

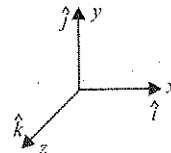


Fig. 2.35

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \text{ and}$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = \hat{j} \cdot \hat{k}$$

$$= \hat{k} \cdot \hat{j} = \hat{k} \cdot \hat{i} = \hat{i} \cdot \hat{k} = 0$$

In component form, the product is expressed as:

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$. Then

$$\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x \hat{i}(B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) + A_y \hat{j}(B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ + A_z \hat{k}(B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\text{So, } \vec{A} \cdot \vec{B} = A_x B_x (\hat{i} \cdot \hat{i}) + A_x B_y (\hat{i} \cdot \hat{j}) + A_x B_z (\hat{i} \cdot \hat{k}) + A_y B_x (\hat{j} \cdot \hat{i}) \\ + A_y B_y (\hat{j} \cdot \hat{j}) + A_y B_z (\hat{j} \cdot \hat{k}) + A_z B_x (\hat{k} \cdot \hat{i}) \\ + A_z B_y (\hat{k} \cdot \hat{j}) + A_z B_z (\hat{k} \cdot \hat{k}) = A_x B_x \\ + A_y B_y + A_z B_z$$

$$\text{or } \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Illustration 2.13 Find the dot product of two vectors

$$\vec{A} = 3\hat{i} + 2\hat{j} - 4\hat{k} \text{ and } \vec{B} = 2\hat{i} - 3\hat{j} - 6\hat{k}.$$

$$\text{Sol. } \vec{A} \cdot \vec{B} = 3 \times 2 + 2 \times (-3) + (-4) \times (-6) = 24$$

Dot product of a vector with itself

A vector is parallel to itself. So, the angle of a vector with itself is zero.

$$\therefore \vec{A} \cdot \vec{A} = AA \cos 0^\circ = A^2 \quad [\cdot \cos 0^\circ = 1]$$

Hence, the dot product of a vector with itself is square of its magnitude.

$$\text{We can also write: } \vec{A} \cdot \vec{A} = |\vec{A}|^2 \Rightarrow |\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}}$$

$$\text{For example: } |\vec{P} + \vec{Q}| = \sqrt{(\vec{P} + \vec{Q}) \cdot (\vec{P} + \vec{Q})} \\ \text{(Taking } \vec{A} = \vec{P} + \vec{Q})$$

$$\Rightarrow |\vec{P} + \vec{Q}|^2 = (\vec{P} + \vec{Q}) \cdot (\vec{P} + \vec{Q})$$

$$\Rightarrow |\vec{P} + \vec{Q}|^2 = P^2 + Q^2 + 2\vec{P} \cdot \vec{Q} = P^2 + Q^2 + 2PQ \cos \theta$$

$$\text{Similarly, } |\vec{P} - \vec{Q}|^2 = P^2 + Q^2 - 2\vec{P} \cdot \vec{Q} \\ = P^2 + Q^2 - 2PQ \cos \theta$$

Important Points

1. Angle between two vectors can be calculated from:

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

2. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ is 90° .

$$\text{Proof: Given } |\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

$$\text{Squaring both sides, we get } |\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

$$\Rightarrow A^2 + B^2 + 2\vec{A} \cdot \vec{B} = A^2 + B^2 - 2\vec{A} \cdot \vec{B}$$

$$\Rightarrow 4\vec{A} \cdot \vec{B} = 0$$

$$\Rightarrow \vec{A} \cdot \vec{B} = 0. \text{ Hence, } \vec{A} \text{ is perpendicular to } \vec{B}.$$

3. If $(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$, then $\vec{A}$ and $\vec{B}$ are equal in magnitude, i.e., $A = B$.

$$\text{Proof: Given } (\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$$

$$\Rightarrow \vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - \vec{B} \cdot \vec{B} = 0 \Rightarrow A^2 - B^2 = 0$$

$$\Rightarrow A^2 = B^2 \Rightarrow A = B. \text{ Hence proved.}$$

4. We can find addition of two vectors using dot product:

In Fig. 2.36, $\vec{R} = \vec{A} + \vec{B}$.

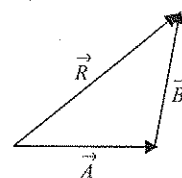


Fig. 2.36

$$\Rightarrow |\vec{R}| = |\vec{A} + \vec{B}|$$

$$\Rightarrow |\vec{R}|^2 = |\vec{A} + \vec{B}|^2$$

$$\Rightarrow R^2 = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = A^2 + B^2 + 2\vec{A} \cdot \vec{B}$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2\vec{A} \cdot \vec{B}} = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

Similarly, we can find subtraction of two vectors also.

Illustration 2.14 If the sum of two unit vectors is a unit vector, then find the magnitude of their difference.

Sol. Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then their sum is

$$\vec{n}_S = \hat{n}_1 + \hat{n}_2 \Rightarrow n_S^2 = n_1^2 + n_2^2 + 2n_1 n_2 \cos \theta \\ = 1 + 1 + 2 \cos \theta$$

Since it is given that $\vec{n}_S$ is a unit vector, so $n_S = 1$. Therefore,

$$1 = 1 + 1 + 2 \cos \theta \Rightarrow \cos \theta = -\frac{1}{2}$$

$$\Rightarrow \theta = 120^\circ$$

Now, the difference vector is $\vec{n}_d = \hat{n}_1 - \hat{n}_2$

$$\Rightarrow n_d^2 = n_1^2 + n_2^2 - 2n_1 n_2 \cos \theta = 1 + 1 - 2 \cos(120^\circ) = 3$$

$$\Rightarrow n_d = \sqrt{3}$$

Illustration 2.15 Find the value of m so that the vector

$3\hat{i} - 2\hat{j} + \hat{k}$ may be perpendicular to the vector $2\hat{i} + 6\hat{j} + m\hat{k}$.

Sol. For the vectors to be perpendicular their dot product has to be zero.

$$\therefore (3\hat{i} - 2\hat{j} + \hat{k}) \cdot (2\hat{i} + 6\hat{j} + m\hat{k}) = 0$$

$$\Rightarrow 6 - 12 + m = 0 \Rightarrow m - 6 = 0 \Rightarrow m = 6$$

Vector or Cross Product

Cross product of two vectors $\vec{A}$ and $\vec{B}$ is equal to the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and sine of the shortest angle between them, i.e., $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$, where $\hat{n}$ is the unit vector which represents the direction of $\vec{A} \times \vec{B}$ and it is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$. It is given by right handed screw rule depicted by Fig. 2.37. Note that $\hat{n}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

$$\text{Magnitude of } \vec{A} \times \vec{B}: |\vec{A} \times \vec{B}| = AB \sin \theta$$

$$\text{From here, we can write: } \vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| \hat{n}$$

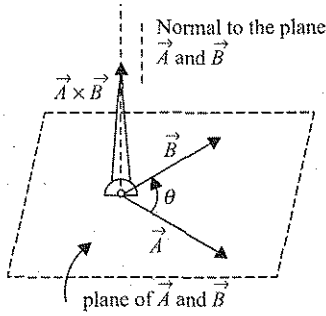


Fig. 2.37

$$\Rightarrow \hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

Unit Vectors and Their Cross Product

$\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors along x -, y -, and z -axis, respectively. The magnitude of each vector is 1 and the angle between any of two unit vectors is 90° . So, $\hat{i} \times \hat{j} = (1)(1) \sin 90^\circ \hat{n} = \hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the plane containing vector $\hat{i}$ and $\hat{j}$.

To find out the resultant of any two unit vectors in a cross product use the following rules (see Fig. 2.38).

1. Multiplication of any two unit vectors in anticlockwise direction gives third unit vector with positive sign.
2. Multiplication of any two unit vectors in clockwise direction gives third unit vector with negative sign.

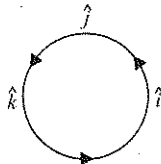


Fig. 2.38

From these rules, we obtain the following results.

From Rule 1:

$$1. \hat{i} \times \hat{j} = \hat{k} \quad 2. \hat{j} \times \hat{k} = \hat{i} \quad 3. \hat{k} \times \hat{i} = \hat{j}$$

From Rule 2:

$$1. \hat{j} \times \hat{i} = -\hat{k} \quad 2. \hat{k} \times \hat{j} = -\hat{i} \quad 3. \hat{i} \times \hat{k} = -\hat{j}$$

CROSS PRODUCT METHOD 1: USING COMPONENT FORM

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= A_x B_x (\hat{i} \times \hat{i}) + A_y B_x (\hat{j} \times \hat{i}) + A_z B_x (\hat{k} \times \hat{i}) \\ &\quad + A_x B_y (\hat{i} \times \hat{j}) + A_y B_y (\hat{j} \times \hat{j}) + A_z B_y (\hat{k} \times \hat{j}) \\ &\quad + A_x B_z (\hat{i} \times \hat{k}) + A_y B_z (\hat{j} \times \hat{k}) + A_z B_z (\hat{k} \times \hat{k}) \end{aligned}$$

[As $\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$, $\hat{k} \times \hat{k} = 0$ and $\hat{i} \times \hat{j} = \hat{k}$,

$$\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{i} = \hat{j}, \hat{i} \times \hat{k} = -\hat{j}, \hat{k} \times \hat{j} = -\hat{i}]$$

$$\begin{aligned} \text{So, we have } \vec{A} \times \vec{B} &= A_y B_x (-\hat{k}) + A_z B_x \hat{j} + A_x B_y \hat{k} \\ &\quad + A_z B_y (-\hat{i}) + A_x B_z (-\hat{j}) + A_y B_z (\hat{i}) \\ &= (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - A_z B_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \end{aligned}$$

CROSS PRODUCT METHOD 2: DETERMINANT METHOD

Cross product of two vectors $\vec{A}$ and $\vec{B}$ can be obtained easily by using the following method.

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \end{aligned}$$

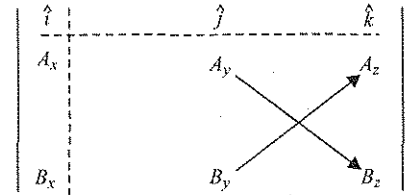


Fig. 2.39

Here, we will use $\hat{i}$, $\hat{j}$, $\hat{k}$ one by one. When $\hat{i}$ is chosen, its corresponding row and column become bound and remaining elements are subtracted after cross multiplication (Fig. 2.39). So, $\hat{i}(A_y B_z - B_y A_z)$, is the component along $\hat{i}$.

Similarly, in the case of $\hat{j}$, the row and column in which it is present become bound and remaining elements are subtracted after cross multiplication (Fig. 2.40).

So, $\hat{i}(A_y B_z - B_y A_z) - \hat{j}(A_x B_z - B_x A_z)$ is the component along $\hat{i}$ and $\hat{j}$.

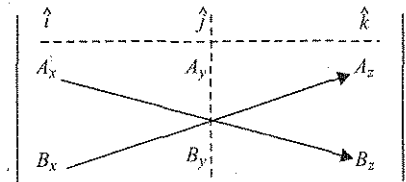


Fig. 2.40

Same argument will follow for $\hat{k}$ as is for $\hat{i}$ and $\hat{j}$ (Fig. 2.41).

$$\begin{aligned} \therefore \vec{A} \times \vec{B} &= \hat{i}(A_y B_z - B_y A_z) - \hat{j}(A_x B_z - B_x A_z) \\ &\quad + \hat{k}(A_x B_y - B_x A_y) \end{aligned}$$

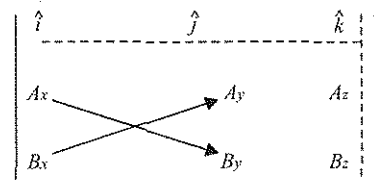


Fig. 2.41

Properties of Cross Product

1. Anticommutative property

The vector product of two vectors is anticommutative.

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} \text{ and } \vec{B} \times \vec{A} = BA \sin \theta (-\hat{n})$$

$$= -AB \sin \theta \hat{n} = -(\vec{A} \times \vec{B})$$

So, $\vec{B} \times \vec{A} = -(\vec{A} \times \vec{B})$. It means $\vec{B} \times \vec{A} \neq \vec{A} \times \vec{B}$.

2. Distributive property

Vector product is distributive, i.e.,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

3. Associative property

$$(\vec{A} + \vec{B}) \times (\vec{C} + \vec{D}) = \vec{A} \times \vec{C} + \vec{A} \times \vec{D} + \vec{B} \times \vec{C} + \vec{B} \times \vec{D}$$

4. Cross product of two parallel vectors

Cross product of the parallel vectors is zero.

As $\theta = 0^\circ$ (for parallel vectors), so

$$(\vec{A} \times \vec{B}) = AB \sin 0^\circ \hat{n} = 0$$

Important Points

1. If two vectors represent the two adjacent sides of a parallelogram, then the magnitude of the cross product will give the area of the parallelogram. Mathematically:

Two vectors $\vec{A}$ and $\vec{B}$ are represented by the two adjacent sides PQ and PS , respectively, of the parallelogram as shown in Fig. 2.42.

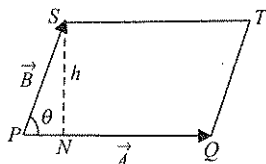


Fig. 2.42

Now, from the magnitude of the cross product:

$$|\vec{A} \times \vec{B}| = AB \sin \theta = Ah = \text{base} \times \text{height of parallelogram} = \text{area of parallelogram.}$$

2. If two vectors represent the two sides of a triangle, then half the magnitude of their cross product will give the area of the triangle.

Consider two vectors $\vec{A}$ and $\vec{B}$ represented by the two sides PQ and PS of triangle PQS (Fig. 2.43).

Using cross product: $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$

Taking magnitude, $|\vec{A} \times \vec{B}| = AB \sin \theta$

$$= A(B \sin \theta) = A \times h = \text{base} \times \text{height}$$

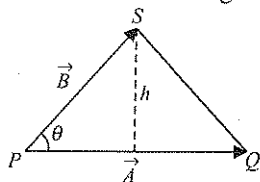


Fig. 2.43

Multiplying by $1/2$ on both sides,

$$\frac{1}{2} |\vec{A} \times \vec{B}| = \frac{1}{2} \times \text{base} \times \text{height} = \text{area of triangle.}$$

Illustration 2.16

Calculate the area of the triangle determined by the two vectors $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = -3\hat{i} + 7\hat{j}$.

Sol. We know that the half of magnitude of the cross product of two vectors gives the area of the triangle.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -3 & 7 & 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(21+12) = 33\hat{k}$$

Taking magnitude $|\vec{A} \times \vec{B}| = \sqrt{33^2} = 33$. So, area of triangle = $\frac{1}{2} |\vec{A} \times \vec{B}| = \frac{33}{2}$ sq. unit.

Illustration 2.17

Calculate the area of the parallelogram when adjacent sides are given by the vectors

$$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}.$$

Sol. We know that the area of the parallelogram is equal to the magnitude of the cross product of given vectors.

$$\text{Now, } \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= \hat{i}(2+9) + \hat{j}(6-1) + \hat{k}(-3-4)$$

$$= 11\hat{i} + 5\hat{j} - 7\hat{k}$$

So, area of parallelogram:

$$|\vec{A} \times \vec{B}| = \sqrt{11^2 + 5^2 + (-7)^2} = \sqrt{195} \text{ sq. unit.}$$

Concept Application Exercise 2.2

1. What is the area of parallelogram whose adjacent sides are given by vectors $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 4\hat{i} + 5\hat{j}$?
2. If the vectors $4\hat{i} + \hat{j} - 3\hat{k}$ and $2m\hat{i} + 6m\hat{j} + \hat{k}$ are perpendicular to each other, then find the value of m .
3. The magnitude of the vector product of two vectors is $\sqrt{3}$ times their scalar product. What is the angle between the two vectors?
4. What is the angle between $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i}$?
5. The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$. If $\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{r} = 4\hat{j} - 3\hat{k}$, then what is the magnitude of $\vec{v}$?
6. Find the magnitude of component of $3\hat{i} - 2\hat{j} + \hat{k}$ along the vector $12\hat{i} + 3\hat{j} - 4\hat{k}$.

Solved Examples

Example 2.1

A car is moving around a circular track with a constant speed v of 20 ms^{-1} (as shown in Fig. 2.44). At different times, the car is at A, B and C, respectively. Find the change in velocity:

1. from A to C,

2. from A to B.

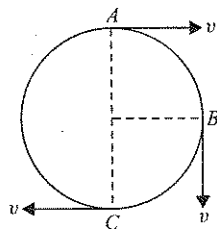


Fig. 2.44

Sol.

1. Change in velocity, as the car moves from A to C,

$$\Delta \vec{v}_{CA} = \vec{v}_C - \vec{v}_A = 20 - (-20) = 40 \text{ ms}^{-1}$$

$$\Delta \vec{v}_{CA} = 40 \text{ ms}^{-1} \text{ in the direction of } \vec{v}_C$$

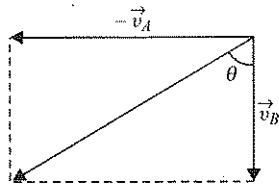


Fig. 2.45

2. Change in velocity, as the car moves from A to B,

$$\Delta \vec{v}_{BA} = \vec{v}_B - \vec{v}_A = \vec{v}_B + (-\vec{v}_A)$$

$$\Delta \vec{v}_{BA} = \sqrt{20^2 + 20^2} = \sqrt{800} \text{ ms}^{-1} = 28.28 \text{ ms}^{-1}$$

$$\text{Also, } \tan \theta = \frac{20}{20} = 1 \Rightarrow \theta = 45^\circ$$

This is the required direction of change in velocity.

Example 2.2 Given $\vec{A} = 0.3\hat{i} + 0.4\hat{j} + c\hat{k}$. Calculate the value of c if A is a unit vector.

Sol. If $\vec{A}$ is a unit vector, then its magnitude must be unity.

$$\Rightarrow \sqrt{a_x^2 + a_y^2 + a_z^2} = 1, \text{ i.e., } \sqrt{(0.3)^2 + (0.4)^2 + c^2} = 1$$

or $0.09 + 0.16 + c^2 = 1$ or $c^2 = 1 - 0.25 = 0.75$ or $c = 0.87$

Example 2.3 Determine that vector which when added to the resultant of $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives a unit vector along y-direction.

Sol. Here, $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$

$$\text{Resultant } \vec{R} = \vec{A} + \vec{B} = (3\hat{i} - 5\hat{j} + 7\hat{k}) + (2\hat{i} + 4\hat{j} - 3\hat{k})$$

$$= 5\hat{i} - \hat{j} + 4\hat{k}$$

Let the vector to be added is $\vec{X}$. So, the unit vector along y-direction $\hat{j} = 5\hat{i} - \hat{j} + 4\hat{k} + \vec{X}$

$$\text{So the required vector } \vec{X} = \hat{j} - (5\hat{i} - \hat{j} + 4\hat{k})$$

$$= -5\hat{i} + 2\hat{j} - 4\hat{k}.$$

Example 2.4 Find the unit vector of

$$\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}.$$

Sol. $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$. We know that $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$,

$$\text{where } |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{2^2 + 3^2 + 2^2} = \sqrt{4 + 9 + 4} = \sqrt{17}$$

$$\therefore \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{i} + 3\hat{j} + 2\hat{k}}{\sqrt{17}}$$

Example 2.5

Find the unit vector of $(\vec{A} + \vec{B})$, where

$$\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{B} = 3\hat{i} - 2\hat{j} - 2\hat{k}.$$

Sol. $(\vec{A} + \vec{B}) = (2\hat{i} - \hat{j} + 3\hat{k}) + (3\hat{i} - 2\hat{j} - 2\hat{k}) = 5\hat{i} - 3\hat{j} + \hat{k} = \vec{C}$ (say)

$$\text{Magnitude of } \vec{C} = C = [5^2 + (-3)^2 + 1^2]^{1/2} = \sqrt{35}$$

$$\text{So, } \hat{C} = \frac{\vec{C}}{C} = \frac{5\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{35}}.$$

Example 2.6

The greatest and least resultant of two forces acting at a point are 10 and 6 N, respectively. If each force is increased by 3 N, find the resultant of new forces when acting at a point at an angle of 90° with each other.

Sol. Let A and B be the two forces

$$\text{Greatest resultant} = A + B = 10 \quad (i)$$

$$\text{Least resultant} = A - B = 6 \quad (ii)$$

Solving equations (i) and (ii), we get, $A = 8 \text{ N}$; $B = 2 \text{ N}$

When each force is increased by 3 N, then

$$A' = A + 3 = 8 + 3 = 11 \text{ N}, B' = B + 3 = 2 + 3 = 5 \text{ N}.$$

As the new forces are acting at an angle of 90° (i.e., $\theta = 90^\circ$),

$$\text{so } R' = \sqrt{A'^2 + B'^2} = \sqrt{(11)^2 + (5)^2} = \sqrt{146} \text{ N}.$$

Example 2.7

Two forces whose magnitudes are in the ratio 3 : 5 give a resultant of 28 N. If the angle of their inclination is 60° , find the magnitude of each force.

Sol. Let A and B be the two forces.

$$\text{Then } A = 3x; B = 5x; R = 28 \text{ N and } \theta = 60^\circ$$

$$\text{Dividing A by B, } A/B = 3/5$$

$$\text{We know that } R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$\Rightarrow 28 = \sqrt{(3x)^2 + (5x)^2 + 2(3x)(5x) \cos 60^\circ}$$

$$\Rightarrow 28 = \sqrt{9x^2 + 25x^2 + 15x^2} = 7x \Rightarrow x = \frac{28}{7} = 4$$

$$\text{Hence, the forces are: } A = 3 \times 4 = 12 \text{ N}, B = 5 \times 4 = 20 \text{ N}.$$

Example 2.8

One of the rectangular components of a velocity of 100 ms^{-1} is 50 ms^{-1} . Find the other component.

Sol. Here, $v = 100 \text{ ms}^{-1}$, Let $v_x = 50 \text{ ms}^{-1}$. $v_x = v \cos \theta$

$$\Rightarrow 50 = 100 \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

$$\therefore v_y = v \sin \theta = 100 \sin 60^\circ = 100 \times \frac{\sqrt{3}}{2} = 50\sqrt{3} \text{ ms}^{-1}$$

Example 2.9

An aeroplane takes off at an angle of 60° to the horizontal. If muzzle velocity of the plane is 200 kmh^{-1} , calculate its horizontal and vertical components.

Sol. Here, $v = 200 \text{ kmh}^{-1}$, $\theta = 60^\circ$.

$$\begin{aligned}\therefore \text{Horizontal component } v_x &= v \cos \theta = 200 \cos 60^\circ \\ &= 200 \times \frac{1}{2} = 100 \text{ kmh}^{-1}\end{aligned}$$

$$\begin{aligned}\text{Vertical component } v_y &= v \sin \theta = 200 \sin 60^\circ \\ &= 200 \times \frac{\sqrt{3}}{2} = 100\sqrt{3} \text{ kmh}^{-1}\end{aligned}$$

Example 2.10 Prove that $(\vec{A} + 2\vec{B}) \cdot (2\vec{A} - 3\vec{B}) = 2A^2 + AB \cos \theta - 6B^2$.

$$\begin{aligned}\text{Sol. } (\vec{A} + 2\vec{B}) \cdot (2\vec{A} - 3\vec{B}) &= 2\vec{A} \cdot \vec{A} - 3\vec{A} \cdot \vec{B} + 4\vec{B} \cdot \vec{A} - 6(\vec{B} \cdot \vec{B}) \\ &= 2(\vec{A} \cdot \vec{A}) - 3\vec{A} \cdot \vec{B} \cos \theta + 4\vec{A} \cdot \vec{B} \cos \theta - 6(\vec{B} \cdot \vec{B}) \\ &= 2A^2 + AB \cos \theta - 6B^2\end{aligned}$$

Example 2.11 A body constrained to move along the z -axis of a coordinate system is subjected to a constant force $\vec{F}$ given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N, where $\hat{i}$, $\hat{j}$, and $\hat{k}$ represent unit vectors along x -, y -, and z -axis of the system, respectively. Calculate the work done by this force in displacing the body through a distance of 4 m along the z -axis.

$$\text{Sol. Displacement} = 4\hat{k}, \text{ Force: } \vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\begin{aligned}\text{Since work } W &\text{ is the scalar product of force and displacement,} \\ \therefore W &= (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot 4\hat{k} = -4(\hat{i} \cdot \hat{k}) + 8(\hat{j} \cdot \hat{k}) + 12(\hat{k} \cdot \hat{k}) \\ W &= 12 \text{ joule, because } \hat{i} \cdot \hat{k} = 0 = \hat{j} \cdot \hat{k} \text{ and } \hat{k} \cdot \hat{k} = 1\end{aligned}$$

Example 2.12

1. Prove that the vectors $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$, and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form a right-angled triangle.
2. Determine the unit vector parallel to the cross product of the vectors $\vec{A} = 3\hat{i} - 2\hat{j} + 10\hat{k}$ and $\vec{B} = 6\hat{i} + 5\hat{j} + 2\hat{k}$.

Sol.

1. The given vectors will constitute a triangle only if one of the given vectors is equal to vector sum of the remaining two vectors. In the given problem, $\vec{B} + \vec{C} = \vec{A}$. So, the given vectors do form a triangle. This triangle will be right angled only if the dot product of two vectors (out of the given three) is zero,

$$\begin{aligned}\vec{A} \cdot \vec{B} &= (3\hat{i} - 2\hat{j} + \hat{k}) \cdot (\hat{i} - 3\hat{j} + 5\hat{k}) = 3(\hat{i} \cdot \hat{i}) + 6(\hat{j} \cdot \hat{j}) \\ &\quad + 5(\hat{k} \cdot \hat{k}) = 3 + 6 + 5 = 14\end{aligned}$$

$$\begin{aligned}\vec{B} \cdot \vec{C} &= (\hat{i} - 3\hat{j} + 5\hat{k}) \cdot (2\hat{i} + \hat{j} - 4\hat{k}) = 2(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) \\ &\quad - 20(\hat{k} \cdot \hat{k}) = 2 - 3 - 20 = -21\end{aligned}$$

$$\begin{aligned}\vec{C} \cdot \vec{A} &= (2\hat{i} + \hat{j} - 4\hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k}) = 6(\hat{i} \cdot \hat{i}) - 2(\hat{j} \cdot \hat{j}) \\ &\quad - 4(\hat{k} \cdot \hat{k}) = 6 - 2 - 4 = 0\end{aligned}$$

Since the dot product of $\vec{C}$ and $\vec{A}$ is zero, therefore it implies that $\vec{C}$ is perpendicular to $\vec{A}$.

2. The unit vector parallel to $(\vec{A} \times \vec{B})$ is given by
$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

So, let us first determine $\vec{A} \times \vec{B}$.

$$\begin{aligned}\text{Now, } \vec{A} \times \vec{B} &= (3\hat{i} - 5\hat{j} + 10\hat{k}) \times (6\hat{i} + 5\hat{j} + 2\hat{k}) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & 10 \\ 6 & 5 & 2 \end{vmatrix} = \hat{i}(-10 - 50) + \hat{j}(60 - 6) + \hat{k}(15 + 30) \\ &= -60\hat{i} + 54\hat{j} + 45\hat{k} \\ \text{Magnitude: } |\vec{A} \times \vec{B}| &= \sqrt{(-60)^2 + (54)^2 + (45)^2} = \sqrt{8541} \\ \text{So, required unit vector: } \hat{n} &= \frac{-60\hat{i} + 54\hat{j} + 45\hat{k}}{\sqrt{8541}}\end{aligned}$$

Example 2.13 A man rows a boat with a speed of 18 kmh^{-1} in the north-west direction. The shoreline makes an angle of 15° south of west. Obtain the components of the velocity of the boat along the shoreline and perpendicular to the shoreline.

Sol. The north-west direction of the boat makes an angle of 60° with the shoreline (Fig. 2.46).

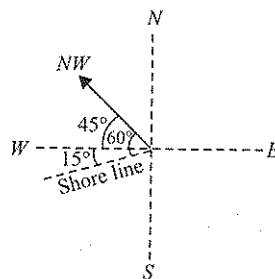


Fig. 2.46

Component of the velocity of boat along the shoreline

$$= 18 \cos 60^\circ \text{ kmh}^{-1} = 9 \text{ kmh}^{-1}$$

Component of the boat velocity along a line normal to the shoreline

$$= 18 \sin 60^\circ \text{ kmh}^{-1} = 18 \times \frac{\sqrt{3}}{2} \text{ kmh}^{-1} = 15.59 \text{ kmh}^{-1}$$

Example 2.14 A point P lies in the xy plane. Its position can be specified by its x , y coordinates or by a radially directed vector $\vec{r} = (x\hat{i} + y\hat{j})$ making an angle θ with the x -axis. Find a vector $\hat{i}_r$ of unit magnitude in the direction of vector $\vec{r}$ and a vector $\hat{i}_\theta$ of unit magnitude normal to the vector $\hat{i}_r$ and lying in the xy plane (Fig. 2.47).

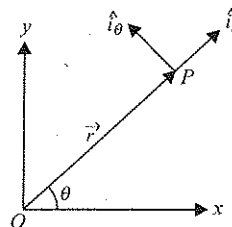


Fig. 2.47

Sol. When a vector is divided by its magnitude, we get a unit vector (Fig. 2.48).

$$\hat{i}_r = \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j}}{r} = \hat{i}\frac{x}{r} + \hat{j}\frac{y}{r}; \quad \cos \theta = \frac{x}{r} \text{ or } x = r \cos \theta$$

$$\text{Again, } \sin \theta = \frac{y}{r} \text{ or } y = r \sin \theta.$$

$$\text{So, we get: } \hat{i}_r = \hat{i} \cos \theta + \hat{j} \sin \theta$$

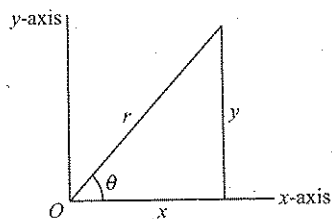


Fig. 2.48

Now, let $\hat{i}_\theta = \hat{i}\alpha + \hat{j}\beta$, where α and β are coefficients to be determined.

Making use of scalar product:

$$\hat{i}_r \cdot \hat{i}_\theta = (\hat{i} \cos \theta + \hat{j} \sin \theta) \cdot (\hat{i}\alpha + \hat{j}\beta) \quad (i)$$

Since dot product of perpendicular vectors is zero,

$$\hat{i}_r \cdot \hat{i}_\theta = 0 \Rightarrow \alpha \cos \theta + \beta \sin \theta = 0$$

$$\Rightarrow \alpha = -\beta \frac{\sin \theta}{\cos \theta}$$

Again, since $\hat{i}_\theta$ is a vector of unit magnitude: $\alpha^2 + \beta^2 = 1$. Put the value of α and get: $\beta = \pm \cos \theta$ and $\alpha = \mp \sin \theta$.

And we get $\hat{i}_\theta = \mp \hat{i} \sin \theta \pm \hat{j} \cos \theta$. Since $\hat{i}_\theta$ should have -ve x component and +ve y component, so finally we have: $\hat{i}_\theta = -\hat{i} \sin \theta + \hat{j} \cos \theta$

EXERCISES

Subjective Type

Solutions on page 2.22

- What is the essential condition for the addition of two vectors?
 - Is addition of any two scalar quantities meaningful?
 - Component of a vector can be a scalar. State true or false.
 - Can two vectors of same magnitude add to give zero resultant vector? If yes, under what conditions?
 - Can two vectors of different magnitudes add to give zero resultant vector? Can three vectors give the zero resultant vector on addition. If yes, under what conditions?
 - Can a rectangular component of a vector have magnitude greater than the vector itself?
 - Can a vector be zero if one of its component is not zero?
 - Can scalar product of two vectors be a negative quantity?
 - State the condition (regarding the value of dot or cross product) for which two vectors are:
 - parallel to each other,
 - perpendicular to each other.
 - Is possession of magnitude and direction sufficient for calling a quantity a vector quantity?
 - Is it necessary that sum of two unit vectors is also a unit vector?
 - What will be the difference in the product of
 - a real number with a vector, and
 - a scalar with a vector.
 - Explain the difference between the following data:
 - 4 (5 kmh⁻¹, east)
 - 4 h (5 kmh⁻¹, east)
- Two equal forces have a resultant equal to one and a half times the either force. Find the angle between the forces.

- At what angle two forces $(P + Q)$ and $(P - Q)$ act so that their resultant is

$$\text{a. } \sqrt{3P^2 + Q^2} \quad \text{b. } \sqrt{2(P^2 + Q^2)}$$

- Two forces 7 and 3 N simultaneously act on a body. What is the value of their (i) maximum resultant, (ii) minimum resultant, and (iii) what will be the resultant if the forces act at right angle to each other?
- Find the resultant force of the following forces which are acting simultaneously upon a particle.
 - 30 N due East
 - 20 N due North
 - 50 N due West
 - 40 N due South
- For the vectors $\vec{A}$ and $\vec{B}$ in Fig. 2.49, use a scale drawing to find the magnitude and direction of

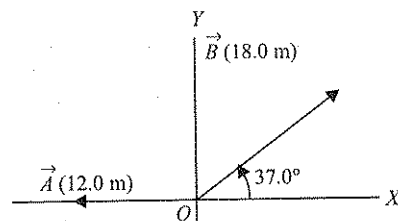


Fig. 2.49

- the vector sum $\vec{A} + \vec{B}$.
 - the vector difference $\vec{A} - \vec{B}$.
 - From your answers to parts (a) and (b), find the magnitude and direction of
 - $-\vec{A} - \vec{B}$
 - $\vec{B} - \vec{A}$
- If two vectors $\vec{a} = 4 \text{ ms}^{-1}$ and $\vec{b} = 7 \text{ ms}^{-1}$ be inclined at an angle of 60° to each other, then determine the direction and magnitude of their resultant.
 - Find a unit vector along and opposite to the vector $3\hat{i} - 4\hat{j} + 12\hat{k}$.
 - If $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$, then find

- a. $3\vec{a} + 2\vec{b} + \vec{c}$
 b. $\vec{a} - 6\vec{b} + 2\vec{c}$
10. Given two vectors $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$ and $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$.
- Find the magnitude of each vector.
 - Write an expression for the vector difference $\vec{A} - \vec{B}$ using unit vectors.
 - Find the magnitude and direction of the vector difference $\vec{A} - \vec{B}$.
 - In a vector diagram show $\vec{A}$, $\vec{B}$, and $\vec{A} - \vec{B}$, and also show that your diagram agrees qualitatively with your answer in part (c).
11. a. Is the vector $(\hat{i} + \hat{j} + \hat{k})$ a unit vector? Justify your answer.
 b. Can a unit vector have any rectangular component with a magnitude greater than unity? Justify your answer.
 c. If $\vec{A} = a(3\hat{i} + 4\hat{j})$, where a is a constant, determine the value of a that makes $\vec{A}$ a unit vector.
12. Resolve the vector $\vec{R} = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + 2\hat{j}$ and $\hat{i} - \hat{j}$ and write down the resolved components.
13. Find the rectangular component of vector $\vec{R} = 2\hat{i} + 3\hat{j}$ along $\vec{A} = \hat{i} + \hat{j}$.
14. Find the angle between each of the following pairs of vectors.
- $\vec{A} = -2.00\hat{i} + 6.00\hat{j}$ and $\vec{B} = 2.00\hat{i} - 3.00\hat{j}$
 - $\vec{A} = 3.00\hat{i} + 5.00\hat{j}$ and $\vec{B} = 10.00\hat{i} + 6.00\hat{j}$
 - $\vec{A} = -4.00\hat{i} + 2.00\hat{j}$ and $\vec{B} = 7.00\hat{i} + 14.00\hat{j}$
15. A cube is placed so that one corner is at the origin and three edges are along the x -, y -, and z -axis of a coordinate system (Fig. 2.50). Use vectors to compute

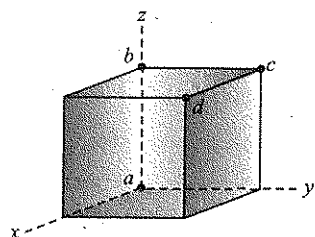


Fig. 2.50

- the angle between the edge along the z -axis (line ab) and the diagonal from the origin to the opposite corner (line ad).
 - the angle between line ac (the diagonal of a face) and line ad .
16. You are given vectors $\vec{A} = 5\hat{i} - 6.5\hat{j}$ and $\vec{B} = 10\hat{i} + 7\hat{j}$. A third vector $\vec{C}$ lies in the $x-y$ plane. Vector $\vec{C}$ is perpendicular to vector $\vec{A}$ and the scalar product of $\vec{C}$ with $\vec{B}$ is 15. From this information, find the components of vector $\vec{C}$.

17. Two vectors $\vec{A}$ and $\vec{B}$ have magnitudes $A = 3.00$ and $B = 3.00$. Their vector product is $\vec{A} \times \vec{B} = -5.00\hat{k} + 2.00\hat{i}$. What is the angle between $\vec{A}$ and $\vec{B}$?
18. Two vectors have magnitudes 5 units and 12 units, respectively. Find their cross product if the angle between them is 30° .
19. Given two vectors $\vec{A} = 3\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} - \hat{k}$. Find the
- area of the triangle whose two sides are represented by the vectors $\vec{A}$ and $\vec{B}$.
 - area of the parallelogram whose two adjacent sides are represented by the vectors $\vec{A}$ and $\vec{B}$.
 - area of the parallelogram whose diagonals are represented by the vectors $\vec{A}$ and $\vec{B}$.
20. On a horizontal flat ground, a person is standing at a point A . At this point, he installs a 5 m long pole vertically. Now, he moves 5 m towards east and then 2 m towards north and reaches at a point B . There he installs another 3 m long vertical pole. A bird flies from the top of first pole to the top of second pole. Find the displacement and magnitude of the displacement of the bird.
21. Find the vector sum of N coplanar forces, each of magnitude F , when each force is making an angle of $\frac{2\pi}{N}$ with that preceding it.
22. Can you find at least one vector perpendicular to $3\hat{i} - 4\hat{j} + 7\hat{k}$?
23. Establish the following inequalities:
- $|\vec{A} + \vec{B}| \leq |\vec{A}| + |\vec{B}|$
 - $|\vec{A} + \vec{B}| \geq ||\vec{A}| - |\vec{B}||$
 - $|\vec{A} - \vec{B}| \leq |\vec{A}| + |\vec{B}|$
 - $|\vec{A} - \vec{B}| \geq ||\vec{A}| - |\vec{B}||$
24. Two forces P and Q acting at a point are such that if P is reversed, the direction of the resultant is turned through 90° , then prove that magnitudes of the forces are equal.
25. Unit vectors $\hat{P}$ and $\hat{Q}$ are inclined at an angle θ , then prove that $|\hat{P} - \hat{Q}| = 2 \sin(\theta/2)$.

Objective Type

Solutions on page 2.25

- The sum and difference of two perpendicular vectors of equal lengths are
 - also perpendicular and of equal length
 - also perpendicular and of different lengths
 - of equal length and have an obtuse angle between them
 - of equal length and have an acute angle between them
- The minimum number of vectors having different planes which can be added to give zero resultant is
 - 2
 - 3
 - 4
 - 5

3. A vector perpendicular to $\hat{i} + \hat{j} + \hat{k}$ is

a. $\hat{i} - \hat{j} + \hat{k}$ b. $\hat{i} - \hat{j} - \hat{k}$
c. $-\hat{i} - \hat{j} - \hat{k}$ d. $3\hat{i} + 2\hat{j} - 5\hat{k}$

4. From Fig. 2.51, the correct relation is

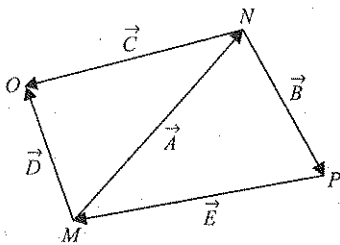


Fig. 2.51

a. $\vec{A} + \vec{B} + \vec{E} = \vec{O}$
b. $\vec{C} - \vec{D} = -\vec{A}$
c. $\vec{B} + \vec{E} - \vec{C} = -\vec{D}$
d. All of the above

5. Out of the following set of forces, the resultant of which cannot be zero

a. 10, 10, 10 b. 10, 10, 20
c. 10, 20, 20 d. 10, 20, 40

6. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to the vector $\vec{A}$ and its magnitude is equal to half of the magnitude of vector $\vec{B}$. The angle between $\vec{A}$ and $\vec{B}$ is

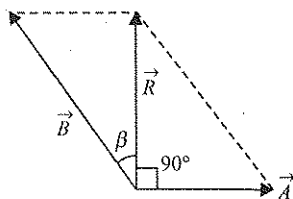


Fig. 2.52

a. 120° b. 150°
c. 135° d. None of these

7. The ratio of maximum and minimum magnitudes of the resultant of two vectors $\vec{a}$ and $\vec{b}$ is 3 : 1. Now, $|\vec{a}|$ is equal to

a. $|\vec{b}|$ b. $2|\vec{b}|$ c. $3|\vec{b}|$ d. $4|\vec{b}|$

8. Two forces, each equal to F , act as shown in Fig. 2.53. Their resultant is

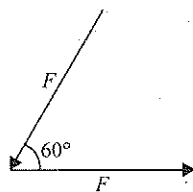


Fig. 2.53

a. $F/2$ b. F c. $\sqrt{3}F$ d. $\sqrt{5}F$

9. Vector $\vec{A}$ is 2 cm long and is 60° above the x -axis in the first quadrant. Vector $\vec{B}$ is 2 cm long and is 60° below the x -axis in the fourth quadrant. The sum $\vec{A} + \vec{B}$ is a vector of magnitude

a. 2 cm along $+y$ -axis
b. 2 cm along $+x$ -axis
c. 2 cm along $-x$ -axis
d. 2 cm along $-y$ -axis

10. What is the angle between two vector forces of equal magnitude such that the resultant is one-third as much as either of the original forces?

a. $\cos^{-1}\left(-\frac{17}{18}\right)$ b. $\cos^{-1}\left(\frac{1}{3}\right)$
c. 45° d. 120°

11. The angle between $\vec{A} + \vec{B}$ and $\vec{A} \times \vec{B}$ is

a. 0 b. $\pi/4$ c. $\pi/2$ d. π

12. The projection of a vector $\vec{r} = 3\hat{i} + \hat{j} + 2\hat{k}$ on the x - y plane has magnitude

a. 3 b. 4 c. $\sqrt{14}$ d. $\sqrt{10}$

13. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is

a. 120° b. 60° c. 90° d. 0°

14. If vectors $\vec{A} = \hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{B} = 5\hat{i}$ represent the two sides of a triangle, then the third side of the triangle can have length equal to

a. 6
b. $\sqrt{56}$
c. Both of the above
d. None of the above

15. Given $|\vec{A}_1| = 2$, $|\vec{A}_2| = 3$ and $|\vec{A}_1 + \vec{A}_2| = 3$. Find the value of $(\vec{A}_1 + 2\vec{A}_2) \cdot (3\vec{A}_1 - 4\vec{A}_2)$

a. 64 b. 60 c. 62 d. 61

16. Three vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ satisfy the relation $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \cdot \vec{C} = 0$. The vector $\vec{A}$ is parallel to

a. $\vec{B}$ b. $\vec{C}$
c. $\vec{B} \cdot \vec{C}$ d. $\vec{B} \times \vec{C}$

17. If $\vec{A} = \vec{B} + \vec{C}$, and the magnitudes of $\vec{A}$, $\vec{B}$, $\vec{C}$ are 5, 4, and 3 units, then angle between $\vec{A}$ and $\vec{C}$ is

a. $\cos^{-1}\left(\frac{3}{5}\right)$ b. $\cos^{-1}\left(\frac{4}{5}\right)$
c. $\sin^{-1}\left(\frac{3}{4}\right)$ d. $\frac{\pi}{2}$

18. Given: $\vec{A} = A \cos \theta \hat{i} + A \sin \theta \hat{j}$. A vector $\vec{B}$ which is perpendicular to $\vec{A}$ is given by

a. $B \cos \theta \hat{i} - B \sin \theta \hat{j}$

- b. $B \sin \theta \hat{i} - B \cos \theta \hat{j}$
 c. $B \cos \theta \hat{i} + B \sin \theta \hat{j}$
 d. $B \sin \theta \hat{i} + B \cos \theta \hat{j}$
19. The angle which the vector $\vec{A} = 2\hat{i} + 3\hat{j}$ makes with y-axis, where $\hat{i}$ and $\hat{j}$ are unit vectors along x- and y-axes, respectively, is
 a. $\cos^{-1}(3/5)$ b. $\cos^{-1}(2/3)$
 c. $\tan^{-1}(2/3)$ d. $\sin^{-1}(2/3)$
20. Given: $\vec{P} = 3\hat{i} - 4\hat{j}$. Which of the following is perpendicular to $\vec{P}$?
 a. $3\hat{i}$ b. $4\hat{j}$
 c. $4\hat{i} + 3\hat{j}$ d. $4\hat{i} - 3\hat{j}$
21. In going from one city to another, a car travels 75 km north, 60 km north-west and 20 km east. The magnitude of displacement between the two cities is (Take $1/\sqrt{2} = 0.7$)
 a. 170 km b. 137 km
 c. 119 km d. 140 km
22. What is the angle between $\vec{A}$ and $\vec{B}$, if $\vec{A}$ and $\vec{B}$ are the adjacent sides of a parallelogram drawn from a common point and the area of the parallelogram is $AB/2$?
 a. 15° b. 30° c. 45° d. 60°
23. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$. What is the angle between $\vec{a}$ and $\vec{b}$?
 a. 0° b. 90°
 c. 60° d. 180°
24. Given: $\vec{A} = 4\hat{i} + 6\hat{j}$ and $\vec{B} = 2\hat{i} + 3\hat{j}$. Which of the following is correct?
 a. $\vec{A} \times \vec{B} = \vec{0}$ b. $\vec{A} \cdot \vec{B} = 24$
 c. $\frac{|\vec{A}|}{|\vec{B}|} = \frac{1}{2}$ d. $\vec{A}$ and $\vec{B}$ are antiparallel
25. Given: $\vec{A} = 2\hat{i} + p\hat{j} + q\hat{k}$ and $\vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$. If $\vec{A} \parallel \vec{B}$, then the values of p and q are, respectively,
 a. $\frac{14}{5}$ and $\frac{6}{5}$ b. $\frac{14}{3}$ and $\frac{6}{5}$
 c. $\frac{6}{5}$ and $\frac{1}{3}$ d. $\frac{3}{4}$ and $\frac{1}{4}$
26. If $\vec{A}$ is perpendicular to $\vec{B}$, then
 a. $\vec{A} \times \vec{B} = \vec{0}$
 b. $\vec{A} \cdot [\vec{A} + \vec{B}] = A^2$
 c. $\vec{A} \cdot \vec{B} = AB$
 d. $\vec{A} \cdot [\vec{A} + \vec{B}] = A^2 + AB$
27. If the angle between vectors $\vec{a}$ and $\vec{b}$ is an acute angle, then the difference $\vec{a} - \vec{b}$ is
 a. the major diagonal of the parallelogram
 b. the minor diagonal of the parallelogram
 c. any of the above
 d. none of the above
28. Given that $\vec{A} + \vec{B} = \vec{C}$. If $|\vec{A}| = 4$, $|\vec{B}| = 5$ and $|\vec{C}| = \sqrt{61}$. The angle between $\vec{A}$ and $\vec{B}$ is
 a. 30° b. 60° c. 90° d. 120°
29. Given vector $\vec{A} = 2\hat{i} + 3\hat{j}$, the angle between $\vec{A}$ and y-axis is
 a. $\tan^{-1}(3/2)$ b. $\tan^{-1}(2/3)$
 c. $\sin^{-1}(2/3)$ d. $\cos^{-1}(2/3)$
30. If $\vec{b} = 3\hat{i} + 4\hat{j}$ and $\vec{a} = \hat{i} - \hat{j}$, the vector having the same magnitude as that of $\vec{b}$ and parallel to $\vec{a}$ is
 a. $\frac{5}{\sqrt{2}}(\hat{i} - \hat{j})$ b. $\frac{5}{\sqrt{2}}(\hat{i} + \hat{j})$
 c. $5(\hat{i} - \hat{j})$ d. $5(\hat{i} + \hat{j})$
31. Choose the wrong statement
 a. Three vectors of different magnitudes may be combined to give zero resultant.
 b. Two vectors of different magnitudes can be combined to give a zero resultant.
 c. The product of a scalar and a vector is a vector quantity.
 d. All of the above are wrong statements.
32. What displacement at an angle 60° to the x-axis has an x-component of 5 m? $\hat{i}$ and $\hat{j}$ are unit vectors in x and y directions, respectively.
 a. $5\hat{i}$ b. $5\hat{i} + 5\hat{j}$
 c. $5\hat{i} + 5\sqrt{3}\hat{j}$ d. All of the above
33. Mark the correct statement
 a. $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$
 b. $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$
 c. $|\vec{a} - \vec{b}| \geq |\vec{a}| + |\vec{b}|$
 d. All of the above
34. Out of the following forces, the resultant of which cannot be 10 N?
 a. 15 N and 20 N b. 10 N and 10 N
 c. 5 N and 12 N d. 12 N and 1 N
35. Which of the following pairs of forces cannot be added to give a resultant force of 4 N?
 a. 2 N and 8 N b. 2 N and 2 N
 c. 2 N and 6 N d. 2 N and 4 N
36. In an equilateral triangle ABC , AL , BM , and CN are medians. Forces along BC and BA represented by them will have a resultant represented by
 a. $2AL$ b. $2BM$ c. $2CN$ d. AC

37. The vector sum of two forces is perpendicular to their vector difference. The forces are
 a. equal to each other
 b. equal to each other in magnitude
 c. not equal to each other in magnitude
 d. cannot be predicted
38. If a parallelogram is formed with two sides represented by vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} + \vec{b}$ represents the
 a. major diagonal when the angle between vectors is acute
 b. minor diagonal when the angle between vectors is obtuse
 c. both of the above
 d. none of the above
39. The resultant $\vec{C}$ of $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$. Also, $|\vec{A}| = |\vec{C}|$. The angle between $\vec{A}$ and $\vec{B}$ is
 a. $\frac{\pi}{4}$ rad
 b. $\frac{3\pi}{4}$ rad
 c. $\frac{5\pi}{4}$ rad
 d. $\frac{7\pi}{4}$ rad
40. Two forces of $F_1 = 500$ N due east and $F_2 = 250$ N due north have their common initial point. $\vec{F}_2 - \vec{F}_1$ is
 a. $250\sqrt{5}$ N, $\tan^{-1}(2)$ W of N
 b. 250 N, $\tan^{-1}(2)$ W of N
 c. zero
 d. 750 N, $\tan^{-1}(3/4)$ N of W

41. The resultant of the three vectors $\vec{OA}$, $\vec{OB}$, and $\vec{OC}$ shown in Fig. 2.54 is

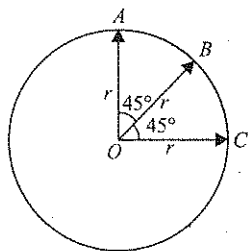


Fig. 2.54

- a. r
 b. $2r$
 c. $r(1 + \sqrt{2})$
 d. $r(\sqrt{2} - 1)$
42. Two vectors $\vec{a}$ and $\vec{b}$ are at an angle of 60° with each other. Their resultant makes an angle of 45° with $\vec{a}$. If $|\vec{b}| = 2$ units, then $|\vec{a}|$ is
 a. $\sqrt{3}$
 b. $\sqrt{3} - 1$
 c. $\sqrt{3} + 1$
 d. $\sqrt{3}/2$
43. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant vector becomes perpendicular to $\vec{P}$. Then, the magnitude of $\vec{R}$ is equal to
 a. $P + Q$
 b. P
 c. $P - Q$
 d. Q

44. A vector $\vec{A}$ when added to the vector $\vec{B} = 3\hat{i} + 4\hat{j}$ yields a resultant vector that is in the positive y-direction and has a magnitude equal to that of $\vec{B}$. Find the magnitude of $\vec{A}$.

a. $\sqrt{10}$ b. 10 c. 5 d. $\sqrt{15}$

45. $ABCDEF$ is a regular hexagon with point O as centre. The value of $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$ is

a. $2\vec{AO}$ b. $4\vec{AO}$ c. $6\vec{AO}$ d. 0

46. In a two dimensional motion of a particle, the particle moves from point A , position vector $\vec{r}_1$, to point B , position vector $\vec{r}_2$. If the magnitudes of these vectors are, respectively, $r_1 = 3$ and $r_2 = 4$ and the angles they make with the x -axis are $\theta_1 = 75^\circ$ and $\theta_2 = 15^\circ$, respectively, then find the magnitude of the displacement vector.

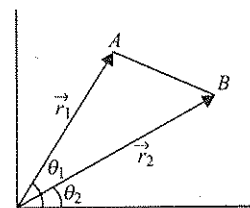


Fig. 2.55

- a. 15 b. $\sqrt{13}$ c. 17 d. $\sqrt{15}$
47. The sum of the magnitudes of two forces acting at a point is 16 N. The resultant of these forces is perpendicular to the smaller force and has a magnitude of 8 N. If the smaller force is of magnitude x , then the value of x is
 a. 2 N b. 4 N c. 6 N d. 7 N
48. The angle between two vectors $\vec{A}$ and $\vec{B}$ is θ . Resultant of these vectors $\vec{R}$ makes an angle $\theta/2$ with $\vec{A}$. Which of the following is true?
 a. $A = 2B$ b. $A = B/2$
 c. $A = B$ d. $AB = 1$
49. The resultant of three vectors 1, 2, and 3 units whose directions are those of the sides of an equilateral triangle is at an angle of
 a. 30° with the first vector
 b. 15° with the first vector
 c. -100° with the first vector
 d. 150° with the first vector
50. A particle moves in the xy plane with only an x -component of acceleration of 2 ms^{-2} . The particle starts from the origin at $t = 0$ with an initial velocity having an x -component of 8 ms^{-1} and y -component of -15 ms^{-1} . The total velocity vector at any time t is
 a. $[(8 + 2t)\hat{i} - 15\hat{j}] \text{ ms}^{-1}$
 b. zero
 c. $2t\hat{i} + 15\hat{j}$
 d. directed along z -axis

51. A unit vector along incident ray of light is $\hat{i}$. The unit vector for the corresponding refracted ray of light is $\hat{r}$. $\hat{n}$ is a unit vector normal to the boundary of the medium and directed towards the incident medium. If m be the refractive index of the medium, then Snell's law (2nd) of refraction is

- $\hat{i} \times \hat{n} = \mu(\hat{n} \times \hat{r})$
- $\hat{i} \cdot \hat{n} = \mu(\hat{r} \cdot \hat{n})$
- $\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$
- $\mu(\hat{i} \times \hat{n}) = \hat{r} \times \hat{n}$

52. The simple sum of two co-initial vectors is 16 units. Their vector sum is 8 units. The resultant of the vectors is perpendicular to the smaller vector. The magnitudes of the two vectors are

- 2 units and 14 units
- 4 units and 12 units
- 6 units and 10 units
- 8 units and 8 units

53. The components of a vector along x - and y -directions are $(n + 1)$ and 1 , respectively. If the coordinate system is rotated by an angle $\theta = 60^\circ$, then the components change to n and 3 . The value of n is

- 2
- $\cos 60^\circ$
- $\sin 60^\circ$
- 3.5

54. Two point masses 1 and 2 move with uniform velocities $\vec{v}_1$ and $\vec{v}_2$, respectively. Their initial position vectors are $\vec{r}_1$ and $\vec{r}_2$, respectively. Which of the following should be satisfied for the collision of the point masses?

- $\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} = \frac{\vec{v}_2 + \vec{v}_1}{|\vec{v}_2 + \vec{v}_1|}$
- $\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$
- $\frac{\vec{r}_2 + \vec{r}_1}{|\vec{r}_2 + \vec{r}_1|} = \frac{\vec{v}_2 + \vec{v}_1}{|\vec{v}_2 + \vec{v}_1|}$
- $\frac{\vec{r}_2 + \vec{r}_1}{|\vec{r}_2 + \vec{r}_1|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

55. What is the resultant of three coplanar forces: 300 N at 0° , 400 N at 30° , and 400 N at 150° ?

- 500 N
- 700 N
- 1,100 N
- 300 N

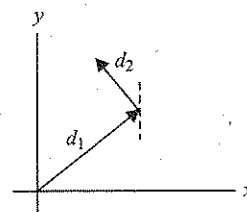


Fig. 2.56

- The signs of x - and y -components of $\vec{d}_1 + \vec{d}_2$ are positive.
 - None of these.
2. Given two vectors $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$. θ is the angle between $\vec{A}$ and $\vec{B}$. Which of the following statements is/are correct?
- $|\vec{A}| \cos \theta \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right)$ is the component of $\vec{A}$ along $\vec{B}$.
 - $|\vec{A}| \sin \theta \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}} \right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.
 - $|\vec{A}| \cos \theta \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}} \right)$ is the component of $\vec{A}$ along $\vec{B}$.
 - $|\vec{A}| \sin \theta \left(\frac{\hat{i} - \hat{j}}{2} \right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.
3. If $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ are two vectors, then the unit vector
- perpendicular to $\vec{A}$ is $\left(\frac{-\hat{j} + \hat{k}}{\sqrt{2}} \right)$
 - parallel to $\vec{A}$ is $\frac{(2\hat{i} + \hat{j} + \hat{k})}{\sqrt{6}}$
 - perpendicular to $\vec{B}$ is $\left(\frac{-\hat{j} + \hat{k}}{\sqrt{2}} \right)$
 - parallel to $\vec{A}$ is $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$
4. If $(\vec{v}_1 + \vec{v}_2)$ is perpendicular to $(\vec{v}_1 - \vec{v}_2)$, then
- $\vec{v}_1$ is perpendicular to $\vec{v}_2$
 - $|\vec{v}_1| = |\vec{v}_2|$
 - $\vec{v}_1$ is null vector
 - the angle between $\vec{v}_1$ and $\vec{v}_2$ can have any value
5. Two vectors $\vec{A}$ and $\vec{B}$ lie in one plane. Vector $\vec{C}$ lies in a different plane. Then, $\vec{A} + \vec{B} + \vec{C}$
- cannot be zero
 - can be zero
 - lies in the plane of $\vec{A}$ or $\vec{B}$
 - lies in a plane different from that of any of the three vectors

Multiple Correct Answers Type

Solutions on page 2.29

1. Which of the following statement is/are correct (see Fig. 2.56)?

- The signs of x -component of $\vec{d}_1$ is positive and that of $\vec{d}_2$ is negative.
- The signs of the y -component of $\vec{d}_1$ and $\vec{d}_2$ are positive and negative, respectively.

ANSWERS AND SOLUTIONS

Subjective Type

1.
 - a. The two vectors should be of the same nature, i.e., a force vector can be added only into another force vector, not in a velocity vector, say.
 - b. No, they should have same nature, i.e., mass can be added into mass, not in length, say.
 - c. False, component of a vector is also a vector.
 - d. Yes, if they are equal and opposite.
 - e. Two vectors of different magnitudes cannot add to give zero resultant. Three vectors of different magnitude can add to give zero resultant if they are coplanar.
 - f. No
 - g. No, a vector can be zero if all components are zero.
 - h. Yes
 - i.
 - i. cross product is zero
 - ii. dot product is zero
 - j. No, it should follow the vector rules of addition multiplication etc. For example, electric current has both magnitude and direction but still it is a vector quantity.
 - k. No
 - l.
 - i. nature of vector remains same, only magnitude may change
 - ii. magnitude of the vector changes
 - iii. • $4 (5 \text{ kmh}^{-1}, \text{ east}) = 20 \text{ kmh}^{-1}, \text{ east}$
Here, we are multiplying a velocity $5 \text{ kmh}^{-1}, \text{ east}$ with a real number 4. The final result obtained is $20 \text{ kmh}^{-1}, \text{ east}$ which is also a velocity. So, nature of vector remains same if it is multiplied with a real number, only magnitude may change.
 - $4 \text{ h } (5 \text{ kmh}^{-1}, \text{ east}) = 20 \text{ km}, \text{ east}$
Here, we multiply velocity with scalar quantity, time 4 h. The result obtained is 20 km, east which is a displacement vector. Here, nature of vector changes.

2. $R^2 = A^2 + B^2 + 2AB \cos \theta$ Given $A = B, R = 3A/2$

$$\Rightarrow \left(\frac{3}{2}A\right)^2 = A^2 + A^2 + 2A^2 \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1}{8} \Rightarrow \theta = 83^\circ$$

3. a. $R = \sqrt{3P^2 + Q^2}$, $A = P + Q$, $B = P - Q$

$$\text{Apply } R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$\Rightarrow 3P^2 + Q^2 = (P + Q)^2 + (P - Q)^2 + 2(P + Q)(P - Q) \cos \theta$$

$$\Rightarrow \cos = 1/2 \Rightarrow \theta = 60^\circ$$

b. Here, $R = \sqrt{2(P^2 + Q^2)}$, $A = P + Q$, $B = P - Q$

$$\text{Apply } R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$\Rightarrow 2(P^2 + Q^2) = (P + Q)^2 + (P - Q)^2 + 2(P + Q)(P - Q) \cos \theta$$

$$\Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

4. a. Resultant is maximum when both vectors act in same direction. $R_{\max} = A + B = 7 + 3 = 10 \text{ N}$

b. Resultant is minimum when both vectors act in opposite direction: $R_{\min} = A - B = 7 - 3 = 4 \text{ N}$

c. If both vectors act at right angle, then

$$R = \sqrt{A^2 + B^2} = \sqrt{7^2 + 3^2} = \sqrt{58} \text{ N}$$

5. Resultant force: $\vec{F} = 30\hat{i} + 20\hat{j} - 50\hat{i} - 40\hat{j}$

$$= 20\hat{i} - 20\hat{j} \quad F = \sqrt{20^2 + 20^2} = 20\sqrt{2} \text{ S - W}$$

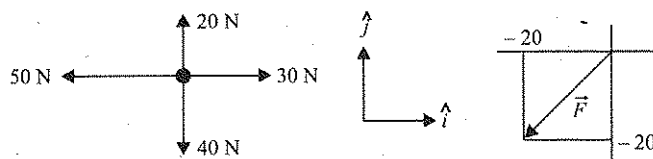


Fig. 2.57

6. a. $\vec{R} = \vec{A} + \vec{B}$

$$R = \sqrt{12^2 + 18^2 + 2 \times 12 \times 18 \cos(180 - 37^\circ)} = 11.1 \text{ m}$$

$$\tan(\alpha - 37^\circ) = \frac{12 \sin(180 - 37^\circ)}{18 + 12 \cos(180 - 37^\circ)}$$

$$\Rightarrow \alpha - 37^\circ = 40.6^\circ \Rightarrow \alpha = 77.6^\circ \text{ with } x\text{-axis (Fig. 2.58)}$$

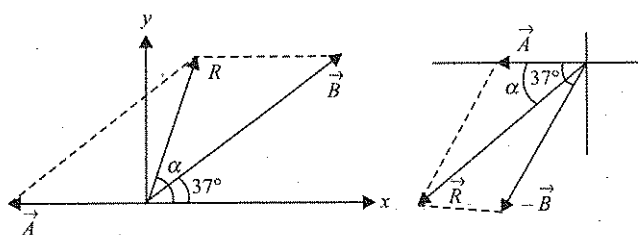


Fig. 2.58

b. $\vec{R} = \vec{A} - \vec{B}$

$$R = \sqrt{12^2 + 18^2 + 2 \times 12 \times 18 \cos 37^\circ} = 28.5$$

$$\tan \alpha = \frac{18 \sin 37^\circ}{12 + 18 \cos 37^\circ} \Rightarrow \alpha = 22^\circ$$

$$\text{Angle with } x\text{-axis} = 180^\circ + 22^\circ = 202^\circ$$

c. $-\vec{A} - \vec{B}$ is opposite to $\vec{A} + \vec{B}$ and having same magnitude as that of $\vec{A} + \vec{B}$.

d. $\vec{B} - \vec{A}$ is opposite to $\vec{A} - \vec{B}$ and having same magnitude as that of $\vec{A} - \vec{B}$.

$$7. R = \sqrt{4^2 + 7^2 + 2 \times 4 \times 7 \cos 60^\circ} = \sqrt{93} \text{ m/s}$$

$$\tan \alpha = \frac{7 \sin 60^\circ}{4 + 7 \cos 60^\circ} = \frac{7\sqrt{3}}{15}$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{7\sqrt{3}}{15} \right)$$

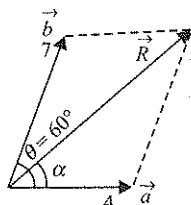


Fig. 2.59

$$8. \text{ Let } \vec{A} = 3\hat{i} - 4\hat{j} + 12\hat{k}, \text{ then } A = \sqrt{3^2 + 4^2 + 12^2} = 13$$

$$\text{Unit vector along } \vec{A} \text{ is } \hat{A} = \frac{\vec{A}}{A} = \frac{3\hat{i} - 4\hat{j} + 12\hat{k}}{13}$$

$$\text{Unit vector opposite to } \vec{A} \text{ is } -\hat{A} = -\frac{\vec{A}}{A}$$

$$= -\left(\frac{3\hat{i} - 4\hat{j} + 12\hat{k}}{13} \right)$$

$$9. \text{ a. } 3\vec{a} + 2\vec{b} + \vec{c} = 3(2\hat{i} - 3\hat{j}) + 2(6\hat{i} + 2\hat{j} - 3\hat{k}) + \hat{i} + \hat{k} = 19\hat{i} - 5\hat{j} - 5\hat{k}$$

$$\text{b. } \vec{a} - 6\vec{b} + 2\vec{c} = 2\hat{i} - 3\hat{j} - 6(6\hat{i} + 2\hat{j} - 3\hat{k}) + 2(\hat{i} + \hat{k}) = -32\hat{i} - 15\hat{j} + 20\hat{k}$$

$$10. \text{ a. } A = \sqrt{4^2 + 3^2} = 5, B = \sqrt{5^2 + 2^2} = \sqrt{29}$$

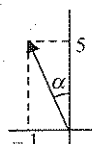


Fig. 2.60

$$\text{b. } \vec{A} - \vec{B} = -\hat{i} + 5\hat{j}$$

$$\text{c. Magnitude: } |\vec{A} - \vec{B}| = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$\tan \alpha = \frac{5}{1} \Rightarrow \alpha = \tan^{-1} \left(\frac{5}{1} \right)$$

d. See Fig. 2.61

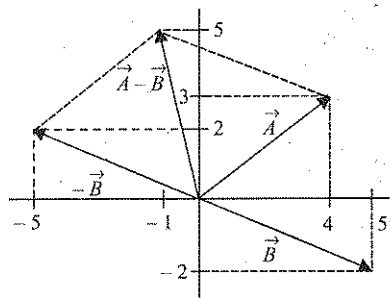


Fig. 2.61

$$11. \text{ a. Let } \vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\text{Magnitude: } |\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Since magnitude is not 1, so $\vec{a}$ is not a unit vector.

b. No, a rectangular component cannot be greater than a vector itself. Since rectangular component, say, $R_x = R \cos \theta$, $\cos \theta$ never becomes greater than 1. So R_x never becomes greater than R .

$$12. \vec{R} = 2\hat{i} + 3\hat{j}, \text{ Let } \vec{A} = \hat{i} + 2\hat{j}, \vec{B} = \hat{i} - \hat{j}$$

Then, we can write $\vec{R} = m\vec{A} + n\vec{B}$,

where $m\vec{A}$ is the component of $\vec{R}$ along $\vec{A}$ and

$n\vec{B}$ is the component of $\vec{R}$ along $\vec{B}$

$$\Rightarrow 2\hat{i} + 3\hat{j} = m\hat{i} + 2m\hat{j} + n\hat{i} - n\hat{j}$$

$$\Rightarrow m + n = 2, 2m - n = 3$$

From these $m = 5/3, n = 1/3$

$$m\vec{A} = \frac{5}{3}(\hat{i} + 2\hat{j}),$$

$$n\vec{B} = \frac{1}{3}(\hat{i} - \hat{j})$$

$$13. \text{ Rectangular component of } \vec{R} \text{ along } \vec{A} \text{ is } (\vec{R} \cdot \hat{A}) \hat{A}$$

$$= \frac{(\vec{R} \cdot \vec{A}) \vec{A}}{A^2} = \frac{(2 \times 1 + 3 \times 1)(\hat{i} + \hat{j})}{(\sqrt{2})^2} = \frac{5}{2}(\hat{i} + \hat{j})$$

$$14. \text{ a. } \vec{A} \cdot \vec{B} = -2 \times 2 + 6 \times (-3) = -22,$$

$$A = \sqrt{2^2 + 6^2} = \sqrt{40}, B = \sqrt{2^2 + 3^2} = \sqrt{13},$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{-22}{\sqrt{40}\sqrt{13}}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{-22}{\sqrt{40}\sqrt{13}} \right)$$

$$\text{b. In the same way as above: } \theta = \cos^{-1} \left[\frac{-22}{\sqrt{40}\sqrt{13}} \right]$$

$$\text{c. } \vec{A} \cdot \vec{B} = -4 \times 7 + 2 \times 14 = 0$$

So, angle between $\vec{A}$ and $\vec{B}$ is 90°

$$15. \text{ a. Let side of the cube is } d, \text{ then } \vec{ab} = d\hat{k}, \vec{ad} = d\hat{i}$$

$$+ d\hat{j} + d\hat{k}; \cos \theta = \frac{\vec{ab} \cdot \vec{ad}}{|\vec{ab}| |\vec{ad}|} = \frac{d^2}{d\sqrt{3}d} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\text{b. } \vec{ac} = d\hat{j} + d\hat{k}, \vec{ad} = d\hat{i} + d\hat{j} + d\hat{k}$$

$$\cos \theta = \frac{\vec{ac} \cdot \vec{ad}}{|\vec{ac}| |\vec{ad}|} = \frac{2d^2}{\sqrt{2}d\sqrt{3}d} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{\sqrt{2}}{\sqrt{3}} \right)$$

16. Since $\vec{C}$ lies at x-y plane, so let us assume $\vec{C} = x\hat{i} + y\hat{j}$,
 Now, $\vec{C} \cdot \vec{A} = 0 \Rightarrow 5x - 6.5y = 0$ (i)
 and $\vec{C} \cdot \vec{B} = 15 \Rightarrow 10x + 7y = 15$ (ii)

From equations (i) and (ii), $y = \frac{3}{4}$, $x = \frac{39}{40}$

17. $|\vec{A} \times \vec{B}| = \sqrt{5^2 + 2^2} = \sqrt{29}$

$\sin \theta = \frac{|\vec{A} \times \vec{B}|}{AB} = \frac{\sqrt{29}}{3 \times 3}$

$\Rightarrow \theta = \sin^{-1} \left[\frac{\sqrt{29}}{9} \right]$

18. $|\vec{A} \times \vec{B}| = AB \sin \theta = 5 \times 12 \times \sin 30^\circ = 30$ units

19. a. Area = $\frac{1}{2} |\vec{A} \times \vec{B}|$

b. Area = $|\vec{A} \times \vec{B}|$

c. Area = $\frac{1}{2} |\vec{A} \times \vec{B}|$

20. $\vec{AP} = 5\hat{k}$, $\vec{AQ} = 5\hat{i} + 2\hat{j} + 3\hat{k}$

Displacement of the bird = $\vec{PQ} = \vec{AQ} - \vec{AP}$
 $= 5\hat{i} + 2\hat{j} - 2\hat{k}$

$PQ = \sqrt{5^2 + 2^2 + 2^2} = \sqrt{33}$ m

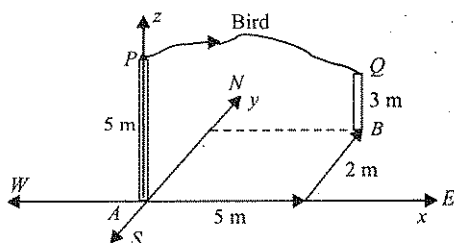


Fig. 2.62

21. Consider a regular polygon of N sides. Each side of the polygon is same. C is the centre of polygon (Fig. 2.63).
 $\theta = \frac{2\pi}{N} \rightarrow \theta$ is the angle subtended at the centre of polygon by any one side.

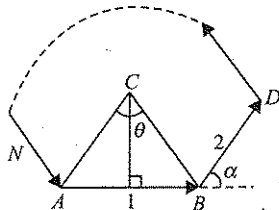


Fig. 2.63

We can prove that $\alpha = \theta$ as: $\angle ABC + \angle CBD + \alpha = 180^\circ$ (i)

But $\angle ABC = \angle CBD = 90^\circ - \frac{\theta}{2}$ (ii)

From equations (i) and (ii), we get $\alpha = \theta$

$\Rightarrow \alpha = \frac{2\pi}{N}$

So, if we have N vectors of equal magnitude and they are arranged in such a way that each vector makes an angle $\frac{2\pi}{N}$ ($= \alpha$) with its preceding vector, we find that head of the last vector will coincide with the tail of first vector. Hence, resultant of all these vectors becomes zero.

22. Let $x\hat{i} + y\hat{j} + z\hat{k}$ is perpendicular to $3\hat{i} - 4\hat{j} + 7\hat{k}$, then their dot product should be zero.
 $\Rightarrow 3x - 4y + 7z = 0$. Take $x = 1$, $y = 2$, $z = 5/7$
 So, $\hat{i} + 2\hat{j} + \frac{5}{7}\hat{k}$ is perpendicular to $3\hat{i} - 4\hat{j} + 7\hat{k}$.

23. a. Sum of any two sides of a triangle is greater than or equal to third side.

$\Rightarrow PR \leq PQ + QR$

$\Rightarrow |\vec{A} + \vec{B}| \leq |\vec{A}| + |\vec{B}|$

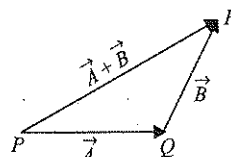


Fig. 2.64

b. $PR + QR \geq PQ \Rightarrow |\vec{A} + \vec{B}| + |\vec{B}| \geq |\vec{A}|$
 $\Rightarrow |\vec{A} + \vec{B}| \geq |\vec{A}| - |\vec{B}| \Rightarrow |\vec{A} + \vec{B}| \geq ||\vec{A}| - |\vec{B}||$ [(c) and (d) parts, do as (a) and (b)]

24. $\vec{R} = \vec{P} + \vec{Q}$, $\vec{S} = -\vec{P} + \vec{Q}$, Given that $\vec{S}$ is $\perp \vec{R}$

So, $\vec{R} \cdot \vec{S} = 0 \Rightarrow (\vec{P} + \vec{Q}) \cdot (-\vec{P} + \vec{Q}) = 0$

$\Rightarrow -P^2 + Q^2 = 0$

$\Rightarrow P^2 = Q^2$

$\Rightarrow P = Q$

Hence proved.

Alternatively: $\beta = 90^\circ - \alpha$, $\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$ (i)

$\tan \beta = \frac{Q \sin (180^\circ - \theta)}{P + Q \cos (180^\circ - \theta)}$

$\Rightarrow \tan (90^\circ - \alpha) = \frac{Q \sin \theta}{P - Q \cos \theta}$

$\Rightarrow \cot \alpha = \frac{Q \sin \theta}{P - Q \cos \theta}$ (ii)

Multiply equations (i) and (ii): $1 = \frac{Q^2 \sin^2 \theta}{P^2 - Q^2 \cos^2 \theta}$

Simplify to get: $P = Q$

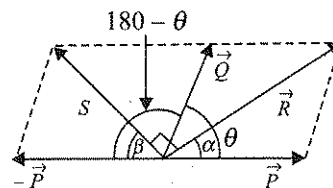


Fig. 2.65

25. $|\hat{P}| = |\hat{Q}| = 1$, because they are unit vectors.

L.H.S. = $|\hat{P} - \hat{Q}| = \sqrt{(\hat{P} - \hat{Q}) \cdot (\hat{P} - \hat{Q})}$
 $= \sqrt{P^2 + Q^2 - 2\hat{P} \cdot \hat{Q}} = \sqrt{1^2 + 1^2 - 2PQ \cos \theta}$
 $= \sqrt{2 - 2 \times 1 \times 1 \cos \theta} = \sqrt{2(1 - \cos \theta)}$
 $= \sqrt{2[2 \sin^2 (\theta/2)]} = 2 \sin (\theta/2) = \text{R.H.S.}$

Objective Type

1. a. Given $|\vec{A}| = |\vec{B}|$ or $A = B$.

$$\text{Sum: } \vec{R} = \vec{A} + \vec{B}$$

$$\Rightarrow |\vec{R}| = \sqrt{A^2 + B^2} = \sqrt{2}A$$

$$\text{Difference: } \vec{S} = \vec{A} - \vec{B}$$

$$\Rightarrow |\vec{S}| = \sqrt{A^2 + B^2} = \sqrt{2}A$$

$$\alpha_1 = 45^\circ, \alpha_2 = 45^\circ$$

Hence, $\vec{R}$ and $\vec{S}$ will be perpendicular and also of equal lengths.

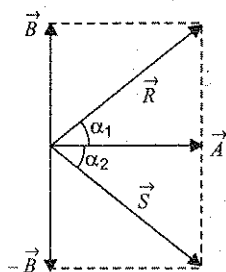


Fig. 2.66

2. c. The minimum number of vectors having different planes which can be added to give zero resultant is 4.

3. d. We see that dot product of $\hat{i} + \hat{j} + \hat{k}$ with $3\hat{i} + 2\hat{j} - 5\hat{k}$ is zero.

4. d. In $\triangle MNO$: $\vec{A} + \vec{C} - \vec{D} = 0 \Rightarrow \vec{C} - \vec{D} = -\vec{A}$

Hence (b) is correct.

$$\text{In } \triangle MNP: \vec{A} + \vec{B} + \vec{E} = 0$$

Hence (a) is correct.

$$\text{In } \square MPNO: -\vec{E} - \vec{B} + \vec{C} - \vec{D} = 0$$

$$\Rightarrow \vec{B} + \vec{E} - \vec{C} = -\vec{D}$$

Hence, (c) is correct.

5. d. For the resultant of some vectors to be zero, they should form a closed figure taken in same order.

6. b. $\cos \beta = \frac{R}{B} = \frac{1}{2} \Rightarrow \beta = 60^\circ$

$$\text{Angle between } \vec{A} \text{ and } \vec{B} = 90^\circ + \beta = 150^\circ$$

7. b. $\frac{a+b}{a-b} = \frac{3}{1}$ or $3a - 3b = a + b$

$$\text{or } 2a = 4b \text{ or } a = 2b$$

8. b. Note that the angle between two forces is 120° and not 60° .

$$R^2 = F^2 + F^2 + 2F^2 \cos 120^\circ$$

$$\text{or } R^2 = 2F^2 + 2F^2 \left(-\frac{1}{2}\right) = F^2 \text{ or } R = F$$

9. b. Here, the angle between two vectors of equal magnitude is 120° . So, resultant has the same magnitude as either of the given vectors. Moreover, it is mid-way between the two vectors, i.e., it is along x -axis.

10. a. $\left(\frac{1}{3}\right)^2 = 1^2 + 1^2 + 2 \times 1 \times 1 \cos \theta$

$$\text{or } \frac{1}{9} = 2(1 + \cos \theta) \text{ or } 1 + \cos \theta = \frac{1}{18}$$

$$\text{or } \cos \theta = \frac{1}{18} - 1 = -\frac{17}{18} \text{ or } \theta = \cos^{-1} \left(-\frac{17}{18}\right)$$

11. c. $\vec{A} + \vec{B}$ will be in the plane containing $\vec{A}$ and $\vec{B}$, whereas $\vec{A} \times \vec{B}$ will be $\perp$ to that plane.

12. d. Consider only x and y components: $\sqrt{3^2 + 1^2} = \sqrt{10}$

13. a.

14. c. Let third side is $\vec{C}$, then $|\vec{C}| = |\vec{A} + \vec{B}|$ or

$$|\vec{C}| = |\vec{A} - \vec{B}|$$

15. a. $A_1 = 2, A_2 = 3, |\vec{A}_1 + \vec{A}_2| = 3$

$$\Rightarrow |\vec{A}_1 + \vec{A}_2|^2 = 9 \Rightarrow A_1^2 + A_2^2 + 2\vec{A}_1 \cdot \vec{A}_2 = 9$$

$$\Rightarrow 2^2 + 3^2 + 2\vec{A}_1 \cdot \vec{A}_2 = 9 \Rightarrow \vec{A}_1 \cdot \vec{A}_2 = -2$$

$$\text{Now, } (\vec{A}_1 + 2\vec{A}_2) \cdot (3\vec{A}_1 + 4\vec{A}_2) = 3A_1^2 - 8A_2^2$$

$$+ 2\vec{A}_1 \cdot \vec{A}_2 = 3(2)^2 - 8(3)^2 + 2(-2) = -64$$

16. d. $\vec{A} \cdot \vec{B} = 0$ (given) $\Rightarrow \vec{A} \perp \vec{B}$

$$\vec{A} \cdot \vec{C} = 0 \text{ (given) } \Rightarrow \vec{A} \perp \vec{C}$$

$\vec{A}$ is perpendicular to both $\vec{B}$ and $\vec{C}$.

We know, from the definition of cross product, that $\vec{B} \times \vec{C}$ is perpendicular to both $\vec{B}$ and $\vec{C}$.

So, $\vec{A}$ is parallel to $\vec{B} \times \vec{C}$.

17. a. $\cos \theta = \frac{C}{A} = \frac{3}{5}$ or $\theta = \cos^{-1} \left(\frac{3}{5}\right)$. See Fig. 2.67.

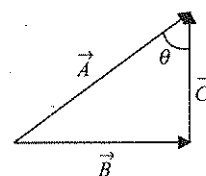


Fig. 2.67

18. b. Clearly, $\vec{B}$ should be either in second quadrant or fourth quadrant. In none of the given options, we have ' $-\hat{i}$ term'.

So, second quadrant is ruled out. Also, $\vec{B}$ should make an angle of $90^\circ - \theta$ with x -axis (Fig. 2.68). So, $\vec{B}$ should be $B \cos(90^\circ - \theta)\hat{i} - B \sin(90^\circ - \theta)\hat{j} = B \sin \theta \hat{i} - B \cos \theta \hat{j}$.

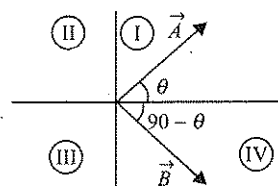


Fig. 2.68

19. c. See Fig. 2.69

$$\tan \beta = \frac{2}{3} \text{ or } \beta = \tan^{-1} \left(\frac{2}{3} \right)$$

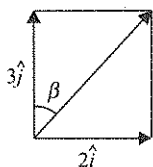


Fig. 2.69

20. c. $\vec{P}$ is in fourth quadrant.

$4\hat{i} + 3\hat{j}$ is in the first quadrant.

Clearly, $4\hat{i} + 3\hat{j}$ can be perpendicular to $\vec{P}$. For confirmation, let us check whether their dot product is zero.

$$(3\hat{i} - 4\hat{j}) \cdot (4\hat{i} + 3\hat{j}) = 12 - 12 = 0$$

This shows that $4\hat{i} + 3\hat{j}$ is perpendicular to $3\hat{i} - 4\hat{j}$.

21. c. $\vec{S} = 75\hat{j} + [60 \cos 45^\circ \hat{j} - 60 \sin 45^\circ \hat{i}] + 20\hat{i}$

$$\vec{S} = (20 - 60 \times 0.7)\hat{i} + (60 \times 0.7 + 75)\hat{j}$$

$$\text{or } \vec{S} = -22\hat{i} + 117\hat{j}$$

$$S = \sqrt{22^2 + 117^2} = \sqrt{484 + 13689} = \sqrt{14173} = 119 \text{ km}$$

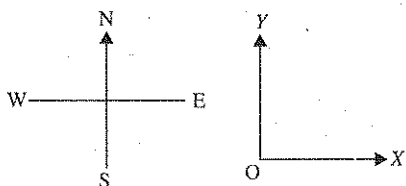


Fig. 2.70

22. b. Area of parallelogram: $|\vec{A} \times \vec{B}| = AB/2$ (given)

$$\Rightarrow AB \sin \theta = AB/2 \Rightarrow \sin \theta = 1/2$$

$$\Rightarrow \theta = 30^\circ$$

23. b. $a^2 + b^2 + 2ab \cos \theta = a^2 + b^2 - 2ab \cos \theta$
or $4ab \cos \theta = 0$

$$\text{But } 4ab \neq 0 \Rightarrow \cos \theta = 0 \text{ or } \theta = 90^\circ$$

Aliter

$(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are the diagonals of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$.

Since $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, therefore the two diagonals of a parallelogram are equal. So, think of square. This leads to $\theta = 90^\circ$.

24. a. $\vec{A} \times \vec{B} = (4\hat{i} + 6\hat{j}) \times (2\hat{i} + 3\hat{j})$

$$= 12(\hat{i} \times \hat{j}) + 12(\hat{j} \times \hat{i}) = 12(\hat{i} \times \hat{j}) - 12(\hat{i} \times \hat{j}) = 0$$

$$\text{Again, } \vec{A} \cdot \vec{B} = (4\hat{i} + 6\hat{j}) \cdot (2\hat{i} + 3\hat{j}) = 8 + 18 = 26$$

$$\text{Again, } \frac{|\vec{A}|}{|\vec{B}|} = \frac{\sqrt{16+36}}{\sqrt{4+9}} \neq \frac{1}{2}$$

$$\text{Also, } \vec{B} = \frac{1}{2}\vec{A}$$

$$\Rightarrow \vec{A} \text{ and } \vec{B} \text{ are parallel and not antiparallel.}$$

$$25. \text{ a. } \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & p & q \\ 5 & 7 & 3 \end{vmatrix} = 0$$

$$\text{or } \hat{i}(3p - 7q) + \hat{j}(5q - 6) + \hat{k}(14 - 5p) = 0$$

$$3p = 7q, 5q - 6 = 0 \text{ or } q = \frac{6}{5}$$

$$14 - 5p = 0 \text{ or } 5p = 14 \text{ or } p = \frac{14}{5}$$

26. b. If $\vec{A}$ is perpendicular to $\vec{B}$, then $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \times \vec{B} \neq 0$

$$27. \text{ b. } \vec{PR} = \vec{a} + \vec{b} \Rightarrow \text{major diagonal}$$

$$\vec{SQ} = \vec{a} - \vec{b} \Rightarrow \text{minor diagonal}$$

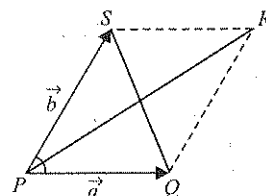


Fig. 2.71

28. b. $\vec{A} + \vec{B} = \vec{C}$

$$(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = \vec{C} \cdot \vec{C}$$

$$\Rightarrow A^2 + B^2 + 2AB \cos \theta = C^2$$

$$\Rightarrow 4^2 + 5^2 = 2 \times 4 \times 5 \cos \theta = 61$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

29. b. $\tan \theta = \frac{2}{3} \Rightarrow \theta = \tan^{-1} \left(\frac{2}{3} \right)$

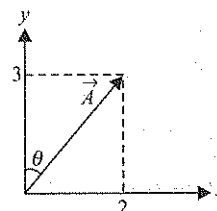


Fig. 2.72

30. a. Let that vector is $\vec{C}$. Then

$$\vec{C} = C\hat{C} = b\hat{a} \Rightarrow \vec{C} = \frac{b\vec{a}}{a} = \frac{5}{\sqrt{2}}(\hat{i} - \hat{j})$$

31. b. For the resultant of two vectors to be zero, they should be equal and opposite.

32. c. In first option (a), vector is along x-axis (Fig. 2.73).

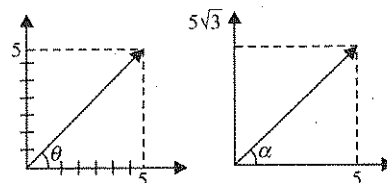


Fig. 2.73

In (b), angle of vector with x-axis

$$\tan \theta = \frac{5}{5} = 1 \Rightarrow \theta = 45^\circ$$

In (c), angle of vector with x-axis

$$\tan \alpha = \frac{5\sqrt{3}}{5} = \sqrt{3} \Rightarrow \alpha = 60^\circ$$

33. b. See Fig. 2.74

$$AC \leq AB + BC \Rightarrow |\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$

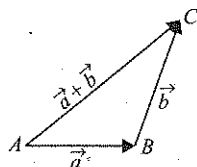


Fig. 2.74

34. d. The resultant of two forces can lie between $A - B$ and $A + B$, i.e., $12 - 1 = 11$ N and $12 + 1 = 13$ N.

35. a. Find min ($A - B$) and maximum ($A + B$) value of each case, then check if 4 N lies between them.

36. b. $\vec{BA} + \vec{BC} = \vec{BD}$

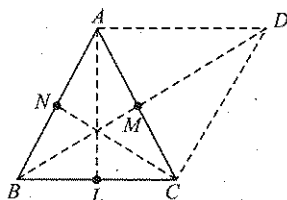


Fig. 2.75

$$\vec{BA} + \vec{BC} = 2\vec{BM}. \text{ Hence, the answer is } 2BM.$$

37. b. $(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$

$$A^2 - B^2 = 0 \Rightarrow A^2 = B^2$$

$$\Rightarrow A = B$$

38. c.

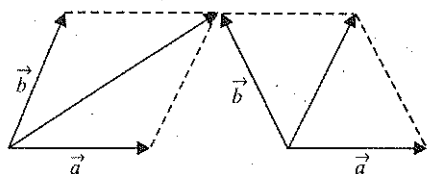


Fig. 2.76

$$\vec{a} + \vec{b} \rightarrow \text{major diagonal, } \vec{a} - \vec{b} \rightarrow \text{minor diagonal}$$

39. b. $\tan \theta' = \frac{C}{A} = 1$

$$\Rightarrow \theta' = 45^\circ = \frac{\pi}{4}$$

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

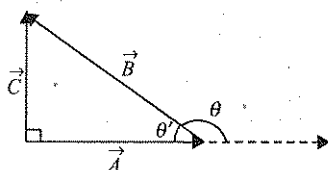


Fig. 2.77

40. a. $\vec{F}_2 - \vec{F}_1 = \vec{F}_2 + (-\vec{F}_1)$
 $= 250$ N due north + 500 N due west

$$\tan \theta = \frac{500}{250} = 2$$

$$|\vec{F}_2 - \vec{F}_1| = \sqrt{(500)^2 + (250)^2} = 250\sqrt{5}$$

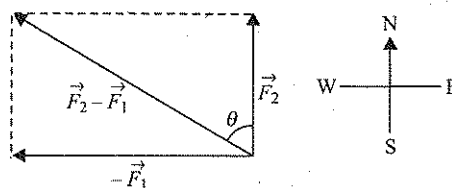


Fig. 2.78

41. c. $\vec{OC}$ and $\vec{OA}$ are equal in magnitude and inclined to each other at an angle of 90° . So, their resultant is $\sqrt{2}r$. It acts mid-way between $\vec{OC}$ and $\vec{OA}$, i.e., along OB .

Now, both r and $\sqrt{2}r$ are along the same line and in the same direction.

$$\therefore \text{resultant} = r + \sqrt{2}r = r(1 + \sqrt{2})$$

42. b. $\tan 45^\circ = \frac{2 \sin 60^\circ}{a + 2 \cos 60^\circ} = \frac{\sqrt{3}}{a + 1}$

$$\text{or } 1 = \frac{\sqrt{3}}{a + 1} \text{ or } a + 1 = \sqrt{3} \text{ or } a = \sqrt{3} - 1$$

43. d. $\tan 90^\circ = \frac{2Q \sin \theta}{P + 2Q \cos \theta} \Rightarrow \infty = \frac{2Q \sin \theta}{P + 2Q \cos \theta}$

$$\Rightarrow P + 2Q \cos \theta = 0$$

$$\text{Now, } R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$\Rightarrow R^2 = Q^2 + P[P + 2Q \cos \theta]$$

$$\Rightarrow R^2 = Q^2 \Rightarrow R = Q$$

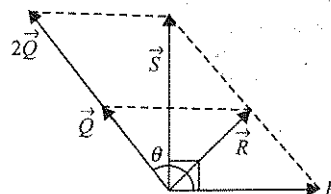


Fig. 2.79

Alternate method:

$$\vec{R} = \vec{P} + \vec{Q} \Rightarrow \vec{P} = \vec{R} - \vec{Q} \text{ and } \vec{S} = \vec{P} + 2\vec{Q}$$

$$= \vec{R} - \vec{Q} + 2\vec{Q}$$

Now, $\vec{S}$ and $\vec{P}$ are perpendicular (Fig. 2.78), so

$$\vec{S} \cdot \vec{P} = 0 \Rightarrow (\vec{R} + \vec{Q}) \cdot (\vec{R} - \vec{Q}) = 0$$

$$\Rightarrow R^2 = Q^2 \Rightarrow R = Q$$

44. a. Given $\vec{C} = |\vec{B}| \hat{j} \Rightarrow \vec{C} = 5\hat{j}$

$$\text{Let } \vec{C} = \vec{A} + \vec{B} = \vec{A} + 3\hat{i} + 4\hat{j}$$

$$\Rightarrow 5\hat{j} = \vec{A} + 3\hat{i} + 4\hat{j}$$

$$\Rightarrow \vec{A} = -3\hat{i} + \hat{j} \Rightarrow |\vec{A}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

45. c. $\vec{AB} + \vec{AF} = \vec{AO} \Rightarrow \vec{AB} = \vec{AO} - \vec{AF}$
 $\vec{AC} = \vec{AB} + \vec{AO}$, $\vec{AD} = 2\vec{AO}$, $\vec{AE} = \vec{AO} + \vec{AF}$

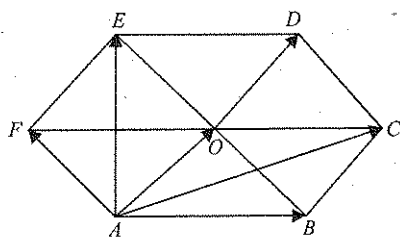


Fig. 2.80

Now, $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$
 $= 5\vec{AO} + \vec{AB} + \vec{AF} = 5\vec{AO} + \vec{AO} = 6\vec{AO}$

46. b. Displacement = $\vec{AB}$, angle between $\vec{r}_1$ and $\vec{r}_2$: $\theta = 75^\circ - 15^\circ = 60^\circ$

From Fig. 2.55, $AB^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos \theta$
 $= 3^2 + 4^2 - 2 \times 3 \times 4 \cos 60^\circ = 13$
 $\Rightarrow AB = \sqrt{13}$

47. c. $x + y = 16$. Also, $y^2 = 8^2 + x^2$

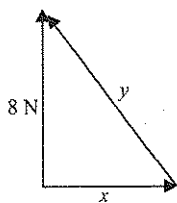


Fig. 2.81

or $y^2 = 64 + (16 - y)^2$ [$\because x = 16 - y$]
or $y^2 = 64 + 256 + y^2 - 32y$
or $32y = 320$ or $y = 10$ N
 $\therefore x + 10 = 16$ or $x = 6$ N

48. c. Graphically:

$\angle ROQ = \theta/2$, $\angle RQO = \theta/2$

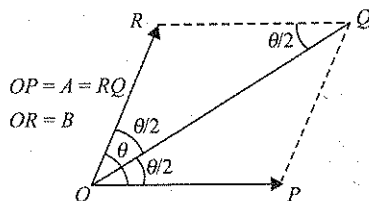


Fig. 2.82

Hence, $\triangle OQR$ is isosceles

$\Rightarrow OR = RQ \Rightarrow B = A$

Analytically: $\tan(\theta/2) = \frac{B \sin \theta}{A + B \cos \theta}$

$\Rightarrow \frac{\sin(\theta/2)}{\cos(\theta/2)} = \frac{2B \sin(\theta/2) \cos(\theta/2)}{A + B(2 \cos^2(\theta/2) - 1)}$

$\Rightarrow A + 2B \cos^2(\theta/2) - B = 2B \cos^2(\theta/2) \Rightarrow A = B$

49. a. $R_x = 1 + 2 \cos 120^\circ + 3 \cos 240^\circ$

$R_y = 2 \sin 120^\circ + 3 \sin 240^\circ$

$= 2 \times \frac{\sqrt{3}}{2} + 3 \times \left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$

$\tan \theta = \frac{R_y}{R_x} = \frac{-\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$

50. a. $u_x = 8 \text{ ms}^{-1}$, $a_x = 2 \text{ ms}^{-2}$

$\vec{v}_x = \vec{u}_x + \vec{a}_x t = 8\hat{i} + 2t\hat{i}$

$\vec{v}_y = -15\hat{j}$, $\vec{V} = \vec{v}_x + \vec{v}_y$, $\vec{V} = [(8 + 2t)\hat{i} - 15\hat{j}] \text{ ms}^{-1}$

51. c. You have to try all the options. Let us discuss the correct option only.

$\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$

(1)(1) $\sin(180^\circ - i) = \mu(1)(1) \sin(180^\circ - r)$
 $\sin i = \mu \sin r$

52. c. $P + Q = 16$

$P^2 + Q^2 + 2PQ \cos \theta = 64$

$\tan 90^\circ = \frac{Q \sin \theta}{P + Q \cos \theta}$ or $\infty = \frac{Q \sin \theta}{P + Q \cos \theta}$

$P + Q \cos \theta = 0$ or $Q \cos \theta = -P$

From equation (ii), $P^2 + Q^2 + 2P(-P) = 64$

or $Q^2 - P^2 = 64$ or $Q - P = \frac{64}{16} = 4$

Adding equations (i) and (iii), we get

$2Q = 20$ or $Q = 10$ units

From equation (i), $P + 10 = 16$ or $P = 6$ units

53. d. The length of the vector is not changed by the rotation of the coordinate axes.

$\sqrt{(n+1)^2 + 1^2} = \sqrt{n^2 + 3^2}$ or $n^2 + 2n + 2 = n^2 + 9$
or $2n = 7$ or $n = 3.5$

54. b. For collision,

$\vec{r}_1 + \vec{v}_1 t = \vec{r}_2 + \vec{v}_2 t$ or $\vec{r}_1 - \vec{r}_2 = (\vec{v}_2 - \vec{v}_1)t$

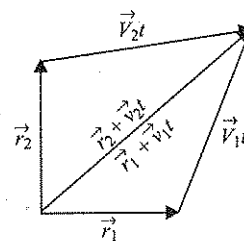


Fig. 2.83

Equating unit vectors, we get

$\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

55. a. Net force along x-axis (Fig. 2.84)

$F_x = F_1 + F_2 \cos 30^\circ - F_3 \cos 30^\circ$

$F_x = 300$ N

$F_y = F_2 \sin 30^\circ + F_3 \sin 30^\circ$

$$= 400 \times \frac{1}{2} + 400 \times \frac{1}{2} = 400 \text{ N}$$

$$\text{Net force: } F = \sqrt{300^2 + 400^2} = 500 \text{ N}$$

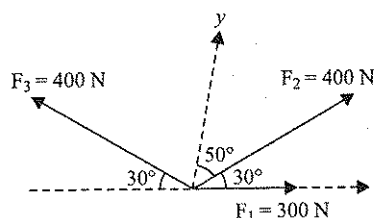


Fig. 2.84

Multiple Correct Answers Type

- a., c.** Both x and y components of $\vec{d}_1$ are positive.
 x component of $\vec{d}_2$ is negative and y component is positive.
 Both x and y components of $\vec{d}_1 + \vec{d}_2$ are positive.
- a., b.** Component of $\vec{A}$ along $\vec{B}$ is $|\vec{A}| \cos \theta$ for θ being the angle between the vectors.
 Also $\hat{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$. So, choice (a) is correct.
 The vector $(\hat{i} - \hat{j})$ is perpendicular to the vector $(\hat{i} + \hat{j})$.

So, the other resolved component is $|\vec{A}| \sin \theta \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}} \right)$.

$$\begin{aligned} \text{3. a., b., c. } \vec{A} \times \vec{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \\ &= \hat{i}(1 - 1) - \hat{j}(2 - 1) + \hat{k}(2 - 1) = -\hat{j} + \hat{k} \end{aligned}$$

Unit vector perpendicular to $\vec{A}$ and $\vec{B}$ is $\left(\frac{-\hat{j} + \hat{k}}{\sqrt{2}} \right)$. So, choices (a) and (c) are correct.

Any vector whose magnitude is K (constant) times $(2\hat{i} + \hat{j} + \hat{k})$ is parallel to $\vec{A}$.

So, unit vector $\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}$ is parallel to $\vec{A}$.

So, choice (b) is correct.

- a., d.** If two vectors are normal to each other, then their dot product is zero.

$$\begin{aligned} (\vec{v}_1 + \vec{v}_2) \cdot (\vec{v}_1 - \vec{v}_2) &= 0 \Rightarrow v_1^2 - v_2^2 = 0 \\ \Rightarrow v_1^2 &= v_2^2 \Rightarrow v_1 = v_2 \text{ or } |\vec{v}_1| = |\vec{v}_2| \end{aligned}$$

- a., d.** The resultant of three vectors is zero only if they can form a triangle. But three vectors lying in different planes cannot form a triangle.

Units and Dimensions

- Systems of Units
- Dimensions of a Physical Quantity
- Dimensional Formulae
- Uses of Dimensional Analysis
- Significant Figures
- Errors in Measurements
- Absolute Errors
- Propagation of Combined Errors

To measure a physical quantity, we need a standard known as unit. For example, if length of some metal rod is measured to be 15 cm, then cm is the unit of length. 15 is the numerical part. So

$$\text{Physical Quantity} = \text{Numerical Part} \times \text{Unit}$$

We have three types of units: Fundamental units, Supplementary units and Derived units as illustrated in Fig. 3.1 below.

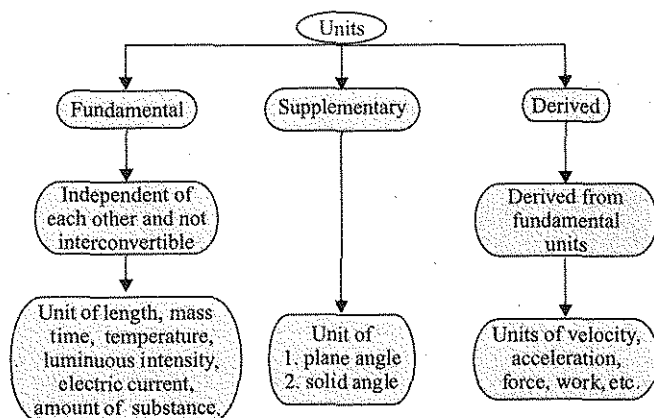


Fig. 3.1

SYSTEMS OF UNITS

It is a complete set of fundamental and derived units. We have four types of systems of units. Generally, a system is named in terms of fundamental units on which it is based.

- M.K.S. system:** In this system length, mass and time are taken as fundamental quantities.
- C.G.S. system:** It is Gaussian system. In this also length, mass and time are taken as fundamental quantities.
- F.P.S. system:** In this also length, mass and time are taken as fundamental quantities. It is British Engineering system.

M.K.S. and C.G.S. systems are also called metric systems or decimal systems, because multiples and submultiples are related by powers of 10. Example: 1 km = 10^3 m

F.P.S. system is not used much now a days because of inconvenient multiplies and submultiples. The disadvantages of C.G.S. system is that many derived units in this system become unnecessarily small.

The main drawback of all the above systems is that they are confined to mechanics only. All the physical quantities appearing in physics cannot be described by these systems. So, we need such a system which takes care of all the physical quantities appearing in physics. S.I. system is such a kind of system.

- S.I. system:** It was introduced in 1971 by General Conference on Weights and Measures. It is also called as rationalised M.K.S. system because it is made by modifying the M.K.S. system. It is nothing but extended M.K.S. system. It is a comprehensive system (see Table 1).

This system contains seven fundamental units and two supplementary units as shown in Table 2. It also contains a large number of derived quantities.

Table 1

M.K.S. System	C.G.S. System	F.P.S. System	S.I. Units
(i) Length m (meter)	(i) Length cm (centimeter)	(i) Length ft (foot)	It is an extended form of M.K.S. system. It includes four more fundamental units (in addition to three basic units), which represent fundamental quantities in electricity, magnetism, heat and light.
(ii) Mass kg (kilogram)	(ii) Mass g (gram)	(ii) Mass (pound)	
(iii) Time s (second)	(iii) Time s (second)	(iii) Time s (second)	

Table 2

A. Fundamental Quantities in S.I. System and Their Units

Sr. No.	Physical Quantity	Name of Unit	Symbol of Unit
1.	Mass	kilogram	kg
2.	Length	meter	m
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Luminous intensity	candela	Cd
6.	Electric current	ampere	A
7.	Amount of substance	mole	mol

B. Supplementary Quantities in S.I. System and Their Units

Sr. No.	Physical Quantity	Name of Unit	Symbol of Unit
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

Advantage of S.I. system is that it assigns only one unit to various forms of a particular physical quantity. For example, unit of all kinds of energy is J in this system. But in M.K.S. system:

Unit of mechanical energy is joule, that of heat energy is calorie, that of electric energy is Wh (watt hour), etc.

DIMENSIONS OF A PHYSICAL QUANTITY

These are the powers to which the fundamental units of mass, length and time have to be raised to represent a derived unit of the physical quantity under consideration. Dimensions of any derived physical quantity can be represented in the form of fundamental units of mass, length and time. Knowing the units, dimensions can be easily written.

To write the dimensions of a physical quantity, we use following symbols for mass, length and time:

Mass — [M]; Length — [L]; and Time — [T].

DIMENSIONAL FORMULAE

Relations which express physical quantities in terms of appropriate powers of fundamental units are known as dimensional formulae. These formulae tell us about:

1. Fundamental units involved to represent a quantity.
2. The nature of their dependence.

Illustration 3.1 Obtain the dimensions of acceleration.

Sol. We know that acceleration:

$$a = \frac{v}{t} = \frac{s/t}{t} = \frac{s}{t^2} \quad (S - \text{distance}, t - \text{time})$$

$$[a] = \frac{[L]}{[T]^2} = [L^1 T^{-2}] = [M^0 L^1 T^{-2}]$$

So, the dimensions of acceleration are 0 in mass, +1 in length and -2 in time.

Some more examples:

1. **Force:** Force = mass $\times$ acceleration = $[M] \times [L^1 T^{-2}]$
 $= [M^1 L^1 T^{-2}]$
2. **Momentum:** Momentum = mass $\times$ Velocity
 $= [M] \times [L^1 T^{-1}] = [M^1 L^1 T^{-1}]$
3. **Work:** Work = force $\times$ distance = $[M^1 L^1 T^{-2}] \times [L]$
 $= [M^1 L^2 T^{-2}]$

USES OF DIMENSIONAL ANALYSIS

1. Conversion of units of a quantity from one system to another.
2. To check the accuracy of formulae.
3. Derivation of formulae.

Conversion of Units of a Quantity from One System to Another

Physical quantities can be converted from one system of units to another. Due to this conversion, the numerical part of physical quantity changes but the dimensions and the overall quantity remain the same.

Suppose a physical quantity has the dimensional formula $M^a L^b T^c$.

Let N_1 and N_2 be the numerical values of a quantity in the two systems of units, respectively.

In first system, Physical Quantity $Q = N_1 M_1^a L_1^b T_1^c = N_1 U_1$

In second system, Same Quantity $Q = N_2 M_2^a L_2^b T_2^c = N_2 U_2$

A physical quantity remains the same irrespective of the system of measurement, i.e.,

$$Q = N_1 U_1 = N_2 U_2 \Rightarrow N_1 M_1^a L_1^b T_1^c = N_2 M_2^a L_2^b T_2^c$$

$$\Rightarrow N_2 = N_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

So, knowing the quantities on the right hand side the value of N_2 can be obtained.

Illustration 3.2 Convert 1 joule into erg.

Sol. Joule: S.I. system, erg: C.G.S. system

$$\text{Work} = \text{force} \times \text{distance} = \text{mass} \times \text{acceleration} \times \text{length}$$

$$= \text{mass} \times \frac{\text{length}}{(\text{time})^2} \times \text{length}$$

$$\text{Dimensions of work} = [W] = [M^1 L^2 T^{-2}]$$

$$\therefore a = 1, b = 2, c = -2.$$

Now,

S.I. system	$M_1 = 1 \text{ kg}$	$L_1 = 1 \text{ m}$	$T_1 = 1 \text{ s}$	$N_1 = 1 \text{ cm}$
C.G.S. system	$M_2 = 1 \text{ g}$	$L_2 = 1 \text{ cm}$	$T_2 = 1 \text{ s}$	$N_2 = ?$

$$\therefore \text{using } N_2 = N_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c, \text{ we have}$$

$$N_2 = 1 \left[\frac{1 \text{ kg}}{1 \text{ g}} \right]^1 \left[\frac{1 \text{ m}}{1 \text{ cm}} \right]^2 \left[\frac{1 \text{ s}}{1 \text{ s}} \right]^{-2}$$

$$= 1 \left[\frac{1000 \text{ g}}{1 \text{ g}} \right]^1 \left[\frac{100 \text{ cm}}{1 \text{ cm}} \right]^2 = 10^7$$

$$\text{So, } 1 \text{ joule} = 10^7 \text{ erg.}$$

Illustration 3.3 Convert 54 kmh⁻¹ into ms⁻¹.

Sol. Let $v = 54 \text{ kmh}^{-1} = n_2 \text{ ms}^{-1}$

$$[v] = L T^{-1}, a = 0, b = 1, c = -1$$

$$n_2 = 54 \left[\frac{\text{kg}}{\text{g}} \right]^0 \left[\frac{\text{km}}{\text{m}} \right]^1 \left[\frac{\text{h}}{\text{s}} \right]^{-1}$$

$$= 54 \times 1 \times 1000 \times [3600]^{-1} = \frac{54 \times 1000}{3600} = 15$$

$$\text{Hence, } 54 \text{ kmh}^{-1} = 15 \text{ ms}^{-1}.$$

To Check the Accuracy of Formulae

The accuracy of the expression of any physical quantity can be checked by using the *principle of homogeneity*. According to this principle, dimensions of various quantities as a whole on both sides of an expression (related to a physical quantity) are equal.

Illustration 3.4 Check the accuracy of the relation

$v^2 - u^2 = 2as$, where v and u are final and initial velocities, a is acceleration and s is the distance.

Sol. We have $v^2 - u^2 = 2as$

Checking the dimensions on both sides, we get

$$\text{L.H.S} = [L T^{-1}]^2 - [L T^{-1}]^2 = [L^2 T^{-2}] - [L^2 T^{-2}] = [L^2 T^{-2}]$$

$$\text{R.H.S} = [L^1 T^{-2}][L] = [L^2 T^{-2}]$$

Comparing L.H.S. and R.H.S., we find L.H.S. = R.H.S.
Hence, the formula is dimensionally correct.

Illustration 3.5 Check whether the relation $S = ut$

$+ \frac{1}{2}at^2$ is dimensionally correct or not, where symbols have their usual meaning.

Sol. We have $S = ut + \frac{1}{2}at^2$. Checking the dimensions on both sides, L.H.S. = $[M^0L^1T^0]$,

$$\begin{aligned}\text{R.H.S.} &= [LT^{-1}][T] + [LT^{-2}][T^2] \\ &= [M^0L^1T^0] + [M^0L^1T^0] = [M^0L^1T^0]\end{aligned}$$

Comparing the L.H.S. and R.H.S., we find L.H.S. = R.H.S.

Hence, the formula is dimensionally correct.

Limitations

1. Sometimes an equation which is dimensionally correct may not be correct actually.

Example: $S = ut + at^2$, $T = \pi\sqrt{\frac{l}{g}}$.

So, mere dimensional correctness does not indicate that an equation must be correct physically also. But if an equation is dimensionally wrong, then certainly it must be wrong, although it may also have exceptions like:

$$D_n = u + \frac{a}{2}(2n - 1).$$

2. Two different physical quantities may have same dimensional formula. For example, work and torque have same dimensions though they are different physical quantities.

Note: According to principle of homogeneity, the quantities having same dimensions only be added or subtracted. So, dimensionally: $L + L = L$ and $L - L = L$.

Derivation of Formulae

If we know the factors on which a quantity depends, then we can derive its formula using the principle of homogeneity.

Illustration 3.6 For a particle to move in a circular orbit uniformly, centripetal force is required which in fact depends upon mass (m), velocity (v) and radius (r) of the circle. Express centripetal force in terms of these quantities.

Sol. According to provided information, let $F \propto m^a v^b r^c$.

$$\Rightarrow F = km^a v^b r^c \quad (\text{i})$$

where k is a dimensionless constant of proportionality and a , b , c are the constant powers of m , v , r , respectively.

$$\Rightarrow [M^1L^1T^{-2}] = [M^a(LT^{-1})^bL^c]$$

$$\Rightarrow [M^1L^1T^{-2}] = [M^aL^{b+c}T^{-b}]$$

Now, using principle of homogeneity (i.e., comparing the power of like quantities on both side), we have

$$a = 1 \quad (\text{ii}) \quad b + c = 1 \quad (\text{iii}) \quad b = 2 \quad (\text{iv})$$

Using (ii), (iii) and (iv), we have $a = 1$, $b = 2$, $c = -1$

Using these values in (i), we get $F = km^1v^2r^{-1}$

$$\Rightarrow F = k \frac{mv^2}{r}, \text{ which is the desired relation.}$$

Illustration 3.7 Find out the unit and dimensions of the constants a and b in the Van der Waal's equation $\left(p + \frac{a}{V^2}\right)(V - b) = RT$, where p is pressure, V is volume, R is gas constant and T is temperature.

Sol. We can add and subtract only like quantities.

So, dimensions of $\frac{a}{V^2} = \text{dimensions of } P$

$\Rightarrow$ Dimensions of $a = \text{dimensions of } P \times \text{dimensions of } V^2$

$\Rightarrow [a] = [M^1L^{-1}T^{-2}] \times [L^3]^2 = [M^1L^5T^{-2}]$ and

Dimensions of $b = \text{dimensions of } V$

$\Rightarrow [b] = [V] = [M^0L^3T^0]$

Unit of $a = \text{unit of } p \times \text{unit of } V^2 = \frac{\text{N}}{\text{m}^2} \times \text{m}^6 = \text{Nm}^4$.

Unit of $b = \text{unit of } V = \text{m}^3$.

Illustration 3.8 Derive an expression for the time period of a simple pendulum of length l .

Sol. Let $t \propto m^a l^b g^c \Rightarrow t = km^a l^b g^c$

$$M^0L^0T^{-1} = M^aL^b[LT^{-2}]^c$$

$$\Rightarrow M^0L^0T^{-1} = M^aL^{b+c}T^{-2c}$$

Comparing the powers of M , L and T on both sides: $a = 0$, $b + c = 0$, $-2c = 1$

$\Rightarrow a = 0$, $b = 1/2$ and $c = -1/2$. Putting these values, we get

$$t = km^0 \frac{l^{1/2}}{g^{1/2}} \Rightarrow t = k\sqrt{\frac{l}{g}}, \text{ which is the required relation.}$$

Useful Tips

- All of the following have the same dimensional formula $[M^0L^0T^{-1}]$:
Frequency, angular frequency, angular velocity and velocity gradient.
- All of the following quantities are dimensionless:
Angle, solid angle, T-ratios, strains, Poisson's ratio, relative density, relative permittivity, refractive index and relative permeability.
- Following three quantities have the same dimensional formula $[M^0L^2T^{-2}]$:
Square of velocity, gravitational potential, and latent heat.
- Following quantities have the same dimensionless formula $[ML^2T^{-2}]$:
Work, energy, torque and heat.
- Following have the same dimensional formula $[MLT^{-1}]$: Momentum and impulse.
- Both acceleration and gravitational field intensity have the same dimensional formula $[M^0LT^{-2}]$.
- Force, weight, thrust and energy gradient have the same dimensional formula $[MLT^{-2}]$.
- Entropy, gas constant, Boltzmann constant and thermal capacity have the same dimensions in mass, length and time.
- Light year, radius of gyration and wavelength have the same dimensional formula $[M^0LT^0]$.
- Surface tension and spring constant have the same dimensional formula $[ML^0T^{-2}]$.
- Planck's constant and angular momentum have the same dimensional formula $[ML^2T^{-1}]$.

- Rydberg constant and propagation constant have the same dimensional formula $[M^0 L^{-1} T^0]$.
- Following quantities have the same dimensional formula $[ML^{-1} T^{-2}]$:
Pressure, stress, moduli of elasticity and energy density.
- It is not possible for a quantity to have dimensions but no units.
- It is possible for a quantity to have units but no dimensions.
- A quantity has same dimensions in different systems of units.

Concept Application Exercise 3.1

1. Is light year a unit of time?
2. Does magnitude of a quantity change with change in the unit of system?
3. All constants are dimensionless. Comment.
4. What is the difference between nm, mN and Nm?
5. Name three physical quantities which have same dimensions.
6. Justify $L + L = L$ and $L - L = L$.
7. Displacement of a particle is given by:
 $x = A^2 \sin^2 Kt$, where t denotes the time. What is the unit of K ?
8. In the equation $\left(P + \frac{a}{V^2}\right)(V - b) = \text{constant}$, what is the unit of a ?
9. According to quantum mechanics light travels in the form of packets and energy associated with each packet is $E = hf$, where h is Planck's constant and f is the frequency. What is the dimensional formula of h ?
10. The velocity v (in cm s^{-1}) of a particle is given in terms of time t (in sec) by the equation: $v = at + \frac{b}{t + c}$. What are the dimensions of a , b and c ?
11. What is the dimensional formula of magnetic flux?
12. A force F is given by $F = at + bt^2$, where t is the time. What are the dimensions of a and b ?
13. Let us redefine 1 N as the force of attraction between two particles, each of mass 1 kg, separated by 1 m. Then, what is the new value of universal constant of gravitation?
14. In a hypothetical new system of measurement, the gravitational force between two particles, each of mass 1 kg, separated by 1 km is taken as a unit of force. Let us call this new unit of force 'notwen'. How many newton will be there in one 'notwen'? Given: $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$.
15. Let us consider a new hypothetical system of measurement in which the unit of energy is called eluoj. Suppose, in this system, the gravitational force of attraction between two particles, each of mass 1 kg, separated by 1 km is taken as a unit of force. Then, how many joule are contained in one eluoj? Given: $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$.

SIGNIFICANT FIGURES

When we measure a physical quantity, generally the measured value does not come out to be accurate. It may contain some error. When the measured value is expressed as a number, then some digits which it contains are known reliably plus the first digit which is unreliable.

For example: Let us measure the length of a glass plate using a scale. Let this length lies somewhere between 2.6 and 2.7 cm, say 2.63 cm. Here, digits 2 and 6 are reliable, but digit 3 is unreliable. Now, the reliable digits and first unreliable digit are known as significant figures or significant digits. Thus, the measurement 2.63 cm contains three significant figures.

These significant figures are desired when the observations of any experiment have to be recorded and then to be used in calculations. Here, the knowledge of significant figures is helpful.

Rules for Counting Significant Figures

For a Number Greater Than 1

1. All non-zero digits are significant. There may be decimal point in between and location of decimal does not matter. Example:
 - a. 2357 has four significant figures
 - b. 312 has three significant figures
 - c. 325.23 has five significant figures
 - d. 32.523 has five significant figures
2. All zeros between two non-zero digits are significant. Location of decimal does not matter. Example:
 - a. 2307 has four significant figures
 - b. 320.03 has five significant figures
 - c. 32.003 has five significant figures
3. If the number is without decimal part, then the terminal or trailing zeros are not significant. Example:
 - a. 23500 has three significant figures
 - b. 53000 has two significant figures
4. Trailing zeros in the decimal part are significant. Example:
 - a. 3.700 has four significant figures
 - b. 2.50 has three significant figures

Note: If trailing zeros come from some measurement then they may be significant as illustrated below.

Experiment 1: Let some length is measured to be 1500 mm. According to rule (3), it should have two significant figures. But we can write $1500 \text{ mm} = 1.500 \text{ m}$. Then, according to rule (4) it should have four significant figures. Now, the question arises: Does the change of unit change the number of significant figures? The answer is No. In fact, the measured length 1500 mm will have four significant figures.

Experiment 2: Now, let same length is measured again and this time the value comes out to be 1.5 m. This measurement has two

significant figures. We can write $1.5 \text{ m} = 1500 \text{ mm}$. Here, 1500 mm will have two significant figures.

From the above experiments, we learn that only originally measured quantity will indicate the correct number of significant figures.

So, in order to avoid confusion in counting the number of significant figures, we usually express a measured quantity in scientific notation. By this, the number of significant figures are clearly mentioned and do not change on changing the units. For illustrations, see the table below.

If original measured quantity is 1,500 mm	If original measured quantity is 1.5 m	If original measured quantity is 150 cm
1,500 mm $= 1.500 \times 10^3 \text{ mm}$ $= 1.500 \text{ m}$ $= 1.500 \times 10^2 \text{ cm}$ $= 1.500 \times 10^{-3} \text{ km}$ All of the above contain four significant figures.	1.5 m $= 1.5 \times 10^3 \text{ mm}$ $= 1.5 \times 10^2 \text{ cm}$ $= 1.5 \times 10^{-3} \text{ km}$ All of the above contain two significant figures.	150 cm $= 1.50 \times 10^3 \text{ mm}$ $= 1.50 \text{ m}$ $= 1.50 \times 10^2 \text{ cm}$ $= 1.50 \times 10^{-3} \text{ km}$ All of the above contain three significant figures.

For a Number Less Than 1

Any zero to the right of a non-zero digit is significant. All zeros between decimal point and first non-zero digit are not significant. Example:

- 0.0074 has two significant figures
- 0.00704 has three significant figures
- 0.007040 has four significant figures
- 0.07040 has four significant figures

Illustration 3.9

Write the number of significant figures in the following:

- 0.053
- 50.00
- 0.0500
- 5.7×10^6
- 5.70×10^6
- 2400
- 2400 kg
- 0.0305090

Sol.

- a. 2 b. 4 c. 3 d. 2 e. 3 f. 2 g. 4 h. 6

Rules for Rounding off the Uncertain Digits

When we do the calculations using measured values, the result may contain more than one uncertain digits which should be rounded off. The following rules are used for rounding off:

- If digit to be dropped is less than 5, then the preceding digit remains unchanged. Example:
 - 6.32 after rounding off becomes 6.3
 - 5.934 after rounding off becomes 5.93

- If digit to be dropped is more than 5, then the preceding digit is increased by one. Example:
 - 7.86 after rounding off becomes 7.9
 - 5.937 after rounding off becomes 5.94
- If digit to be dropped is 5:
 - If it is only 5 or 5 followed by zero, then the preceding digit is raised by one if it is odd and left unchanged if it is even. Example:
 - 4.750 after rounding off becomes 4.8
 - 4.75 after rounding off becomes 4.8
 - 4.650 after rounding off becomes 4.6
 - 4.65 after rounding off becomes 4.6
 - If 5 is further followed by a non-zero digit, the preceding digit is raised by one. Example:
 - 15.352 after rounding off becomes 15.4
 - 9.853 after rounding off becomes 9.9

Note: In intermediate steps during multistep calculations, we should retain one digit more than the significant digits and at the end of the calculation, round off to proper significant figures.

Illustration 3.10

Round off to two significant figures:

- 0.05857
- 0.05837
- 5.07×10^6
- 5.01×10^6

Sol.

- 0.059
- 0.058
- 5.1×10^6
- 5.0×10^6

Significant Figures in Calculations

When we do the calculations using measured values, the result cannot be more accurate than any of the measured value. The result must possess the accuracy level as that of original measurements. So, to have proper accuracy in the final result we need to follow some rules during different arithmetical operations.

- Addition and subtraction:** The number of decimal places in the final result of any of these operations has to be equal to the smallest number of decimal places in any of the terms involved in calculations. Example:
 - Sum of terms 2.29 and 62.7 is 64.99. After rounding off to one place of decimal it will become 65.0.
 - Subtraction of 62.7 from 82.29 gives 19.59. After rounding off to one place of decimal it will become 19.6.

Note: During the subtraction of quantities of nearly equal magnitude, accuracy is almost destroyed, e.g., $3.28 - 3.23 = 0.05$. Result 0.05 has only one significant figure whereas original measurements have three significant figures each.

So, it is advised to measure the difference directly instead of measuring the quantities first and then finding their difference.

- 2. Multiplication and division:** In these operations, the number of significant figures in the result is the same as the smallest number of significant figures in any of the factors. Example:
- $1.3 \times 1.2 = 1.56$. After rounding off to two significant figures it becomes 1.6.
 - $\frac{3500}{7.52} = 465.42$. As 3500 has minimum number of significant figures, i.e., two, so the quotient must have two significant figures. So, $465.42 = 470$ (after rounding off).
 - If we divide 3500 m by 7.52, 3500 m has four significant figures, then final result should be 465 (after rounding off to three significant figures).

Illustration 3.11 Subtract 2.5×10^4 from 3.9×10^5 with due regard to significant figures.

Sol. Let $x = 2.5 \times 10^4 = 25,000$, $y = 3.9 \times 10^5 = 3,90,000$
 $y - x = 3,90,000 - 25,000 = 3,65,000 = 3.65 \times 10^5$
 $= 3.6 \times 10^5$ (rounded off to one place of decimal)

Illustration 3.12 Calculate area enclosed by a circle of diameter 1.06 m to correct number of significant figures.

Sol. Here, $r = \frac{1.06}{2} = 0.53$ m.

Area enclosed $= \pi r^2 = 3.14(0.53)^2 = 0.882026 \text{ m}^2$
 $= 0.882 \text{ m}^2$
 (rounded off to three significant figures)

ERRORS IN MEASUREMENTS

The difference in the true value and the measured value of a quantity is called *error*.

One basic thing on which every branch of science depends is measurement. There are always many factors which influence the measurement. These factors always introduce error (may be small, whatever be the level of accuracy). So, no measurement is perfect. We can only minimize the errors using best methods and techniques, but we cannot eliminate them permanently.

Types of errors: We have three types of errors.

1. Systematic errors, 2. Random errors, and 3. Gross errors.

Systematic Errors

Errors whose causes are known are called systematic errors. These errors can be minimized by applying some corrections. These are of various types:

Errors due to External Factors

These errors are due to fluctuation in atmospheric conditions like temperature, pressure, humidity, etc.

Errors due to Imperfection

These are introduced due to negligence of facts, e.g., error in weighing of a body arising out of buoyancy is usually ignored.

Instrumental Errors

These errors are introduced due to improper designing and manufacturing defects of instrument. Often there may be zero error. For example, a meter scale may be worn off at the end of zero mark. The instrumental errors can be reduced by using more accurate instruments and applying zero correction, when required.

Personal Errors

These errors are introduced due to lack of proper care on part of the observer. For example, lack of proper setting of the apparatus, recording the reading without applying proper precautions and so on.

Random Errors

The causes of such errors are not known precisely. Hence, it is not possible to eliminate the random errors, e.g., same person repeating the same experiment may get different readings each time. These errors are also known as chance errors.

These are minimized by repeating the experiment and taking the arithmetic mean of all the observations. The mean value should be close to the accurate value.

Gross Errors

These errors arise on account of sheer carelessness of the observer. For example:

1. Reading an instrument without setting it properly.
2. Taking the observations wrongly without caring for the sources of errors.
3. Recording the observations wrongly.
4. Using wrong values of the observations in calculations.

These errors can be minimized only if the observer is sincere and mentally alert.

ABSOLUTE ERRORS

It is the magnitude of the difference between the true value and the measured value of a physical quantity. Let $x_1, x_2, x_3, \dots, x_n$ be n observations recorded corresponding to a physical quantity x , then mean value x_m is given by:

$$x_m = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$
 As the correct value of x is not known, so x_m is taken to be the absolute value or correct value.

The *absolute error* in various observations are given by

$$\Delta x_1 = |x_m - x_1|, \quad \Delta x_2 = |x_m - x_2|, \dots, \Delta x_n = |x_m - x_n|$$

The arithmetic mean of these absolute errors is called *mean absolute error*. Let us denote it by $\overline{\Delta x_m}$.

$$\text{Then, } \overline{\Delta x_m} = \frac{\Delta x_1 + \Delta x_2 + \dots + \Delta x_n}{n} = \frac{1}{n} \sum_{i=1}^n |\Delta x_i|$$

Now, the measurement is likely to be in between $x_m - \overline{\Delta x_m}$ and $x_m + \overline{\Delta x_m}$. Therefore, the final result of the measured physical quantity can be written as $x = x_m \pm \overline{\Delta x_m}$

Relative Error and Percentage Error

Relative error is defined as the ratio of mean absolute error to the mean value of the quantity measured. When it is multiplied by 100, it becomes percentage error.

$$\text{Relative error} = \frac{\text{mean absolute error}}{\text{real value (mean value)}} = \frac{\overline{\Delta x_m}}{x_m}$$

$$\text{Percentage error} = \frac{\overline{\Delta x_m}}{x_m} \times 100$$

Illustration 3.15 Repeated measurements of a certain quantity in an experiment gave the following values: 1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30 and 1.33. Calculate the mean value, mean absolute error, the relative error and the percentage error.

Sol. Here, mean value:

$$x_m = \frac{1.29 + 1.33 + 1.34 + 1.35 + 1.32 + 1.36 + 1.30 + 1.33}{8}$$

$$= 1.3275 = 1.33 \text{ (rounded off to two places of decimal).}$$

Absolute errors in measurement are:

$$\Delta x_1 = |1.33 - 1.29| = 0.04; \quad \Delta x_2 = |1.33 - 1.33| = 0.00;$$

$$\Delta x_3 = |1.33 - 1.34| = 0.01; \quad \Delta x_4 = |1.33 - 1.35| = 0.02;$$

$$\Delta x_5 = |1.33 - 1.32| = 0.01; \quad \Delta x_6 = |1.33 - 1.36| = 0.03;$$

$$\Delta x_7 = |1.33 - 1.30| = 0.03; \quad \Delta x_8 = |1.33 - 1.33| = 0.00.$$

Mean absolute error:

$$\overline{\Delta x_m} = \frac{0.04 + 0.00 + 0.01 + 0.02 + 0.01 + 0.03 + 0.03 + 0.00}{8}$$

$$= 0.0175$$

$$= 0.02 \text{ (rounded off to two places of decimal).}$$

$$\text{Relative error} = \pm \frac{\overline{\Delta x_m}}{x_m} = \pm \frac{0.02}{1.33} = \pm 0.01503 = \pm 0.02$$

(rounded off to two places of decimal)

$$\text{Percentage error} = \pm 0.01503 \times 100 = \pm 1.503 = \pm 1.5\%.$$

PROPAGATION OF COMBINATION OF ERRORS

When we do calculations using measured values which themselves contain error, definitely there will also be error in the final result. To calculate the net error in the final result, we should know how errors propagate in different mathematical operations.

Error in Summation

Maximum absolute error in the sum of two quantities is equal to sum of the absolute errors in the individual quantities. Suppose $x = a + b$.

Let Δa = absolute error in measurement of a , Δb = absolute error in measurement of b and Δx = absolute error in calculation of x , i.e., sum of a and b .

Thus, value of the maximum absolute error in x is given by $\Delta x = \pm(\Delta a + \Delta b)$.

Error in Difference

Maximum absolute error in difference of two quantities is equal to sum of the absolute errors in the individual quantities. Suppose $x = a - b$.

Let Δa = absolute error in measurement of a , Δb = absolute error in measurement of b and Δx = absolute error in calculation of x , i.e., difference of a and b .

Thus, value of the maximum absolute error in x is given by $\Delta x = \pm(\Delta a + \Delta b)$.

Error in Product

Maximum fractional error or relative error in product of quantities is equal to sum of the fractional or relative errors in the individual quantities. Suppose $x = a \times b$.

Let Δa = absolute error in measurement of a , Δb = absolute error in measurement of b and Δx = absolute error in calculation of x , i.e., product of a and b .

So, the maximum possible value of fractional error in product of quantities is given by $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$.

Error in Division

Maximum value of fractional or relative error in division of quantities is equal to sum of the fractional or relative errors in the individual quantities. Suppose $x = \frac{a}{b}$.

Let Δa = absolute error in measurement of a , Δb = absolute error in measurement of b and Δx = absolute error in calculation of x , i.e., division of a and b .

So, the maximum possible value of fractional error in division of quantities is given by $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$.

Error in Power of a Quantity

Fractional error or relative error in the quantity is equal to sum of fractional or relative error of the individual quantities multiplied by their powers.

As the error multiplies n times, therefore, in any formula, the quantity with maximum power should be measured with highest degree of accuracy, i.e., with least error.

Consider a quantity $x = \frac{a^m}{b^n}$.

Let Δa = absolute error in measurement of a , Δb = absolute error in measurement of b and Δx = absolute error in calculation of x . Then, fractional or relative error in x is given by

$$\frac{\Delta x}{x} = \pm \left[m \left(\frac{\Delta a}{a} \right) + n \left(\frac{\Delta b}{b} \right) \right]$$

Note: If a set of experimental data is specified to n significant figures, a result obtained by combining the data will also be valid to n significant figures.

When two or more experimentally obtained numbers are multiplied, the percentage uncertainty of the final result is equal to square root of the sum of the squares of the percentage uncertainties of the original numbers.

Illustration 3.14 The initial and final temperatures of water as recorded by an observer are $(40.6 \pm 0.2)^\circ\text{C}$ and $(78.3 \pm 0.3)^\circ\text{C}$. Calculate the rise in temperature with proper error limits.

Sol. Here, $\theta_1 = (40.6 \pm 0.2)^\circ\text{C}$, $\theta_2 = (78.3 \pm 0.3)^\circ\text{C}$
 Rise in temp: $\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7^\circ\text{C}$.
 Error in θ : $\Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2) = \pm(0.2 + 0.3) = \pm 0.5^\circ\text{C}$
 Hence, rise in temperature = $(37.7 \pm 0.5)^\circ\text{C}$.

Illustration 3.15 The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate area of the rectangle with error limits.

Sol. Here, $l = (5.7 \pm 0.1)$ cm, $b = (3.4 \pm 0.2)$ cm
 Area: $A = l \times b = 5.7 \times 3.4 = 19.38 \text{ cm}^2 = 19 \text{ cm}^2$
 (rounding off to two significant figures)
 $\therefore \frac{\Delta A}{A} = \pm \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} \right) = \pm \left(\frac{0.1}{5.7} + \frac{0.2}{3.4} \right)$
 $= \pm \left(\frac{0.34 + 1.14}{5.7 \times 3.4} \right) = \pm \frac{1.48}{19.38}$
 $\Rightarrow \Delta A = \pm \frac{1.48}{19.38} \times A = \pm \frac{1.48}{19.38} \times 19.38$
 $= \pm 1.48 = \pm 1.5$
 (rounding off to two significant figures)
 So, Area = $(19.0 \pm 1.5) \text{ cm}^2$.

Illustration 3.16 The distance covered by a body in time (5.0 ± 0.6) s is (40.0 ± 0.4) m. Calculate the speed of the body. Also, determine the percentage error in the speed.

Sol. Here, $s = 40.0 \pm 0.4$ m and $t = 5.0 \pm 0.6$ s
 $\therefore \text{Speed } v = \frac{s}{t} = \frac{40.0}{5.0} = 8.0 \text{ ms}^{-1}$ (As $v = \frac{s}{t}$)
 $\therefore \frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t}$
 Here, $\Delta s = 0.4$ m, $s = 40.0$ m, $\Delta t = 0.6$ s, $t = 5.0$ s.
 $\therefore \frac{\Delta v}{v} = \frac{0.4}{40.0} + \frac{0.6}{5.0} = 0.13$
 $\Rightarrow \Delta v = 0.13 \times 8.0 = 1.04$
 Hence, $v = (8.0 \pm 1.04) \text{ ms}^{-1}$
 $\therefore \text{Percentage error} = \left(\frac{\Delta v}{v} \times 100 \right) = 0.13 \times 100 = 13\%$.

Accuracy and Precision

Accuracy tells us how close is the measured value to the true value. Precision indicates from which instrument, the measurement is taken.

Concept Application Exercise 3.2

- Which of the following length measurement is most precise and why?
 - 2.0 cm
 - 2.00 cm
 - 2.000 cm
- In a number without decimal, what is the significance of zeros on the right of non-zero digits?

- A research worker takes 100 observations in an experiment. If he repeats the same experiment by taking 500 observations, how is the probable error affected?
- Which quantity in a given formula should be measured most accurately? Why?
- A body travels uniformly a distance of (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Find the velocity of the body within error limits and the percentage error.
- Error in the measurement of radius of a sphere is 1%. Find the error in the measurement of volume.
- Given $R_1 = 5.0 \pm 0.2 \Omega$, $R_2 = 10.0 \pm 0.1 \Omega$. What is the total resistance in parallel with possible % error?
- The value of resistance is 10.845 ohm and the current is 3.23 ampere. On multiplying them, we get the potential difference = 35.02935 V. What is the value of potential difference in terms of significant figures?
- The length of one rod is 2.53 cm and that of the other is 1.27 cm. The least count of measuring instrument is 0.01 cm. If the two rods are put together end to end, find the combined length.
- A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. What is the maximum percentage error in the measurement of density?
- The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate by using the formula $P = \frac{F}{l^2}$. If the maximum errors in the measurement of force and length are 4% and 2%, respectively, then what is the maximum error in the measurement of pressure?
- The density of a cube is measured by measuring its mass and the length of its sides. If the maximum errors in the measurement of mass and length are 3% and 2%, respectively, then find the maximum error in the measurement of the density of cube.

Vernier Callipers: Its Use to Measure Length, Internal and External Diameters and Depth of a Cylindrical Vessel

1. Description: Vernier callipers and screw gauge are used to measure the length of objects, thickness of wires, diameter of cylinders, etc. upto an accuracy of $1/10^{\text{th}}$ or $1/100^{\text{th}}$ of a mm.

A Vernier Callipers consists of a main scale M graduated in cm and mm over which is an auxiliary scale (or vernier scale) V which can slide along the length of main scale M . The divisions of the vernier scale being either slightly longer or slightly smaller than the divisions of the main scale.

The main scale has two fixed jaws A and C as shown in Fig. 3.2 while B and D are the jaws of vernier scale. The position of vernier scale is fixed with the help of screw S . In general, the vernier scale has 10 divisions over a length of 9 mm.

We can measure the external diameter of an object, internal diameter of a hollow object or depth of some vessel using vernier callipers. Diameter (external) of an object can be determined by placing the object in between the jaws *A* and *B* while internal diameter can be measured by inserting *C* and *D* in the object.

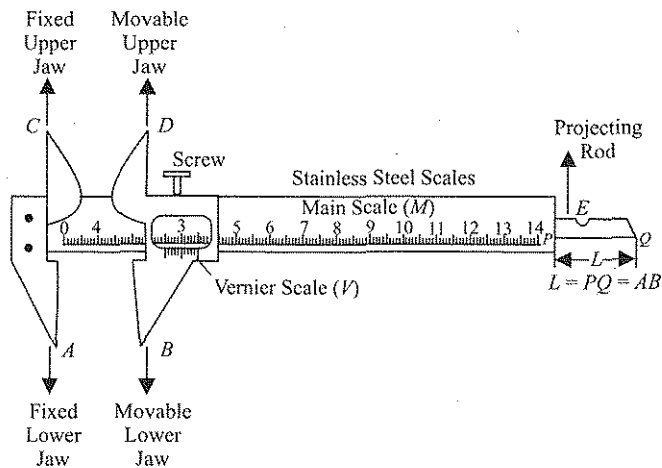


Fig. 3.2

While performing the experiment using a vernier callipers first of all the jaws of vernier scale, i.e., *A* and *B* touch while straight edges of *C* and *D* touch. If the instrument is free from zero error, then zero of main scale coincides with the zero of vernier scale.

After noticing whether the instrument is having zero error or not, hold the object whose external diameter is to be computed between jaws *A* and *B* as shown in the figure. Now, the most important task is to measure the diameter, i.e., to take the readings of vernier caliper.

To measure the depth of the vessel, a metallic strip *E* is connected to the back of *M* and vernier scale. When jaws *A* and *B* touch each other, the edge of *E* touches the edge of *M*. When the vernier is separated from *M*, *E* moves outwards.

2. Vernier constant: The difference between one main scale division and one vernier division is called the vernier constant or the least count of the vernier because it is the smallest length that can be measured accurately with its help.

To find the vernier constant (or least count):

- Find the magnitude of the smallest division of the main scale.
- Count the total number of divisions on the vernier scale.
- Slide the movable jaw that the zero mark (the first division) of the vernier scale coincides with any of the main scale divisions.
- Find the number of scale divisions which coincide with the total number of vernier division.

If *m* main scale divisions (MSD) are equal to *n* vernier scale divisions (VSD), then we can write

$mx = ny$, where *x* is the length of 1 MSD and *y* is the length of 1 VSD.

Now, Vernier constant = 1 MSD – 1 VSD = $x - y = x - \frac{m}{n}x$

$$= x \left(1 - \frac{m}{n} \right)$$

In general, if *m* vernier scale divisions are equal to (*n* – 1) main scale divisions, i.e., $m = n - 1$, then we have

$$\begin{aligned} \text{Vernier constant} &= \text{least count} = x \left(1 - \frac{m}{n} \right) \\ &= \left(1 - \frac{n-1}{n} \right) x = \frac{1 \text{ MSD}}{n} \end{aligned}$$

If the *p*th vernier division coincides with any one of the main scale division, then

$$\text{Fraction to be added} = \frac{p}{n}$$

Hence, the measured value of length: *L* = complete main scale reading before zero of the V.S. mark + $\frac{p}{n}$.

3. Zero error: In a correctly adjusted instrument, the zero of the vernier coincides with the zero of the main scale when the two jaws *A* and *B* are brought in contact. If it is not so, the instrument has a zero error which may be positive or negative according as the zero of vernier scale lies to the right or left of the zero of the main scale when its two jaws are brought in close contact with each other.

Screw Gauge: Its Use to Measure Thickness/Diameter of Thin Sheet or Wire

1. Description: It works on the principle of micrometer screw and its diagram showing the constant is shown in the Fig. 3.3. *H* is the linear scale or pitch scale while *E* is the cap scale. A hollow cylindrical cap *k* is capable to rotate over *H* (hub or linear scale) when the screw is rotated. When zero of pitch scale coincides with zero of cap scale, then the instrument is free from zero error.

The wire whose diameter has to be measured is held between *A* and *B*.

Reading of screw gauge is,

$$\text{the measured diameter} = N + n \times \text{Least count}$$

where *N* is the division of linear scale beyond which the edge of the cap lies, and *n* is the division of circular scale which lies over reference line. If some zero error is there, subtract it from the above reading.

When the zero of circular scale advances beyond the reference line, the zero error is negative and if it is left behind the reference line, the zero error is positive. In positive zero error, zero of the linear scale is not hidden from the circular scale, while in negative zero error, zero of the linear scale is hidden from circular scale.

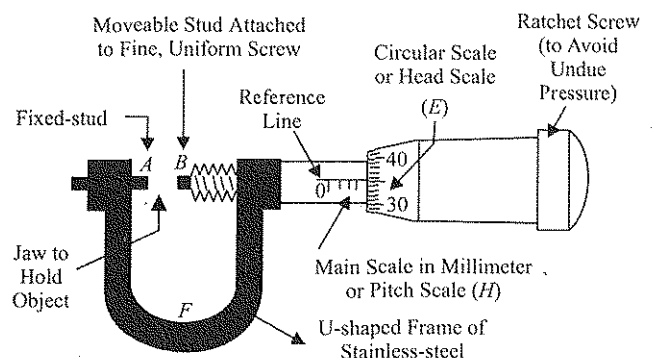


Fig. 3.3

2. Pitch of the screw gauge: The pitch of screw is defined as the distance through which the screw moves forward or backward parallel to its axis when one complete rotation is given to the circular cap. To find pitch:

- Rotate the circular scale H and coincide the zero mark with the reference line. Notice the reading on the pitch scale.
- Give four complete rotations to the circular scale and note the reading on the pitch scale again.
- Calculate the pitch of the screw by dividing the distance moved by the number of rotations.

$$\text{Pitch of the screw} = \frac{\text{distance travelled on the pitch scale}}{\text{number of rotations}}$$

Note: The pitch of a screw gauge is usually 0.5 mm or 1 mm.

3. Least count of the screw gauge: The least count of the screw gauge is defined as the distance through which the screw moves backward or forward when the cap is rotated through one division on the circular scale. Note the number of divisions on the circular scale H . Then,

$$\text{Least count} = \frac{\text{pitch of the screw}}{\text{no. of rotations on the circular scale}}$$

4. Zero correction: In some instruments, when the jaws A and B are brought in contact without applying any undue pressure, the zero of the circular scale does not coincide with the reference line. In some instruments, zero mark goes beyond the reference line, while in others it is left behind when the two studs A and B are in contact with each other without undue pressure.

To find zero correction, count the number of divisions on the circular scale that the zero mark has advanced beyond or is left behind the reference line. Multiply this number with the least count. This is the zero correction. If the zero of the circular scale has advanced beyond the reference line, the zero correction is *positive* but if it is left behind the reference line, it is *negative*.

Remember zero above, add and zero below, subtract.

5. Backlash error: With constant use a little play always creeps in between the nut and the screw, due to which the screw does not move forward or backward for a little distance when the head is rotated. This error is called the backlash error. To avoid this, the screw is well greased and is always turned in the same direction while taking an observation.

6. To read a screw gauge:

- Find the zero correction.
- Find the least count.
- Insert the wire/sheet between the jaws A and B . Rotate screw R till it is held tightly without any pressure. Note the reading of main scale and circular scale. Then

$$\text{Diameter/thickness } d = \text{main scale reading in millimeter} + \text{circular scale reading} \times \text{least count}$$

(Apply the zero correction)

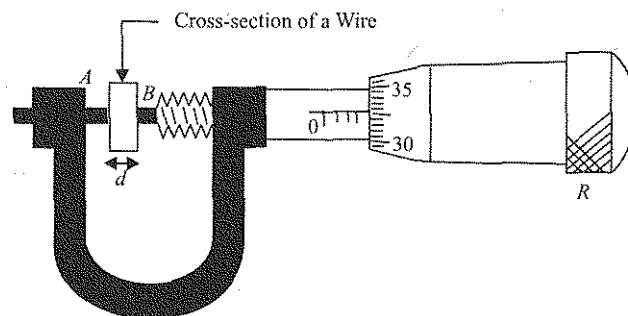


Fig. 3.4

Concept Application Exercise 3.3

- Each division on the main scale is 1 mm. Out of the following which of the vernier scales will give vernier constant equal to 0.1 mm?
 - 90 mm divided into 100 divisions
 - 90 mm divided into 10 divisions
 - 90 mm divided into 1000 divisions
 - None of these
- A vernier callipers has 20 divisions on the vernier scale which coincide with 19 on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of:

a. 0.5 mm	b. 1 mm
c. 2 mm	d. $\frac{1}{4}$ mm
- The length of one solid thin cylinder is 2.25 cm and that of another cylinder is 1.31 cm. The vernier constant of calliper is 0.01 cm. If the two cylinders are put together end to end, the combined length will be expressed as:
 - (3.56 ± 0.005) cm
 - (3.56 ± 0.001) cm
 - (3.56 ± 0.002) cm
 - (3.56 ± 0.10) cm
- A screw gauge has 1.0 mm pitch and 200 divisions on the circular scale. What is the least count of the instrument?
 - 5×10^{-3} mm
 - 4×10^{-3} mm
 - 6×10^{-3} mm
 - 2×10^{-2} mm

Solved Examples

Example 3.1 A famous relation in physics relates moving mass m to the rest mass m_0 of a particle in terms of its speed v and the speed of light c . (This relation first arose as

a consequence of special theory of relativity due to Albert Einstein.) A boy recalls the relation almost correctly but forgets where to put the constant c . He writes, $m = \frac{m_0}{(1 - v^2)^{1/2}}$.

Guess where to put the missing c ?

Sol. According to the principle of homogeneity of dimensions, powers of M , L , T on either side of the formula must be equal. For this, on R.H.S., the denominator $(1 - v^2)^{1/2}$ should be dimensionless. Here, v^2 is not dimensionless. So, there is something missing. Therefore, instead of $(1 - v^2)^{1/2}$ we should write $(1 - v^2/c^2)^{1/2}$. Hence, the correct formula would be $m = \frac{m_0}{(1 - v^2/c^2)^{1/2}}$

Example 3.2 The velocity of a body is given by the equation

$$v = \frac{b}{t} + ct^2 + dt^3.$$

The dimensional formula of b is

- a. $[M^0 L T^0]$ b. $[ML^0 T^0]$
c. $[M^0 L^0 T]$ d. $[MLT^{-1}]$

Sol. a. In the equation, left hand side has the dimension of velocity. Thus, from the principle of homogeneity each term in the right hand side should have the dimensions of velocity.

$$\left[\frac{b}{t}\right] = [v] \text{ or } [b] = [vt] = [L]$$

Example 3.3 The force F is given in terms of time t and displacement x by the equation

$F = A \cos Bx + C \sin Dt$. The dimensional formula of D/B is

- a. $[M^0 L^0 T^0]$ b. $[M^0 L^0 T^{-1}]$
c. $[M^0 L^{-1} T^0]$ d. $[M^0 L^1 T^{-1}]$

Sol. d. In the given equation Bx and Dt should be dimensionless.

$$[B] = [x^{-1}] \text{ and } [D] = [T^{-1}] \Rightarrow \left[\frac{D}{B}\right] = \left[\frac{x}{T}\right] = [\text{velocity}]$$

Example 3.4 The velocity of a body which has fallen freely under gravity varies as $g^p h^q$, where g is the acceleration due to gravity and h is the height through which it has fallen. The value of p and q are

- a. $-\frac{1}{2}, \frac{1}{2}$ b. $\frac{1}{2}, -\frac{1}{2}$
c. $\frac{1}{2}, \frac{1}{2}$ d. $-\frac{1}{2}, -\frac{1}{2}$

Sol. c. Here, we are given $v = kg^p h^q$. Dimensions of left hand side and right hand side terms should be same.

$$\therefore [M^0 L T^{-1}] = [L T^{-2}]^p [L]^q = [L^{p+q} T^{-2p}]$$

Comparing powers of L and T , $p + q = 1$ and $-2p = -1$

Solving them, we get $p = 1/2$ and $q = 1/2$

Putting these values, we get $v = k\sqrt{gh}$, which is the required relation.

Example 3.5 The wavelength associated with a moving particle depends upon p^{th} power of its mass m , q^{th} power of its velocity v and r^{th} power of Planck's constant h . Then, the correct set of values of p , q , and r is

- a. $p = 1, q = -1, r = 1$
b. $p = 1, q = 1, r = 1$
c. $p = -1, q = -1, r = -1$
d. $p = -1, q = -1, r = 1$

Sol. d. Given $\lambda = km^p v^q h^r$. The dimensions of right hand side and left hand side terms should be equal.

$$\text{So } [M^0 L T^0] = [M]^p [L T^{-1}]^q [M L^2 T^{-1}]^r$$

$$\text{or } [M^0 L T^0] = [M^{p+r}] [L^{q+2r}] [T^{-q-r}]$$

Now, comparing powers of M , L and T , we get

$$p + r = 0 \text{ (i), } \quad q + 2r = 1 \text{ (ii), } \quad -q - r = 0 \text{ (iii)}$$

After solving, $p = -1, q = -1$ and $r = 1$.

Putting these values, we get

$$\lambda = k \frac{h}{mv}, \text{ which is the required relation.}$$

Example 3.6 If P represents radiation pressure, c represents speed of light and Q represents radiation striking unit area per second, then non-zero integers x , y , and z such that $P^x Q^y c^z$ is dimensionless are

- a. $x = 1, y = 1, z = -1$
b. $x = 1, y = -1, z = 1$
c. $x = -1, y = 1, z = 1$
d. $x = 1, y = 1, z = 1$

Sol. b. Dimensions of

$$P^x Q^y c^z = [ML^{-1} T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z.$$

As it is dimensionless, so

$$[ML^{-1} T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z = [M^0 L^0 T^0] \text{ or}$$

$$[M^{x+y} L^{-x+z} T^{-2x-3y-z}] = [M^0 L^0 T^0]$$

Comparing powers of M , L and T , we get

$$x + y = 0, \quad -x + z = 0, \quad -2x - 3y - z = 0$$

Solving, $x = 1, y = -1, z = 1$.

Example 3.7 A physical quantity x is calculated from the relation $x = \frac{a^2 b^3}{c \sqrt{d}}$. If percentage error in a, b, c and d are 2%, 1%, 3%, and 4%, respectively, what is the percentage error in x ?

Sol. Given $x = \frac{a^2 b^3}{c \sqrt{d}}$

$$\therefore \frac{\Delta x}{x} \times 100 = \pm \left[2 \frac{\Delta a}{a} + 3 \frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2} \frac{\Delta d}{d} \right]$$

$$= \pm \left[2 \times 2\% + 3 \times 1\% + 3\% + \frac{1}{2} \times 4\% \right] = \pm 12\%$$

Example 3.8 The length and breadth of a field are measured as: $l = (120 \pm 2)$ m and $b = (100 \pm 5)$ m, respectively. What is the area of the field?

Sol. Here, $\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = \left(\frac{2}{120} + \frac{5}{100} \right) = 0.0667$

$$\Delta A = 0.0667 \times A$$

$$\text{Now, } A = lb = 120 \times 100 = 12000 \text{ m}^2$$

$$\Rightarrow \Delta A = 0.0667 \times 12000 = 800.4 \text{ m}^2$$

$$\text{Area of the field} = A \pm \Delta A = 12000 \pm 800.4$$

$$= (1.2 \pm 0.08) \times 10^4 \text{ m}^2.$$

Example 3.9 In an experiment of simple pendulum, time period measured was 50 s for 25 vibrations when the length of the simple pendulum was taken 100 cm. If the least count of stop watch is 0.1 s and that of meter scale is 0.1 cm, calculate the maximum possible error in the measurement of value of g .

Sol. The time period of a simple pendulum is given by

$$T = 2\pi \sqrt{\frac{l}{g}} \text{ or } T^2 = \frac{4\pi^2 l}{g} \text{ or } g = \frac{4\pi^2 l}{T^2}$$

As 4 and π are constants, maximum permissible error in g is given by $\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$

Here, $\Delta l = 0.1$ cm, $l = 1$ m = 100 cm, $\Delta T = 0.1$ s, $T = 50$ s.

$$\therefore \frac{\Delta g}{g} = \frac{0.1}{100} + 2 \left(\frac{0.1}{50} \right) = \frac{0.1}{100} + \left(\frac{0.1}{25} \right)$$

$$\text{or } \frac{\Delta g}{g} \times 100 = \left[\frac{0.1}{100} + \frac{0.1}{25} \right] \times 100 = 0.1 + 0.4 = 0.5\%.$$

Example 3.10 If $X = A \times B$ and ΔX , ΔA and ΔB are maximum absolute errors in X , A and B , respectively, then the maximum relative error in X is given by

a. $\Delta X = \Delta A + \Delta B$

b. $\Delta X = \Delta A - \Delta B$

c. $\frac{\Delta X}{X} = \frac{\Delta A}{A} - \frac{\Delta B}{B}$

d. $\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

Sol. d. When two quantities are multiplied, their maximum relative errors are added up.

$$\text{Hence: } \frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Example 3.11 If $X = \frac{A}{B}$ and ΔX , ΔA and ΔB are the maximum absolute errors in X , A and B , respectively, then the maximum fractional error in X is given by

a. $\Delta X = \Delta A + \Delta B$

b. $\Delta X = \Delta A - \Delta B$

c. $\frac{\Delta X}{X} = \frac{\Delta A}{A} - \frac{\Delta B}{B}$

d. $\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

Sol. d. When two quantities are divided, their maximum fractional or relative errors are added up.

$$\text{Hence: } \frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Example 3.12 A physical quantity is represented by $X = M^a L^b T^{-c}$. If percentage errors in the measurement of M , L and T are α , β and γ , respectively, then total percentage error is

a. $(\alpha a + \beta b - \gamma c)$

b. $(\alpha a + \beta b + \gamma c)$

c. $(\alpha a - \beta b - \gamma c)$

d. zero

Sol. b. $\frac{\Delta x}{x} \times 100 = a \frac{\Delta M}{M} \times 100 + b \frac{\Delta L}{L} \times 100$

$$+ c \frac{\Delta T}{T} \times 100 = a\alpha + b\beta + c\gamma$$

Example 3.13 Given: potential difference, $V = (8 \pm 0.5)$ V and current, $I = (2 \pm 0.2)$ A. The value of resistance R in Ω is

a. $4 \pm 16.25\%$

b. $4 \pm 6.25\%$

c. $4 \pm 10\%$

d. $4 \pm 8\%$

Sol. a. We know that $R = V/I = 8/2 = 4 \Omega$.

$$\text{Percentage error in } V \text{ is } \frac{0.5}{8} \times 100 = 6.25.$$

$$\text{Percentage error in } I \text{ is } \frac{0.2}{2} \times 100 = 10.$$

Now, add up the percentage errors and get the answer.

Example 3.14 Given: resistance, $R_1 = (8 \pm 0.4) \Omega$ and resistance, $R_2 = (8 \pm 0.6) \Omega$. What is the net resistance when R_1 and R_2 are connected in series?

a. $(16 \pm 0.4) \Omega$

b. $(16 \pm 0.6) \Omega$

c. $(16 \pm 1.0) \Omega$

d. $(16 \pm 0.2) \Omega$

Sol. c. When resistances are connected in series, then net resistance: $R = R_1 + R_2$

$$R = (8 + 8 \pm 0.4 \pm 0.6) \Omega = (16 \pm 1.0) \Omega.$$

Example 3.15 The focal length f of a mirror is given by $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where u and v represent object and image distances, respectively.

a. $\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v}$

b. $\frac{\Delta f}{f} = \frac{\Delta u}{v} + \frac{\Delta v}{u}$

c. $\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} - \frac{\Delta(u+v)}{u+v}$

d. $\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u}{u+v} + \frac{\Delta v}{u+v}$

Sol. d. $f = \frac{uv}{u+v}$, $\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta(u+v)}{u+v}$
 $= \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u}{u+v} + \frac{\Delta v}{u+v}$

Example 3.16 The following observations were taken for determining surface tension of water by capillary tube method: Diameter of capillary, $D = 1.25 \times 10^{-2}$ m and rise of water in capillary, $h = 1.45 \times 10^{-2}$ m. Taking $g = 9.80 \text{ ms}^{-2}$ and using the relation $T = (rgh/2) \times 10^3 \text{ Nm}^{-1}$, what is the possible error in measurement of surface tension T ?

a. 2.4%

b. 15%

c. 1.6%

d. 0.15%

Sol. c. Percentage error in T ,

$$\frac{\Delta T}{T} \% = \frac{\Delta r}{r} \% + \frac{\Delta g}{g} \% + \frac{\Delta h}{h} \% = \frac{0.01}{1.25} \times 100 + \frac{0.01}{9.80} \times 100 + \frac{0.01}{1.45} \times 100$$

$$= 0.8 + 0.1 + 0.7 = 1.6\%$$

Example 3.17 If force, velocity and time are taken as fundamental quantities, find the dimensions of work.

a. FVT

b. FVT^2

c. F^0VT^{-1}

d. FV^2T^{-1}

Sol. a. $W = ML^2T^{-2} = [F]^a[V]^bT^c$

$$\Rightarrow ML^2T^{-2} = [MLT^{-2}]^a [LT^{-1}]^b T^c$$

$$\Rightarrow a = 1, a + b = 2 \Rightarrow b = 1 \text{ and } c - 2a - b = -2$$

$$\Rightarrow c = 1. \text{ So, } [W] = FVT.$$

Example 3.18 A screw gauge having 100 equal divisions and a pitch of length 1 mm is used to measure the diameter of a wire of length 5.6 cm. The main scale reading is 1 mm

and 47th circular division coincides with the main scale. Find the curved surface area of the wire in cm^2 to appropriate significant figure. (Use $\pi = \frac{22}{7}$.) (IIT-JEE, 2004)

Sol. Least count = $\frac{1 \text{ mm}}{100} = 0.01 \text{ mm}$.

Diameter = MSR + CSR = 1 mm + 47 (0.01) mm = 1.47 mm.

Surface area = $\pi D l = \frac{22}{7} \times 1.47 \times 56 \text{ mm}^2 = 2.58724 \text{ cm}^2$
 $= 2.6 \text{ cm}^2$

Example 3.19 In a Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data. (IIT-JEE, 2004)

Sol. Maximum percentage error in Y is given by

$$Y = \frac{W}{\pi D^2} \times \frac{L}{4}$$

$$\left(\frac{\Delta Y}{Y}\right)_{\max} = 2 \left(\frac{\Delta D}{D}\right) + \frac{\Delta x}{x} + \frac{\Delta L}{L}$$

$$= 2 \left(\frac{0.001}{0.05}\right) + \left(\frac{0.001}{0.125}\right) + \left(\frac{0.1}{110}\right)$$

$$= 0.0489.$$

So, maximum percentage error = 4.89%

Example 3.20 The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures. (IIT-JEE, 2005)

Sol. Least count of vernier calliper

$$= \frac{1 \text{ division of main scale}}{\text{number of divisions in vernier scale}} = \frac{1}{10} = 0.1 \text{ mm}$$

The side of cube = 10 mm + 1 $\times$ 0.1 mm = 1.01 cm,

Now, density = $\frac{\text{mass}}{\text{volume}} = \frac{2.736 \text{ g}}{(1.01)^3 \text{ cm}^3} = 2.66 \text{ g cm}^{-3}$

(to correct significant figures)

EXERCISES

Subjective Type

Solutions on page 3.26

- Write the number of significant figures in the following
 - 0.053
 - 50.00
 - 0.0500
 - 5.7×10^6
 - 5.70×10^6
 - 2400
 - 2400 kg
 - 0.0305090
- Round off to two significant figures
 - 0.05857
 - 0.05837
 - 5.07×10^6
 - 5.01×10^6
- With due regard of significant figures, add the following
 - 953 and 0.324
 - 953 and 0.625
 - 953.0 and 0.324
 - 953.0 and 0.374
- With due regard of significant figures, subtract
 - 0.35 from 7
 - 0.65 from 7
 - 0.35 from 7.0
 - 0.65 from 7.0
- A diamond weighs 3.71 g. It is put into a box weighing 1.4 kg. Find the total weight of the box and diamond to correct number of significant figures.
- Calculate the area enclosed by a circle of radius 0.56 m to correct number of significant figures.
 - Calculate the area enclosed by a circle of diameter 1.12 m to correct number of significant figures.
- Add 3.8×10^{-6} to 4.2×10^{-5} with due regard to significant figures.
 - Subtract 3.2×10^{-6} from 4.7×10^{-4} with due regard to significant figures.
 - Subtract 1.5×10^3 from 4.8×10^4 with due regard to significant figures.
- The length, breadth, and thickness of a metal sheet are 4.234 m, 1.005 m, and 2.01 cm, respectively. Give the area and volume of the sheet to correct number of significant figures.
- The diameter of a sphere is 3.34 m. Calculate its volume with due regard to significant figures.
- Solve with due regard to significant figures:

$$\frac{5.42 \times 0.6753}{0.085}$$
- In an experiment, refractive index of glass was observed to be 1.45, 1.56, 1.54, 1.44, 1.54, and 1.53. Calculate
 - Mean value of refractive index.
 - Mean absolute error.

- Fractional error.
 - Percentage error.
 - Express the result in terms of absolute error and percentage error.
- Two plates have lengths measured as (1.9 ± 0.3) m and (3.5 ± 0.2) m. Calculate their combined length with error limits.
 - The initial and final temperatures of a liquid are measured to be $67.7 \pm 0.2^\circ\text{C}$ and $76.3 \pm 0.3^\circ\text{C}$. Calculate the rise in temperature with error limits.
 - The sides of a rectangle are (10.5 ± 0.2) cm and (5.2 ± 0.1) cm. Calculate its perimeter with error limits.
 - The length and breadth of a rectangle are 5.7 ± 0.1 cm and 3.4 ± 0.2 cm. Calculate area of the rectangle with error limits.
 - A body travels uniformly a distance of 13.8 ± 0.2 m in a time 4.0 ± 0.3 s. Calculate its velocity with error limits. What is the percentage error in velocity?
 - The radius of a sphere is measured to be 2.1 ± 0.5 cm. Calculate its surface area with error limits.
 - Calculate the percentage error in specific resistance, $\rho = \pi r^2 R / l$, where r = radius of wire = 0.26 ± 0.02 cm, l = length of wire = 156.0 ± 0.1 cm, R = resistance of wire = 64 ± 2 ohm.
 - Time period of a pendulum is given by $T = 2\pi \sqrt{\frac{L}{g}}$. Length of pendulum = 20 cm and is measured upto 1 mm accuracy. Time period is about 0.6 s. The time of 100 oscillations is measured with a watch of 1/10 s resolution. What is the accuracy in the determination of g ?

Objective Type

Solutions on page 3.27

- Which of the following is not measured in units of energy?
 - Couple $\times$ angle turned through
 - Moment of inertia $\times$ (angular velocity)²
 - Force $\times$ distance
 - Impulse $\times$ time
- The unit of surface tension may be expressed as
 - joule metre
 - newton metre
 - joule metre⁻²
 - newton metre⁻²
- The equation of the stationary wave is

$$y = 2A \sin\left(\frac{2\pi ct}{\lambda}\right) \cos\left(\frac{2\pi x}{\lambda}\right)$$

Which of the following statements is wrong?

- The unit of ct is same as that of λ .
- The unit of x is same as that of λ .

- c. The unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda t$.
 d. The unit of c/λ is same as that of x/λ .
4. Given that: $y = A \sin \left[\left(\frac{2\pi}{\lambda} \right) (ct - x) \right]$, where y and x are measured in the unit of length. Which of the following statements is true?
 a. The unit of λ is same as that of x and A .
 b. The unit of λ is same as that of x but not of A .
 c. The unit of c is same as that of $2\pi/\lambda$.
 d. The unit of $(ct - x)$ is same as that of $2\pi/\lambda$.
5. The dimensional representation of Planck's constant is identical to that of
 a. torque
 b. work
 c. stress
 d. angular momentum
6. The dimensional representation of latent heat is identical to that of
 a. internal energy
 b. angular momentum
 c. gravitational potential
 d. electric potential
7. The dimensions of shear modulus of rigidity are
 a. $M^1 L^1 T^{-2}$
 b. $M^1 L^1 T^{-1}$
 c. $ML^2 T^2$
 d. $ML^{-1} T^{-2}$
8. Out of the following, the only pair that does not have identical dimensions is
 a. angular momentum and Planck's constant
 b. moment of inertia and moment of a force
 c. work and torque
 d. impulse and momentum
9. The dimensions of self-induction are
 a. $MLT^{-2} A^{-2}$
 b. $ML^2 T^{-1} A^{-2}$
 c. $ML^2 T^{-2} A^{-2}$
 d. $ML^2 T^{-2} A^{-1}$
10. The dimensional formula for latent heat is
 a. $M^0 L^2 T^{-2}$
 b. MLT^{-2}
 c. $ML^2 T^{-2}$
 d. $ML^2 T^{-1}$
11. Which of the following have same dimension?
 a. Thermal capacity
 b. Universal gas constant
 c. Boltzmann's constant
 d. All of the above
12. Dimensional representation of coefficient of friction is
 a. $[ML^2 T^{-2}]$
 b. $[MLT^{-2}]$
 c. $[M^0 L^0 T^0]$
 d. $[MLT^{-1}]$
13. In the relation: $\frac{dy}{dt} = 2\omega \sin(\omega t + \phi_0)$, the dimensional formula for $(\omega t + \phi_0)$ is
 a. MLT
 b. MLT^0
 c. $ML^0 T^0$
 d. $M^0 L^0 T^0$
14. A physical quantity depends upon five factors, all of which have dimensions. Then, method of dimensional analysis
 a. can be applied.
 b. cannot be applied.
 c. depends upon factors involved.
 d. both a. and c.
15. A student when discussing the properties of a medium (except vacuum) writes:
 Velocity of light in vacuum = velocity of light in medium
 This formula is
 a. dimensionally correct
 b. dimensionally incorrect
 c. numerically incorrect
 d. both (a) and (c)
16. Given that T stands for time period and l stands for the length of simple pendulum. If g is the acceleration due to gravity, then which of the following statements about the relation $T^2 = (l/g)$ is correct?
 a. It is correct both dimensionally as well as numerically.
 b. It is correct neither dimensionally nor numerically.
 c. It is correct dimensionally but not numerically.
 d. It is correct numerically but not dimensionally.
17. Suppose refractive index μ is given as

$$\mu = A + \frac{B}{\lambda^2}$$
 where A and B are constants and λ is wavelength, then dimensions of B are same as that of
 a. wavelength
 b. volume
 c. pressure
 d. area
18. A physical quantity x depends on quantities y and z as follows: $x = Ay + B \tan(Cz)$, where A , B and C are constants. Which of the following do not have the same dimensions?
 a. x and B
 b. C and z^{-1}
 c. y and B/A
 d. x and A
19. If L and R denote inductance and resistance, respectively, then the dimensions of L/R are
 a. $M^1 L^0 T^0 Q^{-1}$
 b. $M^0 L^0 T Q^0$
 c. $M^0 L^1 T^{-1} Q^0$
 d. $M^{-1} L T^0 Q^{-1}$
20. The best method of reducing random error is
 a. to change the instrument used for measurement
 b. to take help of experienced observer

39. The dimensional formula for magnetizing field H is
 a. $[M^0 L^{-1} T^0 A]$ b. $[M^0 L T^{-3} A]$
 c. $[M^0 L T A^{-1}]$ d. $[M^0 L^1 T^{-1} A]$
40. The dimensions of intensity of a wave are
 a. $[ML^2 T^{-3}]$ b. $[ML^0 T^{-3}]$
 c. $[ML^{-2} T^{-3}]$ d. $[M^1 L^2 T^3]$
41. The dimensions of formula of capacitance is
 a. $[M^{-1} L^{-2} T A^2]$ b. $[M^{-1} L^{-2} T^3 A^2]$
 c. $[M^{-1} L^{-2} T^4 A^2]$ d. $[M^{-1} L^{-2} T^2 A^2]$
42. What are the dimensions of Gas constant?
 a. $[MLT^{-2} K^{-1}]$ b. $[M^0 L T^{-2} K^{-1}]$
 c. $[ML^2 T^{-2} K^{-1} \text{ mol}^{-1}]$ d. $[M^0 L^2 T^{-2} K^{-1}]$
43. If the order of magnitude of 499 is 2, then order of magnitude of 501 will be
 a. 4 b. 2 c. 1 d. 3
44. The order of magnitude of 0.00701 is
 a. -2 b. -1 c. 2 d. 1
45. The order of magnitude of 379 is
 a. 1 b. 2 c. 3 d. 4
46. If $X = a + b$, the maximum percentage error in the measurement of X will be
 a. $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100\%$
 b. $\left(\frac{\Delta a}{a+b} - \frac{\Delta b}{a+b}\right) \times 100\%$
 c. $\left(\frac{\Delta a}{a+b} + \frac{\Delta b}{a+b}\right) \times 100\%$
 d. $\left(\frac{\Delta a}{a} \times \frac{\Delta b}{b}\right) \times 100\%$
47. Which of the following is the most precise instrument for measuring length?
 a. Metre rod of least count 0.1 cm.
 b. Vernier callipers of least count 0.01 cm.
 c. Screw gauge of least count 0.001 cm.
 d. Data is not sufficient to decide.
48. The number of significant figures in $5.69 \times 10^{15} \text{ kg}$ is
 a. 1 b. 2 c. 3 d. 18
49. The position x of a particle at time t is given by $x = \frac{V_0}{a}(1 - e^{-at})$, where V_0 is constant and $a > 0$. The dimensions of V_0 and a are
 a. $M^0 L T^{-1}$ and T^{-1}
 b. $M^0 L T^0$ and T^{-1}
 c. $M^0 L T^{-1}$ and $L T^{-2}$
 d. $M^0 L T^{-1}$ and T
50. The time dependence of a physical quantity P is given by $P = P_0 e^{-\alpha t^2}$, where α is a constant and t is time. Then, constant α is
 a. dimensionless
 b. has dimensions of T^{-2}
 c. has dimensions of P
 d. has dimensions of T^2
51. Which of the following have got same dimensions?
 1. Latent heat
 2. Energy per unit mass
 3. Gravitational potential
 4. Specific heat.
 a. 4 and 3 b. 1 and 4
 c. 1, 2 and 3 d. None of these.
52. Of the following quantities which one has the dimensions different from the remaining three?
 1. Energy density
 2. Force per unit area
 3. Product of charge per unit volume and voltage
 4. Angular momentum per unit mass.
 a. 1 b. 2 c. 3 d. 4
53. The frequency f of vibrations of a mass m suspended from a spring of spring constant k is given by $f = C m^x k^y$, where C is a dimensionless constant. The values of x and y are, respectively
 a. $\frac{1}{2}, \frac{1}{2}$ b. $-\frac{1}{2}, -\frac{1}{2}$
 c. $\frac{1}{2}, -\frac{1}{2}$ d. $-\frac{1}{2}, \frac{1}{2}$
54. If C , the velocity of light, g the acceleration due to gravity and P the atmospheric pressure be the fundamental quantities in M.K.S. system, then the dimensions of length will be same as that of
 a. C/g b. C/P
 c. PCg d. C^2/g
55. The quantities A and B are related by the relation $A/B = m$, where m is the linear density and A is force. The dimensions of B will be
 a. same as that of pressure
 b. same as that of work
 c. same as that of momentum
 d. same as that of latent heat
56. The dimensions of $\frac{1}{2} \epsilon_0 E^2$ (ϵ_0 = permittivity of free space and E = electric field) are
 a. $[ML^2 T^{-1}]$ b. $[ML^{-1} T^{-2}]$
 c. $[ML^2 T^{-2}]$ d. $[MLT^{-1}]$
57. A physical quantity X is represented by $X = (M^x L^{-y} T^{-z})$. The maximum percentage errors in the measurement of M , L and T , respectively, are $a\%$, $b\%$, and $c\%$. The maximum percentage error in the measurement of X will be

- a. $(ax + by - cz)\%$
 b. $(ax - by - cz)\%$
 c. $(ax + by + cz)\%$
 d. $(ax - by + cz)\%$
58. The velocity of transverse wave in a string is $v = \sqrt{\frac{T}{m}}$, where T is the tension in the string and m is mass per unit length. If $T = 3.0$ kgf, mass of string is 2.5 g and length of string is 1.000 m, then the percentage error in the measurement of velocity is
 a. 0.5 b. 0.7 c. 2.3 d. 3.6
59. Write the dimensions of a/b in the relation $P = \frac{a - t^2}{bx}$, where P is the pressure, x is the distance and t is the time.
 a. $M^{-1}L^0T^{-2}$ b. ML^0T^{-2}
 c. ML^0T^2 d. MLT^{-2}
60. Write the dimensions of $a \times b$ in the relation $E = \frac{b - x^2}{at}$, where E is the energy, x is the displacement and t is time.
 a. ML^2T b. $M^{-1}L^2T^1$
 c. ML^2T^{-2} d. MLT^{-2}
61. If the velocity of light C , the universal gravitational constant G and Planck's constant h be chosen as fundamental units, the dimensions of mass in this system are
 a. $h^{1/2}C^{1/2}G^{-1/2}$ b. $h^{-1}C^{-1}G$
 c. hCG^{-1} d. hCG
62. If the relation $V = \frac{\pi Pr^4}{8nl}$, where the letters have their usual meanings, the dimensions of V are
 a. $M^0L^3T^0$ b. $M^0L^3T^{-1}$
 c. $M^0L^{-3}T^{-1}$ d. $M^1L^3T^0$
63. The length l , breadth b and thickness t of a block of wood were measured with the help of a measuring scale. The results with permissible errors are: $l = 15.12 \pm 0.01$ cm, $b = 10.15 \pm 0.01$ cm and $t = 5.28 \pm 0.01$ cm. The percentage error in volume upto proper significant figures is
 a. 0.28% b. 0.36%
 c. 0.48% d. 0.64%
64. The relative density of the material of a body is found by weighing it first in air and then in water. If the weight of the body in air is $W_1 = 8.00 \pm 0.05$ N and weight in water is $W_2 = 6.00 \pm 0.05$ N, then the relative density, $\rho_r = \frac{W_1}{W_1 - W_2}$ with the maximum permissible error is
 a. $4.00 \pm 0.62\%$ b. $4.00 \pm 0.82\%$
 c. $4.00 \pm 3.2\%$ d. $4.00 \pm 5.62\%$
65. The length l , breadth b and thickness t of a block of wood are measured with the help of a meter scale. The results after calculating the errors are given as $l = 15.12 \pm 0.01$ cm, $b = 10.15 \pm 0.01$ cm, $t = 5.28 \pm 0.01$ cm. The percentage error in volume upto proper significant figures is
 a. 0.36% b. 0.28%
 c. 0.48% d. 0.64%
66. The number of particles crossing a unit area perpendicular to the X -axis in a unit time is given by $n = -D \left(\frac{n_2 - n_1}{x_2 - x_1} \right)$, where n_1 and n_2 are the number of particles per unit volume at $x = x_1$ and $x = x_2$, respectively, and D is the diffusion constant. The dimensions of D are
 a. $[M^0LT^{-2}]$ b. $[M^0L^2T^{-4}]$
 c. $[M^0L^2T^{-2}]$ d. $[M^0L^2T^{-1}]$
67. If E , M , J and G , respectively, denote energy, mass, angular momentum and gravitational constant, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of
 a. time b. angle
 c. mass d. length
68. If L , R , C and V , respectively, represent inductance, resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as that of
 a. Charge b. 1/Charge
 c. Current d. 1/Current
69. The moment of inertia of a body rotating about a given axis is 12.0 kgm² in the S.I. system. What is the value of the moment of inertia in a system of units in which the units of length is 5 cm and the unit of mass is 10 g?
 a. 2.4×10^3 b. 6.0×10^3
 c. 5.4×10^5 d. 4.8×10^5
70. If the velocity (V), acceleration (A) and force (F) are taken as fundamental quantities instead of mass (M), length (L) and time (T), the dimensions of Young's modulus (Y) would be
 a. FA^2V^{-4} b. FA^2V^{-5}
 c. FA^2V^{-3} d. FA^2V^{-2}
71. A gas bubble formed from an explosion under water oscillates with a period proportional to $P^a d^b E^c$, where P is the static pressure, d is the density of water and E is the energy of explosion. Then, a , b , c are, respectively
 a. 1, 1, 1 b. $\frac{1}{3}, \frac{1}{2}, \frac{-5}{6}$
 c. $\frac{-5}{6}, \frac{1}{2}, \frac{1}{3}$ d. $\frac{1}{2}, \frac{-5}{6}, \frac{1}{3}$

72. The percentage error in the measurement of mass and speed are 2% and 3%, respectively. How much will be the maximum error in the estimation of K.E. obtained by measuring mass and speed?
 a. 5% b. 1% c. 8% d. 11%
73. An experiment measures quantities a , b , c and then X is calculated from $X = \frac{a^{1/2}b^2}{c^3}$. If the percentage errors in a , b and c are $\pm 1\%$, $\pm 3\%$, and $\pm 2\%$, respectively, then the percentage error in X can be
 a. 12.5% b. 7% c. 1% d. 4%
74. The resistance of a metal is given by $R = \frac{V}{I}$, where V is potential difference and I is the current. In a circuit the potential difference across resistance is $V = (8 \pm 0.5)$ V and current in resistance, $I = (4 \pm 0.2)$ A. What is the value of resistance with its percentage error?
 a. $(2 \pm 5.6\%) \Omega$ b. $(2 \pm 0.7\%) \Omega$
 c. $(2 \pm 35\%) \Omega$ d. $(2 \pm 11.25\%) \Omega$
75. Which of the following product of e , h , μ , G (where μ is the permeability) be taken so that the dimensions of the product are same as that of speed of light?
 a. $he^{-2}\mu^{-1}G^0$ b. $h^2eG^0\mu$
 c. $h^0e^2G^{-1}\mu$ d. $hGe^{-2}\mu^0$
76. Which of the following does not have the dimensions of velocity? (Given, ϵ_0 = permittivity of free space; μ_0 = permeability of free space; v = frequency; λ = wavelength; P = pressure; ρ = density; k = wave number; and ω = angular frequency.)
 a. ωk b. $v\lambda$
 c. $\frac{1}{\sqrt{\epsilon_0\mu_0}}$ d. $\sqrt{\frac{P}{\rho}}$
77. The mass of a liquid flowing per second per unit area of cross section of a tube is proportional to P^x and v^y , where P is the pressure difference and v is the velocity. Then, the relation between x and y is
 a. $x = y$ b. $x = -y$
 c. $y^2 = x$ d. $y = -x^2$
78. A physical quantity x is calculated from $x = \frac{ab^2}{\sqrt{c}}$. Calculate the percentage error in measuring x when the percentage errors in measuring a , b , c are 4, 2, and 3 percent, respectively.
 a. 7% b. 9% c. 11% d. 9.5%
79. In the formula $X = 3YZ^2$, X and Z have the dimensions of capacitance and magnetic induction, respectively. The dimensions of Y in M.K.S. system are
 a. $M^{-3}L^{-2}T^{-2}A^{-4}$ b. $ML^{-2}A$
 c. $M^{-3}L^{-2}T^8A^4$ d. $M^{-3}L^{-2}T^4A^4$
80. A quantity x is given by $\epsilon_0 L \frac{\Delta V}{\Delta t}$ where ϵ_0 is permittivity of free space, L is length, ΔV is potential difference and Δt is the time interval. The dimensional formula for x is the same as that of
 a. resistance b. charge
 c. voltage d. current
81. Given that $Y = a \sin \omega t + bt + ct^2 \cos \omega t$. The unit of abc is same as that of
 a. y b. y/t c. $(y/t)^2$ d. $(y/t)^3$
82. The potential energy of a particle varies with distance x as $U = \frac{Ax^{1/2}}{x^2 + B}$, where A and B are constants. The dimensional formula for $A \times B$ is
 a. $M^1L^{7/2}T^{-2}$ b. $M^1L^{11/2}T^{-2}$
 c. $M^1L^{5/2}T^{-2}$ d. $M^1L^{9/2}T^{-2}$
83. If x and a stand for distance, then for what value of n is the given equation dimensionally correct? The equation is $\int \frac{dx}{\sqrt{a^2 - x^n}} = \sin^{-1} \frac{x}{a}$.
 a. Zero b. 2 c. -2 d. 1
84. The specific resistance ρ of a circular wire of radius r , resistance R and length l is given by $\rho = \frac{\pi r^2 R}{l}$. Given: $r = 0.24 \pm 0.02$ cm, $R = 30 \pm 1 \Omega$, and $l = 4.80 \pm 0.01$ cm. The percentage error in ρ is nearly
 a. 7% b. 9% c. 13% d. 20%
85. Using mass (M), length (L), time (T) and electric current (A) as fundamental quantities, the dimensions of permittivity will be
 a. $[MLT^{-1}A^{-1}]$ b. $[MLT^{-2}A^{-2}]$
 c. $[M^{-1}L^{-3}T^4A^2]$ d. $[M^2L^{-2}T^{-2}A]$
86. Assume that the mass m of the largest stone that can be moved by a flowing river depends upon the velocity v of the water, its density ρ and the acceleration due to gravity g . Then, m is directly proportional to
 a. v^3 b. v^4 c. v^5 d. v^6
87. A spherical body of mass m and radius r is allowed to fall in a medium of viscosity η . The time in which the velocity of the body increases from zero to 0.63 times the terminal velocity (v) is called time constant (τ). Dimensionally, τ can be represented by
 a. $\frac{mr^2}{6\pi\eta}$ b. $\sqrt{\left(\frac{6\pi m r \eta}{g^2}\right)}$
 c. $\frac{m}{6\pi\eta r v}$ d. None of these
88. A student writes four different expressions for the displacement y in a periodic motion as a function of time t . a is amplitude and T is time period, which of the following can be correct?

- a. $y = aT \sin \frac{2\pi t}{T}$
 b. $y = a \sin Vt$
 c. $y = \frac{a}{T} \sin \frac{t}{a}$
 d. $y = \frac{a}{\sqrt{2}} \left[\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T} \right]$
89. The relation $\tan \theta = v^2/rg$ gives the angle of banking of the cyclist going round the curve. Here, v is the speed of the cyclist, r is the radius of the curve and g is acceleration due to gravity. Which of the following statements about the relation is true?
 a. It is both dimensionally as well as numerically correct.
 b. It is neither dimensionally correct nor numerically correct.
 c. It is correct dimensionally but not numerically.
 d. It is correct numerically but not dimensionally.
90. In the relation $P = \frac{\alpha}{\beta} e^{-\alpha z/K\theta}$, P is pressure, Z is distance, K is Boltzmann constant and θ is the temperature. The dimensional formula of β will be
 a. $[M^0 L^2 T^0]$
 b. $[M^1 L^2 T^{-1}]$
 c. $[M^1 L^0 T^{-1}]$
 d. $[M^0 L^2 T^{-1}]$
91. A liquid drop of density r , radius r and surface tension σ oscillates with time period T . Which of the following expressions for T^2 is correct?
 a. $\frac{\rho r^3}{\sigma}$
 b. $\frac{\rho \sigma}{r^3}$
 c. $\frac{r^3 \sigma}{\rho}$
 d. None of these
92. A highly rigid cubical block A of small mass M and side L is fixed rigidly on the other cubical block of same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is given by
 a. $2\pi \sqrt{M\eta L}$
 b. $2\pi \sqrt{(M\eta/L)}$
 c. $2\pi \sqrt{ML/\eta}$
 d. $2\pi \sqrt{M/\eta L}$
93. The mass of a body is 20.000 g and its volume is 10.00 cm³. If the measured values are expressed up to the correct significant figures, the maximum error in the value of density is
 a. 0.001 g cm⁻³
 b. 0.010 g cm⁻³
 c. 0.100 g cm⁻³
 d. None of these
94. The length of a strip measured with a meter rod is 10.0 cm. Its width measured with a vernier callipers is 1.00 cm. The least count of the meter rod is 0.1 cm and that of vernier callipers is 0.01 cm. What will be the error in its area?

- a. $\pm 0.01 \text{ cm}^2$
 c. $\pm 0.11 \text{ cm}^2$
 b. $\pm 0.1 \text{ cm}^2$
 d. $\pm 0.2 \text{ cm}^2$

95. While measuring the acceleration due to gravity by a simple pendulum, a student makes a positive error of 1% in the length of the pendulum and a negative error of 3% in the value of time period. His percentage error in the measurement of g by the relation, $g = 4\pi^2(l/T^2)$ will be
 a. 2%
 b. 4%
 c. 7%
 d. 10%
96. While measuring acceleration due to gravity by a simple pendulum, a student makes a positive error of 2% in the length of the pendulum and a positive error of 1% in the value of time period. His actual percentage error in the measurement of the value of g will be
 a. 3%
 b. 0%
 c. 4%
 d. 5%
97. The relative density of a material is found by weighing the body first in air and then in water. If the weight in air is $(10.0 \pm 0.1) \text{ gf}$ and weight in water is $(5.0 \pm 0.1) \text{ gf}$, then the maximum permissible percentage error in relative density is
 a. 1
 b. 2
 c. 3
 d. 5
98. Dimensional formula of a physical quantity x is $[M^{-1} L^3 T^{-2}]$. The errors in measuring the quantities M , L and T , respectively, are 2%, 3% and 4%. The maximum percentage of error that occurs in measuring the quantity x is
 a. 9
 b. 10
 c. 14
 d. 19
99. The heat generated in a circuit is given by $Q = I^2 R t$, where I is current, R is resistance and t is time. If the percentage errors in measuring I , R and t are 2%, 1%, and 1%, respectively, then the maximum error in measuring heat will be
 a. 2%
 b. 3%
 c. 4%
 d. 6%
100. The internal and external diameters of a hollow cylinder are measured with the help of a vernier callipers. Their values are $4.23 \pm 0.01 \text{ cm}$ and $3.87 \pm 0.01 \text{ cm}$, respectively. The thickness of the wall of the cylinder is
 a. $0.36 \pm 0.02 \text{ cm}$
 b. $0.18 \pm 0.02 \text{ cm}$
 c. $0.36 \pm 0.01 \text{ cm}$
 d. $0.18 \pm 0.01 \text{ cm}$

Multiple Correct Answers Type

Solutions on page 3.32

- Which of the following pairs have the same dimensions?
 - Torque and work
 - Angular momentum and Planck's constant
 - Energy and Young's modulus
 - Light year and wavelength
- Which of the following pairs have different dimensions?
 - Frequency and angular velocity.
 - Tension and surface tension.

- c. Density and energy density.
d. Linear momentum and angular momentum.
3. Pressure is dimensionally
- force per unit area
 - energy per unit volume
 - momentum per unit area per second
 - momentum per unit volume
4. Which of following pairs have the same dimensions? (L = inductance, C = capacitance, R = resistance)
- $\frac{L}{R}$ and CR
 - LR and CR
 - $\frac{L}{R}$ and $\sqrt{LC}$
 - RC and $\frac{1}{LC}$
5. Choose the correct statement(s)
- A dimensionally correct equation must be correct.
 - A dimensionally correct equation may be correct.
 - A dimensionally incorrect equation must be incorrect.
 - A dimensionally incorrect equation may be correct.
6. Which of the following pairs have the same dimensions?
- $\frac{h}{e}$ and magnetic flux
 - $\frac{h}{e}$ and electric flux
 - electric flux and $\frac{q}{\epsilon_0}$
 - electric flux and $\mu_0 I$
7. The values of measurement of a physical quantity in 5 trials were found to be 1.51, 1.53, 1.53; 1.52 and 1.54. Then
- average absolute error is 0.01
 - relative error is 0.01
 - percentage error is 0.01%
 - percentage error is 1%
8. If S and V are one main scale and one vernier scale and $n - 1$ divisions on the main scale are equivalent to n divisions of the vernier, then
- Least count is $\frac{S}{n}$
 - Vernier constant is $\frac{S}{n}$
 - The same vernier constant can be used for circular verniers also
 - The same vernier constant cannot be used for circular verniers
9. Consider three quantities: $x = \frac{E}{B}$, $y = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ and $z = \frac{l}{CR}$. Here, l is the length of a wire, C is the capacitance and R is a resistance. All other symbols have usual meanings. Then

- x and y have the same dimensions
- x and z have the same dimensions
- y and z have the same dimensions
- None of the above three pairs have the same dimensions

10. The velocity, acceleration and force in two systems of units are related as under:

$$\text{i. } v' = \frac{\alpha^2}{\beta} v \quad \text{ii. } a' = (\alpha\beta) a \quad \text{iii. } F' = \left(\frac{1}{\alpha\beta}\right) F$$

All the primed symbols belong to one system and unprimed ones belong to the other system. α and β are dimensionless constants. Which of the following is/are correct?

- a. Length standards of the two systems are related by:

$$L' = \left(\frac{\alpha^3}{\beta^3}\right) L.$$

- b. Mass standards of the two systems are related by:

$$m' = \left(\frac{1}{\alpha^2 \beta^2}\right) m.$$

- c. Time standards of the two systems are related by:

$$T' = \left(\frac{\alpha}{\beta^2}\right) T.$$

- d. Momentum standards of the two systems are related

$$\text{by: } P' = \left(\frac{1}{\beta^3}\right) P.$$

Assertion-Reasoning Type

Solutions on page 3.33

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has 4 choices (a), (b), (c), and (d) out of which **ONLY ONE** is correct.

- Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
- Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
- Statement I is True, Statement II is False.
- Statement I is False, Statement II is True.

1. **Statement I:** Mass, length and time are fundamental quantities.

Statement II: Mass, length and time are independent of one another.

2. **Statement I:** Light year and wavelength have same dimensions.

Statement II: Light year represents time while wavelength represents distance.

3. **Statement I:** The product RC and $\frac{L}{R}$ have same dimensions.

Statement II: Both RC and $\frac{L}{R}$ have dimensions of time.

4. **Statement I:** Linear momentum and impulse have same dimensions $[MLT^{-1}]$.

Statement II: Impulse is equal to final momentum.

5. **Statement I:** Angular momentum of an orbiting electron may be represented in terms of Planck's constant.

Statement II: Angular momentum of an orbiting electron is an integral multiple of $\frac{h}{2\pi}$.

6. **Statement I:** Å (Angstrom) and AU are different units of length.

Statement II: Å (Angstrom) is a small unit of length while AU is a big unit of length.

7. **Statement I:** The number of significant figures in 0.001 is 1 while in 0.100 it is 3.

Statement II: Zeros before a non-zero significant digit are not counted while zeros after a non-zero significant digit are counted.

8. **Statement I:** If error in measurement of mass is 2% and that in measurement of velocity is 5%, then error in measurement of kinetic energy is 6%.

Statement II: Error in kinetic energy is

$$\frac{\Delta K}{K} = \left(\frac{\Delta m}{m} + 2 \frac{\Delta v}{v} \right).$$

Comprehensive Type

Solutions on page 3.33

For Problems 1–5

The van der Waal's equation of state for some gases can be expressed as

$$\left(P + \frac{a}{V^2} \right) (V - b) = RT$$

where P is the pressure, V is the molar volume and T is the absolute temperature of the given sample of gas and a , b and R are constants.

1. The dimensions of a are

- a. ML^5T^{-2} b. $ML^{-1}T^{-2}$
c. L^3 d. L^6

2. The dimensions of constant b are

- a. ML^5T^{-2} b. $ML^{-1}T^{-2}$
c. L^3 d. L^6

3. Which of the following does not have the same dimensional formula as that for RT ?

- a. PV b. Pb c. $\frac{a}{V^2}$ d. $\frac{ab}{V^2}$

4. The dimensional representation of ab/RT is

- a. ML^5T^{-2} b. $M^0L^3T^0$
c. $ML^{-1}T^{-2}$ d. None of these

5. The dimensional formula of RT is same as that of

- a. energy b. force
c. specific heat d. latent heat

For Problems 6–8

Dimensional methods provide three major advantages in verification, derivation and changing the system of units. Any empirical formula that is derived based on this method has to be verified and proportionality constants found by experimental means. The

presence or absence of certain factors—non dimensional constants or variables—cannot be identified by this method. So, every dimensionally correct relation cannot be taken as perfectly correct.

6. If α kg, β meter and γ second are the fundamental units, 1 calorie can be expressed in new units as [1 cal = 4.2 J]

- a. $\alpha^{-1} \beta^2 \gamma$ b. $\alpha^{-1} \beta^{-2} \gamma$
c. $4.2 \alpha^{-1} \beta$ d. $4.2 \alpha^{-1} \beta^{-2} \gamma^2$

7. Time period of oscillation of a drop depends on surface tension σ , density of the liquid ρ and radius r . The relation is

- a. $\sqrt{\frac{\rho r^2}{\sigma}}$ b. $\sqrt{\frac{r^2}{\rho \sigma}}$
c. $\sqrt{\frac{r^3 \rho}{\sigma}}$ d. $\sqrt{\frac{\rho \sigma}{r^3}}$

8. Energy of a S.H.M. is dependent on mass m , frequency f and amplitude A of oscillation. The relation is

- a. $\frac{Mf}{A^2}$ b. MfA^{-2}
c. Mf^2A^{-2} d. Mf^2A^2

For Problems 9–11

Accuracy of measurement also lies in the way the result is expressed. The number of digits to which a value is to be expressed is one digit more than number of sure numbers. Rules do exist to deal with number of digits after an operation is carried out on the given values. The error can be minimised by many trials and using the correct methods and instruments.

9. If length and breadth are measured as 4.234 and 1.05 m, the area of the rectangle is

- a. 4.4457 m² b. 4.45 m²
c. 4.446 m² d. 0.4446 m²

10. The order of magnitude of 147 is

- a. 1 b. 2 c. 3 d. 4

11. Number of significant figures can reduce in

- a. addition b. subtraction
c. multiplication d. division

Matching Column Type

Solutions on page 3.33

1. If R is resistance, L is inductance, C is capacitance, H is latent heat and s is specific heat, then match the quantity given in column I with the dimensions given in column II.

Column I	Column II
i. LC	a. L^2T^{-2}
ii. LR	b. $L^2T^{-2}K^{-1}$
iii. H	c. T^{-2}
iv. s	d. $M^2L^4T^{-5}A^{-4}$

2. There are four vernier scales; whose specifications are given in column I and the least count is given in column II. Match the columns I and II with correct specification and corresponding least count (s = value of main scale division, n = number of marks on vernier)

Column I	Column II
i. $s = 1 \text{ mm}, n = 10$	a. 0.05 mm
ii. $s = 0.5 \text{ mm}, n = 10$	b. 0.01 mm
iii. $s = 0.5 \text{ mm}, n = 20$	c. 0.1 mm
iv. $s = 1 \text{ mm}, n = 100$	d. 0.025 mm

3. Match the columns

Column I	Column II
i. Backlash error	a. Always subtracted
ii. Zero error	b. Least count = $1 \text{ MSD} - 1 \text{ VSD}$
iii. Vernier callipers	c. May be -ve or +ve
iv. Error in Screw gauge	d. Due to loose fittings

4. Using significant figures, match the following

Column I	Column II
i. 0.12345	a. 5
ii. 0.12100 cm	b. 4
iii. $47.23 \div 2.3$	c. 1
iv. 3×10^8	d. 2

5. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II**.

Column I	Column II
i. $GM_e M_s$	a. (volt) (coulomb) (metre)
ii. $3RT/M$	b. (kilogram) (metre) ³ (second) ⁻²
iii. $F^2/q^2 B^2$	c. (meter) ² (second) ⁻²
iv. GM_e/R_e	d. (farad) (volt) ² (kg) ⁻¹

where G is universal gravitational constant; M_e , mass of the earth; M_s , mass of sun; R_e , radius of the earth; R , universal gas constant; T , absolute temperature; M , molar mass; F , force; q , charge; B , magnetic field. (IIT-JEE, 2007)

Archives

Solutions on page 3.34

Fill in the Blanks Type

1. Planck's constant has dimensions _____. (IIT-JEE, 1985)
2. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction, respectively. The dimensions of Y in M.K.S. system are _____.

(IIT-JEE, 1988)

3. The dimensions of electrical conductivity are _____. (IIT-JEE, 1997)

4. The equation of state for real gas is given by $\left(P + \frac{a}{V^2}\right) \times (v - b) = RT$. The dimensions of the constant a are _____. (IIT-JEE, 1997)

Single Correct Answer Type

1. The dimensions of $\left(\frac{1}{2}\right) \epsilon_0 E^2$ (ϵ_0 : permittivity of free space, E : electric field) are

- a. MLT^{-1} b. ML^2T^{-2}
c. $ML^{-1}T^{-2}$ d. ML^2T^{-1}

(IIT-JEE, 2000)

2. A quantity X is given by $\epsilon_0 L \frac{\Delta V}{\Delta t}$, where ϵ_0 is the permittivity of the free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of

- a. resistance b. charge
c. voltage d. current

(IIT-JEE, 2001)

3. A cube has a side of length $1.2 \times 10^{-2} \text{ m}$. Calculate its volume.

- a. $1.7 \times 10^{-6} \text{ m}^3$
b. $1.73 \times 10^{-6} \text{ m}^3$
c. $1.70 \times 10^{-6} \text{ m}^3$
d. $1.732 \times 10^{-6} \text{ m}^3$

(IIT-JEE, 2003)

4. Pressure depends on distance as $P = \frac{\alpha}{\beta} \exp\left(-\frac{\alpha z}{k\theta}\right)$, where α , β are constants, z is distance, k is Boltzmann's constant and θ is temperature. The dimensions of β are

- a. $M^0 L^0 T^0$ b. $M^{-1} L^{-1} T^{-1}$
c. $M^0 L^2 T^0$ d. $M^{-1} L^1 T^2$

(IIT-JEE, 2004)

5. A wire of length $l = 6 \pm 0.06 \text{ cm}$ and radius $r = 0.5 \pm 0.005 \text{ cm}$ has mass $m = 0.3 \pm 0.003 \text{ g}$. Maximum percentage error in density is

- a. 4 b. 2 c. 1 d. 6.8

(IIT-JEE, 2004)

6. Which of the following set have different dimensions?

- a. Pressure, Young's modulus, stress
b. Emf, potential difference, electric potential
c. Heat, work done, energy
d. Dipole moment, electric flux, electric field

(IIT-JEE 2005)

7. In a screw gauge, the zero of main scale coincides with fifth division of circular scale in first figure. The circular divisions of screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball in second figure is

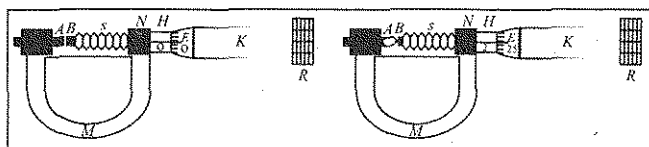


Fig. 3.5

- a. 2.25 mm b. 2.20 mm
c. 1.20 mm d. 1.25 mm

(IIT-JEE, 2006)

8. A student performs an experiment for determination of $g \left[= \frac{4\pi^2 L}{T^2} \right]$, $L \approx 1$ m, and he commits an error of ΔL . For T he takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 s. For which of the following data, the measurement of g will be most accurate?

- a. $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 20$
b. $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 50$
c. $\Delta L = 0.5$, $\Delta T = 0.01$, $n = 20$
d. $\Delta L = 0.5$, $\Delta T = 0.05$, $n = 50$

(IIT-JEE, 2006)

9. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of 0.01 mm. Take $g = 9.8 \text{ m/s}^2$ (exact). The Young's modulus obtained from the reading is

- a. $(2.0 \pm 0.3) \times 10^{11} \text{ Nm}^{-2}$
b. $(2.0 \pm 0.2) \times 10^{11} \text{ Nm}^{-2}$
c. $(2.0 \pm 0.1) \times 10^{11} \text{ Nm}^{-2}$
d. $(2.0 \pm 0.05) \times 10^{11} \text{ Nm}^{-2}$

(IIT-JEE, 2007)

10. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the following table. Least count for length = 0.1 cm, Least count for time = 0.1 s.

Student	Length of Pendulum (cm)	Number of Oscillations (n)	Total Time for n Oscillations (s)	Time Period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

11. If E_I , E_{II} and E_{III} are the percentage errors in g , i.e., $\left(\frac{\Delta g}{g} \times 100 \right)$ for students I, II, and III, respectively, then

- a. $E_I = 0$
b. E_I is minimum
c. E_I and E_{II}
d. E_{II} is maximum

(IIT-JEE, 2008)

Multiple Correct Answers Type

1. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair(s).

- a. Torque and work
b. Angular momentum and work
c. Energy and Young's modulus
d. Light year and wavelength

(IIT-JEE, 1986)

2. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If $M = \text{mass}$, $L = \text{length}$, $T = \text{time}$ and $I = \text{electric current}$, then

- a. $[\epsilon_0] = M^{-1} L^{-3} T^2 I$
b. $[\epsilon_0] = M^{-1} L^{-3} T^4 I^2$
c. $[\mu_0] = M L T^{-2} I^{-2}$
d. $[\mu_0] = M L^2 T^{-1} I$

(IIT-JEE, 1998)

3. The S.I. unit of inductance, the henry can be written as

- a. weber/ampere b. volt-sec/amp
c. joule/(ampere)² d. ohm-second

4. Which of the following pairs have the same dimensions?

- a. Reynold number and coefficient of friction
b. Curie and frequency of light wave
c. Latent heat and gravitational potential
d. Planck's constant and torque

(IIT-JEE, 1995)

Subjective

1. Give the M.K.S. units for each of the following quantities,

- a. Young's modulus
b. Magnetic induction
c. Power of a lens

(IIT-JEE, 1980)

2. A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$, where p is the static pressure, d is the density of water and E is the total energy of the explosion. Find the values of a , b and c .

(IIT-JEE, 1981)

3. Write the dimensions of the following in terms of mass, time, length and charge.

- a. Magnetic flux b. Rigidity modulus

(IIT-JEE, 1982)

4. Match the physical quantities given in column I with dimensions expressed in terms of mass (M), length (L) time (T), and charge (Q) given in column II.

Column I

Angular momentum
Torque
Inductance
Latent heat
Capacitance
Resistivity

Column II

ML^2T^{-2}
 ML^2T^{-1}
 $M^{-1}L^{-2}T^2Q^2$
 ML^2Q^{-2}
 $ML^3T^{-1}Q^{-2}$
 L^3T^{-2}

(IIT-JEE, 1983)

5. Column I gives three physical quantities. Select the appropriate units for the choices given in Column II. Some of the physical quantities may have more than one choice correct.

(IIT-JEE, 1990)

Column I

Capacitance
Magnetic induction
Inductance

Column II

ohm-second
coulomb (volt)⁻¹
coulomb²-joule⁻¹
newton (ampere metre)⁻¹
volt-second (ampere)⁻¹

6. The n^{th} division of main scale coincides with $(n+1)^{\text{th}}$ divisions of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier.

(IIT-JEE, 2003)

ANSWERS AND SOLUTIONS**Subjective Type**

- 2
 - 4
 - 3
 - 2
 - 3
 - 2
 - 4
 - 6
- 0.059
 - 0.058
 - 5.1×10^6
 - 5.0×10^6
- 953
 - 954
 - 953.3
 - 953.4
- 7
 - 6
 - 6.6
 - 6.4
- $3.17 \text{ g} = 0.00371 \text{ kg}$
 $1.4 + 0.00371 = 1.40371 \text{ kg} \sim 1.4 \text{ kg}$ [correct upto one place of decimal]
- $A = \pi r^2 = \frac{22}{7}(0.56)^2 = 0.9856 \text{ m}^2 \sim 0.99 \text{ m}^2$
 (two significant figures)
- $3.8 \times 10^{-6} + 4.2 \times 10^{-5} = 0.38 \times 10^{-5} + 4.2 \times 10^{-5}$
 $= 4.58 \times 10^{-5} \sim 4.6 \times 10^{-5}$
 [upto one place of decimal]
 - $4.7 \times 10^{-4} - 3.2 \times 10^{-6} = 4.7 \times 10^{-4} - 0.032 \times 10^{-4}$
 $= 4.668 \times 10^{-4} \sim 4.7 \times 10^{-4}$ [one decimal place]
 - $4.8 \times 10^4 - 1.5 \times 10^3$
 $= 4.8 \times 10^4 - 0.15 \times 10^4 = 4.65 \times 10^4$
 $\sim 4.6 \times 10^4$ [one decimal place]
- $l = 4.234 \text{ m}, b = 1.005 \text{ m}, t = 2.01 \text{ cm} = 0.0201 \text{ m}$
 $\text{Area} = 2[lb + bt + lt]$
 $= 2[4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234]$
 $= 8.7209478 \text{ m}^2 \sim 8.72 \text{ m}^2$ (three significant figures).
 $\text{Volume} = lbt = 4.234 \times 1.005 \times 0.0201$
 $= 0.0855289 \text{ m}^3 = 0.0855 \text{ m}^3$
 (three significant figures).
- $V = \frac{4}{3}\pi \left(\frac{3.34}{2}\right)^3 = \frac{4}{3} \times \frac{22}{7} \left(\frac{3.34}{2}\right)^3 = 19.5169 \text{ m}^3$
 $= 19.5 \text{ m}^3$
 (up to three significant figures)

$$10. \frac{5.42 \times 0.6753}{0.085} = 43.06030 = 43$$

(up to two significant figures)

$$11. \text{ a. } \mu_m = \frac{1.45 + 1.56 + 1.54 + 1.44 + 1.54 + 1.53}{6} = 1.51$$

b. absolute error in each measurement

$$\Delta\mu_1 = |1.45 - 1.51| = 0.06, \quad \Delta\mu_2 = |1.56 - 1.51| = 0.05$$

$$\Delta\mu_3 = |1.54 - 1.51| = 0.03, \quad \Delta\mu_4 = |1.44 - 1.51| = 0.07$$

$$\Delta\mu_5 = |1.54 - 1.51| = 0.03$$

$$\Delta\mu_6 = |1.53 - 1.51| = 0.02$$

$$\Delta\mu_m = \frac{0.06 + 0.05 + 0.03 + 0.07 + 0.03 + 0.02}{6}$$

$$= \frac{0.26}{6} = 0.0433 \sim 0.04$$

$$\text{c. Fractional error } \frac{\Delta\mu_m}{\mu} = \frac{0.04}{1.51} = 0.02649 = 0.03$$

d. Percentage error = 3 %

$$\text{e. } \mu = 1.51 \pm 0.04 \text{ or } 1.51 \pm 3\%$$

$$12. \text{ a. } l = l_1 + l_2 = (1.9 \pm 0.3) + (3.5 \pm 0.2) = (5.4 \pm 0.5) \text{ m}$$

$$\text{b. } \Delta T = T_2 - T_1 = (76.3 \pm 0.3) - (67.7 \pm 0.2)$$

$$= 8.6 \pm 0.5^\circ\text{C}$$

$$13. l = 10.5 \pm 0.2 \text{ cm}, b = 5.2 \pm 0.1 \text{ cm}$$

$$P = 2l + 2b = 31.4 \text{ cm}, \Delta P = 2(\Delta l + \Delta b) = 0.6 \text{ cm}$$

So, $P = 31.4 \pm 0.6 \text{ cm}$

$$14. A = lb = 5.7 \times 3.4 = 19.38 \text{ cm}^2 \sim 19 \text{ cm}^2$$

(two significant figures)

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} \Rightarrow \Delta A = \left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right)(19.38)$$

$$\Delta A = 1.48 \sim 1.5$$

(two significant figures)

$$\text{So, area} = (19 \pm 1.5) \text{ cm}^2$$

$$15. v = \frac{S}{t} = \frac{13.8}{4} = 3.45 \text{ ms}^{-1} \sim 3.4$$

(two significant figures)

$$\Delta v = \left[\frac{0.2}{13.8} + \frac{0.3}{4} \right] 3.45 = 0.3087 \sim 0.31$$

(two significant figures)

$$v = (3.4 \pm 0.31) \text{ ms}^{-1}$$

$$\text{Percentage error} = \frac{0.31}{3.4} \times 100 = 9.11\% \sim 9\%$$

$$16. r = 2.1 \pm 0.5 \text{ cm} \Rightarrow A = 4\pi r^2 = 4 \times \frac{22}{7} (2.1)^2$$

$$= 4 \times 13.86 = 55.44 \text{ cm}^2 \sim 55.4 \text{ cm}^2$$

$$\frac{\Delta A}{A} = 2 \frac{\Delta r}{r} \Rightarrow \Delta A = \frac{2 \times 0.5}{2.1} \times 55.44 = 26.4$$

$$\text{Area} = 55.4 \pm 26.4 \text{ cm}^2$$

$$17. P = \frac{\pi r^2 R}{l} = \frac{22}{7} \times \frac{(0.26)^2 \times 64}{156}$$

$$\frac{\Delta P}{P} = \frac{2(0.02)}{0.26} + \frac{2}{64} + \frac{0.1}{156}$$

$$\frac{\Delta P}{P} \times 100 = 18.57 \sim 18.6\%$$

$$18. T^2 = 4\pi^2 \frac{l}{g} \Rightarrow g = 4\pi^2 \frac{l}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2 \frac{\Delta T}{T} = \frac{0.1}{20} + 2 \left(\frac{1}{600} \right) = \frac{1}{600}$$

$$\frac{\Delta g}{g} \times 100 = 0.833 \sim 0.8\%$$

Objective Type

- d.** Couple $\tau \times$ angle $d\theta = dW$
 $\frac{1}{2} I \omega^2 =$ kinetic energy and $F dx = dW$
- c.** Unit of surface tension is Nm^{-1} . Also,
 $\text{Jm}^{-2} = \text{Nmm}^{-2} = \text{Nm}^{-1}$.
- d.** Here, $(2\pi ct/\lambda)$ as well as $(2\pi x/\lambda)$ are dimensionless.
 So, unit of ct is same as that of λ . Unit of x is same as that of λ . Also, $\left[\frac{2\pi ct}{\lambda} \right] = \left[\frac{2\pi x}{\lambda} \right] = M^0 L^0 T^0$
 Hence, $\left[\frac{2\pi c}{\lambda} \right] = \left[\frac{2\pi x}{\lambda t} \right]$. In the option (d), $\frac{x}{\lambda}$ is unitless. This is not the case with c/λ .
- b.** Here, unit of y and A will be same and that of x and λ will be same. $\frac{2\pi}{\lambda}(ct - x)$ is dimensionless. Here, $\frac{ct}{\lambda}$ and $\frac{x}{\lambda}$ are dimensionless. Unit of ct is same as that of λ or x .
- d.** Angular momentum, $J = mvr$

$$[J] = [MLT^{-1}L] = [ML^2T^{-1}]$$

This is same as that of Planck's constant.

$$6. \text{ c. Latent heat, } L = \frac{\text{heat}}{\text{mass}}, [L] = \frac{[ML^2T^{-2}]}{[M]}$$

$$= [L^2T^{-2}]$$

$$\text{Gravitational potential, } V = \frac{\text{work done}}{\text{mass}}$$

$$\Rightarrow [V] = \frac{[ML^2T^{-2}]}{[M]} = [L^2T^{-2}]$$

$$7. \text{ d. } \eta = \frac{\text{tangential stress}}{\text{shearing strain}} = \frac{T}{\theta} = \frac{F/A}{x/L}$$

$$\Rightarrow [\eta] = \frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$$

$$8. \text{ b. [Moment of inertia]} = [ML^2]$$

$$[\text{Moment of force}] = [ML^2T^{-2}]$$

$$9. \text{ c. Induced e.m.f. } |e| = L \frac{dI}{dt}$$

$$[L] = \frac{[e]}{[dI/dt]} = \frac{[W/q]}{[dI/dt]} = \frac{[ML^2T^{-2}/AT]}{[AT^{-1}]}$$

$$= [ML^2T^{-2}A^{-2}]$$

$$10. \text{ a. As } Q = mL,$$

$$[L] = \frac{[Q]}{[m]} = \frac{[ML^2T^{-2}]}{[M]}$$

$$= [M^0L^2T^{-2}]$$

$$11. \text{ d. All the four have same dimensions, i.e., } [ML^2T^{-2}K^{-1}].$$

$$12. \text{ c. Coefficient of friction is a dimensionless quantity.}$$

$$13. \text{ d. Here, } (\omega t + \phi_0) \text{ is dimensionless because it is an argument of a trigonometric function.}$$

$$14. \text{ b. If a quantity depends upon more than three factors, each having dimensions, the method of dimensional analysis cannot be applied. It is because applying principle of homogeneity will give only three equations.}$$

$$15. \text{ d. The formula can be written as}$$

$$\frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}} = 1$$

This formula is dimensionally correct as both the sides are dimensionless. Numerically, this ratio is equal to refractive index which is > 1 . Hence, the equation is numerically incorrect.

$$16. \text{ c. The correct relation for time period of simple pendulum is } T = 2\pi(l/g)^{1/2}. \text{ So, the given relation is numerically incorrect as the factor of } 2\pi \text{ is missing. But it is correct dimensionally.}$$

$$17. \text{ d. As } \mu = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}},$$

hence, μ is dimensionless. Thus, each term on the R.H.S. of given equation should be dimensionless,

i.e., $\frac{B}{\lambda^2}$ is dimensionless, i.e., B should have dimension of λ^2 , i.e., cm^2 , i.e., area.

18. d. $[x] = [Ay] = [B] \Rightarrow [y] = [B/A]$

Also, $[x] \neq [A]$ and $[Cz] = \text{dimensionless}$
 $\Rightarrow [C] = [z^{-1}]$.

19. b. Dimension of L/R is same as that of time.

20. c. Random errors are reduced by making large number of observations and taking mean of all the results.

21. c. All the choices are equivalent but the answer must possess three significant digits as significant digits do not change on conversion from one system to another. So, appropriate choice is (c).

22. c. $\frac{A}{B} = \frac{\text{Force}}{\text{Force}} = [M^0 L^0 T^0]$

$Ct = \text{angle} \Rightarrow C = \frac{\text{angle}}{\text{time}} = \frac{1}{T} = T^{-1}$

$Dx = \text{angle} \Rightarrow D = \frac{\text{angle}}{\text{distance}} = \frac{1}{L} = L^{-1}$

$\therefore \frac{C}{D} = \frac{T^{-1}}{L^{-1}} = [M^0 L T^{-1}]$

23. d. Surface tension = $\frac{\text{force}}{\text{length}} = \frac{MLT^{-2}}{L} = [MT^{-2}]$

Surface energy = $\frac{\text{work done}}{\text{increase in area}}$

$= \frac{ML^2 T^{-2}}{L^2} = [MT^{-2}]$

$\therefore$ Surface tension and surface energy have same dimensions.

24. b. Resistivity = $\frac{\text{resistance} \times \text{area}}{\text{length}}$
 $= \frac{ML^2 T^{-3} A^{-2} \times L^2}{L} = [ML^3 T^{-3} A^{-2}]$

25. a. Electric potential = $\frac{\text{work}}{\text{charge}}$
 $= \frac{ML^2 T^{-2}}{AT} = [ML^2 T^{-3} A^{-1}]$

26. d. Moment of force = $F \times \perp \text{ distance} = [ML^2 T^{-2}]$
 Momentum = mass $\times$ velocity = MLT^{-1}

27. c. Pressure $\times$ Volume gives work = $[ML^2 T^{-2}]$

28. c. Relation between E_k and p is $E_k = \frac{p^2}{2m}$. When p is doubled, E_k becomes four times. So, E_k increases by 300.

29. d. Rate of change of velocity is equal to acceleration which is a vector quantity and all others are scalar quantities.

30. b. $(92.0 + 2.15) \text{ cm} = 94.15 \text{ cm}$. Rounding off to first decimal place, we get 94.2 cm.

31. c. $n = \frac{1}{2l} \sqrt{\frac{T}{m}} \Rightarrow n^2 = \frac{1}{4l^2} \frac{T}{m}$

$m = \frac{T}{4l^2 n^2} = \left[\frac{MLT^{-2}}{L^2 \times T^{-2}} \right] = [ML^{-1}]$

32. b. $y = r \sin(\omega t - kx)$

Here, $\omega t = \text{angle} \Rightarrow \omega = \frac{1}{T} = T^{-1}$

Similarly, $kx = \text{angle} \Rightarrow k = \frac{1}{x} = L^{-1}$

$\therefore \frac{\omega}{k} = \frac{T^{-1}}{L^{-1}} = LT^{-1}$

Or simply $\frac{\omega}{k}$ represents wave velocity

$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = v$, where f is frequency.

33. d. We know that speed of e.m. wave is

$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow \mu_0 \epsilon_0 = \frac{1}{C^2} = [L^{-2} T^2]$

34. c. Momentum = force $\times$ time = Ns

35. d. Momentum, $p = mv = MLT^{-1} = ML^{-3} L^4 T^{-4} T^3 = DV^4 F^{-3}$

36. d. $AT^2 = LT^{-2} \times T^2 = [M^0 L T^0]$

37. b. In a product, percentage errors are added up.

38. c. 0.08076 has least number of significant figures, i.e., 4.

39. a. $H = \frac{B}{\mu_0}$

40. b. Intensity of wave

$= \frac{\text{energy}}{\text{area} \times \text{time}} = \frac{ML^2 T^{-2}}{L^2 T} = [ML^0 T^{-3}]$

41. c. Capacitance, $C = \frac{\text{charge}}{\text{potential}} = \frac{AT}{ML^2 T^{-3} A^{-1}}$
 $= [M^{-1} L^{-2} T^4 A^2]$

42. c. $PV = nRT \Rightarrow R = \frac{PV}{nT} = \frac{ML^{-1} T^{-2} \times L^3}{\text{mol} \times K}$
 $= [ML^2 T^{-2} K^{-1} \text{ mol}^{-1}]$

43. d. $501 = 0.501 \times 10^3 \Rightarrow$ order of magnitude of 501 is 3.

44. a. $0.00701 = 0.701 \times 10^{-2}$
 $\therefore$ Order of magnitude of 0.00701 is -2.

45. b. $379 = 3.79 \times 10^2 \Rightarrow$ Order of magnitude of 379 is 2.

46. c. $X = a + b \Rightarrow \Delta X = \Delta a + \Delta b$
 Now, $\frac{\Delta X}{X} \times 100 = \frac{(\Delta a + \Delta b)}{a + b} \times 100$

$= \left(\frac{\Delta a}{a + b} + \frac{\Delta b}{a + b} \right) \times 100$

47. c. Screw gauge has minimum least count of 0.001 cm, hence it is most precise instrument.

48. c. $5.69 \times 10^{15} \text{ kg}$ has 3 significant figures as the power of 10 are not considered for significant figures.

49. a. Here, at is dimensionless

$\Rightarrow a = \frac{1}{t} = \left[\frac{1}{T} \right] = [T^{-1}]$

$$x = \frac{V_0}{a} \text{ and } V_0 = xa = [LT^{-1}] = [M^0LT^{-1}].$$

50. b. Here, αt^2 is a dimensionless. Therefore, $\alpha = \frac{1}{t^2}$ and α has the dimensions of T^{-2} .

51. c. Latent heat = $\frac{Q}{m} = \frac{\text{energy}}{\text{mass}}$

$$\text{Gravitational pot.} = \frac{U}{m}$$

52. d. Energy density = $\frac{\text{energy}}{\text{volume}} = [ML^{-1}T^{-2}]$

$$\text{Force/area} = ML^{-1}T^{-2}$$

$$[\text{charge/volume}] \times [\text{voltage}] = \frac{Q}{\text{vol.}} \times \frac{W}{Q}$$

$$= \frac{\text{work}}{\text{volume}} = ML^{-1}T^{-2}$$

The dimensions of (4) are different,

$$\text{i.e., } \frac{ML^2T^{-1}}{M} = M^0L^2T^{-1}$$

Hence, correct choice is (d).

53. d. $f = Cm^xk^y$. Writing dimensions on both sides:

$$[M^0L^0T^{-1}] = M^x[ML^0T^{-2}]^y$$

$$[M^0L^0T^{-1}] = [M^{x+y}T^{-2y}]$$

Comparing dimensions on both sides, we have

$$0 = x + y \text{ and } -1 = -2y \Rightarrow y = \frac{1}{2}, x = -\frac{1}{2}$$

Aliter. Remembering that frequency of oscillation of loaded spring is

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} (k)^{1/2} m^{-1/2}$$

$$\text{which gives } x = -\frac{1}{2} \text{ and } y = \frac{1}{2}.$$

54. d. $\frac{C^2}{g} = \frac{L^2T^{-2}}{LT^{-2}} = [L]$

55. d. $\frac{A}{B} = m, B = \frac{A}{m} = \frac{\text{force}}{\text{linear density}} = \frac{MLT^{-2}}{ML^{-1}}$

$$\therefore B = [M^0L^2T^{-2}]$$

$$\text{Latent heat} = \frac{\text{heat energy}}{\text{mass}} = \frac{ML^2T^{-2}}{M} = [M^0L^2T^{-2}]$$

Thus, B has same dimensions as that of latent heat.

56. b. $\frac{1}{2}\epsilon_0 E^2$ is the expression for electrostatic energy density, i.e., the energy stored per unit volume in a parallel plate capacitor.

$$\therefore \frac{1}{2}\epsilon_0 E^2 = \frac{\text{energy}}{\text{volume}}$$

$$= \frac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$$

$$\text{Alternatively, } \epsilon_0 = \frac{1}{4\pi F} \times \frac{q_1 q_2}{r^2} \text{ and } E = \frac{F}{q}$$

$$\therefore \frac{1}{2}\epsilon_0 E^2 = \frac{1}{8\pi F} \frac{q_1 q_2}{r^2} \times \frac{F^2}{q^2} = \frac{F}{r^2} = \frac{MLT^{-2}}{L^2}$$

57. c. $X = M^x L^y T^{-z}$

$$\therefore \frac{\Delta X}{X} \times 100 = x \left(\frac{\Delta M}{M} \times 100 \right) + y \left(\frac{\Delta L}{L} \times 100 \right) + z \left(\frac{\Delta T}{T} \times 100 \right) \text{ (Errors are always added)}$$

$$\therefore \frac{\Delta X}{X} \times 100 = (ax + by + cz) \text{ per cent}$$

58. d. $v = \sqrt{\frac{T}{m}} = \left[\frac{m'g}{\frac{M}{l}} \right]^{1/2} = \left[\frac{m'lg}{M} \right]^{1/2}$

It follows from here,

$$\frac{\Delta v}{v} = \frac{1}{2} \left[\frac{\Delta m}{m} + \frac{\Delta l}{l} + \frac{\Delta M}{M} \right]$$

$$= \frac{1}{2} \left[\frac{0.1}{3.0} + \frac{0.001}{1.000} + \frac{0.1}{2.5} \right]$$

$$= \frac{1}{2} [0.03 + 0.001 + 0.04] = 0.036$$

$$\therefore \text{Percentage error in the measurement} = 0.036 \times 100 = 3.6$$

59. b. Here, 'a' has the same dimensions as t^2

$$a = [T^2], b = \frac{T^2}{Px} = \frac{T^2}{ML^{-1}T^{-2}L^1} = \frac{T^4}{M} = [M^{-1}T^4]$$

$$\text{Now, } \frac{a}{b} = \frac{T^2}{M^{-1}T^4} = ML^0T^{-2}$$

60. b. Here, b and $x^2 = L^2$ have same dimensions.

$$\text{Also, } a = \frac{x^2}{E \times t} = \frac{L^2}{(ML^2T^{-2})T} = M^{-1}T^1$$

$$a \times b = [M^{-1}L^2T^1]$$

Hence, correct answer is (b).

61. a. $h = [ML^2T^{-1}], G = [M^{-1}L^3T^{-2}], C = [LT^{-1}]$

$$\therefore h^{1/2}G^{-1/2}C^{1/2}$$

$$= M^{1/2}LT^{-1/2} \times M^{1/2}L^{-3/2}T^1 \times L^{1/2}T^{-1/2}$$

$$= ML^0T^0 = \text{Mass}$$

62. b. $V = \frac{\pi Pr^4}{8nl} = \frac{ML^{-1}T^{-2}L^4}{ML^{-1}T^{-1}L} = M^0L^3T^{-1}$

63. b. Percentage error in volume

$$= \frac{0.01}{15.12} \times 100 + \frac{0.01}{10.15} \times 100 + \frac{0.01}{5.28} \times 100$$

$$= 0.07 + 0.10 + 0.19 = 0.36$$

64. d. Relative density, $\rho_r = \frac{W_1}{W_1 - W_2}$

$$= \frac{8.00}{8.00 - 6.00} = 4.00$$

$$\begin{aligned} \frac{\Delta \rho_r}{\rho_r} \times 100 &= \frac{\Delta W_1}{W_1} \times 100 + \frac{\Delta(W_1 - W_2)}{W_1 - W_2} \times 100 \\ &= \frac{0.05}{8.00} \times 100 + \frac{0.05 + 0.05}{2} \times 100 = 5.26\% \end{aligned}$$

$$\therefore \rho_r = 4.00 \pm 5.62\%$$

65. a. $V = lbt$

$$\begin{aligned} \frac{\Delta V}{V} \times 100 &= \frac{\Delta l}{l} \times 100 + \frac{\Delta b}{b} \times 100 + \frac{\Delta t}{t} \times 100 \\ &= \frac{0.01}{15.12} \times 100 + \frac{0.01}{10.15} \times 100 + \frac{0.01}{5.28} \times 100 \\ &= 0.36\% \end{aligned}$$

$$\begin{aligned} 66. \text{ d. } n &= -\frac{D(n_2 - n_1)}{x_2 - x_1} \Rightarrow T^{-1}L^{-2} = \frac{D(L^{-3})}{L} \\ \Rightarrow D &= \frac{T^{-1}L^{-2} \times L}{L^{-3}} \Rightarrow D = [M^0L^2T^{-1}] \end{aligned}$$

$$67. \text{ b. } \frac{EJ^2}{M^5G^2} = \frac{[ML^2T^{-2}][ML^2T^{-1}]^2}{M^5 \times [M^{-1}L^3T^{-2}]^2} = M^0L^0T^0$$

This comes out to be dimensionless and angle is also dimensionless.

$$68. \text{ d. } \frac{L}{RCV} = \frac{L}{T \left(L \frac{dI}{dt} \right)} = \frac{1}{dI} = \frac{1}{\text{current}}$$

[As RC = time constant T and pot. diff. $V = L \frac{dI}{dt}$]

$$69. \text{ d. } n_2 = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c$$

Dimensional formula of moment of inertia = $[ML^2T^0]$

$$\therefore a = 1, b = 2, c = 0$$

$$n_1 = 12.0, M_1 = 1 \text{ kg}, M_2 = 10 \text{ g}$$

$$L_1 = 1 \text{ m}, L_2 = 5 \text{ cm}, T_1 = 1 \text{ s}, T_2 = 1 \text{ s}$$

$$n_2 = 12.0 \left(\frac{1 \text{ kg}}{10 \text{ g}} \right)^1 \left(\frac{1 \text{ m}}{5 \text{ cm}} \right)^2 \left(\frac{1 \text{ sec}}{1 \text{ sec}} \right)^0$$

$$= 12 \times \left(\frac{1000 \text{ g}}{10 \text{ g}} \right)^1 \left(\frac{100 \text{ cm}}{5 \text{ cm}} \right)^2 \times 1$$

$$= 12 \times 100 \times 400 = 4.8 \times 10^5$$

70. a. Let $Y = [V^a A^b F^c]$

$$\begin{aligned} [ML^{-1}T^{-2}] &= [LT^{-1}]^a [LT^{-2}]^b [MLT^{-2}]^c \\ ML^{-1}T^{-2} &= M^c L^{a+b+c} T^{-a-2b-2c} \end{aligned}$$

$$\therefore c = 1, a + b + c = -1, -a - 2b - 2c = -2$$

On solving, we get $a = -4, b = 2$ and $c = 1$

71. c. Let $T \propto P^a d^b E^c$

Writing dimensions on both sides.

$$[M^0L^0T] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$$

$$[M^0L^0T] = M^{a+b+c} L^{-a-3b+2c} T^{-2a-2c}$$

$$\text{Thus, } a + b + c = 0, -a - 3b + 2c = 0, -2a - 2c = 0$$

On solving these equations, we get

$$a = -\frac{5}{6}, b = \frac{1}{2} \text{ and } c = \frac{1}{3}$$

72. c. Kinetic energy, $E = \frac{1}{2}mv^2$

$$\begin{aligned} \frac{\Delta E}{E} \times 100 &= \frac{\Delta m}{m} \times 100 + 2 \frac{\Delta v}{v} \times 100 \\ &= 2 + 2 \times 3 = 8\% \end{aligned}$$

73. a. $X = \frac{a^{1/2}b^2}{c^3}$

$$\begin{aligned} \frac{\Delta X}{X} \times 100 &= \frac{1}{2} \frac{\Delta a}{a} \times 100 + 2 \frac{\Delta b}{b} \times 100 + 3 \frac{\Delta c}{c} \times 100 \\ &= \frac{1}{2} \times 1 + 2 \times 3 + 3 \times 2 = 12.5\% \end{aligned}$$

74. d. $R = \frac{V}{I} = \frac{8}{4} = 2 \text{ ohm}$

$$\begin{aligned} \frac{\Delta R}{R} \times 100 &= \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 \\ &= \frac{0.5}{8} \times 100 + \frac{0.2}{4} \times 100 = 11.25\% \end{aligned}$$

$$\Rightarrow R = (2 \pm 11.25)\Omega$$

75. a. Here $v = e^a h^b \mu^c G^d$. Taking the dimensions

$$M^0 L T^{-1} A^0 = [AT^{-1}]^a [ML^2 T^{-1}]^b [MLT^{-2} A^{-2}]^c [M^{-1} L^3 T^{-2}]^d$$

There will be 4 simultaneous equations by equating the dimensions of M, L, T and A . These are $a - 2c = 0$, $a - b - 2c - 2d = -1$, $b + c - d = 0$ and $2b + c + 3d = 1$

Solving for 'a', 'b', 'c' and 'd' we get

$$a = -2, b = 1, c = -1, d = 0$$

$$\text{Thus, } v = e^{-2} h \mu^{-1} G^0$$

76. a. Here, $\omega \times k = T^{-1}L^{-1}$ which are not the dimensions of velocity. All others have got the dimensions of velocity.

$$\begin{aligned} 77. \text{ b. } \frac{M}{At} \propto P^x v^y &\Rightarrow ML^{-2}T^{-1} \\ &= [ML^{-1}T^{-2}]^x [L^1T^{-1}]^y \\ &= M^x L^{-x+y} T^{-2x-y} \end{aligned}$$

$$x = 1, -x + y = -2 \text{ and } -2x - y = -1$$

From here, we get $y = -1$. Thus, $x = -y$

$$\begin{aligned} 78. \text{ d. Here, } \frac{\Delta x}{x} \times 100 &= \frac{\Delta a}{a} \times 100 + 2 \frac{\Delta b}{b} \times 100 \\ &\quad + \frac{1}{2} \frac{\Delta c}{c} \times 100 \\ &= [4 + 2 \times 2 + 1/2 \times 3]\% = 9.5\% \end{aligned}$$

$$\begin{aligned} 79. \text{ c. Here, } X &= \frac{Q}{V}. \text{ But } V = \frac{W}{Q} \\ \therefore X &= \frac{Q^2}{W}. \text{ Now, } Z = B = \frac{F}{IL} \\ \therefore Y &= \frac{X}{3Z^2} = \frac{1}{3} \frac{Q^2}{W} \times \frac{I^2 L^2}{F^2} \end{aligned}$$

$$= \frac{A^2 T^2 A^2 L^2}{[ML^2 T^{-2}][M^2 L^2 T^{-4}]} = [M^{-3} L^{-2} T^8 A^4]$$

80. d. $\epsilon_0 L \frac{\Delta V}{\Delta t} = C \frac{\Delta V}{\Delta t}$ ($\because \epsilon_0 L = \text{capacity}$)
 $= \frac{q}{V} \times \frac{\Delta V}{\Delta t} = \frac{q}{\Delta t} = \text{Current}$

81. d. $y = a \sin \omega t + bt + ct^2 \cos \omega t$

Here, $a = y; b = y/t; c = y/t^2$

$\therefore abc = y \times y/t \times y/t^2 = (y/t)^3$

82. b. Here, x^2 has the dimensions of L^2 , $B = [L^2]$

Also, $ML^2 T^{-2} = \frac{AL^{1/2}}{L^2}$ or $A = ML^{7/2} T^{-2}$

$\therefore A \times B = ML^{11/2} T^{-2}$

83. b. Here, x^n has the same dimensions as a^2 . Thus, $n = 2$ will make the expression dimensionally correct.

84. d. Required percentage $= 2 \times \frac{0.02}{0.24} \times 100 + \frac{1}{30} \times 100 + \frac{0.01}{4.80} \times 100 = 16.7 + 3.3 + 0.2 = 20\%$

85. c. By Coulomb's law $F = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$
 $\epsilon_0 = \frac{q_1 q_2}{4\pi \times F \times r^2}$. Taking dimensions

$\epsilon_0 = \frac{(AT)(AT)}{ML^1 T^{-2} \times L^2} = [M^{-1} L^{-3} T^4 A^2]$

86. d. $m \propto v^a \rho^b g^c$

$ML^0 T^0 \propto (LT^{-1})^a (ML^{-3})^b (LT^{-2})^c$

Comparing the powers of M , L and T and solving, we get $b = 1, c = 3, a = 6 \Rightarrow m \propto v^6$

87. d. $\left[\frac{mr^2}{6\pi\eta} \right] = \left[\frac{ML^2}{ML^{-1}T^{-1}} \right] = [L^3 T]$

As we have $[\eta] = [ML^{-1}T^{-1}]$

$\left[\left(\frac{6\pi m r \eta}{g^2} \right)^{1/2} \right] = \left[\left(\frac{MLML^{-1}T^{-1}}{L^2 T^{-4}} \right)^{1/2} \right]$

$\left[\frac{m}{6\pi\eta r v} \right] = \left[\frac{M}{ML^{-1}T^{-1}LLT^{-1}} \right] = [L^{-1}T^2]$

Thus, none of the given expressions have the dimensions of time.

88. d. Since L.H.S. is displacement, so R.H.S. should have dimensions of displacement. In (a), aT does not have the dimension of displacement. Also argument of a trigonometric function should be dimensionless. In (b), argument is not dimensionless and in (c), a/T does not have the dimensions of displacement. Hence, the correct choice is (d).

89. a. Here, $[\tan \theta] = \left[\frac{v^2}{rg} \right] = M^0 L^0 T^0$. Also, in actual expression for the angle of banking of a road, there is no numerical factor involved. Therefore, the relation is both numerically and dimensionally correct.

90. a. Argument of exponential term must be dimensionless,

i.e., $\left[\frac{\alpha Z}{K\theta} \right] = [M^0 L^0 T^0]$

Now, $[K\theta] = [\text{Energy}] = [ML^2 T^{-2}]$

$\Rightarrow [\alpha] = \frac{[K\theta]}{[Z]} = \frac{[ML^2 T^{-2}]}{[L]} = [MLT^{-2}]$

Hence, $[\beta] = \frac{[\alpha]}{[P]} = \frac{[MLT^{-2}]}{[ML^{-1}T^{-2}]} = [M^0 L^2 T^0]$

91. a. Let $T^2 = \rho^a r^b \sigma^c$

$T^2 = (ML^{-3})^a (L)^b (MT^{-2})^c$
 $= M^{a+c} L^{-3a+b} T^{-2c}$

$\Rightarrow a + c = 0, 3a + b = 0$ and $2c = 2$

Hence, $a = 1, b = 3$ and $c = -1$

$T^2 = \rho r^3 \sigma^{-1} = \frac{\rho r^3}{\sigma}$

92. d. $[\eta] = [ML^{-1}T^{-2}]$

Hence, $\left[\sqrt{\frac{M}{\eta L}} \right] = \sqrt{\frac{[M]}{[ML^{-1}T^{-2}][L]}} = [T]$

93. d. Maximum error in measuring mass = 0.001 g, because least count is 0.001 g. Similarly, maximum error in measuring volume is 0.01 cm³

$\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{0.001}{20.000} + \frac{0.01}{10.00}$

$= (5 \times 10^{-5}) + (1 \times 10^{-3}) = 1.05 \times 10^{-3}$

$\Rightarrow \Delta \rho = (1.05 \times 10^{-3}) \times \rho$

$= 1.05 \times 10^{-3} \times \frac{20.000}{10.00} = 0.002 \text{ g cm}^{-3}$

94. d. $\Delta A = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} \right) A$

$= \pm \left(\frac{0.1}{10.0} + \frac{0.01}{1.00} \right) \times (10.0 \times 1.00) \text{ cm}^2$

$= \pm 0.02 \times 10 = \pm 0.2 \text{ cm}^2$

95. c. Given that $\frac{\Delta l}{l} \times 100 = +1\%$

and $\frac{\Delta T}{T} \times 100 = -3\%$

Percentage error in the measurement of

$g = \left[\frac{4\pi^2 l}{T^2} \right] = 100 \times \frac{\Delta l}{l} - 2 \times \frac{\Delta T}{T} \times 100$
 $= [1\% - 2 \times (-3\%)] = 7\%$

96. b. $T = 2\pi \sqrt{\frac{L}{g}}$ or $T^2 = 4\pi^2 \frac{L}{g}$

$\Rightarrow g = 4\pi^2 \frac{L}{T^2}; \frac{\Delta g}{g} = \frac{\Delta L}{L} - 2 \frac{\Delta T}{T}$

$\frac{\Delta g}{g} \times 100 = \frac{\Delta L}{L} \times 100 - 2 \frac{\Delta T}{T} \times 100$

Actual % error in $g = \frac{\Delta L}{L} \times 100 - 2 \frac{\Delta T}{T} \times 100$
 $= +2\% - 2 \times 1\% = 0\%$

97. d. Relative density = $\frac{W_a}{W_a - W_w}$, $\rho = \frac{W_a}{w}$

where ρ is relative density, W_a weight in air and w is loss in weight. $\frac{\Delta \rho}{\rho} = \frac{\Delta W_a}{W_a} - \frac{\Delta w}{w}$

For maximum error, $\frac{\Delta \rho}{\rho} = \frac{\Delta W_a}{W_a} + \frac{\Delta w}{w}$

For maximum percentage error,

$$\frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta W_a}{W_a} \times 100 + \frac{\Delta w}{w} \times 100$$

Given $\Delta W_a = 0.1$ gf and $W_a = 10.0$ gf

$w = 10.0 - 5.0 = 5.0$ gf

$\Delta w = \Delta W_a + \Delta W_w = 0.1 + 0.1 = 0.2$ gf

$$\frac{\Delta \rho}{\rho} \times 100 = \left(\frac{0.1}{10.0} \right) \times 100 + \left(\frac{0.2}{5.0} \right) \times 100$$

$$= 1 + 4 = 5$$

98. d. $X = M^{-1}L^3T^{-2}$

$$\frac{\Delta X}{X} = \frac{\Delta M}{M} + 3 \frac{\Delta L}{L} + 2 \frac{\Delta T}{T} = 2 + 3 \times 3 + 2 \times 4 = 19$$

99. d. Required error is $2 \times 2\% + 1\% + 1\%$, i.e., 6%.

100. b. Subtract 3.87 from 4.23 and then divide by 2.

Multiple Correct Answers Type

1. a., b., d. Torque = $\vec{r} \times \vec{F}$, Work = $\vec{r} \cdot \vec{F}$

Both have dimensions $[ML^2T^{-2}]$.

Angular momentum and Planck's constant have same dimensions $[ML^2T^{-1}]$.

Light year and wavelength have same dimension $[L]$.

2. b., c., d. Frequency and angular velocity have same dimensions $[T^{-1}]$.

Tension has dimension of force and surface tension has dimension of (force/length). Density has dimensions $\left(\frac{\text{mass}}{\text{volume}} \right)$ and energy density has dimensions $\left(\frac{\text{energy}}{\text{volume}} \right)$. Angular momentum has dimensions of (linear momentum $\times$ distance).

3. a., b., c. Pressure has dimensions $[ML^{-1}T^{-2}]$, force per area, energy per unit volume and momentum per unit area per second have same dimension $[ML^{-1}T^{-2}]$.

4. a., c. $\frac{L}{R}$, CR and $\sqrt{LC}$ all have dimensions of time $[T]$.

5. b., d. A dimensionally correct equation may or may not be correct. For example, $s = ut + at^2$ is dimensionally correct, but not correct actually.

A dimensionally incorrect equation may be correct also:

For example, $s = u + \frac{a}{2}(2n - 1)$ is a correct equation, but not correct dimensionally.

6. a., c. Dimensions of magnetic flux = magnetic field $\times$ area

$$= \left[\frac{F}{iL} \right] \times [\text{Area}] = \frac{MLT^{-2}}{AL} L^2 = [ML^2T^{-2}A^{-1}]$$

Dimensions of $\frac{h}{e} = \left[\frac{\text{Planck's constant}}{\text{charge}} \right]$

$$= \left[\frac{ML^2T^{-1}}{AT} \right] = [ML^2T^{-2}A^{-1}]$$

From Gauss theorem, electric flux = $\frac{q}{\epsilon_0}$

7. a., b., d. Mean value

$$= \frac{1.51 + 1.53 + 1.53 + 1.52 + 1.54}{5} = 1.53$$

Absolute errors are: $(1.53 - 1.51 = 0.02)$,

$(1.53 - 1.53 = 0.00)$, $(1.53 - 1.53 = 0.00)$,

$(1.53 - 1.52 = 0.01)$ and $(1.54 - 1.53 = 0.01)$

Mean absolute error

$$= \frac{0.02 + 0.00 + 0.00 + 0.01 + 0.01}{5} = \frac{0.04}{5}$$

$$= 0.008 \approx 0.01$$

So, choice (a) is correct.

$$\text{Relative error} = \frac{0.01}{1.53} = 0.00653 \approx 0.01$$

$$\% \text{ Error} = \frac{0.01}{1.53} \times 100 = 1\%$$

8. a., b., c. Least count = 1 MSD - 1 VSD = $S - V$

$$= S - \left(\frac{n-1}{n} \right) S = \frac{S}{n} [\because nV = (n-1)S]$$

It is also called vernier constant.

So, choices (a) and (b) are correct

Choice (d) is wrong and choice (c) is correct, since for all vernier scales similar approach can be used.

9. a., b., d. Unit of $x = \frac{\text{unit of } E}{\text{unit of } B} = \text{unit of velocity}$

Because $E = vB$

$$y = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \rightarrow \text{velocity of light}$$

$$\text{Unit of } z = \frac{\text{unit of } l}{\text{unit of } RC} = \frac{\text{length}}{\text{time}} = \text{velocity}$$

10. a., b., c., d. Velocity = length/time, acc = length/(time)²

$$\Rightarrow \text{Length} = \frac{(\text{velocity})^2}{\text{acc.}}, \text{ i.e., } L' = \frac{v^2}{a'} \text{ and } L = \frac{v^2}{a}$$

$$\Rightarrow \frac{L'}{L} = \left(\frac{v'}{v} \right)^2 \left(\frac{a}{a'} \right) = \left(\frac{\alpha^2}{\beta} \right)^2 \frac{1}{\alpha \beta} = \alpha^3 / \beta^3$$

$$\text{Now, } m' = \frac{F'}{a'} \text{ and } m = \frac{F}{a}$$

$$\Rightarrow \frac{m'}{m} = \frac{F'}{F} \frac{a}{a'} = \frac{1}{\alpha \beta} \times \frac{1}{\alpha \beta} = \frac{1}{\alpha^2 \beta^2}$$

$$\text{Time} = \text{vel./acc.}, \text{ i.e., } T' = \frac{v'}{a'} \text{ and } T = \frac{v}{a}$$

$$\Rightarrow \frac{T'}{T} = \frac{v'}{v} \frac{a}{a'} = \frac{\alpha^2}{\beta} \frac{1}{\alpha \beta} = \frac{\alpha}{\beta^2}$$

Momentum = mass $\times$ velocity, i.e.,

$$P' = m'v', \text{ and } P = mv$$

$$\Rightarrow \frac{P'}{P} = \frac{m'v'}{mv} = \frac{1}{\alpha^2 \beta^2} \frac{\alpha^2}{\beta} = \frac{1}{\beta^3}$$

Assertion-Reasoning Type

1. a. Fundamental quantity is that quantity which does not depend upon other quantities. Since mass, length and time are independent of one another, so they are fundamental quantities.
2. c. Light year and wavelength both have same dimensions of length.
3. a. Both RC and $\frac{L}{R}$ have dimensions of time. Both are the time constants of RC circuits and LR circuits, respectively.
4. c. Impulse is equal to change in momentum. Dimensionally both are same.
5. a. Angular momentum of an orbiting electron in n^{th} orbit is given by $n \frac{h}{2\pi}$.
6. a. $\text{\AA}$ is equal to 10^{-10} m, whereas 1 AU is the distance between sun and earth.
7. a. Zeros before a non-zero significant digit are not counted while zeros after a non-zero significant digit are counted.
8. d. $\frac{\Delta K}{K} \times 100 = \left(\frac{\Delta m}{m} + 2 \frac{\Delta v}{v} \right) \times 100 = 2 + 2 \times 5 = 12\%$

Comprehensive Type

For Problems 1–5

1. a., 2. c., 3. c., 4. d., 5. a.

Sol.

$$1. \text{ a. } [a] = [PV^2] = \left[\frac{FV^2}{A} \right] = \frac{[MLT^{-2}L^6]}{[L^2]} = [ML^5T^{-2}]$$

$$2. \text{ c. } [b] = [V] = [L^3]$$

3. c. According to given equation,

$$PV + \frac{a}{V} - Pb - \frac{ab}{V^2} = RT$$

As dimensions of all the terms on L.H.S. must be equal to dimensions of term RT on R.H.S., hence a/V^2 does not have the same dimensional formula as that of RT .

$$4. \text{ d. } \left[\frac{ab}{V^2} \right] = [RT] \Rightarrow \left[\frac{ab}{RT} \right] = [V^2] = [L^6]$$

$$5. \text{ a. } [RT] = [PV] = \left[\frac{F}{A} \cdot V \right] = [ML^2T^{-2}]$$

So dimensional formula of RT is same as that of energy.

For Problems 6–8

6. d., 7. c., 8. d

Sol.

$$6. \text{ d. } 1 \text{ cal} = 4.2 \text{ J} = 4.2 \text{ kgm}^2\text{s}^{-2} = n_2(\alpha \text{ kg})(\beta \text{ m})^2(\gamma \text{ s})^{-2}$$

$$\Rightarrow n_2 = 4.2\alpha^{-1}\beta^{-2}\gamma^2$$

$$\text{So, } 1 \text{ cal} = (4.2 \alpha^{-1} \beta^{-2} \gamma^2) \text{ new units.}$$

7. c. Let $T \propto \sigma^a \rho^b r^c$

$$M^0 L^0 T = (MT^{-2})^a (ML^{-3})^b L^c$$

$$\text{Equating the powers of } M: 0 = a + b \Rightarrow b = -a$$

$$L: 0 = -3b + c \Rightarrow c = 3b$$

$$T: 1 = -2a \Rightarrow a = -\frac{1}{2}, b = \frac{1}{2}, c = \frac{3}{2}$$

$$\Rightarrow T = k \sqrt{\frac{r^3 \rho}{\sigma}}$$

8. d. $E \propto m^a f^b A^c$

$$ML^2T^{-2} = m^a T^{-b} L^c$$

$$\therefore a = 1, b = 2, c = 2$$

For Problems 9–11

9. b., 10. b., 11. b

Sol.

$$9. \text{ b. } A = lb = 4.4457 \text{ m}^2$$

One should retain only three significant figures.

10. b. For x to be order of magnitude, $0.5 < \frac{147}{10^x} \leq 5$ should be satisfied.

11. b. To find say difference between 20.17 and 20.15, we get, 0.02 (i.e., one significant figure only).

Matching Column Type

1. i. $\rightarrow$ c., ii. $\rightarrow$ d., iii. $\rightarrow$ a., iv. $\rightarrow$ b.

$$(i) [LC] = \left[\frac{L}{R} RC \right] = T^2$$

$$(ii) [LR] = \left[\frac{L}{R} R^2 \right] = T[R^2] = T \left[\frac{V}{I} \right]^2 = T \left[\frac{W}{QI} \right]^2$$

$$= T \left(\frac{W}{I^2 t} \right)^2 = T \left(\frac{ML^2T^{-2}}{A^2T} \right)^2$$

$$= M^2L^4T^{-5}A^{-4}$$

$$(iii) [H] = \left[\frac{\text{Heat}}{\text{Mass}} \right] = \frac{ML^2T^{-2}}{M} = L^2T^{-2}$$

$$(iv) [s] = \left[\frac{\text{Heat}}{\text{Mass} \times \text{temperature}} \right] = \frac{ML^2T^{-2}}{MK}$$

$$= L^2T^{-2}K^{-1}$$

2. i. $\rightarrow$ c., ii. $\rightarrow$ a., iii. $\rightarrow$ d., iv. $\rightarrow$ b.

We know that least count is given by s/n , so

$$(i) \text{ least count} = s/n = 1/10 = 0.1 \text{ mm}$$

$$(ii) \text{ least count} = s/n = 0.5/10 = 0.05 \text{ mm}$$

$$(iii) \text{ least count} = s/n = 0.5/20 = 0.025 \text{ mm}$$

$$(iv) \text{ least count} = s/n = 1/100 = 0.01 \text{ mm}$$

3. i. $\rightarrow$ d., ii. $\rightarrow$ a., c., iii. $\rightarrow$ b., iv. $\rightarrow$ c., d.

Backlash error is caused by loose fittings, wear and tear etc., in the screw mechanisms.

Motion in One Dimension

- Frame of Reference
- Rest of Motion
- Position Vector and Displacement Vector
- Uniform Motion
- Accelerated Motion
- Use of Differentiation and Integration in One-Dimensional Motion
- One-Dimensional Motion in Vertical Line
- Graphs in Motion in One Dimension

FRAME OF REFERENCE

Frame of reference is the frame in which an observer sits and makes the observations. Frame of reference is of two types:

1. *Inertial frame of reference*
2. *Non-Inertial frame of reference*

A frame of reference which is either at rest or moving with constant velocity is known as inertial frame of reference. A frame of reference moving with some acceleration is known as non-inertial frame of reference (Fig. 4.1).

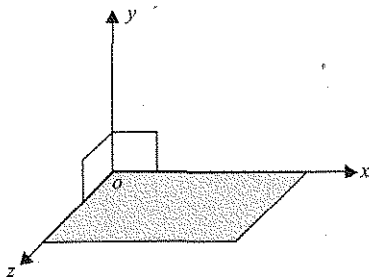


Fig. 4.1

To take the observations, we attach a Cartesian-coordinate system with the frame of reference. Cartesian-coordinate system consists of three mutually perpendicular axis x , y , and z , meeting at common point O . O is known as the origin. When a particle moves in space, its position at an instant can be described with the help of its three position coordinates (x, y, z) , which change with time during the motion of particle. At one time, one, two or all three position coordinates may change. Accordingly we have three types of motions.

Motion in One Dimension

The motion of a particle is said to be in one dimension if only one out of the three coordinates specifying the position of the particle changes with time. For example, motion along a straight line.

Motion in Two Dimensions

The motion of a particle is said to be in two dimensions if any two out of the three coordinates specifying the position of the object change with time. In such a motion, the object moves in a plane. For example, projectile motion, circular motion.

Motion in Three Dimensions

The motion of an object is said to be in three dimensions if all the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a space. For example, random motion of a gas molecule.

There is no rule or restriction on the choice of a frame. We can choose a frame of reference to describe the situation under study according to our convenience. For example, if we are in a train it is convenient to choose a frame attached to our compartment. The coordinates of box kept on the berth do not change with time and we say that the suitcase is at rest in train frame, but electric

poles, trees, etc. change their coordinates and we say that they are moving in train frame.

REST AND MOTION

A body is said to be at rest when it does not change its position with time; on the other hand, if the position of a body continually changes with time, it is said to be in motion. To visualise the state of rest or motion of a body leads us to the concept of reference frame. Description of the state of a body requires a second object, with respect to which the state must be specified, otherwise it would have no sense. The inherent meaning of the above statement is that when we speak of a body in rest or in motion we always say this in comparison with a second body, which is known as the reference body. To locate the position of a body relative to the reference body, a system of coordinates fixed on the reference body is constructed. This is known as **reference frame**. If two cars A and B move side by side in same direction with same speed, it would appear to the passengers of the cars that they are mutually at rest with respect to each other. Obviously relative to a reference frame on A , B is at rest. The reverse is also true. Absolute rest or absolute motion is unknown. All motions are relative.

Trajectory: The path followed by a point object during its motion is called its trajectory. Shape of path depends upon closer reference frame.

Further, all the motions around us can be broadly divided into three types of motion viz., translatory motion, rotatory motion and oscillatory motion, or a combination of them.

Translatory Motion

When a body moves in such a way that the linear distance covered by each particle of the body is same during the motion, then the motion is said to be translatory or translation.

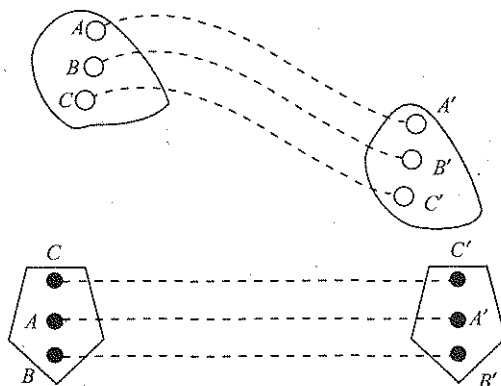


Fig. 4.2

Translatory motion can again be of two types viz., curvilinear or rectilinear, accordingly as the paths of every constituent particles are similarly curved or straight line paths. Here it is important that the body does not change its orientation (Fig. 4.2).

Rotatory Motion

When a body moves in such way that every constituent particle of it describes the same angular displacement about a particular axis of rotation then the motion is said to be rotatory or rotational (Fig. 4.3).

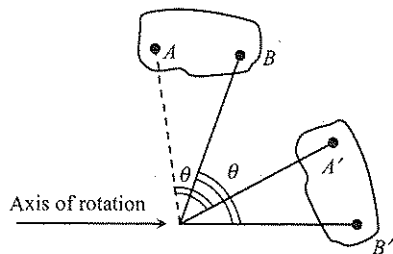


Fig. 4.3

Oscillatory Motion

The motion in which an object repeats its motion about a fixed point either to and fro or back and forth is called oscillatory motion. Such a motion always takes place within well defined boundaries which are called extreme points.

POSITION VECTOR AND DISPLACEMENT VECTOR

Position Vector

Let A is a point in space whose coordinates are (x, y, z) , then we know that its position vector w.r.t. the origin of coordinate system is given by: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ (Fig. 4.4).

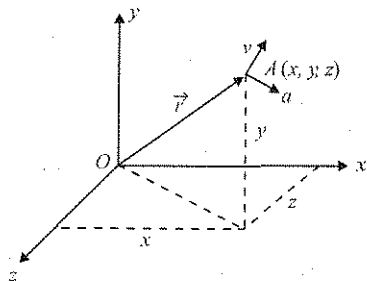


Fig. 4.4

Displacement Vector

It is a vector joining the initial position of the particle to its final position after an interval of time. Mathematically, it is equal to the change in position vector (Fig. 4.5).

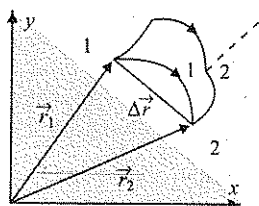


Fig. 4.5

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$. Here $\Delta\vec{r}$ is displacement vector from point 1 to 2.

Displacement and Distance

Displacement is the vector drawn from initial position to final position and its magnitude is equal to the shortest distance between the initial and final positions.

To find displacement, we can use:

$$\text{Displacement, } \vec{s} = \int \vec{v} dt = \int (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) dt$$

Distance is the length of actual path travelled by a body during its motion in a given interval of time.

To find distance, we can use:

$$\text{Distance, } d = \int |\vec{v}| dt = \int \sqrt{v_x^2 + v_y^2 + v_z^2} dt$$

- Distance is a scalar quantity and displacement is a vector quantity.
- Distance between a given set of initial and final positions can have infinite values but displacement is unique.
- Displacement can be negative, zero or positive, but distance is never negative. When a body returns to its initial position of starting, its displacement is zero but distance or path length is not zero.
- Magnitude of displacement can never be greater than distance.
- In uniform motion, displacement = distance.
- Displacement is zero if particle returns to initial position.
- Distance does not decrease with time and never be a zero for a moving body.
- Displacement can decrease with time, can be zero, or even negative if body is returning to its initial position, reaches the initial position and moves back from the initial position. So magnitude of displacement is not equal to the distance, however it can be so if the motion is along a straight line without change in direction.

In general distance $\geq$ displacement.

Illustration 4.1 A particle is moving in a circle. What is its displacement when it covers (i) half the circle (ii) full circle?

Sol. (i) equal to diameter (ii) zero

First we will discuss motion in one dimension

Sign convention: Any vector quantity directed towards right will be taken as positive and that directed towards left will be taken as negative (Fig. 4.6).

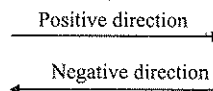


Fig. 4.6

Velocity (Instantaneous Velocity)

Time rate of change of position (x) or displacement (s) at any instant of time (t) is known as *Instantaneous Velocity* or simply *Velocity* at that instant of time. It is denoted by v (Fig. 4.7).

Quantitatively, velocity = $\frac{\text{Displacement}}{\text{Time}}$

Mathematically, $v = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt}$ or $v = \frac{ds}{dt}$

[also $v = \left(\frac{ds}{dt} \right)$]

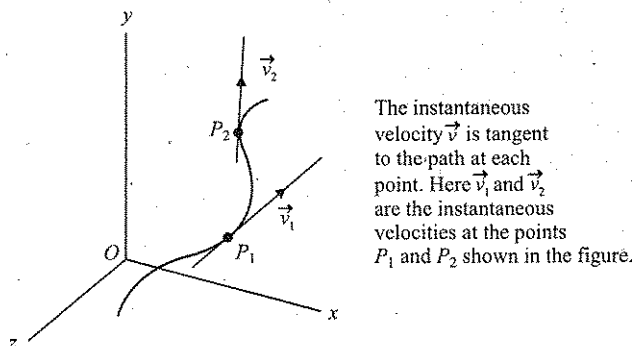


Fig. 4.7

Velocity is a vector quantity, it can be positive, zero or negative. According to our sign convention if the particle is going towards right, velocity will be taken as positive and if the particle is going towards left, velocity will be taken as negative.

In general if the particle moves in space, then $\vec{r}$ will change and time rate of change of position vector is known as velocity (Fig. 4.8). Thus

$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} = \frac{d}{dt} [x\hat{i} + y\hat{j} + z\hat{k}] \\ &= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}\end{aligned}$$

$$\Rightarrow \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

where

$$v_x = \frac{dx}{dt},$$

$$v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

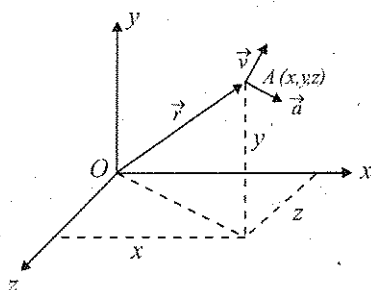


Fig. 4.8

Note: Velocity is also defined as time rate of change of displacement, $\vec{v} = \frac{d\vec{s}}{dt}$

Magnitude of velocity (known as speed), $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$.

Uniform Velocity

A particle is said to be moving with uniform velocity if it covers equal displacements in equal intervals of time, however small these intervals may be.

Uniform velocity is that velocity in which the body continuously moves in the same direction with constant speed.

Variable Velocity

A particle is said to be moving with variable velocity if it covers equal displacements in unequal intervals of time, or unequal displacements in equal intervals of time. Here velocity changes in either magnitude or direction or both.

Average Velocity

It is the ratio of the total displacement (s) to the total time interval (t) in which the displacement occurs.

$$v_{av} = \frac{s}{t} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

Average velocity is a vector quantity. Its direction is same as that of displacement.

If at any time t_1 position vector of the particle is $\vec{r}_1$ and at time t_2 position is $\vec{r}_2$ then for this interval, $\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ (Fig. 4.9).

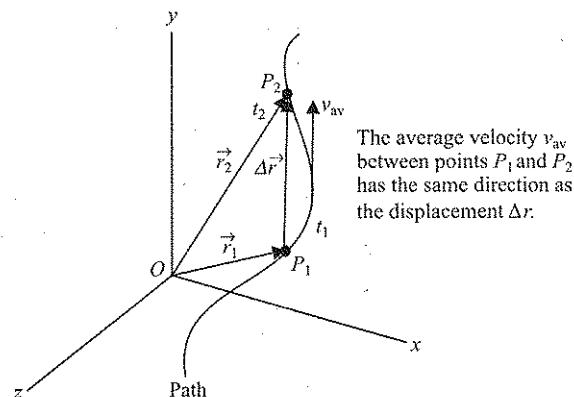


Fig. 4.9

To find average velocity we need to know only the total displacement from initial position to final position and need not consider the nature of motion between initial and final positions.

Physical meaning of average velocity: It is that uniform velocity with which if the object is made to move, it will cover the same displacement in a given time as it does with its actual velocity in the same time.

Speed (Instantaneous Speed)

The magnitude of the velocity at any instant of time is known as *Instantaneous Speed* or simply *speed* at that instant of time.

It is denoted by v . Quantitatively, speed = $\frac{\text{Distance}}{\text{Time}}$

Mathematically, it is the time rate at which the distance is being travelled by the particle.

- Speed is a scalar quantity. It can never be negative.
- Instantaneous speed is the speed of a particle at a particular instant of time.

Uniform Speed

A particle is said to be moving with uniform speed if it covers equal distances in equal intervals of time, however small these intervals may be.

Uniform speed is the speed which always remains constant. Here the body may change its direction of motion.

Variable Speed

A particle is said to be moving with variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time.

Average Speed

It is the ratio of total distance (d) travelled by the particle to the total time taken (t) in which this distance is travelled.

$$v_{av} = \frac{\text{Total distance}}{\text{Total time taken}} = \frac{d}{t}$$

If motion takes place in same direction then average speed and average velocity are same.

In Fig. 4.10, a particle goes from A to C. Distances, velocities and time taken are shown.

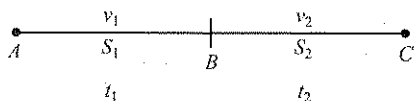


Fig. 4.10

$$S_1 = v_1 t_1 \Rightarrow t_1 = \frac{S_1}{v_1}; S_2 = v_2 t_2 \Rightarrow t_2 = \frac{S_2}{v_2}$$

$$v_{av} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} \quad (1)$$

$$= \frac{S_1 + S_2}{\frac{S_1}{v_1} + \frac{S_2}{v_2}} \quad (2)$$

1. If $t_1 = t_2 = t$, then $v_{av} = \frac{v_1 + v_2}{2}$: average speed is equal to arithmetical mean of individual speeds.
2. If $S_1 = S_2 = S$, then $v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$: average speed is equal to harmonic mean of individual speeds.

UNIFORM MOTION

A particle is said to be in uniform motion if it covers equal displacements in equal intervals of time, however small these intervals may be.

Features of Uniform Motion

- Uniform motion is a straight line motion with constant velocity.
- In uniform motion displacement and distance are equal.
- The average and instantaneous velocities have same values in uniform motion.
- No net force is required for an object to be in uniform motion.
- The velocity in uniform motion does not depend upon the time interval.
- The velocity in uniform motion is independent of choice of origin.

Unit of velocity or speed: ms^{-1} in SI system and cms^{-1} in CGS system.

Dimensional formula of velocity or speed: $[M^0 L T^{-1}]$

Illustration 4.2 A car travelled the first third of a distance d at a speed of 10 kmh^{-1} , the second third at a speed of 20 kmh^{-1} and the last third at a speed of 60 kmh^{-1} . Determine the average speed of the car (Fig. 4.11).

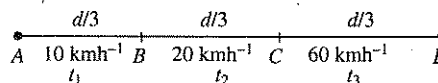


Fig. 4.11

Sol. Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$\begin{aligned} \frac{AB + BC + CD}{t_1 + t_2 + t_3} &= \frac{d/3 + d/3 + d/3}{\frac{d/3}{10} + \frac{d/3}{20} + \frac{d/3}{60}} \\ &= \frac{d}{\frac{d}{30} + \frac{d}{60} + \frac{d}{180}} = 18 \text{ km/h} \end{aligned}$$

Illustration 4.3 A ship moves due east at 12 kmh^{-1} for 1 h and then turns exactly towards south to move for an hour at 5 kmh^{-1} . Calculate its average speed and average velocity for the given motion (Fig. 4.12).

Sol. Time taken to complete the journey = 2 h

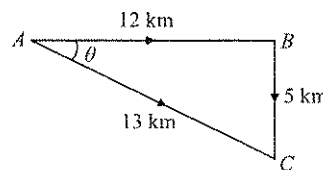


Fig. 4.12

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{12 + 5}{2} \\ &= 8.5 \text{ km/h} \end{aligned}$$

$$\begin{aligned} \text{Average velocity} &= \frac{\text{Total displacement travelled}}{\text{Total time taken}} = \frac{13}{2} \\ &= 6.5 \text{ km/hr} \end{aligned}$$

Direction of average velocity: $\tan \theta = \frac{5}{12}$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{5}{12} \right) = 22^\circ 37' \text{ S of E}$$

ACCELERATED MOTION

The motion of an object is said to be accelerated motion if its velocity changes with time.

Example: A vehicle moving on a crowded road.

An accelerated motion is a kind of non-uniform motion.

Average Acceleration

It is the ratio of total change in velocity to the total time taken in which this change in velocity takes place (Fig. 4.13).

$$a_{av} = \frac{\text{Total change in velocity}}{\text{Total time taken}} = \frac{\Delta v}{\Delta t}$$

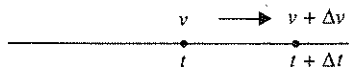


Fig. 4.13

To find average acceleration, we need to know only the total change in velocity from initial position to final position. We need not consider how the motion takes place between these two points.

If velocity of the particle at an instant t_1 is $\vec{v}_1$ and at instant t_2 is $\vec{v}_2$, then the average acceleration is mathematically given by

$$(\text{Fig. 4.14}): \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

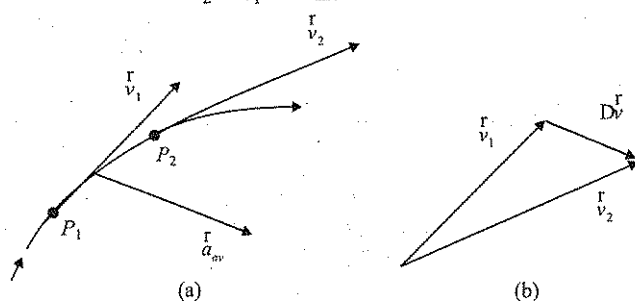


Fig. 4.14

Points to Remember

Change in velocity = final velocity – initial velocity

Acceleration (Instantaneous Acceleration)

Time rate of change of velocity at any instant of time is known as *Instantaneous Acceleration* or simply *Acceleration*. $a = \frac{dv}{dt}$.

Actually, acceleration at an instant is defined as the limit of the average acceleration as the time interval Δt around that instant becomes infinitesimally small (Fig. 4.15).

$$a = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta v}{\Delta t} \right) = \frac{dv}{dt}$$

In general, $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

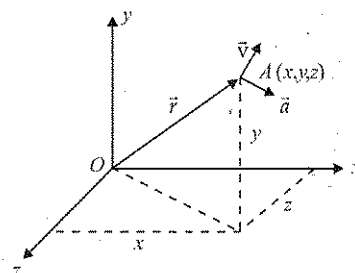


Fig. 4.15

Differentiating $\vec{v}$ w.r.t. time, we get acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} [v_x \hat{i} + v_y \hat{j} + v_z \hat{k}]$$

$$\Rightarrow \vec{a} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k} \Rightarrow \vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

where $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$, $a_z = \frac{dv_z}{dt}$

Magnitude of acceleration, $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Instantaneous acceleration $\vec{a}$ at point P is shown in Fig. 4.16. Vector $\vec{v}$ is tangent to the path; vector $\vec{a}$ points toward the concave side of the path, $0 \leq \theta \leq \pi$

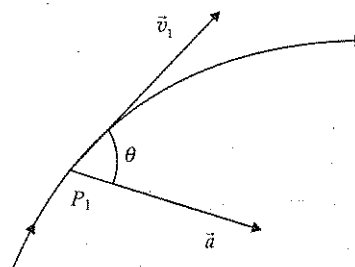


Fig. 4.16

- $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$. Acceleration is also defined as second time derivative of position.
- Also, $a = v \frac{dv}{dx}$.
- Acceleration is a vector quantity. It can be positive, zero or negative.
- Acceleration is in the direction of increasing velocity, e.g., let a particle is going from A to B. In both the cases the particle is going towards right, so velocity in both the cases is positive at any instant of time. In first case velocity is increasing towards right, so acceleration is towards right and positive. In second case, velocity is decreasing towards right (or increasing towards left), so acceleration is towards left and negative (Fig. 4.17).
- Negative acceleration is also known as retardation or deceleration. Deceleration and acceleration have opposite directions.

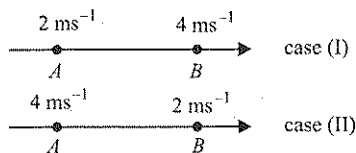


Fig. 4.17

Uniform Acceleration

An object is said to be moving with uniform acceleration if equal change in velocity takes place in equal intervals of time, however small these intervals may be.

In uniform acceleration velocity changes with a uniform rate.

Variable Acceleration

A particle is said to be moving with variable acceleration if its velocity changes equally in unequal intervals of time, or unequally in equal intervals of time.

Average Velocity in Uniformly Accelerated Motion

Generally, Average velocity = $\frac{\text{Total displacement}}{\text{Total time taken}}$ but if the motion is uniformly accelerated, then average velocity can also be written as

$$\text{Average velocity} = \frac{\text{Initial velocity} + \text{Final velocity}}{2} = \frac{u + v}{2} \quad (\text{For constant acceleration})$$

Illustration 4.4 A uniformly accelerated body in a straight line has a velocity of 2 ms^{-1} at any time. After some time its velocity becomes 4 ms^{-1} . Find the average velocity.

$$\text{Sol. } v_{\text{av}} = \frac{u + v}{2} = \frac{2 + 4}{2} = 3 \text{ ms}^{-1}$$

Illustration 4.5 A car travelling at 20 ms^{-1} takes a U-turn in 20 s without changing its speed. What is the average acceleration of the car?

Sol. Initial velocity = $v_1 = +20 \text{ m/s}$,

Final velocity = $v_2 = -20 \text{ m/s}$

$$\begin{aligned} \text{Average acceleration} &= \frac{\text{change in velocity}}{\text{time taken}} = \frac{v_2 - v_1}{t} \\ &= \frac{-20 - 20}{20} = -2 \text{ m/s}^2 \end{aligned}$$

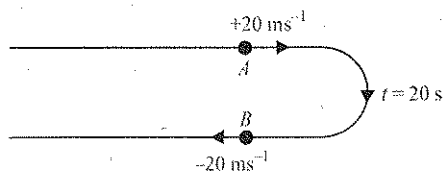


Fig. 4.18

Negative sign indicates that average acceleration is towards left during the time interval given.

Formulae for Uniformly Accelerated Motion in a Straight Line

Let a particle starts from point A at $t = 0$ and reaches at point B at $t = t$. Initial velocity at point A is u and final velocity at

point B is v . Let the particle covers a displacement s during this motion with constant acceleration a (Fig. 4.19).

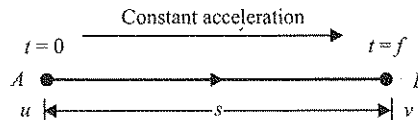


Fig. 4.19

1. Velocity-Time Relation

$$a = \frac{v - u}{t - 0} \Rightarrow v = u + at \quad (1)$$

2. Displacement-Time Relation

$$\begin{aligned} v_{\text{av}} &= \frac{s}{t} = \frac{u + v}{2} \\ \Rightarrow \frac{s}{t} &= \frac{u + v}{2} \\ s &= \left(\frac{u + v}{2} \right) t = \left(\frac{u + u + at}{2} \right) t \Rightarrow s = ut + \frac{1}{2} at^2 \quad (2) \end{aligned}$$

3. Displacement-Velocity Relation

$$\begin{aligned} s &= v_{\text{av}} t = \left(\frac{u + v}{2} \right) \left(\frac{v - u}{a} \right) = \frac{v^2 - u^2}{2a} \\ \Rightarrow v^2 - u^2 &= 2as \quad (3) \end{aligned}$$

The above equations (1), (2), and (3) are known as *equations of motion* for uniform acceleration.

Illustration 4.6 A particle moving with uniform acceleration from A to B along a straight line has velocities v_1 and v_2 at A and B, respectively. If C is the mid-point between A and B then determine the velocity of the particle at C.

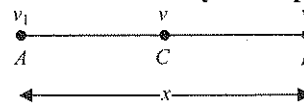


Fig. 4.20

Sol. Let v be the velocity of the particle at C. Assume acceleration of the particle to be a and distance between A and B to be x . To find the velocity at point C. Consider the motion from A to C:

$$\text{Apply } v^2 - u^2 = 2as, \text{ we get } v^2 - v_1^2 = 2a \frac{x}{2}$$

$$\Rightarrow v^2 - v_1^2 = ax \quad (1)$$

Applying the same equation from C to B, we get

$$v_2^2 - v^2 = 2a \frac{x}{2}$$

$$\Rightarrow v_2^2 - v^2 = ax \quad (2)$$

$$\text{From equation (1) and (2), we get } v = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

Illustration 4.7 Two trains P and Q moving along parallel tracks with same uniform speed of 20 m/s . The driver of train P decides to overtake train Q and accelerates his

train by 1 ms^{-2} . After 50 s, the train P crosses the engine of the train Q . Find out what was the distance between the two trains initially provided the length of each train is 400 m.

Sol. Let the initial distance between two train is x .

Distance travelled by train P in 50 s: $S_P = ut + \frac{1}{2}at^2$

$$= 20 \times 50 + \frac{1}{2} \times 1 \times 50^2 = 2,250 \text{ m}$$

Distance travelled by train Q in 50 s:

$$S_Q = ut = 20 \times 50 = 1000 \text{ m}$$

Now, $S_P - S_Q = 2250 - 1000 = 1,250 \text{ m}$. This distance must be equal to initial distance between the trains plus the sum of length of two trains. $\Rightarrow x + 800 = 1250 \Rightarrow x = 450 \text{ m}$

Displacement Travelled by a Particle in n^{th} Second of its Motion in Uniformly Accelerated Motion

Let a particle starts from a point A at $t=0$ and travels along the straight line $ABC \dots DEF$.

At $t=1 \text{ s}$ the particle arrives at point B , at $t=2 \text{ s}$ the particle arrives at point C and in the last, the particle arrives at point F at $t=ns$ as shown in Fig. 4.21. Our aim is to calculate the displacement in the n^{th} second (say D_n) which is equal to EF here.

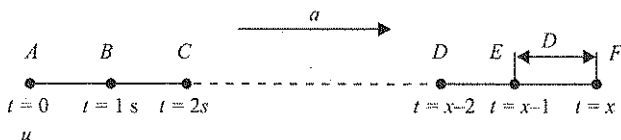


Fig. 4.21

Let a = uniform acceleration of the particle,

u = initial velocity of object at point A

Now, $EF = AF - AE$ (1)

AF is displacement travelled in $t = n$ seconds, so

$$AF = un + \frac{1}{2}an^2$$

AE is displacement travelled in $t = n - 1$ seconds, so

$$AE = u(n-1) + \frac{1}{2}a(n-1)^2$$

Put the values of AF and AE in (1), we get

$$EF = [un + \frac{1}{2}an^2] - [u(n-1) + \frac{1}{2}a(n-1)^2]$$

After simplifying, we get $EF = u + \frac{a}{2}(2n-1)$

$$\Rightarrow D_n = u + \frac{a}{2}(2n-1)$$

Illustration 4.8 A particle having initial velocity 10 ms^{-1} is moving with constant acceleration 4 ms^{-2} . The particle is moving in a straight line. Find the distance travelled by the particle in 5^{th} second of its motion.

Sol. Given: $u = 10 \text{ ms}^{-1}$, $a = 4 \text{ ms}^{-2}$, $n = 5$

$$\text{Using } D_n = u + \frac{a}{2}(2n-1),$$

$$D_n = 10 + \frac{4}{2}[2 \times 5 - 1] \Rightarrow D_n = 10 + 18 = 28 \text{ m}$$

Illustration 4.9 A particle starts from rest with a constant acceleration $a = 1 \text{ m/s}^2$.

1. Determine the velocity after 2 s.
2. Calculate the distance travelled in 3 s.
3. Find the distance travelled in the third second.
4. If the particle was initially moving with a velocity of 5 m/s, then find the distance travelled in the 3^{rd} second.

Sol.

1. We need velocity-time relation to solve this question.

Formula used, $v = u + at$, So $v = 0 + (1)(2) = 2 \text{ m/s}$

2. Here $u = 0$, $t = 3 \text{ s}$, $a = 1 \text{ m/s}^2$,

$$s = ut + \frac{1}{2}at^2 = 0 \times 3 + \frac{1}{2} \times 1 \times (3)^2 = 4.5 \text{ m}$$

3. $D_n = u + \frac{a}{2}(2n-1) = 0 + \frac{1}{2}(2 \times 3 - 1) = 2.5 \text{ m}$

4. Here $u = 5 \text{ m/s}$,

$$D_n = u + \frac{a}{2}(2n-1) = 5 + \frac{1}{2}(2 \times 3 - 1) = 7.5 \text{ m}$$

Illustration 4.10 Consider a particle initially moving with a velocity of 5 m/s starts decelerating at a constant rate of 2 m/s^2 .

1. Determine the time at which the particle becomes stationary.
2. Find the distance travelled in the 2^{nd} second.

Sol. Here $u = 5 \text{ m/s}$, $a = -2 \text{ m/s}^2$

1. $v = u + at \Rightarrow 0 = 5 - 2t \Rightarrow t = 2.5 \text{ s}$
2. $D_n = u + \frac{a}{2}(2n-1) = 5 - \frac{2}{2}[2 \times 2 - 1] = 2 \text{ m}$

Concept Application Exercise 4.1

1. State the following statements as True or False:

For ordinary terrestrial experiments:

- a. a child revolving in a giant wheel is a non-inertial observer.
- b. a driver in a sports car moving with a constant high speed of 200 km/h on a straight road is in inertial frame.
- c. the pilot of an aeroplane which is taking off is a non-inertial observer.
- d. a cyclist negotiating a sharp turn is an example of inertial frame.
- e. the guard of a train which is slowing down to stop at a station is an inertial frame of reference.

2. a. If velocity of a body is zero, does it mean that acceleration is also zero? (Yes/No)
- b. If acceleration of a body is zero, does it mean that velocity is also zero? (Yes/No)

- c. If a body travels with a uniform acceleration a_1 for time t_1 and with uniform acceleration a_2 for a time t_2 , then average acceleration is given by

$$a = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2} \text{ (Yes/No)}$$
- d. If a body starts from rest and moves with a uniform acceleration, the displacements travelled by the body in 1 s, 2 s, 3 s, ..., etc. are in the ratio of 1 : 4 : 9 ..., etc. (True/False)
- e. For a body moving with uniform acceleration, the displacement travelled by the body in successive seconds is in the ratio of 1 : 3 : 5 : 7 ... (True/False)
3. Say Yes or No:
- Can an object moving towards north have acceleration towards south?
 - Can an object reverse the direction of its motion even though it has constant acceleration?
 - Can an object reverse the direction of its acceleration even though it continues to move in the same direction?
 - Average speed is the magnitude of average velocity.
 - At any instant of time, direction of change in velocity and acceleration direction are different.
4. Can a body have
- zero instantaneous velocity and yet be accelerating?
 - zero average speed but non-zero average velocity?
 - negative acceleration and yet be speeding up?
 - magnitude of average velocity be equal to average speed?
5. A body covered a distance of 5 m along a semicircular path. Find the ratio of distance to displacement.
6. A particle travels from point A to B on a circular path of radius $15/\pi$ cm. If the arc length AB be 10 cm, find the displacement.

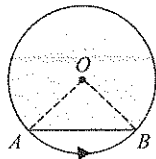


Fig. 4.22

7. A body moves at a speed of 100 m/s for 10 s and then moves at a speed of 200 m/s for 20 s along the same direction. The average speed is _____.
8. A body moves in the southern direction for 10 s at the speed of 10 m/s. It then starts moving in the eastern direction at the speed of 20 m/s for 10 s. The magnitude of average velocity is _____. The average speed is _____. The total displacement will be _____.
9. A body moves from (3, 2) to (7, 6). The initial and final position vectors are _____ and _____. The displacement vector is _____.
10. A car travelling at 108 km/h has its speed reduced to 36 km/h after travelling a distance of 200 m. Find the retardation (assumed uniform) and time taken for this process.

11. The displacement of a body is given to be proportional to square of time elapsed. What is the nature of the acceleration (constant or variable)?
12. A car starts from rest and accelerates uniformly for 10 s to a velocity of 8 m/s. It then runs at a constant velocity and is finally brought to rest in 64 m with a constant retardation. The total distance covered by the car is 584 m. Find the value of acceleration, retardation and total time taken.
13. A body covers 10 m in the 2nd second and 25 m in 5th second of its motion. If the motion is uniformly accelerated, how far will it go in 7th second?
14. A body moving with uniform acceleration in a straight line describes 25 m in 5th second and 33 m in 7th second. Find its initial velocity and acceleration.
15. Two trains, each of length 100 m, moving in opposite directions along parallel lines, meet each other with speeds of 50 km/h and 40 km/h. If their accelerations are 30 cm/s^2 and 20 cm/s^2 , respectively, find the time, they will take to pass each other.

USE OF DIFFERENTIATION AND INTEGRATION IN ONE-DIMENSIONAL MOTION

Let a particle is moving from point A to B. Suppose the particle takes some finite time Δt to cover a finite distance Δx between points C and D (Fig. 4.23). It is not known that how the particle has travelled from C to D. Particle may have travelled with uniform velocity or with variable velocity. Acceleration may or may not have been constant during the motion from C to D. But certainly we can write the average velocity from C to D.

$$v_{av} = \frac{\Delta x}{\Delta t}$$

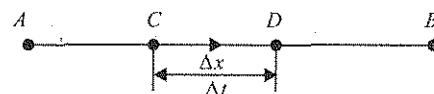


Fig. 4.23

Now what happens if Δt approaches zero, or in other words, Δt is infinitesimally small. When we say Δt approaches zero, it does not mean that Δt is equal to zero. It means that Δt is very-very close to zero. If Δt is very small, then distance covered during this time will also be very small. So Δx also approaches zero and then points C and D will lie very close to each other.

Let us write, $\Delta t = dt$ when Δt approaches zero and $\Delta x = dx$ when Δx approaches zero.

Then average velocity becomes $v_{av} = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

We can also write $v_{av} = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt}$.

When Δt approaches zero then average velocity becomes instantaneous velocity. This we can explain as follows: Since points C and D are very close to each other, we can neglect any change in velocity from C to D , if there is any. We can assume that velocity remains constant for the motion from C to D . Then the average velocity becomes *instantaneous velocity* or simply *velocity*. So $v = v_{av}$ when $\Delta t \rightarrow 0$.

or we can simply write instantaneous velocity as: $v = \frac{dx}{dt}$.

It can be stated as follows: Time rate of change of position is equal to *instantaneous velocity* or simply *velocity*.

Similarly we can define *instantaneous acceleration* or simply *acceleration* as time rate of change of velocity.

$$a = \frac{dv}{dt}$$

Also,

$$a = v \frac{dv}{dx}$$

We have seen that by using differentiation, we can find velocity by differentiating position, and acceleration by differentiating velocity. We can do the reverse also. We can find velocity from acceleration and position from velocity. Here we will have to use *integration*.

Velocity is given by integration of acceleration with respect to time.

$$\text{So, } v = \int a dt + C$$

Position is given by integration of velocity with respect to time.

$$\text{So } x = \int v dt + C$$

Here C is known as constant of integration. Its value depends upon the given conditions. Its value can be different for different conditions.

Derivations of Equations of Motions by Calculus Method

Let a particle starts moving with velocity u at time $t = 0$ along a straight line. The particle has a constant acceleration a .

Let at $t = t$, its velocity becomes v and it covers a displacement of s during this time.

$$v = \int a dt + C_1 \text{ or } v = at + C_1$$

At $t = 0$, $v = u$. This gives $C_1 = u$

Putting C_1 , we get $v = u + at$ (i)

$$x = \int v dt + C_2 \Rightarrow x = \int (u + at) dt + C_2$$

$$x = \int u dt + \int at dt + C_2 = ut + \frac{1}{2}at^2 + C_2$$

At $t = 0$, $x = 0$. This gives $C_2 = 0$

Putting C_2 , we get $x = ut + \frac{1}{2}at^2$ (ii)

$$v \frac{dv}{dx} = a \Rightarrow \int v dv = \int a dx + C_3$$

$$\Rightarrow \frac{v^2}{2} = ax + C_3$$

At $x = 0$, $v = u$. This gives $C_3 = \frac{u^2}{2}$

$$\frac{v^2}{2} = ax + \frac{u^2}{2} \Rightarrow v^2 = u^2 + 2ax \quad \text{(iii)}$$

Illustration 4.11 The displacement of a body as a function of time is given by $s = (3t^2 + 4t + 7)$ m. Calculate the magnitude of its instantaneous velocity and acceleration at $t = 1$ s.

Sol. Given $s = (3t^2 + 4t + 7)$ m. Velocity is the time rate of change of displacement.

$$\text{So } |\vec{v}| = \frac{ds}{dt} = \frac{d}{dt}(3t^2 + 4t + 7) = 6t + 4$$

$$\text{At } t = 1 \text{ s, } v = 6 \times 1 + 4 = 10 \text{ m/s}$$

Now acceleration is the rate of change of velocity:

$$|\vec{a}| = \left| \frac{d\vec{v}}{dt} \right| = \frac{d}{dt}(6t + 4) = 6 \text{ m/s}^2$$

Illustration 4.12 The acceleration of a particle varies with time t seconds according to the relation $a = 6t + 6 \text{ m/s}^2$. Find the velocity and position as a function of time. It is given that the particle starts from origin at $t = 0$ with velocity 2 ms^{-1} .

$$\begin{aligned} \text{Sol. We know that } \int dv &= \int a dt \Rightarrow \int_2^v dv \\ &= \int_0^t (6t + 6) dt \end{aligned}$$

$$\Rightarrow v - 2 = \left[\frac{6t^2}{2} + 6t \right]_0^t \Rightarrow v = 3t^2 + 6t + 2$$

$$\text{now } \int dx = \int v dt \Rightarrow \int_0^x dx = \int_0^t (3t^2 + 6t + 2) dt$$

$$\Rightarrow x = t^3 + 3t^2 + 2t$$

Illustration 4.13 At $t = 0$ a body is started from origin with some initial velocity. The displacement x (m) of the body varies with time t (s) as $x = -(2/3)t^2 + 16t + 2$. Find the initial velocity of the body and also find how long does the body take to come to rest? What is the acceleration of the body when it comes to rest?

$$\begin{aligned} \text{Sol. } v &= \frac{dx}{dt} = \frac{d}{dt} \left(-\frac{2}{3}t^2 + 16t + 2 \right) = -\frac{4}{3}t + 16; \\ \text{at } t &= 0, \end{aligned}$$

$$v = -\frac{4}{3} \times 0 + 16 = 16 \text{ m/s}$$

$$\text{Putting } v = 0, \text{ we get, } -\frac{4}{3}t + 16 = 0 \Rightarrow t = 12 \text{ s}$$

Hence the body takes 12 s to come to rest (momentarily).

$$\text{Now } a = \frac{dv}{dt} = \frac{d}{dt} \left(-\frac{4}{3}t + 16 \right) = -\frac{4}{3} \text{ m/s}^2$$

We see that acceleration is constant, so when the body comes to rest its acceleration is $-\frac{4}{3} \text{ m/s}^2$.

Concept Application Exercise 4.2

1. The position x of a particle varies with time t according to the relation $x = t^3 + 3t^2 + 2t$. Find velocity and acceleration as a function of time.
2. The displacement of a particle along x -axis is given by $x = 3 + 8t + 7t^2$. Obtain its velocity and acceleration at $t = 2$ s.
3. The acceleration a in ms^{-2} of a particle is given by $a = 3t^2 + 2t + 2$, where t is the time. If the particle starts out with a velocity $v = 2 \text{ ms}^{-1}$ at $t = 0$, then find the velocity at the end of 2 s.
4. The displacement x of a particle along the x -axis at time t is given by $x = \frac{a_1}{2}t + \frac{a_2}{3}t^2$. Find the acceleration of the particle.
5. A particle moves along a straight line such that its displacement s at any time t is given by $s = t^3 - 6t^2 + 3t + 4$ m, t being in seconds. Find the velocity of the particle when the acceleration is zero.

ONE-DIMENSIONAL MOTION IN A VERTICAL LINE (MOTION UNDER GRAVITY)

Sign convention: Any vector quantity directed upward will be taken as positive and directed downward will be taken as negative (Fig. 4.24).

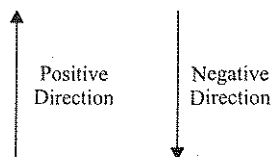


Fig. 4.24

According to this sign convention:

1. s will be taken as positive if final position lies above initial position and negative if final position lies below initial position.
2. velocity(initial or final) will be taken as positive if it is upward and negative if it is downward.
3. acceleration a is always taken to be $-g$.

Illustrations to use sign convention: Look at three different situations in Fig. 4.25 given below. In each of the situations a particle is projected from a point A with initial velocity u and the particle reaches point B after some time.

Earth attracts each and every object towards its centre. As a result of this attraction a constant acceleration is produced in the objects lying close to the earth's surface. This acceleration is always directed downward. This acceleration is called *acceleration due to gravity*. It is denoted by g . For the bodies lying close to the earth's surface, value of $g = 9.8 \text{ ms}^{-2}$ and sometimes to make the calculations easy we take $g = 10 \text{ ms}^{-2}$.

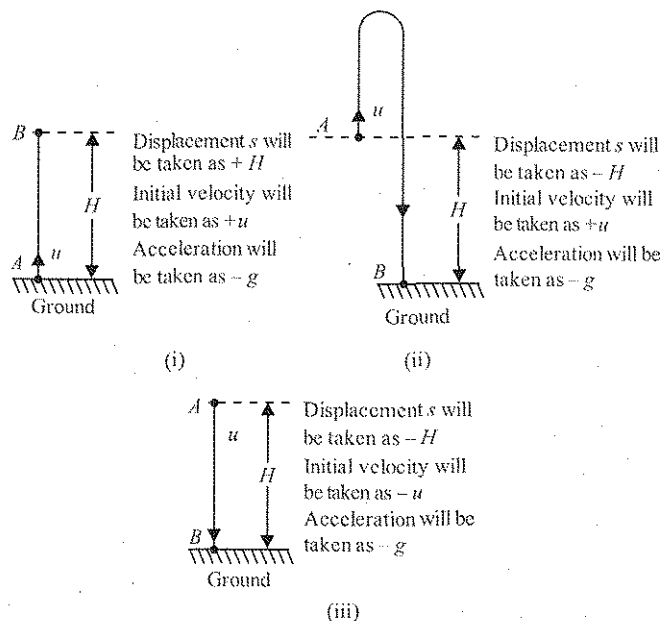


Fig. 4.25

When the motion takes place under the effect of *gravitational attractive force* only, the motion is known as *Free Fall*. Here free fall does not mean that the particle is falling down only. Even if the particle is rising up or is momentarily at rest at highest point, but if only *gravitational force* is acting on it, then motion will be called as *Free Fall*.

In equations of motion we replace a by $-g$ (minus sign because acceleration is always directed downward),

$$\text{we get : } \begin{cases} v = u - gt \\ s = ut - \frac{1}{2}gt^2 \\ v^2 = u^2 - 2gs \end{cases}$$

Some Formulae

1. A body is dropped from a height H . What will be the time taken by the body to reach the ground and velocity just before reaching the ground?

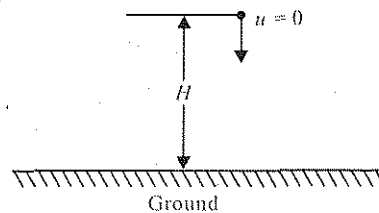


Fig. 4.26

$$\text{Ans: } T = \sqrt{\frac{2H}{g}}, \quad v = \sqrt{2gH}$$

Proof: Let time taken to reach the ground is T and velocity on reaching the ground is v . We have $u = 0$, $s = -H$, $t = T$

using $s = ut + \frac{1}{2}gt^2$, we have

$$-H = 0 \times T - \frac{1}{2}gT^2 \Rightarrow H = \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2H}{g}}$$

Hence time taken to reach the ground, $T = \sqrt{\frac{2H}{g}}$

Note: Time of flight is independent of mass, shape and size of the body in vacuum. But in air if two bodies of same mass and different size are fallen, then the body having more volume will take more time.

Now using $v = u - gt$, we have

$$v = 0 - gT = -g\sqrt{\frac{2H}{g}}$$

$\Rightarrow v = -\sqrt{2gH}$, negative sign indicates that velocity is in downward direction.

Hence velocity on reaching the ground: $v = \sqrt{2gH}$ in downward direction

2. A particle is projected vertically upward from ground with the initial velocity u .

- Find the maximum height, H , the particle will attain and time T that it will take to return to the ground (Fig. 4.27).
- What will be velocity when the particle returns to the ground?
- What will be the displacement and distance travelled by the particle during this time of whole motion.

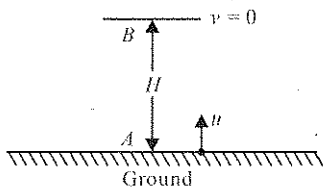


Fig. 4.27

Ans: (a) $H = \frac{u^2}{2g}$, $T = \frac{2u}{g}$

Proof: Consider the motion from A to B:

$s = +H$ (final point lies above the initial point), initial velocity $= u$, final velocity $v = 0$

Let time taken to go from A to B $= t_1$. Using $v = u - gt$

$$\Rightarrow 0 = u - gt_1 \Rightarrow t_1 = \frac{u}{g}$$

$$\text{Using } s = ut - \frac{1}{2}gt^2 \Rightarrow H = u\frac{u}{g} - \frac{1}{2}g\left(\frac{u}{g}\right)^2$$

$$\Rightarrow H = \frac{u^2}{2g}$$

So maximum height attained is $H = \frac{u^2}{2g}$

consider the return motion from B to A:

$s = -H$ (final point lies below the initial point)

$u = 0$ (at point B velocity is zero)

Let time taken to go from B to A $= t_2$. We have

$$t_2 = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2}{g} \frac{u^2}{2g}} = \frac{u}{g}$$

Here t_1 is known as time of ascent and t_2 is known as time of descent. We can see that

$$\text{Time of ascent} = \text{Time of descent} = \frac{u}{g}$$

$$\text{Total time of flight: } T = t_1 + t_2 = \frac{2u}{g}$$

Here time of flight is the time for which the particle remains in air.

Alternative method to find the time of flight:

Consider the motion from A to A:

$s = 0$ (initial and final both points are same),
initial velocity $= u$, time taken $= T$

$$\text{Using } s = ut - \frac{1}{2}gt^2, \text{ we have } 0 = uT - \frac{1}{2}gT^2$$

$$\Rightarrow T = \frac{2u}{g}$$

(b) Magnitude of velocity on returning to the ground will be same as that of initial velocity but direction will be opposite.

Proof: Let v is the velocity on reaching the ground. Then from previous formulae:

$$v = -\sqrt{2gH} = -\sqrt{2g \frac{u^2}{2g}} \Rightarrow v = -u, \text{ hence proved}$$

(c) Displacement travelled $= 0$, distance travelled $= 2H = \frac{u^2}{g}$

Illustration 4.14 A body is dropped from a height of 80 m. Find the time taken by the body to reach the ground and velocity on reaching the ground. Take $g = 10 \text{ m/s}^2$.

Sol. Given $u = 0$, $H = 80 \text{ m}$.

$$\text{Time taken to reach the ground: } T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 80}{10}} = 4 \text{ s}$$

Velocity on reaching the ground:

$$v = \sqrt{2gH} = \sqrt{2 \times 10 \times 80} = 40 \text{ m/s}$$

Illustration 4.15 A ball thrown up is caught by the thrower after 4 s. How high does it go and with what velocity was it thrown? Take $g = 10 \text{ ms}^{-2}$.

Sol. Given $T = 4 \text{ s}$. We know that $T = \frac{2u}{g}$

$$\Rightarrow 4 = \frac{2u}{10} \Rightarrow u = 20 \text{ m/s}$$

$$\text{Also } H = \frac{u^2}{2g} = \frac{20^2}{2 \times 10} = 20 \text{ m}$$

Illustration 4.16 From the top of a building 160 m high, a ball is dropped and at the same time a stone is projected vertically upward from the ground with a velocity of 40 ms^{-1} (Fig. 4.28). Find when and where the ball and the stone will meet. Take $g = 10 \text{ ms}^{-2}$.

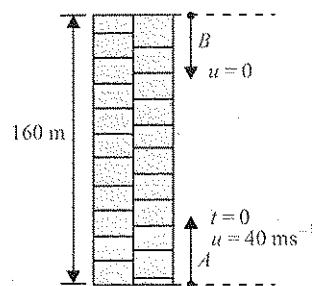


Fig. 4.28

Sol. Let them meet after time t at height x from ground.

For ball: $s = -(160 - x)$, $u = 0$, $a = -g$, then

$$-(160 - x) = 0 \times t - \frac{1}{2}gt^2 \Rightarrow 160 - x = \frac{1}{2}gt^2 \quad (1)$$

For stone: $s = x$, $u = 40$ m/s, $a = -g$, then $x = 40t - \frac{1}{2}gt^2$ (2)

adding equations (1) and (2): $160 = 4t \Rightarrow t = 4$ s

put the value of t in (1): $160 - x = \frac{1}{2} \times 10 \times 4^2$

$$\Rightarrow x = 80 \text{ m}$$

So they meet in the middle of the building at $t = 4$ s

Illustration 4.17 A body is projected vertically upward. If t_1 and t_2 be the times at which it is at a height h above the point of projection while ascending and descending, respectively. Then prove that initial velocity of projection of the body is $\frac{1}{2}g(t_1 + t_2)$ and the value of h is $\frac{1}{2}gt_1t_2$.

Sol. Let the body is projected with initial velocity u and let at any time t the body is at height h from the point of projection.

There will be two possible values of t which are t_1 and t_2 as given in the question.

$$\text{Now we can write } h = ut - \frac{1}{2}gt^2 \Rightarrow gt^2 - 2ut + 2h = 0.$$

This is a quadratic equation having two roots.

$$\text{Sum of roots: } t_1 + t_2 = \frac{-2u}{g} \Rightarrow u = \frac{1}{2}g(t_1 + t_2)$$

$$\text{Product of roots: } t_1t_2 = \frac{2h}{g} \Rightarrow h = \frac{1}{2}gt_1t_2$$

Illustration 4.18 A balloon is at a height of 40 m and is ascending with a velocity of 10 ms^{-1} . A bag of 5 kg weight is dropped from it. When will the body reach the surface of the earth? Given $g = 10 \text{ m/s}^2$.

Sol. Given $s = -40$ m, $u = 10$ m/s, The initial velocity of the bag will be same as that of velocity of balloon.

$$\text{Using } s = ut - \frac{1}{2}gt^2, \text{ we have}$$

$$-40 = 10t - \frac{1}{2}10t^2 \Rightarrow 5t^2 - 10t - 40 = 0$$

$$\Rightarrow (t - 4)(5t + 10) = 0 \Rightarrow t = 4 \text{ s}, -2 \text{ s}$$

Neglecting negative time, time taken by body to reach the ground is $t = 4$ s.

Illustration 4.19 From a point A, 80 m above the ground, a particle is projected vertically upwards with a velocity of 29.4 ms^{-1} . Five seconds later another particle is dropped from a point B, 34.3 m vertically below A. Determine when and where one overtakes the other. Take $g = 9.8 \text{ ms}^{-2}$.

Sol. Let at time t , they reach at level C. Then

$$\text{For A: } -(h + 34.3) = 29.4t - \frac{1}{2}gt^2 \quad (1)$$

and

$$\text{For B: } -h = -\frac{1}{2}g(t - 5)^2 \quad (2)$$

$$\text{From equations (2) - (1) } \Rightarrow 34.3 = -29.4t + \frac{1}{2}g[t^2 - (t^2 - 5)]$$

$$\Rightarrow 7 = -6t + (2t - 5)5$$

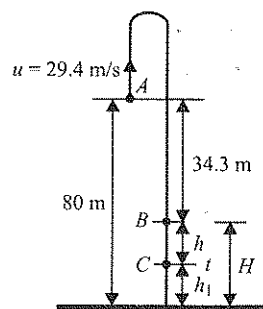


Fig. 4.29

$$\Rightarrow t = 8 \text{ s}$$

$$\text{Now from (2), } h = \frac{1}{2}(9.8)(8 - 5)^2 = 44.1 \text{ m. From figure}$$

$$H = 80 - 34.3 = 45.7 \text{ m}$$

$$\text{So finally: } h_1 = H - h = 45.7 - 44.1 = 1.6 \text{ m}$$

Concept Application Exercise 4.3

- i. State the following statements as True or False:

 - A ball thrown vertically up takes more time to go up than to come down.
 - If a ball starts falling from the position of rest, then it travels a distance of 25 m during the third second of its fall.
 - A packet dropped from a rising balloon first moves upwards and then moves downwards as observed by a stationary observer on the ground.
 - In the absence of air resistance, all bodies fall on the surface of earth with the same rate.

ii. Fill in the blanks

 - When a body is thrown vertically upward, at highest point _____ (both velocity and acceleration are zero/only velocity is zero/only acceleration is zero).
 - If air drag is not neglected, then which is greater: time of ascent or time of descent?
 - A body is projected upward. Up to maximum height, time taken will be greater to travel _____ (first half/second half).
- A ball thrown up from the ground reaches a maximum height of 20 m. Find:

 - its initial velocity.
 - the time taken to reach the highest point.
 - its velocity just before hitting the ground.
 - its displacement between 0.5 s and 2.5 s.
 - the time at which it is 15 m above the ground.
- A balloon is rising up with a velocity of 10 ms^{-1} and a bag is dropped from it when its height from the ground is 40 m. Calculate the time taken by the bag to reach the ground.
- A body is projected from the bottom of a smooth inclined plane with a velocity of 20 m/s. If it is just sufficient to carry it to the top in 4 s, find the inclination and height of the plane.
- A ball is dropped from an elevator at an altitude of 200 m (Fig. 4.30). How much time will the ball take to reach the ground if the elevator is

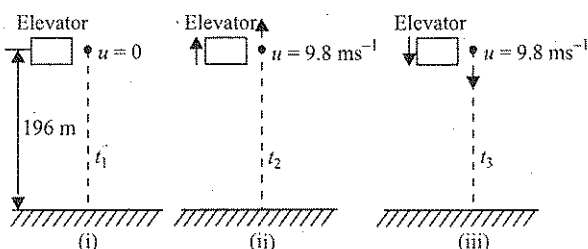


Fig. 4.30

- stationary?
 - ascending with velocity 10 m/s?
 - descending with velocity 10 m/s?
- A particle is projected vertically upwards. Prove that it will be at $3/4$ of its greatest height at times which are in the ratio 1:3.
 - A balloon rises from rest on the ground with constant acceleration $g/8$. A stone is dropped from the balloon when the balloon has risen to a height of H . Find the time taken by the stone to reach the ground.
 - A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2 . He reaches the ground with a speed of 3 m/s. At what height did he bail out?
 - A ball is dropped from the top of the tower of height h . It covers a distance of $h/2$ in the last second of its motion. How long does the ball remain in air?
 - When a ball is thrown up, it reaches to a maximum height h travelling 5 m in the last second. Find the velocity with which the ball should be thrown up.

GRAPHS IN MOTION IN ONE DIMENSION

Graphical analysis is a very potential method of studying the motion of a particle. Indeed the method of analyzing situations graphically can be effectively applied not only to motion but to any field. The variations of two quantities (related) with respect to each other can be demonstrated by means of a graph. The greatest advantage of depicting variables by means of curves lies in the fact that the whole situation can be understood at any instant. In other words, graphs reveal much more than that revealed by a table. In this section, we will study and interpret various types of graphs related to motion such as displacement–time graph, velocity–time graph, etc.

For graphical representation, we require two coordinate (reference) axes, one variable being taken along one axis. The usual practice is to take the independent variable along x -axis and the dependent on along y -axis. In general cases involving time as one of the variables, time, being independent, is usually taken along x -axis.

Graphs play very important role in analyzing a motion. Sometimes it becomes difficult to solve the problems analytically. But with the help of graphs we can solve the problems very easily and without much calculation.

How to Analyse the Graphs and How to Draw the Graphs

In one dimensional motion, generally, we come across position–time (or displacement–time) graph, velocity–time graph, acceleration–time graph, etc.

Whenever we draw a graph, we need an equation involving the variables between which we have to draw the graph. For e.g., to draw position–time graph we generally use the equation $x = ut + \frac{1}{2}at^2$, to draw velocity–time graph we generally use the equation $v = u + at$, etc. *Note that we can use these relations only when acceleration is constant.*

Position–Time Graph of Various Types of Motions of a Particle

Particle is Stationary

Let a particle be at some point P at time $t=0$ which is at a distance x_0 from origin. Since the particle is stationary, so at any further time the particle will remain at point P . Hence position–time graph for a stationary particle is parallel to time axis (Fig. 4.31).

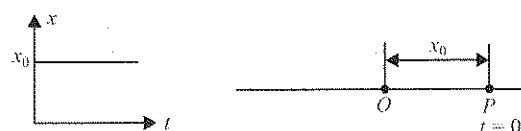


Fig. 4.31

Particle is Moving with Constant Velocity Towards Right

Equation to be used: $x = x_0 + vt$. Graph will be a straight line.

Let the particle be at some point P initially at time $t = 0$ which is at a distance of x_0 from origin. Since the particle is moving towards right so its distance from origin goes on increasing. Hence position–time graph for a particle moving with constant velocity towards right will be a straight line inclined to time axis making an acute angle α (Fig. 4.32).

Recall that $\tan \alpha$ is slope of position–time graph which is equal to velocity of the particle.

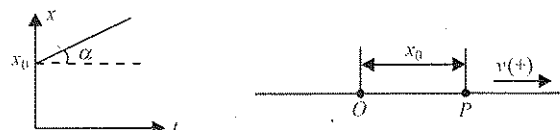


Fig. 4.32

Particle is Moving with Constant Velocity Towards Left

Equation to be used: $x = x_0 - vt$. Graph will be a straight line.

Let the particle be at some point P at time $t = 0$ which is at a distance of x_0 from origin. Since the particle is moving towards left so first its distance from origin goes on decreasing and then its distance from the origin goes on increasing in negative direction. Hence position–time graph for a particle moving with constant velocity towards left will be a straight line inclined to time axis making an obtuse angle α (Fig. 4.33).

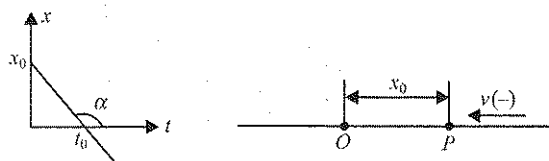


Fig. 4.33

Here $\tan \alpha$, the slope, will be negative which indicates negative velocity.

Particle is Moving with Constant Acceleration Directed Rightward (Positive Acceleration)

Since acceleration is towards right so velocity increases in right direction. Hence slope of position-time graph goes on increasing.

$$\text{Equation to be used: } x = ut + \frac{1}{2}at^2$$

Graph will be a curved line (parabolic)

$$v_1 = \tan \alpha_1, v_2 = \tan \alpha_2$$

It is clear from Fig. 4.34 that as time passes, slope goes on increasing.

$$\text{Now } \alpha_2 > \alpha_1 \Rightarrow \tan \alpha_2 > \tan \alpha_1 \Rightarrow v_2 > v_1$$

So velocity goes on increasing.

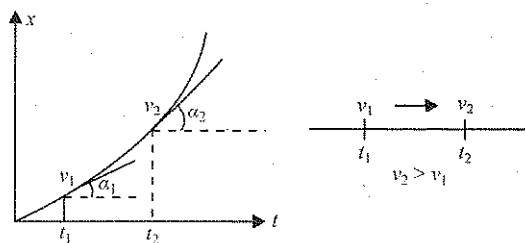


Fig. 4.34

Particle is Moving with Constant Acceleration Directed Leftward (Negative Acceleration)

Since acceleration is towards left so velocity decreases in right direction and increases in left direction.

$$\text{Equation to be used: } x = ut + \frac{1}{2}at^2$$

Graph will be a curved line (parabola)

$$v_1 = \tan \alpha_1, v_2 = \tan \alpha_2$$

It is clear from Fig. 4.35 that as time passes, slope goes on decreasing.

$$\text{Now } \alpha_2 < \alpha_1 \Rightarrow \tan \alpha_2 < \tan \alpha_1 \Rightarrow v_2 < v_1$$

So velocity goes on decreasing.

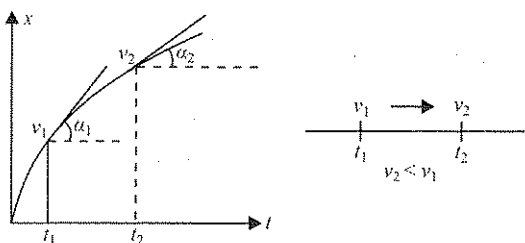


Fig. 4.35

Velocity-Time Graph of Various Types of Motions of a Particle

Particle is Moving with a Constant Velocity

Since velocity is constant, so $a = 0$.

Let at any time velocity of particle is u . Since velocity remains constant, so at any time velocity remains same. Hence velocity-time graph for a particle moving with constant velocity is a straight line parallel to time axis.



Fig. 4.36

Particle is Moving with a Constant Positive Acceleration

$$\text{Equation to be used: } v = u + at$$

As the time passes, velocity goes on increasing. Hence velocity-time graph for a particle moving with constant positive acceleration is a straight line inclined to time axis making an acute angle α . Here $\tan \alpha$ is the slope of velocity-time graph (Fig. 4.37).

Note that the slope of velocity-time graph is equal to acceleration.



Fig. 4.37

Particle is Moving with a Constant Negative Acceleration (Retardation)

$$\text{Equation to be used: } v = u + at$$

As the time passes, the velocity goes on decreasing, at time $t = t_0$ velocity becomes zero and after that it increases in negative direction. Hence velocity-time graph for a particle moving with constant retardation is a straight line inclined to time axis making an obtuse angle α (Fig. 4.38).

Since α is obtuse angle so slope, $\tan \alpha$, becomes negative; hence acceleration is negative.

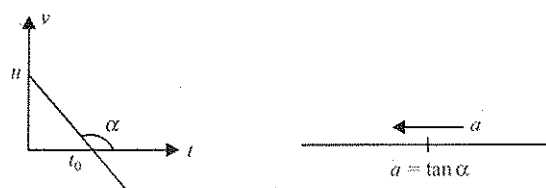


Fig. 4.38

Particle is Moving with Increasing Acceleration

$$\begin{aligned} \Rightarrow \alpha_2 &> \alpha_1 \\ \Rightarrow \tan \alpha_2 &> \tan \alpha_1 \\ \Rightarrow a_2 &> a_1 \end{aligned}$$

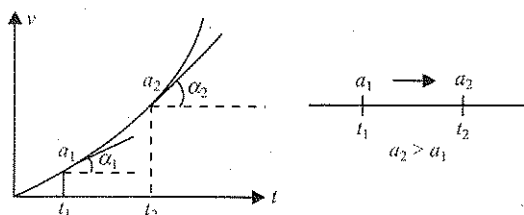


Fig. 4.39

Particle is Moving with Decreasing Acceleration

$$\begin{aligned} \Rightarrow \alpha_2 &< \alpha_1 \\ \Rightarrow \tan \alpha_2 &< \tan \alpha_1 \\ \Rightarrow a_2 &< a_1 \end{aligned}$$

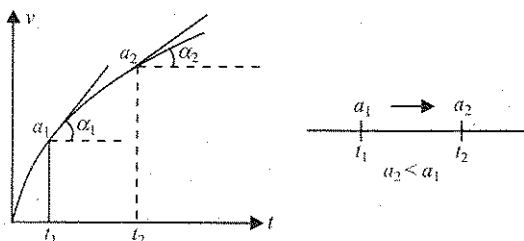


Fig. 4.40

Acceleration-Time Graph of Various Types of Motions of a Particle**Particle is Moving with Constant Acceleration**

Let at time $t = 0$, acceleration is a_0 . Since acceleration is constant, so acceleration at any further time will remain a_0 . Hence acceleration-time graph for a particle moving with constant acceleration is a straight line parallel to time axis (Fig. 4.41).

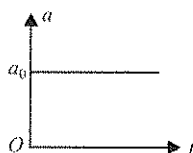


Fig. 4.41

Particle is Moving with Increasing Acceleration at Constant Rate

Acceleration-time graph for a particle moving with increasing acceleration at constant rate is a straight line making an acute angle α with time axis. Graph will be straight line because acceleration is increasing at constant rate (Fig. 4.42).

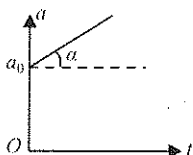


Fig. 4.42

Particle is Moving with Decreasing Acceleration at Constant Rate

Acceleration-time graph for a particle moving with decreasing acceleration at constant rate is a straight line making an obtuse angle α with time-axis (Fig. 4.43).

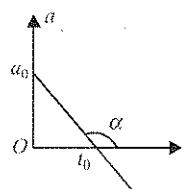


Fig. 4.43

Graph will be straight line because acceleration is decreasing at constant rate. Let at $t = 0$ acceleration is a_0 . At some time $t = t_0$ acceleration becomes zero and then it becomes negative.

Derivation of Equations of Uniformly Accelerated Motion from Velocity-Time Graph

Consider an object is moving with a uniform acceleration ' a ' along a straight line. The initial and final velocities of the object at time $t = 0$ and $t = t$ are u and v , respectively. During time t , let s be the distance travelled by object. In uniformly accelerated motion the velocity-time graph of an object is a straight line, inclined to time axis.

In Fig. 4.44, $OA = u \Rightarrow OE = v$ and $OD = AC = t$

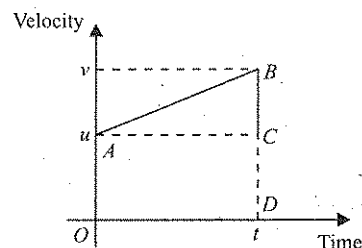


Fig. 4.44

First Equation of Motion

In uniformly accelerated motion, the slope of velocity-time graph represents the acceleration of object.

$$\begin{aligned} \text{i.e., Acceleration} &= \text{slope of graph } AB = \frac{BC}{AC} = \frac{AE}{AC} \\ &= \frac{OE - OA}{AC} \Rightarrow a = \frac{v - u}{t} \end{aligned}$$

$$\Rightarrow v - u = at \quad \text{or} \quad v = u + at$$

Second Equation of Motion

In uniformly accelerated motion, the area included between velocity-time graph and time axis provides us the distance covered by the object in a given interval of time.

Therefore, $s = \text{area of trapezium } ABDO$

$$= \frac{1}{2} (OA + BD) \times OD = \frac{1}{2} (u + v) \times t$$

$$\text{But } v = u + at \Rightarrow s = \frac{1}{2} [u + u + at] \times t$$

$$\Rightarrow s = ut + \frac{1}{2} at^2$$

Third Equation of Motion

$$s = \frac{1}{2}(OA + OB) OD = \frac{1}{2}(DC + OB) OD \quad (i)$$

As $OA = CD$

$$\text{Now, acceleration } a = \text{slope of velocity time graph} = \frac{BC}{AC}$$

$$= \frac{DB - DC}{OD} \text{ or } OD = \frac{DB - DC}{a} \quad (ii)$$

Using equations (i) and (ii)

$$s = \frac{1}{2} (DC + DB) \left(\frac{DB - DC}{a} \right)$$

$$\Rightarrow s = \frac{1}{2} \left(\frac{DB^2 - DC^2}{a} \right)$$

$$\text{or } DB^2 - DC^2 = 2as$$

$$\text{or } v^2 - u^2 = 2as \text{ or } v^2 = u^2 + 2as$$

Illustration 4.20 The position versus time graph for a certain particle moving along the x-axis is shown in Fig. 4.45. Find the average velocity in the time intervals (i) 0 to 2 s (ii) 2 s to 4 s (iii) 4 s to 7 s.

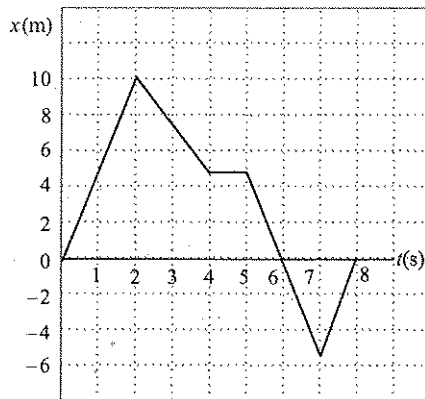


Fig. 4.45

Sol. We are given the values of time, but the values of the position are to be obtained from the graph corresponding to the given time interval under consideration. We know that slope of the displacement-time graph represents velocity.

Formula to be used: $v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$ where x_1 and x_2 are the initial and final positions, respectively and t_1 and t_2 are initial and final values of time, respectively.

1. Here $x_1 = 0$ m, $x_2 = 10$ m, $t_1 = 0$ s, $t_2 = 2$ s

Therefore the required average velocity is given by

$$v_{av} = \frac{10 - 0}{2 - 0} = 5 \text{ ms}^{-1}$$

2. Here $x_1 = 10$ m, $x_2 = 5$ m, $t_1 = 2$ s, $t_2 = 4$ s

So the required value of average velocity is given by

$$v_{av} = \frac{5 - 10}{4 - 2} = -2.5 \text{ ms}^{-1}$$

3. Here $x_1 = 5$ m, $x_2 = -5$ m, $t_1 = 4$ s, $t_2 = 7$ s

So the required value of average velocity is given by

$$v_{av} = \frac{-5 - 5}{7 - 4} = \frac{-10}{3} = -3.3 \text{ ms}^{-1}$$

Illustration 4.21 The velocity versus time curve of a moving point is shown in figure. Find the retardation of the particle.

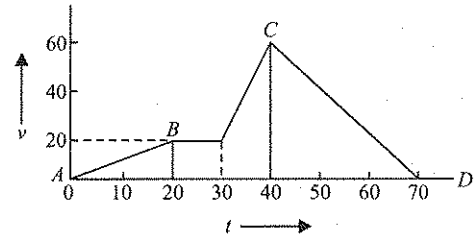


Fig. 4.46

Sol. The slope of velocity-time graph represents acceleration or retardation of the particle during motion. If slope is positive it represents acceleration and if slope is negative it represents retardation. The section CD of the graph represents retardation and magnitude of retardation is

$$|a| = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{60}{(70 - 40)} = 2 \text{ m/s}^2$$

Illustration 4.22 The velocity versus time graph of a linear motion is shown in Fig. 4.47. Find the distance and displacement from the origin after 8 s.

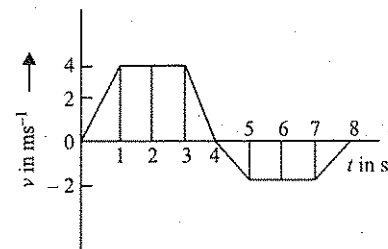


Fig. 4.47

Sol. The area under velocity-time graph represents distance and displacement of the particle.

1. Distance travelled by the particle is the total length of the path travelled by the particle. Hence distance travelled by the particle

$S = \text{Total sum of area under the graph}$

$$= \frac{1}{2}(2 + 4) \times 4 + \frac{1}{2}(2 + 4) \times 2 = 12 + 6 = 18 \text{ m}$$

2. The displacement of the particle is the change of position. The displacement of the particle is positive for area above the time axis and negative for the area below the time axis. Hence displacement of the particle is

$$\Delta x = \frac{1}{2}(2 + 4) \times 4 - \frac{1}{2}(2 + 4) \times 2 = 12 - 6 = 6 \text{ m}$$

Concept Application Exercise 4.4

1. a. What can you say about velocity in each of the following position-time graphs?

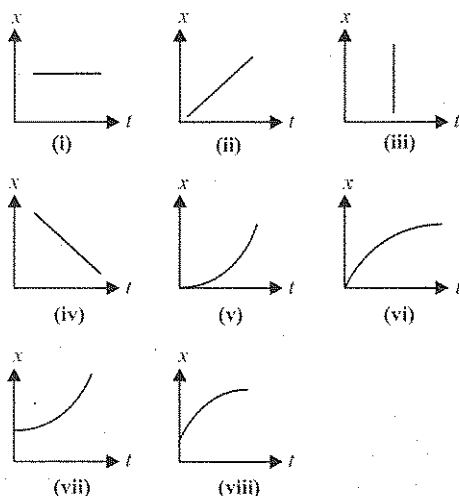


Fig. 4.48

- b. The slope of velocity-time graph is equal to acceleration. (True/False)
- c. What does the area under acceleration-time graph represent?
- d. Can velocity-time graph be parallel to velocity axis? (Yes/No). Why?
- e. What is the slope of $v-t$ graph in uniform motion?
2. a. A ball is thrown vertically upward. After some time it returns to the thrower. Draw the *velocity-time* graph and *speed-time* graph.
- b. A ball is dropped from some height. After rebounding from the floor it ascends to the same height. Draw the *velocity-time* graph and *speed-time* graph.
3. A body starts at $t = 0$ with velocity u and travels along a straight line. The body has a constant acceleration a . Draw the acceleration-time graph, velocity-time graph and displacement-time graph from $t = 0$ to $t = 10$ s for the following cases:
- $u = 8 \text{ ms}^{-1}$, $a = 2 \text{ ms}^{-2}$
 - $u = 8 \text{ ms}^{-1}$, $a = -2 \text{ ms}^{-2}$
 - $u = -8 \text{ ms}^{-1}$, $a = 2 \text{ ms}^{-2}$
 - $u = -8 \text{ ms}^{-1}$, $a = -2 \text{ ms}^{-2}$
4. See Fig. 4.49 and find the average acceleration in first 20 s.

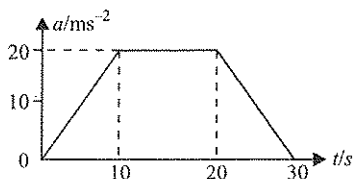


Fig. 4.49

(Hint: Area under $a-t$ graph is equal to change in velocity)

5. At $t = 0$, a particle starts from rest and moves along a straight line, whose acceleration-time graph is shown in Fig. 4.50.

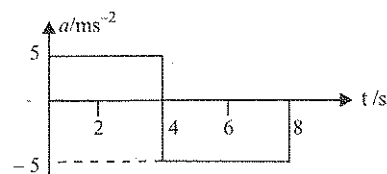


Fig. 4.50

Convert this graph into velocity-time graph. From the velocity-time graph, find the maximum velocity attained by the particle. Also find from $v-t$ graph, the displacement and distance travelled by the particle from 2 to 6 s.

6. Answer the following question giving reasons in brief: Is the time variation of position, shown in the Fig. 4.51, observed in nature? (IIT-JEE, 1979)

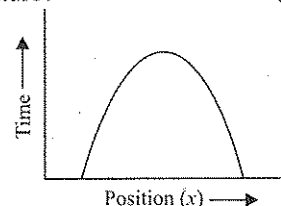


Fig. 4.51

Solved Examples

Example 4.1 A ball is thrown upwards with an initial velocity of 10 ms^{-1} . Considering highest point as the origin and vertically downward direction as positive direction, find the signs of position, velocity and acceleration of the object under motion during its upward and downward journey.

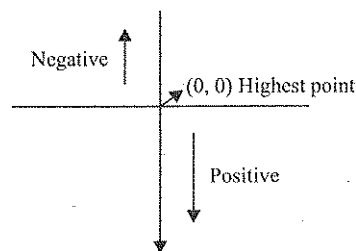


Fig. 4.52

Sol. As the particle always remains below $(0, 0)$ and downward direction is positive, so position x will remain positive during both upwards and downwards motion.

During upward motion, velocity is negative and during downward motion velocity is positive.

Acceleration is positive during both upward and downward motion. It is because acceleration due to gravity (g) is downward and our downward direction is positive.

Example 4.2 A car takes 20 s to move around a roundabout of radius 14 m. Calculate

1. average speed, and
 2. magnitude of average velocity.
- Sol.**

1. Average speed is the ratio of total distance to total time taken.

So, distance covered in one revolution = $2\pi r$

$$= 2 \times \frac{22}{7} \times 14 = 88 \text{ m,}$$

$$\text{Average speed} = 88/20 = 4.4 \text{ m/s}$$

2. In one complete revolution displacement of car is zero.

$$|\vec{v}_{av}| = \frac{\text{Displacement}}{\text{Time}} = \frac{0}{10} = 0 \text{ ms}^{-1}$$

Example 4.3 A train travels from city A to city B with a constant speed of 10 ms^{-1} and returns back to city A with a constant speed of 20 ms^{-1} . Find its average speed during the entire journey.

Sol. Given $v_A = 10 \text{ ms}^{-1}$, $v_B = 20 \text{ ms}^{-1}$

Average speed is always calculated from the ratio of total distance to the total time taken.

Average speed is not the arithmetic average of speeds in a journey.

Consider, the distance between the two cities A and B = $x \text{ m}$.

Time taken by the train to travel from A to B = $\frac{x}{10} = t_1$ (say)

Time taken to come back from B to A = $\frac{x}{20} = t_2$ (say)

$$\begin{aligned} \therefore \text{average speed} &= \frac{\text{Total distance}}{\text{Total time}} = \frac{x + x}{t_1 + t_2} = \frac{2x}{\frac{x}{10} + \frac{x}{20}} \\ &= \frac{2}{\frac{1}{10} + \frac{1}{20}} = \frac{2}{\frac{2+1}{20}} = \frac{2 \times 20}{3} \\ &= \frac{40}{3} \text{ m/s} \end{aligned}$$

Example 4.4 Consider a particle initially moving with a velocity of 5 ms^{-1} which starts decelerating at a constant rate of 2 ms^{-2} .

1. Determine the time at which the particle becomes stationary.
2. Find the distance travelled in the 2nd second.

Sol.

1. Here $u = 5 \text{ ms}^{-1}$, $v = 0$, $a = -2 \text{ ms}^{-2}$, $t = ?$

We have to find the time when the particle becomes stationary, i.e., it comes to rest. It means the final velocity will be zero. Symbol 'a' carries a negative sign as the statement involves retardation or deceleration.

Formula used: $v = u + at$ (Formula that relates the initial and final velocities to time.)

So, we have $0 = 5 - 2t \Rightarrow t = 2.5 \text{ s}$

2. Here $u = 5 \text{ m/s}$, $a = -2 \text{ ms}^{-2}$, $n = 2$, $D_n = ?$

Formula used: For the distance travelled in the n th second the relation used is

$$D_n = u + \frac{a}{2}(2n - 1)$$

Putting the values, $D_n = 5 - \frac{2}{2} [2 \times 2 - 1] = 2 \text{ m}$, which is the required distance travelled.

Example 4.5 A particle starts from rest with a constant acceleration $a = 1 \text{ m/s}^2$.

1. Determine the velocity after 2 s.
2. Calculate the distance travelled in 3 s.
3. Find the distance travelled in the third second.
4. If the particle was initially moving with a velocity of 5 m/s , then find the distance travelled in the third second.

Sol.

1. We need the velocity-time relation to solve this question.

Formula used: $v = u + at$ So we have $v = 0 + (1)(2) = 2 \text{ m/s}$

2. Here $u = 0$, $t = 3 \text{ s}$, $a = 1 \text{ m/s}^2$, $s = ?$

We need the distance-time relation. In a question where motion is under acceleration and it is not circular or rotatory, the distance-time relation which is used is $s = ut + \frac{1}{2}at^2$.

$$\text{So, we have } s = 0 + \frac{1}{2} \times 1 \times (3)^2 = 4.5 \text{ m}$$

3. Here $n = 3$, $D_n = ?$

Again we need the distance-time relation. But here the distance has to be determined in a particular second. So in such a case the formula used is $D_n = u + \frac{a}{2}(2n - 1)$.

So, we have $D_n = 0 + \frac{1}{2}(2 \times 3 - 1) = 2.5 \text{ m}$, which is the required distance travelled.

4. Here $u = 5 \text{ m/s}$

So, we have $D_n = 5 + \frac{1}{2}(2 \times 3 - 1) = 7.5 \text{ m}$ which is the required distance travelled.

Example 4.6 Two balls of different masses m_1 and m_2 are dropped from two different heights h_1 and h_2 , respectively. Find the ratio of time taken by the two balls to drop through these distances.

Sol. Analysis of situation: As the two balls have been dropped so the initial velocity in both the cases must be zero. Also the two heights involved are different. Hence the time taken in both the cases will be different due to the same initial velocity.

$$\text{Formula used: } y = ut + \frac{1}{2}at^2$$

Now for ball 1: $y = h_1$, $t = t_1$, as the motion is under gravity, so $a = g$ (taking the downward direction as positive)

$$\begin{aligned} h_1 &= 0t + \frac{1}{2}gt_1^2 \Rightarrow h_1 = \frac{1}{2}gt_1^2 \\ &\Rightarrow t_1 = \sqrt{\frac{2h_1}{g}} \end{aligned} \quad (i)$$

and for ball 2: $y = h_2$, $t = t_2$, as the motion is under gravity, so $a = g$ (taking the downward direction as positive)

$$h_2 = 0t_2 + \frac{1}{2}gt_2^2 \Rightarrow t_2 = \sqrt{\frac{2h_2}{g}} \quad (ii)$$

From equations (i) and (ii), we have $\frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$

This expression conveys that the time of fall is independent of the mass.

Example 4.7 A child throws a ball up with an initial velocity of 20 ms^{-1} . Find out the maximum height that the ball can achieve and how long will it take to come back to the child's hands?

Sol. Initial velocity = 20 m/s , final velocity = 0 . The final velocity is zero as ball must be temporarily at rest at the maximum height. We take the point of projection as the origin and the upward direction as positive direction.

We need [Maximum height = s = ?, Time = ?]

Formula used : $v^2 - u^2 = 2as$

which gives $0^2 - 20^2 = 2(-10)s \Rightarrow s = 20 \text{ m}$

Here we have taken $a = -g = -10 \text{ ms}^{-2}$ because the upward direction is positive. So the maximum height attained is 20 m .

To find time, formula used: $v = u + at$. Let us use this formula from bottom to top.

It gives: $0 = 20 - 10t \Rightarrow t = 2 \text{ s}$. It means the ball went up to the top in 2 s . For motion under gravity the time of ascent and the time of descent are exactly equal. (Provided air resistance is negligible.) So the total time of travel = $(2 + 2) \text{ s} = 4 \text{ s}$.

Example 4.8 A balloon is at a height of 40 m and is ascending with a velocity of 10 ms^{-1} . A bag of 5 kg weight is dropped from it. When will the bag reach the surface of the earth? Given $g = 10 \text{ ms}^{-2}$.

Sol. Here we can take point of dropping as origin and downward direction as positive. Given, $h = +40 \text{ ms}^{-1}$, $u = -10 \text{ ms}^{-1}$, $a = g = +10 \text{ ms}^{-2}$



Fig. 4.53

The initial velocity of the bag will be same as that of the velocity of balloon. As it moves in opposite direction of the balloon, so it will have negative sign with it. As we have values of h , u and g , so time can be obtained by the distance formula.

$\therefore$ formula used: $\vec{S} = \vec{ut} + \frac{1}{2} \vec{at}^2$ (where symbols have their usual meaning).

$$\text{So } 40 = -10t + \frac{1}{2} 10t^2 \Rightarrow 5t^2 - 10t - 40 = 0$$

$$\Rightarrow 5t^2 - 20t + 10t - 40 = 0$$

$$5t(t - 4) + 10(t - 4) = 0 \Rightarrow (t - 4)(5t + 10) = 0$$

$$\Rightarrow t = 4 \text{ s and } -2 \text{ s}$$

As time cannot be negative, so time taken by body to reach the ground is $t = 4 \text{ s}$.

Example 4.9 Two trains P and Q are moving on parallel tracks with a uniform speed of 20 ms^{-1} . The driver of train P decides to overtake train Q and accelerates the

train by 1 ms^{-1} . After 50 s , train P crosses the engine of the train Q . Find out what was the distance between the trains initially, provided the length of each train is 400 m .

Sol. Let the initial distance between two trains = $x \text{ m}$

First of all we need to find out the total distance the two trains travelled in 50 s as the difference of these two distances must be equal to the initial distance between the two trains plus the sum of length of the two trains.

$$\text{Distance travelled by train } P, S_P = ut + \frac{1}{2} at^2$$

$$\Rightarrow 20 \times 50 + \frac{1}{2} \times 1 \times 50^2 = 2250 \text{ m}$$

Distance travelled by train Q ,

$$S_Q = ut + \frac{1}{2} at^2 = 20 \times 50 + \frac{1}{2} \times 0 \times 50^2 = 1000 \text{ m}$$

$$\text{Now, } S_P - S_Q = 2250 - 1000 = x + 800$$

$$\Rightarrow x = 450 \text{ m}$$

Example 4.10 A ball is dropped from the top of a high building at $t = 0$. At a later time $t = t_0$, a second ball is thrown downward with initial speed v_0 . Obtain an expression for the time t at which the two balls meet.

Sol. Let the distance covered by the first ball = y_1 (for vertical distance symbol y is preferred).

Similarly the distance covered by the second ball = y_2 .

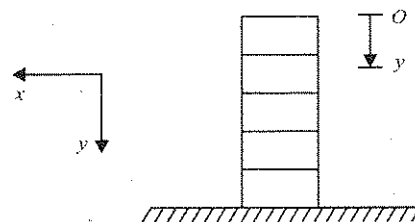


Fig. 4.54

We can use the condition that at a particular time (as given) distances covered by both the balls are same. But the caution is that the first ball has covered y_1 in t_0 time and second ball has covered y_2 in $(t - t_0)$ time.

Let us assume that the origin be at the top of the building with downward direction positive. If the two balls meet at time t , then

$$\text{For } y_1, t = t_0, v_0 = 0, a = g \Rightarrow y_1 = \frac{1}{2} g t_0^2$$

$$\text{For } y_2, t = t - t_0, \text{ initial speed} = v_0, a = g$$

$$\Rightarrow y_2 = v_0(t - t_0) + \frac{1}{2} g (t - t_0)^2$$

From the given condition,

$$y_1 = y_2 \Rightarrow \frac{1}{2} g t_0^2 = v_0(t - t_0) + \frac{1}{2} g (t - t_0)^2$$

After simplifying and solving, we get

$$\Rightarrow t = \left[\frac{v_0 - g t_0 / 2}{v_0 - g t_0} \right] t_0$$

Example 4.11 A balloon starts rising upward with a constant acceleration a and after time t_0 second, a packet

is dropped from it which reaches the ground after t second (Fig. 4.55). Determine the value of t .

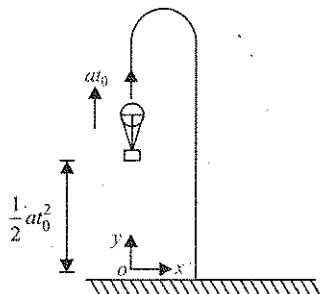


Fig. 4.55

Sol. Analysis of situation: $t = 0$ is the time when the balloon started rising up. At $t = t_0$ when the packet is dropped, the balloon is moving up with velocity $v = 0 + at_0 = at_0$. Hence initial velocity of the packet will be $v_0 = at_0$ (upward). As the balloon has started rising upwards with constant acceleration, a , so after t_0 seconds its height from the ground is $y_0 = \frac{1}{2}at_0^2$.

For packet: $s = ut - \frac{1}{2}gt^2 \Rightarrow -\frac{1}{2}at_0^2 = at_0t - \frac{1}{2}gt^2$

$$\Rightarrow gt^2 - 2at_0t - at_0^2 = 0$$

Solving the quadratic equation, we get

$$t = \frac{at_0}{g} \left[1 + \sqrt{1 + \frac{g}{a}} \right]$$

Example 4.12 A particle, moving with uniform acceleration from A to B along a straight line, has velocities v_1 and v_2 at A and B , respectively. If C is the mid-point between A and B then determine the velocity of the particle at C .

Sol. Let v be the velocity of the particle at C . Assume acceleration of the particle to be a and distance between A and B to be x .

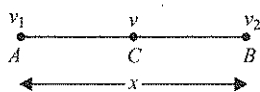


Fig. 4.56

To find the velocity at point C , we can easily express the desired result in terms of given and assumed quantities, i.e., v_1 , v_2 and x . The formula that easily relates these quantities is $v^2 - u^2 = 2as$.

Now from A to C : $v^2 - v_1^2 = 2a \frac{x}{2}$

From C to B : $v_2^2 - v^2 = 2a \frac{x}{2}$

$$\text{Solve to get: } v = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

Example 4.13 A ball, thrown from the top of a building which is 50 m high, is given an initial velocity of 20.0 ms^{-1} straight upward. On its backward journey the ball just misses the edge of the roof on its way down, as shown in Fig. 4.57. Considering the position (of projection of the ball) as origin and time at $A(t_A) = 0$, determine (a) the time

at which the ball reaches its maximum height, (b) the maximum height, (c) the time at which the ball returns to the height from which it was thrown, (d) the velocity of the ball at this instant, (e) the velocity and position of the ball at $t = 5.00$ s.

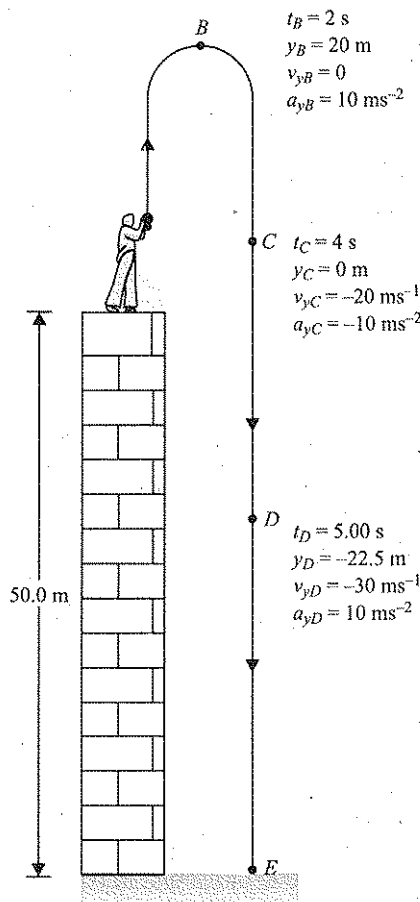


Fig. 4.57

Sol. (a) Analysis of situation: At position (B) velocity of the ball must be zero. It means velocity has changed by 20 m/s as it is momentarily at rest at (B) . Whenever an object goes up in the air, it is under the negative effect of gravity; its velocity decreases by 10 m/s in each second (since the retardation or deceleration is 10 ms^{-2}). So the ball must take 2 s to move from (A) to (B) .

Formula used: $v_{yB} = v_{yA} + a_y t$ [As we have to find time with given initial and final velocities].

$v_{yB} = 0$ (velocity at B), $v_{yA} = 20 \text{ m/s}$ (velocity at A),

$$a_y = -g = -10 \text{ m/s}^2, t = t_B = ?$$

$$\therefore 0 = 20 - 10t \Rightarrow t = 2 \text{ s.}$$

(b) Analysis of situation: The ball is thrown from (A) . At (B) the velocity of the ball becomes zero. The time taken to reach from (A) to (B) is 2 s. Hence the length AB will be equal to maximum height.

Formula to be used is $y = y_0 + v_0 t + \frac{1}{2}a_y t^2$

Here it will be in the following form:

$$y_{\text{max}} = y_B = y_A + v_{yA}t + \frac{1}{2}a_y t^2$$

Substituting values, $y_A = 0$, $v_{yA} = 20 \text{ ms}^{-1}$,

$$a_y = -g = -10 \text{ m/s}^2, t = 2 \text{ s},$$

$$\text{we have } y_B = 0 + (20.0)(2) + \frac{1}{2}(-10)(2)^2 = (40 - 20) = 20 \text{ m}$$

Alternatively:

Because the average velocity for this first interval is 10 m/s (the average of 20 m/s and 0 m/s) and the ball travels for about 2 s , we expect the ball to travel about 20 m .

(c) There is no reason to believe that the ball's motion from (B) to (C) is anything other than the reverse of its motion from (A) to (B). The motion from (A) to (C) is symmetric. Thus, the time needed for it to go from (A) to (C) should be twice the time needed for it to go from (A) to (B). Note that A and C are at same level. Formula to be used is $y = y_0 + v_0 t + \frac{1}{2} a_y t^2$.

Where $y = y_C = 0$, $y_0 = y_A = 0$, $v_{yA} = 20 \text{ m/s}$,
 $a_y = -g = -10 \text{ m/s}^2$, $t = t_C$

$$y_C = y_A + v_{yA} t + \frac{1}{2} a_y t^2$$

Substituting these values, we have $0 = 0 + 20.0 t_C - 5 t_C^2$

This is a quadratic equation and so there must be two solutions for $t = t_C$. The equation can be factored to give $t_C(20.0 - 5t_C) = 0$

One solution is $t_C = 0$, corresponding to the time the ball starts its motion. The other solution is $t_C = 4 \text{ s}$. When the ball is back at the height from which it was thrown (position C), the y coordinate is again zero.

Note: It is double the value we calculated for t_B .

(d) **Analysis of Situation:** Again, we expect everything at (C) to be the same as it is at (A), except that the velocity is now in the opposite direction. The value of t found in (C) can be inserted into equation $v_f = v_i + at$ to give

Formulae: $v_{yC} = v_{yA} + a_y t = 20.0 + (-10 \times 4) = -20.0 \text{ m/s}$

The velocity of the ball when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction (indicated by $-ve$ sign).

(e) **Analysis of Situation:** To find velocity and position at $t = 5.00 \text{ s}$, we can use any position during motion as the initial condition provided we have required values of velocity and time at that position. We can also consider B as our new initial position besides A.

Case (i) When B is the initial position, then again using $v_y = u_y + a_y t$

Time of travel $= (5.00 - 2) \text{ s} = 3 \text{ s}$ and $v_{yB} = 0$,

$$a_y = -g = -10 \text{ m/s}^2$$

$$v_{yD} = v_{yB} + a_y t \Rightarrow v_{yD} = -10 \times 3 = -30 \text{ m/s}$$

Case (ii) When A is initial position,

then $t = 5.00 \text{ s}$, $v_{yA} = 20 \text{ m/s}$, $a_y = -g = -10 \text{ m/s}^2$

$$v_{yD} = v_{yA} + a_y t = 20 - 10 \times 5 = -30 \text{ m/s}$$

Position of ball at $t_D = 5.00 \text{ s}$ w.r.t. $t_A = 0$, let us further extend the idea of choosing initial time. Here we can consider t_C as our new initial time.

So time of travel between C and D $= 5 - 4 = 1 \text{ s}$, $y_C = 0$

$$a = -g = -10 \text{ m/s}^2, \quad v_{yC} = -20 \text{ m/s}$$

$$\begin{aligned} \Rightarrow y_D &= y_C + v_{yC} t + \frac{1}{2} a_y t^2 \\ &= 0 - 20(1) + \frac{1}{2}(-10)(1)^2 = -20 - 5 = -25 \text{ m} \end{aligned}$$

Example 4.14 Plot the graphs for the variation of the velocity with time in each of the following cases:

1. when the acceleration has constant value.
2. when the acceleration increases with time.
3. when the acceleration decreases with time.

Sol.

1. When the acceleration has a constant value, then velocity will change linearly with time. Hence graph is a straight line starting from origin.

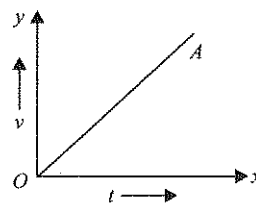


Fig. 4.58

2. When acceleration is increasing with time, the slope of $V-T$ graph will also increase with time. Due to this, the graph will be a concave upward curve (Fig. 4.59).

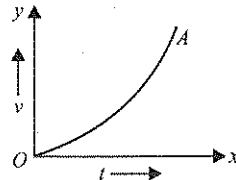


Fig. 4.59

3. When acceleration is decreasing with time, the slope of $V-T$ graph also decreases accordingly. Hence the $V-T$ graph will be a concave downward curve (Fig. 4.60).

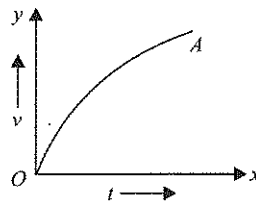


Fig. 4.60

Example 4.15 An object is in uniform motion along a straight line. What will be the position-time graph for the motion of the object if:

1. $x_0 = +ve$, $v = +ve$
2. $x_0 = +ve$, $v = -ve$
3. $x_0 = -ve$, $v = +ve$

4. both x_0 and v are negative? The letters x_0 and v represent position of the object at time $t = 0$ and uniform velocity of the object, respectively.

Sol. The equation of motion of an object at any time t moving with uniform velocity along a straight line is given by $x = x_0 + vt$.

1. When $x_0 = +ve$, $v = +ve$

In this case x will take positive values with the passage of time, so the graph has to be a straight line in first quadrant.

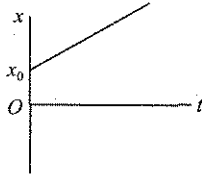


Fig. 4.61

2. When $x_0 = +ve$, $v = -ve$

Here with increasing value of t , the value of x will go on decreasing and will take negative values. So graph is a straight line starting from highest value of x in question and will move on into fourth quadrant.

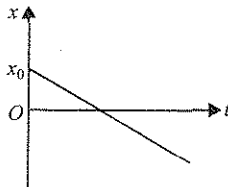


Fig. 4.62

3. When $x_0 = -ve$ and $v = +ve$

In this case, value of x will go on increasing with time as (vt) will go on taking higher value. So the graph is straight line that will start from most negative value of x in question and will cross over to first quadrant where x has $+ve$ values.

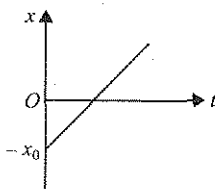


Fig. 4.63

4. When $x_0 = -ve$ and $v = -ve$

Values of x will go on decreasing with time x_0 and vt both will result in negative values. So the straight line will start from a negative value of x_0 and will keep on taking negative values in the fourth quadrant.

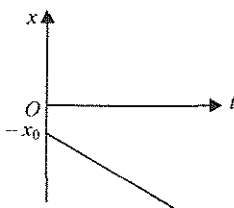


Fig. 4.64

Example 4.16 Consider the following v_x-t graph to be parabolic. Plot acceleration-time graph and analyse the motion of the particle from A to E.

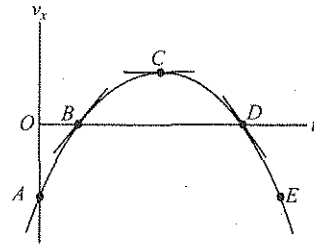


Fig. 4.65

Sol. Acceleration-time graph is as follows: We know that slope of velocity-time graph is equal to acceleration. Slope at A is maximum, decreases linearly and becomes zero at C and then starts increasing in negative side and becomes maximum negative at E.

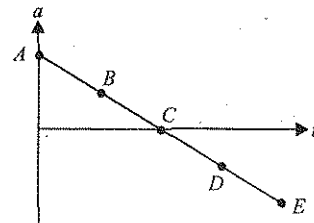
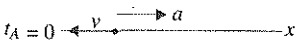
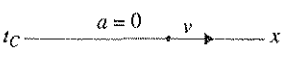


Fig. 4.66

So, at A acceleration is maximum, at C zero and at E acceleration is negative maximum.

Analysis of motion:

A	$v_x < 0$; positive slope, so $a_x > 0$	moving in $-x$ direction, slowing down	$t_A = 0$ 
B	$v_x = 0$; positive slope, so $a_x > 0$	instantaneously at rest, about to move in $+x$ direction	t_B 
C	$v_x > 0$; zero slope, so $a_x = 0$	moving in $+x$ direction, at maximum speed	t_C 
D	$v_x = 0$; negative slope, so $a_x < 0$	instantaneously at rest, about to move in $-x$ direction	t_D 
E	$v_x < 0$; negative slope, so $a_x < 0$	moving in $-x$ direction, speeding up	t_E 

1. Between points A and B

Acceleration is positive and decreasing in magnitude. Also magnitude of velocity is decreasing, because velocity and acceleration are in opposite direction.

2. At point B

The velocity is instantaneously zero at this moment. So particle is at rest momentarily. But the particle is still accelerating as the slope has non-zero value. Acceleration is positive and decreasing in magnitude. From here the particle will start moving along positive x direction.

3. Between points B and C

Velocity is positive and increasing. So the acceleration is positive but decreasing in magnitude.

4. At point C

Velocity at point C is maximum as graph peaks at this point. At this point slope is zero, therefore the acceleration is zero.

5. Between points C and D

Velocity goes on decreasing between C and D. Due to negative slope acceleration is also negative. Acceleration is increasing in magnitude. Velocity and acceleration are in opposite direction.

6. At point D

Velocity is zero at this point. But slope is still negative, so the acceleration is negative. Particle will start moving in negative direction. Acceleration is increasing in magnitude.

7. Between points D and E

Particle is moving in -ve direction. The slope of graph is negative in this region. It means the acceleration is negative; but magnitude of slope is increasing, it means the magnitude of acceleration is increasing. Velocity and acceleration are in same direction.

Example 4.17 Consider the following $x-t$ graph to be parabolic. Draw velocity-time graph and acceleration-time graph and analyse the motion of the particle regarding its velocity and acceleration.

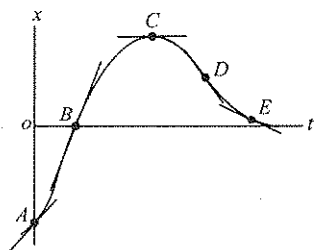


Fig. 4.67

Sol. Velocity-time graph and acceleration-time graph are as shown:

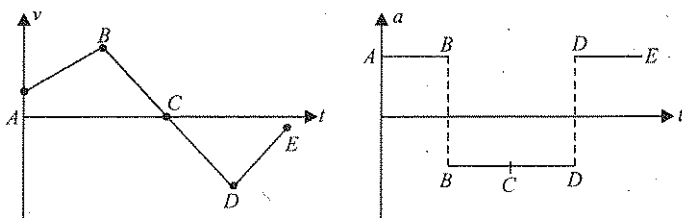


Fig. 4.68

Analysis of motion:

1. Between points A and B:

Between these points, x has negative value, it means particle is to the left of origin. But value of x is decreasing, so the particle is moving towards origin. It means velocity of the particle is positive.

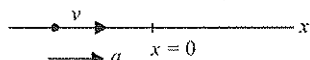


Fig. 4.69

Between these points, slope of $x-t$ graph is increasing, it means magnitude of velocity is increasing. It means velocity and acceleration both are in same direction. So acceleration is positive. Note that acceleration is constant between A and B.

Also the graph is concave up, so the acceleration is positive.

2. At point B :

Particle is at origin at this point. At this point slope of $x-t$ graph is maximum, so velocity is maximum at this point. Before point B acceleration is positive, but after point B acceleration will be negative as the slope of $x-t$ graph will start decreasing after this (Fig. 4.70).

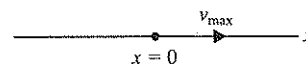


Fig. 4.70

We cannot define acceleration at point B.

3. Between points B and C:

Between these points, x has positive value, it means particle is to the right of origin. But value of x is increasing, so the particle is moving away from origin. It means velocity of the particle is positive.

Between these points, slope of $x-t$ graph is decreasing, it means magnitude of velocity is decreasing. It means velocity and acceleration are in opposite directions. So acceleration is negative. Note that acceleration is constant between B and C.

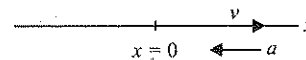


Fig. 4.71

Also the graph is concave down, so acceleration is negative.

4. At point C:

Particle is maximum away from origin at this point. At this point slope of $x-t$ graph is zero, so velocity is zero. But acceleration is negative as the graph is concave down. At this point, the particle will change its direction of motion.

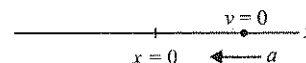


Fig. 4.72

So the particle will start moving towards origin.

5. Between points C and D:

Between these points, x has positive value, it means particle is to the right of origin. But value of x is decreasing, so the particle is moving towards origin. It means velocity of the particle is negative.

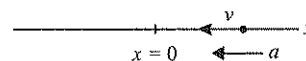


Fig. 4.73

Between these points, magnitude of slope of $x-t$ graph is increasing, it means magnitude of velocity is increasing, but in negative direction. So acceleration is negative. Note that acceleration is constant between C and D .

Also the graph is concave down, so acceleration is negative.

6. At point D :

Particle is to the right of origin at this point. At this point slope of $x-t$ graph is negative maximum, so velocity is maximum at this point in negative direction. Before point D acceleration is negative, but after point D acceleration will be positive (as the graph is concave up after D). We cannot define acceleration at point D .

7. Between points D and E :

Between these points, x has positive value, it means particle is to the right of origin. But value of x is decreasing, so the particle is moving towards origin. It means velocity of the particle is negative.

Between these points, magnitude of slope of $x-t$ graph is decreasing, it means magnitude of velocity is decreasing, but in negative direction. So acceleration is positive. Note that acceleration is constant between D and E .

Also the graph is concave up, so acceleration is positive.

Example 4.18 A rubber ball is released from a height of about 1.5 m. It is caught after three bounces. Sketch graphs of its position, velocity and acceleration as functions of time. Provided positive y -direction as upward direction.

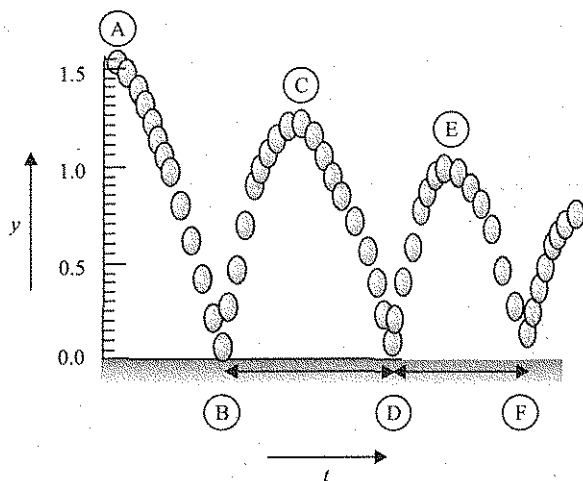


Fig. 4.74

Sol.

Checks the following points:

1. Position and time graph will have the same shape as given graph.
2. Slope of position-time graph gives velocity. Hence change in velocity can be observed as per the slope of position-time graph.
3. Slope of velocity-time graph gives acceleration.
4. Acceleration in free fall remains constant with time.

Note:

- Velocity is changing at points B, C, D, E, F . At B, D, F velocity changes suddenly from negative to positive and at C, E velocity changes smoothly from positive to negative.
- At B, D, F velocity changes very quickly so the acceleration must be very large.

Position-time graph: The diagram given itself conveys position-time graph (Fig. 4.75).

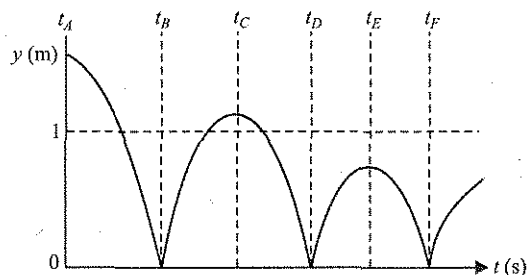


Fig. 4.75

Velocity-time graph: As the slope of $v-t$ graph changes thrice from negative to positive during the bounces, so $V-t$ graph must observe sharp changes at these points (Fig. 4.76).

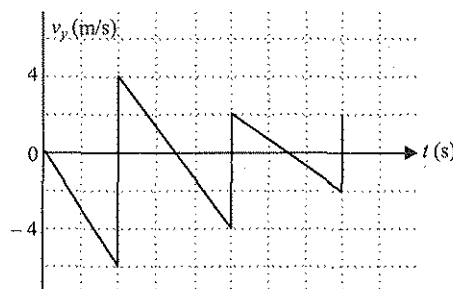


Fig. 4.76

Acceleration-time graph: Slope of $V-t$ graph remains same (-10 m/s^2) till bounces as velocity holds, i.e., it has single value during free fall. At the time of contact with floor it changes substantially during a very short time interval. So the acceleration will be large and positive which has to be represented in the form of straight upward lines (Fig. 4.77).

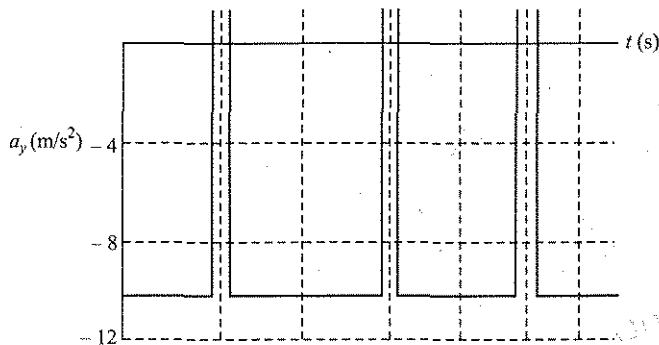


Fig. 4.77

EXERCISES

Subjective Type

Solutions on page 4.46

1. Fig. 4.78 shows a particle starting from point A, travelling upto B with a speed s , then upto point C with a speed $2s$ and finally upto A with a speed of $3s$. Determine its average speed.

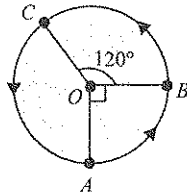


Fig. 4.78

2. A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speeds of 4.5 m/s and 7.5 m/s, respectively. Find the average speed of the particle during this motion.
3. Find the ratio of the distance moved by a freely falling body from rest in 4th and 5th second of its journey.
4. Two balls of different masses (one lighter and other heavier) are thrown vertically upwards with the same speed. Which one will pass through the point of projection in their downward direction with the greater speed?
5. A car runs at a constant speed on a circular track of radius 200 m, taking 62.8 s on each lap. Find the average velocity and average speed on each lap.
6. A train accelerates from the rest for time t_1 at a constant rate α and then it retards at the constant rate β for time t_2 and comes to rest. Find the ratio: t_1/t_2 .
7. An athlete swims the length of 50 m pool in 20 s and makes the return trip to the starting position in 22 s. Determine his average velocity in
- the first half of the swim.
 - the second half of the swim.
 - the round trip.
8. A train stops at two stations d distance apart and takes time t on the journey from one station to the other. Assuming that its motion is first of uniform acceleration α and then immediately of uniform retardation β , show that
- $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{t^2}{2d}.$$
9. The speed of a train increases at a constant rate α from zero to v and then remains constant for an interval and finally decreases to zero at a constant rate β . If l be the total distance described, prove that the total time taken is

$$\frac{l}{v} + \frac{v}{2} \left[\frac{1}{\alpha} + \frac{1}{\beta} \right].$$

10. A train of 150 m length is going towards north direction at a speed of 10 m/s. A bird flies at a speed of 5 m/s towards south direction parallel to the railway track. Find the time taken by the bird to cross the train.
11. A car starts moving with constant acceleration and covers the distance between two points 180 m apart in 6 s. Its speed as it passes the second point is 45 m/s. Find
- its acceleration.
 - its speed when it was at the first point.
 - the distance from the first point when it was at rest.
12. From a lift moving upward with a uniform acceleration a , a man throws a ball vertically upwards with a velocity v relative to the lift. The ball comes back to the man after a time t . Show that $a + g = 2v/t$.
13. A balloon is ascending vertically with an acceleration of 1 m/s^2 . Two stones are dropped from it at an interval of 2 s. Find the distance between them 1.5 s after the second stone is released.
14. A train starts from station A with uniform acceleration a_1 for some distance and then goes with uniform retardation a_2 for some more distance to come to rest at station B. The distance between stations A and B is 4 km and the train takes 1/15 h to complete this journey. If accelerations are in km per minute unit then show that: $\frac{1}{a_1} + \frac{1}{a_2} = 2$.
15. A stone is let to fall from a balloon ascending with an acceleration f . After t time a second stone is dropped. Prove that the distance between the stones after time t' since the second stone is dropped is $\frac{1}{2}(f + g)t(t + 2t')$.
16. A stone falling from the top of a vertical tower has descended x m when another is let to fall from a point y m below the top. If they fall from rest and reach the ground together, show that the height of tower is $\frac{(x + y)^2}{4x}$ m.
17. Divide a plane 10 m long and 5 m high into three parts so that a body starting from rest takes equal time to slide down these parts. Also find the time taken then.
18. The driver of a car moving at 30 m/s suddenly sees a truck that is moving in the same direction at 10 m/s and is 60 m ahead. The maximum deceleration of the car is 5 m/s^2 .
- Will the collision occur if the driver's reaction time is zero? If so, when?
 - If the car driver's reaction time of 0.5 s is included, what is the minimum deceleration required to avoid the collision?
19. A steel ball is dropped from the roof of a building. A man standing in front of a 1 m high window in the building notes that the ball takes 0.1 s to fall from the top to bottom of the window. The ball continues to fall and strike the ground. On striking the ground the ball gets rebounded with the same speed with which it hits the ground. If the ball reappears at the bottom of the window 2 s after passing the bottom of the window on the way down, find the height of the building.

20. A particle is dropped from the top of a tower h m high and at the same moment another particle is projected upward from the bottom. They meet when the upper one has descended a distance h/n . Show that the velocities of the two when they meet are in the ratio $2:(n-2)$ and that the initial velocity of the particle projected up is $\sqrt{(1/2)ngh}$.
21. An elevator whose floor to the ceiling distance is 2.50 m, starts ascending with a constant acceleration of 1.25 m/s^2 . One second after the start, a bolt begins falling from the elevator. Calculate:
- free fall time of the bolt.
 - the displacement and distance covered by the bolt during the free fall in the reference frame of ground.
22. In a car race, car A takes a time t less than car B at the finish and passes the finishing point with a velocity v more than the car B . Assuming that the cars start from rest and travel with constant accelerations a_1 and a_2 , respectively, show that $v = (\sqrt{a_1 a_2}) t$.
23. A stone is dropped from the top of a cliff of height h . n second later a second stone is projected downward from the same cliff with a vertically downward velocity u . Show that the two stones will reach the bottom of the cliff together, if $8h(u - gn)^2 = gn^2(2u - gn)^2$.
What can you say about the limiting value of n ?
24. A particle moving in a straight line is observed to be at a distance 'a' from a marked point initially, at a distance 'b' after an interval of n seconds, to be at a distance 'c' after $2n$ seconds and at a distance 'd' after $3n$ seconds. Prove that if the acceleration is uniform, $d - a = 3(c - b)$ and that the acceleration is equal to $\frac{c + a - 2b}{n^2}$.
25. Two motor cars start from A simultaneously and reach B after 2 h. The first car travelled half the distance at a speed of 30 kmh^{-1} and the other half at a speed of 60 kmh^{-1} . The second car started from rest and covered the entire distance with a constant acceleration. At what instant of time were the speeds of both the vehicles same? Will one of them overtake the other?
26. A train of length $l = 350 \text{ m}$ starts moving rectilinearly with constant acceleration $a = 3.0 \times 10^{-2} \text{ ms}^{-2}$. $t = 30 \text{ s}$ after start, the locomotive headlight is switched on (event 1), and 60 s after this event the tail signal light is switched on (event 2).
- Find the distance between these events in the reference frame fixed to the train and to the earth.
 - How and at what constant velocity v relative to the earth must a certain reference frame R move for the two events to occur in it at the same point?
27. a. Can position–time graph be a straight line parallel to position axis? (Yes/No)
b. Area enclosed by velocity–time graph is equal to _____ (displacement/distance).
c. Area enclosed by speed–time graph is equal to _____ (displacement/distance).
d. For a one dimensional motion, a velocity–time graph is shown in Fig. 4.79. Convert this graph into a speed–time graph.

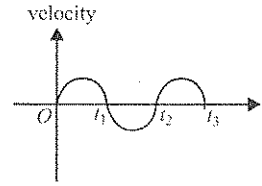


Fig. 4.79

- The slope of position/displacement–time graph is equal to _____ (Velocity/Speed).
- The slope of distance–time graph is equal to _____ (Velocity/Speed).
- A position–time graph is shown. Is it possible practically? (Yes/No)

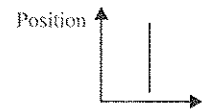
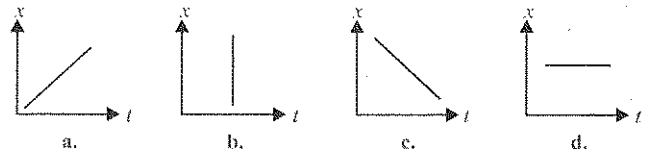


Fig. 4.80

- h. Which of the following position–time graphs represent negative velocity?



- i. A position–time graph is shown in Fig 4.81. What does the slope of line segment PQ indicate. (instantaneous velocity/Average velocity/Nothing)

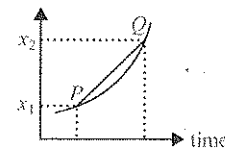
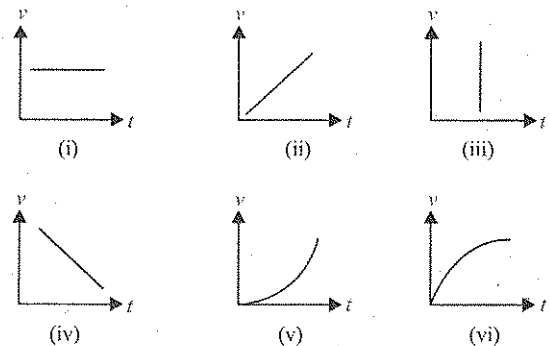
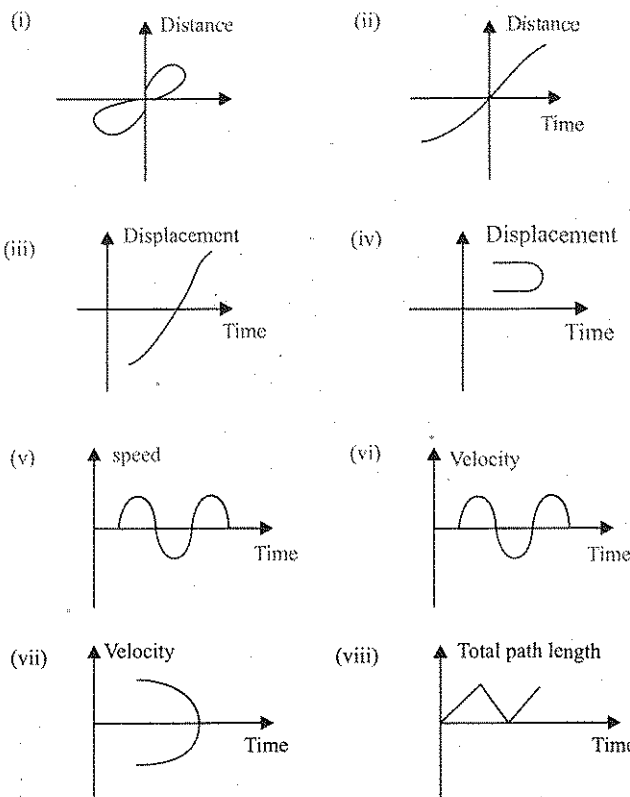


Fig. 4.81

- j. What can you say about acceleration in each of the following velocity–time graphs?



28. You are given some graphs in below. Look at them carefully and find out which of the graphs will represent the one-dimensional motion.



29. Figure 4.82 given below shows the displacement–time graph for a particle moving along a straight line path.

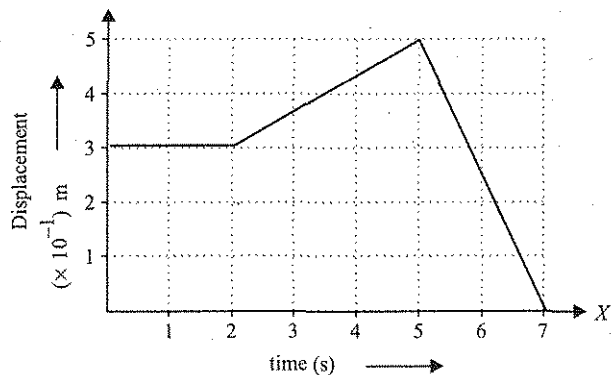


Fig. 4.82

State True or False.

- Time during which the particle was at rest is 0 s to 2 s.
 - The maximum velocity of the particle is -2.5 m/s^2 .
30. A position–time graph for a particle moving along the x axis is shown in Fig. 4.83. State True or False.
- The average velocity in the time interval $t = 1.5 \text{ s}$ to $t = 4.0 \text{ s}$ is -2.5 m/s .
 - The instantaneous velocity at $t = 2.00 \text{ s}$ by measuring the slope of the tangent line shown in the graph is -7.4 m/s . The velocity is zero at $t = 4 \text{ s}$.
31. You are given the position–time graph of three different bodies A, B, and C (Fig. 4.84). Find which will have greater velocity and which will have least velocity.

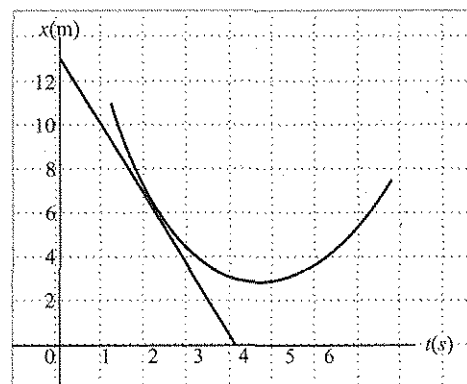


Fig. 4.83

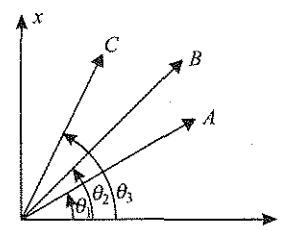


Fig. 4.84

32. The diagram given below represents the motion of a biker in the form of position–time graph (Fig. 4.85).

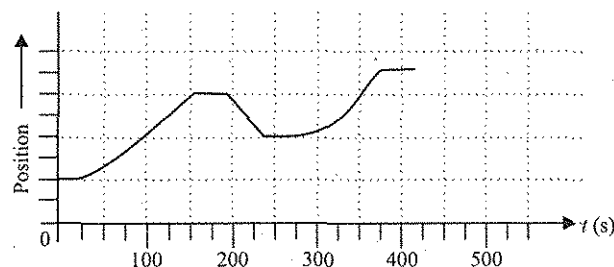


Fig. 4.85

- The time intervals during which the biker was stationary are _____.
 - The time intervals during which the bike is moving in the positive x direction are _____.
 - The time interval for which bike is moving in negative x direction is _____.
33. A physics professor leaves her house and walks along the sidewalk towards campus. After 5 min it starts raining and she returns home. Her distance from her house as a function of time is shown in Fig. 4.86.

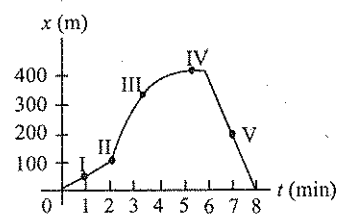


Fig. 4.86

At which of the labelled points is her velocity

- zero?
- constant and positive.
- constant and negative.
- increasing in magnitude.
- decreasing in magnitude.

34. The graph shown in Fig. 4.87 shows the velocity of a police officer's motorcycle plotted as a function of time.

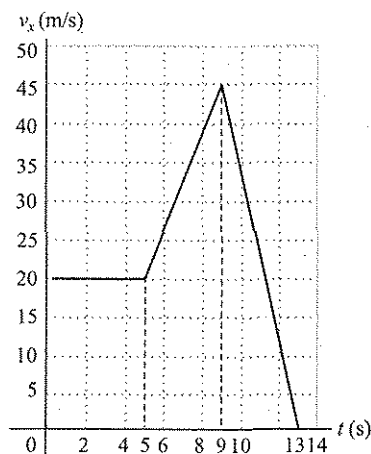


Fig. 4.87

- The instantaneous accelerations at $t = 3$ s, at $t = 7$ s, and at $t = 11$ s are _____.
 - The distances covered by the officer in the first 5 s, first 9 s and first 13 s are _____.
35. A cat walks in a straight line, which we shall call the x -axis with the positive direction to the right. As an observant physicist, you make measurements of this cat's motion and construct a graph of the cat's velocity as a function of time (as shown in Fig. 4.88).

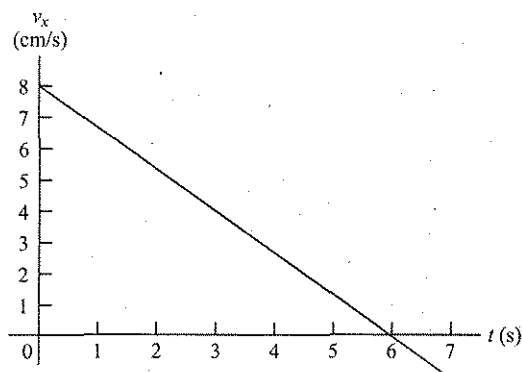


Fig. 4.88

- The cat's velocities at $t = 4.0$ s and at $t = 7.0$ s are _____.
 - The cat's acceleration at $t = 3$ s is _____.
 - The distance that the cat moves during the first 4.5 s and from $t = 0$ to $t = 7.5$ s are _____.
 - Sketch clear graphs of the cat's acceleration and position as functions of time, assuming that the cat started at the origin.
36. Starting at $x = 0$, a particle moves according to the graph of v vs. t shown in Fig. 4.89. Sketch a graph of the instantaneous

acceleration a vs. t , indicating numerical values at significant points of the graph.

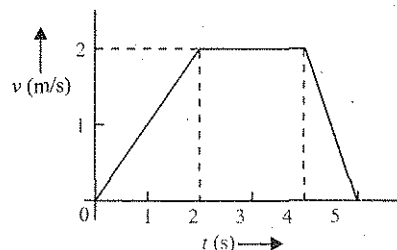


Fig. 4.89

37. The velocity-time graph of a particle moving in a straight line is shown in the Fig. 4.90 given below. Find the displacement and the distance travelled by the particle in 6 s.

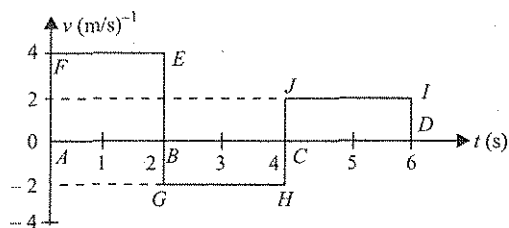


Fig. 4.90

38. Fig. 4.91 shows a graph of the acceleration of a model railroad locomotive moving on the x -axis. Graph its velocity and coordinate as functions of time if $x = 0$ and $v_x = 0$ at $t = 0$.

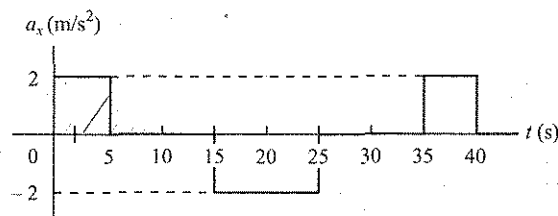


Fig. 4.91

- A woman starts from her home at 9.00 a.m., walks with a speed of 5 km/h on a straight road up to her office 2.5 km away, stays at the office up to 5.00 p.m. and returns home by an auto with a speed of 25 km/h. Plot the position-time graph of the woman taking her home as origin.
- A runner jogs along a straight road (the $+x$ direction) for 30 min, travelling a distance of 6 km. She then turns around and walks back towards her starting point for 20 min, travelling 2 km during this time. State True or False:
 - Final displacement of the runner relative to her starting position is 4 km
 - Her average speed for the entire trip is 0.16 km/m.
 - The average velocity for the entire trip is 0.16 km/m.
 - The runner's average velocity while jogging is 0.4 km/m.
 - Her average velocity while walking is 0.1 km/m.
- At the instant the traffic light turns green, a car that has been waiting at an intersection starts ahead with a constant acceleration of 3.20 m/s^2 . At the same instant a truck, travelling

with a constant speed of 20.0 m/s, overtakes and passes the car.

- The car overtakes the truck at a distance _____ from its starting point.
 - Speed of the car when it overtakes the truck _____.
 - Sketch an $x-t$ graph of the motion of both vehicles. Take $x = 0$ at the intersection.
 - Sketch a v_x-t graph of motion of both the vehicles.
42. A particle moves along the x axis. Its x coordinate varies with time according to the expression $x = -4t + 2t^2$, where x is in m and t is in s. The position-time graph for this motion is shown in Fig. 4.92. Note that the particle moves in the negative x direction for the first second of motion, is at rest at moment $t = 1$ s and then heads back in the positive x direction for $t > 1$ s.

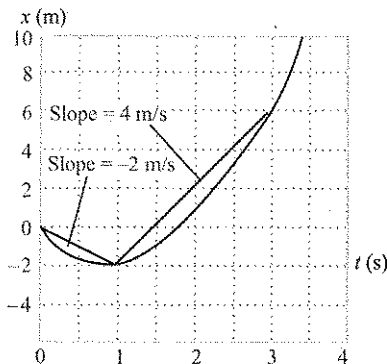


Fig. 4.92

- Displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s are _____.
 - The average velocity in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s are _____.
 - Instantaneous velocity of the particle at $t = 2.5$ s is _____.
43. The acceleration of a particle varies with time as shown in Fig. 4.93.

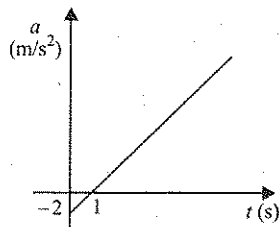


Fig. 4.93

- Find an expression for velocity in terms of t . Assume that $v = 0$ at $t = 0$.
- Calculate the displacement of the particle in the time interval from $t = 2$ s to $t = 4$ s.

Objective Type

Solutions on page 4.52

- cannot be zero
 - depends upon the particle
- If the distance covered is zero, then displacement
 - must be zero
 - may or may not be zero
 - cannot be zero
 - depends upon the particle
- The numerical value of the ratio of average velocity to average speed is
 - always less than one
 - always equal to one
 - always more than one
 - equal to or less than one
- The numerical value of the ratio of instantaneous velocity to instantaneous speed is
 - always less than one
 - always equal to one
 - always more than one
 - equal to or less than one
- A body moves 4 m towards east and then 3 m north. The displacement and distance covered by the body are
 - 7 m, 6 m
 - 6 m, 5 m
 - 5 m, 7 m
 - 4 m, 3 m
- The location of a particle is changed. What can we say about the displacement and distance covered by the particle?
 - Both cannot be zero
 - One of the two may be zero
 - Both must be zero
 - Both must be equal
- The angle between velocity and acceleration during the retarded motion is
 - 180°
 - 40°
 - 45°
 - 0°
- A particle moves with uniform velocity. Which of the following statements about the motion of the particle is true?
 - Its speed is zero
 - Its acceleration is zero
 - Its acceleration is opposite to the velocity
 - Its speed may be variable.
- The magnitude of the displacement is equal to the distance covered in a given interval of time if the particle
 - moves with constant acceleration along any path
 - moves with constant speed
 - moves in same direction with constant velocity or with variable velocity
 - moves with constant velocity
- A moving body is covering the displacement directly proportional to the square of the time. The acceleration of the body is
 - increasing
 - decreasing
 - zero
 - constant
- The magnitude of average velocity is equal to the average speed when a particle moves
 - on a curved path
 - in the same direction
 - with constant acceleration
 - with constant retardation

- If displacement of a particle is zero, the distance covered
 - must be zero
 - may or may not be zero

12. If a particle moves with a constant velocity
- its acceleration is positive
 - its acceleration is negative
 - its acceleration is zero
 - its speed is zero
13. The displacement s of a particle is proportional to the first power of time t , i.e., $s \propto t$; then the acceleration of the particle is
- infinite
 - zero
 - a small finite value
 - a large finite value
14. The ratio of the average velocity of a train during a journey to the maximum velocity between two stations is
- $= 1$
 - > 1
 - < 1
 - $> \text{ or } < 1$
15. The distance travelled by a particle in a straight line motion is directly proportional to $t^{1/2}$, where t = time elapsed. What is the nature of motion?
- Increasing acceleration
 - Decreasing acceleration
 - Increasing retardation
 - Decreasing retardation
16. An athlete completes half a round of a circular track of radius R , then the displacement and distance covered by the athlete are
- $2R$ and πR
 - πR and $2R$
 - R and $2\pi R$
 - $2\pi R$ and R
17. If two balls of same density but different masses are dropped from a height of 100 m, then (neglect air resistance)
- both will come together on the earth
 - both will come late on the earth
 - first will come first and second after that
 - second will come first and first after that
18. If a body starts from rest, the time in which it covers a particular displacement with uniform acceleration is
- inversely proportional to the square root of the displacement
 - inversely proportional to the displacement
 - directly proportional to the displacement
 - directly proportional to the square root of the displacement
19. Check up only the correct statement in the following.
- A body has a constant velocity and still it can have a varying speed.
 - A body has a constant speed but it can have a varying velocity.
 - A body having constant speed cannot have any acceleration.
 - None of these.
20. The position x of a particle varies with time (t) as $x = at^2 - bt^3$. The acceleration at time t of the particle will be equal to zero, where t is equal to
- $\frac{2a}{3b}$
 - $\frac{a}{b}$
 - $\frac{a}{3b}$
 - zero
21. A person travels along a straight road for the first half time with a velocity v_1 and the second half time with a velocity v_2 . Then the mean velocity $\bar{v}$ is given by
- $\bar{v} = \frac{v_1 + v_2}{2}$
 - $\frac{2}{\bar{v}} = \frac{1}{v_1} + \frac{1}{v_2}$
 - $\bar{v} = \sqrt{v_1 v_2}$
 - $\bar{v} = \sqrt{\frac{v_2}{v_1}}$
22. A person travels along the straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2 . Then average velocity is given by
- $v_1 v_2$
 - $\frac{v_2^2}{v_1^2}$
 - $\frac{(v_1 + v_2)}{2}$
 - $\frac{2v_1 v_2}{(v_1 + v_2)}$
23. A body covers one-third of the distance with a velocity v_1 , the second one-third of the distance with a velocity v_2 and the remaining distance with a velocity v_3 . The average velocity is
- $\frac{v_1 + v_2 + v_3}{3}$
 - $\frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$
 - $\frac{v_1 v_2 + v_2 v_3 + v_3 v_1}{3}$
 - $\frac{v_1 v_2 v_3}{3}$
24. Between the two stations, a train accelerates from rest uniformly at first, then moves with constant velocity and finally retards uniformly to come to rest. If the ratio of the time taken be 1:8:1 and the maximum speed attained be 60 km/h, then what is the average speed over the whole journey?
- 48 km/h
 - 52 km/h
 - 54 km/h
 - 56 km/h
25. A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speeds of 4.5 m/s and 7.5 m/s, respectively. The average speed of the particle during this motion is
- 4.0 m/s
 - 5.0 m/s
 - 5.5 m/s
 - 4.8 m/s
26. Two balls of different masses m_a and m_b are dropped from two different heights, viz., a and b . The ratio of times taken by the two to drop through these distances is
- $a:b$
 - $b:a$
 - $\sqrt{a} : \sqrt{b}$
 - $a^2:b^2$
27. If two balls of same density but different masses are dropped from a height of 100 m, then
- both will come together on the earth
 - both will come late on the earth
 - first will come first and second after that
 - second will come first and first after that
28. A body starting from rest moving with uniform acceleration has a displacement of 16 m in first 4 s and 9 m in first 3 s. The acceleration of the body is
- 1 ms^{-2}
 - 2 ms^{-2}
 - 3 ms^{-2}
 - 4 ms^{-2}

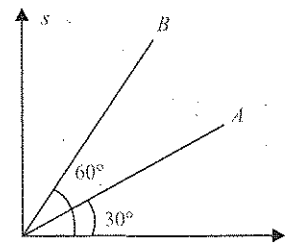


Fig. 4.94

29. The velocity acquired by a body moving with uniform acceleration is 30 ms^{-1} in 2 s and 60 ms^{-1} in 4 s. The initial velocity is
 a. zero b. 2 ms^{-1} c. 3 ms^{-2} d. 10 ms^{-2}
30. A particle moves along x -axis in such a way that its position coordinate (x) varies with time (t) according to the expression $x = 2 - 5t + 6t^2$. Its initial velocity is
 a. -3 m/s b. -5 m/s c. 2 m/s d. 3 m/s
31. A particle starts from the origin with a velocity of 10 m/s and moves with a constant acceleration till the velocity increases to 50 m/s . At that instant, the acceleration is suddenly reversed. What will be the velocity of the particle when it returns to the starting point?
 a. zero b. 10 m/s c. 50 m/s d. 70 m/s
32. A particle is moving along x -axis whose instantaneous speed is $v^2 = 108 - 9x^2$. The acceleration of particle is
 a. $-9x \text{ m/s}^2$
 b. $-18x \text{ m/s}^2$
 c. $\frac{-9x}{2} \text{ m/s}^2$
 d. None of these
33. A ball is released from the top of a tower of height h . It takes time T to reach the ground. What is the position of the ball (from ground) after time $T/3$?
 a. $h/9 \text{ m}$ b. $7h/9 \text{ m}$ c. $8h/9 \text{ m}$ d. $17h/18 \text{ m}$
34. If x denotes displacement in time t and $x = a \cos t$, then the acceleration is
 a. $a \cos t$ b. $-a \cos t$ c. $a \sin t$ d. $-a \sin t$
35. Taxis leave the station X for station Y every 10 min. Simultaneously, a taxi also leaves the station Y for station X every 10 min. The taxis move at the same constant speed and go from X and Y or vice-versa in 2 h. How many taxis coming from the other side will meet each taxi enroute from Y and X ?
 a. 24 b. 23 c. 12 d. 11
36. The velocity-time relation of an electron starting from rest is given by $v = kt$ where $k = 2 \text{ m/s}^2$. The distance traversed in first 3 s is
 a. 9 m b. 16 m c. 27 m d. 36 m
37. When the speed of a car is u , the minimum distance over which it can be stopped is s . If the speed becomes nu , what will be the minimum distance over which it can be stopped during the same time?
 a. s/n b. ns c. s/n^2 d. n^2s
38. A thief is running away on a straight road in a jeep moving with a speed of 9 ms^{-1} . A policeman chases him on a motor cycle moving at a speed of 10 ms^{-1} . If the instantaneous separation of the jeep from the motor cycle is 100 m, how long will it take for the policeman to catch the thief?
 a. 1 s b. 19 s c. 90 s d. 100 s
39. The displacement-time graph of two bodies A and B is shown in Fig. 4.94. The ratio of velocity of A (v_A) to velocity of B (v_B) is
 a. $1/\sqrt{3}$ b. $\sqrt{3}$ c. $1/3$ d. 3
40. A body is released from the top of a tower of height $H \text{ m}$. After 2 s it is stopped and then instantaneously released. What will be its height after next 2 s?
 a. $(H - 5) \text{ m}$ b. $(H - 10) \text{ m}$
 c. $(H - 20) \text{ m}$ d. $(H - 40) \text{ m}$
41. A stone is dropped from the top of a tower of height h . After 1 s another stone is dropped from the balcony 20 m below the top. Both reach the bottom simultaneously. What is the value of h ? Take $g = 10 \text{ ms}^{-2}$.
 a. 3125 m b. 312.5 m c. 31.25 m d. 25.31 m
42. A train 100 m long travelling at 40 ms^{-1} starts overtaking another train 200 m long travelling at 30 ms^{-1} . The time taken by the first train to pass the second train completely is
 a. 30 s b. 40 s c. 50 s d. 60 s
43. A person is throwing two balls into the air one after the other. He throws the second ball when first ball is at the highest point. If he is throwing the balls every second, how high do they rise?
 a. 5 m b. 3.75 m c. 2.50 m d. 1.25 m
44. A stone thrown upwards with speed u attains maximum height h . Another stone thrown upwards from the same point with speed $2u$ attains maximum height H . What is the relation between h and H ?
 a. $2h = H$ b. $3h = H$ c. $4h = H$ d. $5h = H$
45. A body dropped from the top of a tower covers a distance $7x$ in the last second of its journey, where x is the distance covered in first second. How much time does it take to reach the ground?
 a. 3 s b. 4 s c. 5 s d. 6 s
46. An engine of a train moving with uniform acceleration passes an electric pole with velocity u and the last compartment with velocity v . The middle part of the train passes past the same pole with a velocity of
 a. $\frac{u+v}{2}$ b. $\frac{u^2+v^2}{2}$
 c. $\sqrt{\frac{u^2+v^2}{2}}$ d. $\sqrt{\frac{v^2-u^2}{2}}$
47. The relation between time t and distance x is

$$t = \alpha x^2 + \beta x$$
 where α and β are constants. The retardation is
 a. $2\alpha v^3$ b. $2\beta v^3$ c. $2\alpha\beta v^3$ d. $2\beta^2 v^3$
48. The displacement x of a particle moving in one dimension under the action of a constant force is related to time t by the equation $t = \sqrt{x} + 3$, where x is in m and t is in s. Find the

- displacement of the particle when its velocity is zero.
- zero
 - 12 m
 - 6 m
 - 18 m
49. The velocity of light emitted by a source S , observed by an observer O , who is at rest with respect to S , is c . If the observer moves away from S with velocity v , the velocity of light as observed will be
- $c + v$
 - $c - v$
 - $\sqrt{1 - \frac{v^2}{c^2}}$
 - c
50. The x and y coordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$, where x and y are in m and t in s. The acceleration of the particle at 5 s is
- zero
 - 8 m/s^2
 - 20 m/s^2
 - 40 m/s^2
51. The distances moved by a freely falling body (starting from rest) during 1st, 2nd, 3rd, ..., n^{th} second of its motion are proportional to
- even numbers
 - odd numbers
 - all integral numbers
 - squares of integral numbers
52. A wooden block is dropped from the top of a cliff 100 m high and simultaneously a bullet of mass 10 gm is fired from the foot of the cliff upwards with a velocity of 100 m/s. The bullet and wooden block will meet after a time
- 10 s
 - 0.5 s
 - 1 s
 - 7 s
53. A drunkard is walking along a straight road. He takes 5 steps forward and 3 steps backward and so on. Each step is 1 m long and takes 1 s. There is a pit on the road 11 m away from the starting point. The drunkard will fall into the pit after
- 29 s
 - 21 s
 - 37 s
 - 31 s
54. A stone is dropped from a certain height which can reach the ground in 5 s. It is dropped after 3 s of its fall and then it again released. The total time taken by the stone to reach the ground will be
- 6 s
 - 6.5 s
 - 7 s
 - 7.5 s
55. A body travels 200 cm in the first 2 s and 220 cm in the next 4 s with deceleration. The velocity of the body at the end of the 7th second is
- 5 cm/s
 - 10 cm/s
 - 15 cm/s
 - 20 cm/s
56. A body starts from rest and travels a distance S with uniform acceleration, then moves a distance $2S$ uniformly and finally comes to rest after moving further $5S$ under uniform retardation. The ratio of average velocity to maximum velocity is
- $2/5$
 - $3/5$
 - $4/7$
 - $5/7$
57. A body sliding on a smooth inclined plane requires 4 s to reach the bottom, starting from rest at the top. How much time does it take to cover one fourth the distance starting from rest at the top?
- 1 s
 - 2 s
 - 4 s
 - 16 s
58. B_1 , B_2 and B_3 are three balloons ascending with velocities v , $2v$ and $3v$, respectively. If a bomb is dropped from each when they are at the same height, then
- bomb from B_1 reaches ground first
 - bomb from B_2 reaches ground first
 - bomb from B_3 reaches ground first
 - they reach the ground simultaneously
59. A particle is dropped from rest from a large height. Assume g to be constant throughout the motion. The time taken by it to fall through successive distances of 1 m each will be
- all equal, being equal to $\sqrt{2/g}$ second
 - in the ratio of the square roots of the integers 1, 2, 3, ...
 - in the ratio of the difference in the square roots of the integers, i.e., $\sqrt{1}$, $(\sqrt{2} - \sqrt{1})$, $(\sqrt{3} - \sqrt{2})$, $(\sqrt{4} - \sqrt{3})$, ...
 - in the ratio of the reciprocals of the square roots of the integers, i.e., $\frac{1}{\sqrt{1}}$, $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{3}}$, ...
60. A person moves 30 m north and then 20 m towards east and finally $30\sqrt{2}$ m in south-west direction. The displacement of the person from the origin will be
- 10 m along north
 - 10 m along south
 - 10 m along west
 - zero
61. An object accelerates from rest to a velocity 27.5 m/s in 10 sec. Find the distance covered by the object during next 10 s.
- 412.5 m
 - 137.5 m
 - 550 m
 - 275 m
62. A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of
- 3 s
 - 5 s
 - 7 s
 - 9 s
63. A stone is dropped from a rising balloon at a height of 76 m above the ground and reaches the ground in 6 s. What was the velocity of the balloon when the stone was dropped? Take $g = 10 \text{ m/s}^2$.
- $(52/3) \text{ m/s}$ upward
 - $(52/3) \text{ m/s}$ downward
 - 3 m/s
 - 9.8 m/s
64. A body is dropped from a height of 39.2 m. After it crosses half distance, the acceleration due to gravity ceases to act. The body will hit the ground with velocity (Take $g = 10 \text{ m/s}^2$)
- 19.6 m/s
 - 20 m/s
 - 1.96 m/s
 - 196 m/s
65. Two trains, one travelling at 15 ms^{-1} and other at 20 ms^{-1} , are heading towards one another along a straight track. Both the drivers apply brakes simultaneously when they are 500 m apart. If each train has a retardation of 1 ms^{-2} , the separation after they stop is
- 192.5 m
 - 225.5 m
 - 187.5 m
 - 155.5 m
66. Two cars are moving in same direction with a speed of 30 km/h. They are separated by a distance of 5 km. What is the speed of a car moving in opposite direction if it meets the two cars at an interval of 4 min?
- 60 km/h
 - 15 km/h
 - 30 km/h
 - 45 km/h
67. Two trains A and B, 100 m and 60 m long, are moving in opposite directions on parallel tracks. The velocity of shorter train is 3 times that of the longer one. If the trains take 4 s to cross each other, the velocities of the trains are
- $V_A = 10 \text{ m/s}$, $V_B = 30 \text{ m/s}$
 - $V_A = 2.5 \text{ m/s}$, $V_B = 7.5 \text{ m/s}$

- c. $V_A = 20 \text{ m/s}$, $V_B = 60 \text{ m/s}$
 d. $V_A = 5 \text{ m/s}$, $V_B = 15 \text{ m/s}$
68. Two trains each travelling with a speed of 37.5 km/h are approaching each other on the same straight track. A bird that can fly at 60 km/h flies off from one train when they are 90 km apart and heads directly for the other train. On reaching the other train it flies back to the first and so on. Total distance covered by the bird is
 a. 90 km b. 54 km c. 36 km d. 72 km
69. Between two stations a train starting from rest first accelerates uniformly, then moves with constant velocity and finally retards uniformly to come to rest. If the ratio of the time taken be $1:8:1$ and the maximum speed attained be 60 km/h , then what is the average speed over the whole journey?
 a. 48 km/h b. 52 km/h c. 54 km/h d. 56 km/h
70. A ball is thrown upwards with speed v from the top of a tower and it reaches the ground with speed $3v$. What is the height of the tower?
 a. $\frac{v^2}{g}$ b. $\frac{2v^2}{g}$ c. $\frac{4v^2}{g}$ d. $\frac{8v^2}{g}$
71. A ball is dropped into a well in which the water level is at a depth h below the top. If the speed of sound be c , then the time after which the splash is heard will be given by
 a. $h \left[\sqrt{\frac{2}{gh}} + \frac{1}{c} \right]$
 b. $h \left[\sqrt{\frac{2}{gh}} - \frac{1}{c} \right]$
 c. $h \left[\frac{2}{g} + \frac{1}{c} \right]$
 d. $h \left[\frac{2}{g} - \frac{1}{c} \right]$
72. If a particle travels n equal distances with speeds $v_1, v_2, \dots, v_n$, then the average speed $\bar{V}$ of the particle will be
 a. $\bar{V} = \frac{v_1 + v_2 + \dots + v_n}{n}$
 b. $\bar{V} = \frac{nv_1 v_2 \dots v_n}{v_1 + v_2 + \dots + v_n}$
 c. $\frac{1}{\bar{V}} = \frac{1}{n} \left(\frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n} \right)$
 d. $\bar{V} = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$
73. A ball is thrown from the top of a tower in vertically upward direction. Velocity at a point $h \text{ m}$ below the point of projection is twice of the velocity at a point $h \text{ m}$ above the point of projection. Find the maximum height reached by the ball above the top of the tower.
 a. $2h$ b. $3h$ c. $(5/3)h$ d. $(4/3)h$
74. A juggler keeps on moving four balls in the air throwing the balls after regular intervals. When one ball leaves his hand (speed = 20 ms^{-1}) the position of other balls (height in m) will be (Take $g = 10 \text{ ms}^{-2}$)
 a. $10, 20, 10$ b. $15, 20, 15$
 c. $5, 15, 20$ d. $5, 10, 20$
75. An elevator in which a man is standing is moving upwards with a speed of 10 m/s . If the man drops a coin from a height of 2.45 m from the floor of elevator, it reaches the floor of the elevator after a time ($g = 9.8 \text{ m/s}^2$)
 a. $\sqrt{2} \text{ s}$ b. $1/\sqrt{2} \text{ s}$ c. 2 s d. $1/2 \text{ s}$
76. A particle slides from rest from the topmost point of a vertical circle of radius r along a smooth chord making an angle θ with the vertical. The time of descent is
 a. least for $\theta = 0$
 b. maximum for $\theta = 0$
 c. least for $\theta = 45^\circ$
 d. independent of θ
77. A body is thrown vertically upwards from A , the top of a tower. It reaches the ground in time t_1 . If it is thrown vertically downwards from A with the same speed, it reaches the ground in time t_2 . If it is allowed to fall freely from A , then the time it takes to reach the ground is given by
 a. $t = \frac{t_1 + t_2}{2}$ b. $t = \frac{t_1 - t_2}{2}$
 c. $t = \sqrt{t_1 t_2}$ d. $t = \frac{t_1}{t_2}$
78. Two cars A and B are travelling in the same direction with velocities V_A and V_B ($V_A > V_B$). When car A is at a distance s behind car B , the driver of car A applies the brakes producing a uniform retardation a ; there will be no collision when
 a. $s < \frac{(V_A - V_B)^2}{2a}$ b. $s = \frac{(V_A - V_B)^2}{2a}$
 c. $s \geq \frac{(V_A - V_B)^2}{2a}$ d. $s \leq \frac{(V_A - V_B)^2}{2a}$
79. Four persons are initially at the four corners of a square whose side is equal to d . Each person now moves with a uniform speed V in such a way that the first moves directly towards the second, the second directly towards the third, the third directly towards the fourth and the fourth directly towards the first. The four persons will meet after a time equal to
 a. d/V b. $2d/3V$ c. $2d/\sqrt{3}V$ d. $d/\sqrt{3}V$
80. The deceleration experienced by a moving motor boat, after its engine is cut-off is given by $\frac{dv}{dt} = -kv^3$, where k is constant. If v_0 is the magnitude of the velocity at cut-off, the magnitude of the velocity at a time t after the cut-off is
 a. $v_0/2$ b. v_0
 c. $v_0 e^{-kt}$ d. $\frac{v_0}{\sqrt{(2v_0^2 kt + 1)}}$
81. For motion of an object along x -axis, the velocity v depends on the displacement x as $v = 3x^2 - 2x$. What is the acceleration at $x = 2 \text{ m}$.
 a. 48 ms^{-2} b. 80 ms^{-2}
 c. 18 ms^{-2} d. 10 ms^{-2}
82. A stone is dropped from the 25^{th} storey of a multistoried building and reaches the ground in 5 s . In the first second, it passes through how many storeys of the building? ($g = 10 \text{ m/s}^2$).
 a. 1 b. 2 c. 3 d. None of these
83. A body is projected upwards with a velocity u . It passes through a certain point above the ground after t_1 . The time after which the body passes through the same point during

the return journey is

- a. $\left(\frac{u}{g} - t_1^2\right)$ b. $2\left(\frac{u}{g} - t_1\right)$
 c. $3\left(\frac{u^2}{g} - t_1\right)$ d. $3\left(\frac{u^2}{g^2} - t_1\right)$

84. A balloon is moving vertically with a velocity of 4 m/s. When it is at a height of h , a body is gently released from it. If it reaches the ground in 4 s, the height of the balloon, when the body is released, is
 a. 80 m b. 96 m c. 64 m d. 78 m
85. A parachutist drops first freely from an aeroplane for 10 s and then parachute opens out. Now he descends with a net retardation of 2.5 m/s^2 . If he bails out of the plane at a height of 2495 m and $g = 10 \text{ m/s}^2$, his velocity on reaching the ground will be
 a. 5 m/s b. 10 m/s c. 15 m/s d. 20 m/s
86. A police party is chasing a dacoit in a jeep which is moving at a constant speed v . The dacoit is on a motor cycle. When he is at a distance x from the jeep he accelerates from rest at a constant rate. Which of the following relations is true, if the police is able to catch the dacoit?
 a. $v^2 \leq \alpha x$ b. $v^2 \leq 2\alpha x$
 c. $v^2 \geq 2\alpha x$ d. $v^2 \geq \alpha x$
87. A train is moving at a constant speed V . Its driver observes another train in front of him on the same track and moving in the same direction with constant speed v . If the distance between the trains be x , what should be the minimum retardation of the train so as to avoid collision?
 a. $\frac{(V+v)^2}{2x}$ b. $\frac{(V-v)^2}{2x}$
 c. $\frac{x}{(V+v)^2}$ d. $\frac{x}{(V-v)^2}$
88. A moving car possesses average velocities of 5 ms^{-1} , 10 ms^{-1} and 15 ms^{-1} in the first, second and third seconds, respectively. What is the total distance covered by the car in these 3 s?
 a. 15 m b. 30 m c. 55 m d. None of these
89. The average velocity of a body moving with uniform acceleration after travelling a distance of 3.06 m is 0.34 ms^{-1} . If the change in velocity of the body is 0.18 ms^{-1} during this time, its uniform acceleration is
 a. 0.01 m/s^2 b. 0.02 m/s^2
 c. 0.03 m/s^2 d. 0.04 m/s^2
90. Water drops fall from a tap on the floor 5 m below at regular intervals of time, the first drop striking the floor when the fifth drop begins to fall. The height of the third drop from the ground, at the instant when the first drop strikes the ground, will be ($g = 10 \text{ ms}^{-2}$)
 a. 1.25 m b. 2.15 m c. 2.75 m d. 3.75 m
91. Drops of water fall at regular intervals from the roof of a building of height $H = 16 \text{ m}$, the first drop strikes the ground at the same moment when the fifth drop detaches itself from the roof. The distances between the different drops in air as the first drop reaches the ground are
 a. 1m, 5m, 7m, 3m b. 1m, 3m, 5m, 7m
 c. 1m, 3m, 7m, 5m d. None of the above

92. A point moves in a straight line so that its displacement x m at time t s is given by $x^2 = 1 + t^2$. Its acceleration in m/s^2 at time t s is

- a. $\frac{1}{x^3}$ b. $\frac{-t}{x^3}$
 c. $\frac{1}{x} - \frac{t^2}{x^3}$ d. $\frac{1}{x} - \frac{1}{x^2}$

93. A point moves with uniform acceleration and v_1 , v_2 and v_3 denote the average velocities in the three successive intervals of time t_1 , t_2 and t_3 . Which of the following relations is correct?

- a. $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$
 b. $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$
 c. $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_1 - t_3)$
 d. $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$

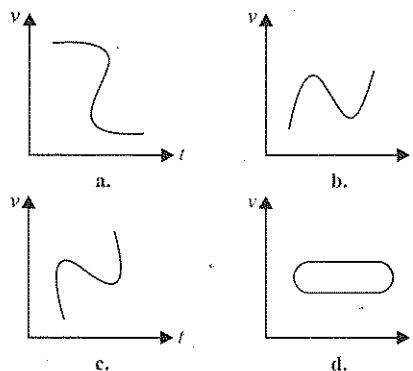
94. A 2 m wide truck is moving with a uniform speed $v_0 = 8 \text{ m/s}$ along a straight horizontal road. A pedestrian starts to cross the road with a uniform speed v when the truck is 4 m away from him. The minimum value of v so that he can cross the road safely is

- a. 2.62 m/s b. 4.6 m/s
 c. 3.57 m/s d. 1.414 m/s

Graphical Concepts

Solutions on page 4.58

1. Which of the following velocity–time graphs shows a realistic situation for a body in motion?



2. The velocity–time graph of a body moving in a straight line is shown in Fig. 4.95. The displacement of the body in 10 s is

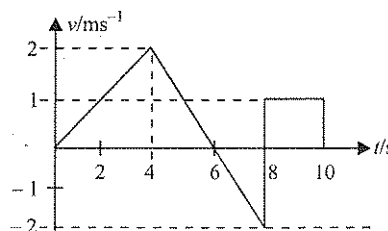


Fig. 4.95

- a. 4 m b. 6 m c. 8 m d. 10 m
3. The velocity–time graph of a body is shown in Fig. 4.96. The displacement covered by the body in 8 s is

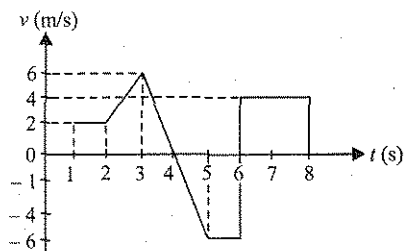


Fig. 4.96

- a. 9 m b. 12 m c. 10 m d. 28 m
4. The variation of velocity of a particle moving along a straight line is shown in Fig. 4.97. The distance travelled by the particle in 12 s is

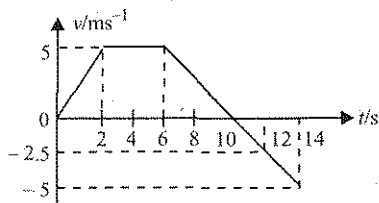


Fig. 4.97

- a. 37.5 m b. 32.5 m c. 35.0 m d. None of these
5. The graph shows the variation of velocity of a rocket with time. The maximum height attained by the rocket is

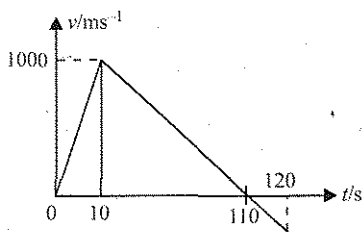


Fig. 4.98

- a. 1.1 km b. 5 km c. 55 km d. none of these
6. From the velocity-time graph, given in Fig. 4.99 of a particle moving in a straight line, one can conclude that

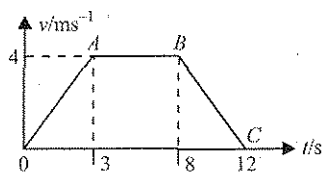


Fig. 4.99

- a. its average velocity during the 12 s interval is $24/7 \text{ ms}^{-1}$
- b. its velocity for the first 3 s is uniform and is equal to 4 ms^{-1}
- c. the body has a constant acceleration between $t = 3 \text{ s}$ and $t = 8 \text{ s}$
- d. the body has a uniform retardation from $t = 8 \text{ s}$ to $t = 12 \text{ s}$

7. The velocity-time graph of a particle moving in a straight line is shown in Fig 4.100. The acceleration of the particle at $t = 9 \text{ s}$ is

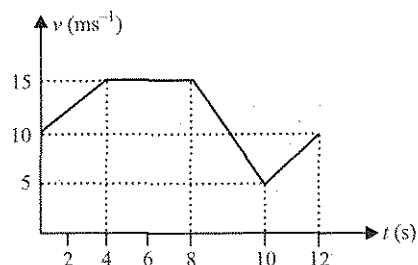


Fig. 4.100

- a. zero b. 5 m/s^2 c. -5 m/s^2 d. -2 m/s^2
8. The velocity-time graph of a body is shown in Fig. 4.101. It indicates that

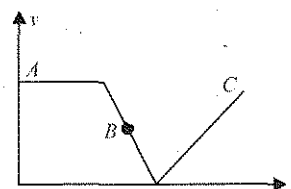


Fig. 4.101

- a. at B force is zero
- b. at B there is a force but towards motion
- c. at B there is a force which opposes motion
- d. none of the above is true
9. The velocity-time graph of a body is given in Fig. 4.102. The maximum acceleration in ms^{-2} is

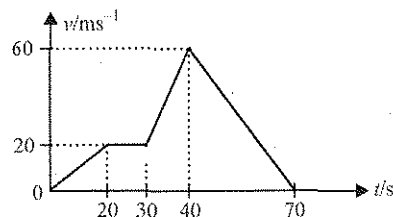
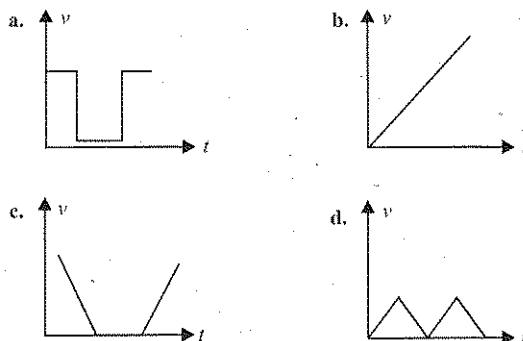


Fig. 4.102

- a. 4 b. 3 c. 2 d. 1
10. Which of the following velocity-time graphs is not possible practically?



11. The velocity–time graph of a body is shown in Fig. 4.103. The ratio of average acceleration during the intervals OA and AB is

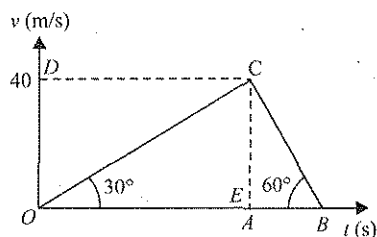


Fig. 4.103

- a. 1 b. $1/2$ c. $1/3$ d. 3
12. On the displacement–time graph, two straight lines make angles 60° and 30° , with time axis as shown in Fig. 4.104. The ratio of the velocities represented by them is

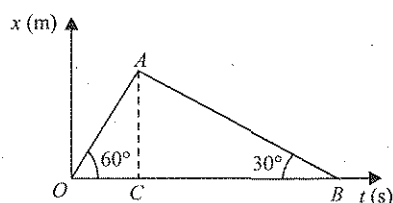


Fig. 4.104

- a. 1:2 b. 1:3 c. 2:1 d. 3:1
13. Displacement–time graph of a body is shown in Fig. 4.105.

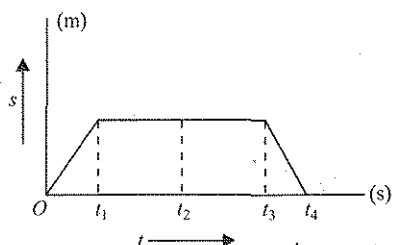
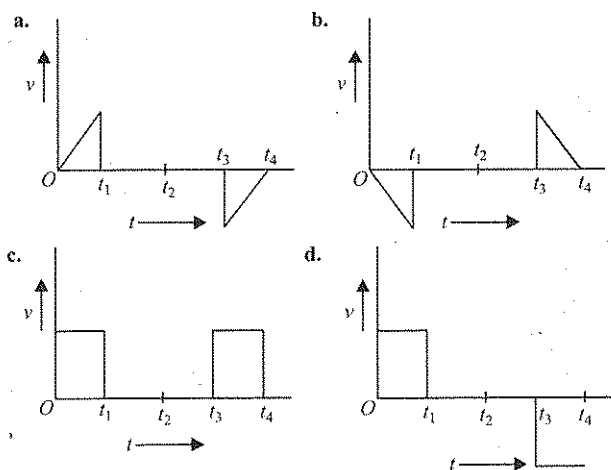
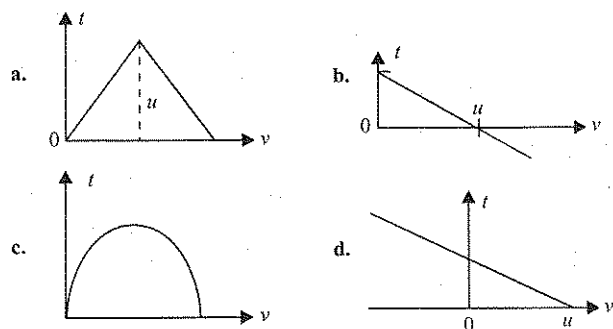


Fig. 4.105

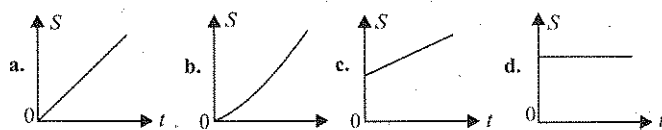
Velocity–time graph of the motion of the body will be



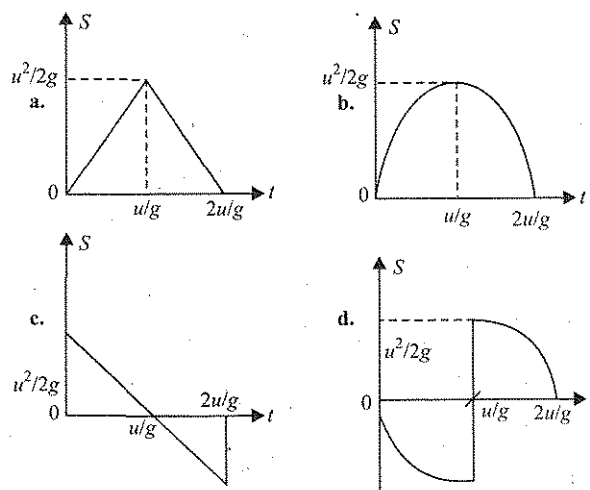
14. An object is thrown up vertically. The velocity–time graph for the motion of the particle is



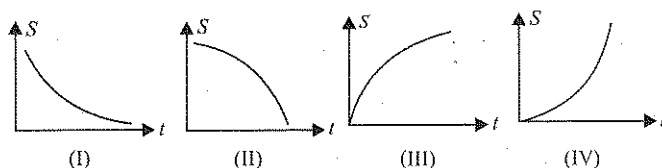
15. From a high tower, at time $t = 0$, one stone is dropped from rest and simultaneously another stone is projected vertically up with an initial velocity. The graph of distance S between the two stones plotted against time t will be



16. An object is vertically thrown upwards. Then the displacement–time graph for the motion is



17. The acceleration will be positive in which of the following graphs,



- a. (I) and (III) b. (I) and (IV)
c. (II) and (IV) d. None of these
18. The graph (Fig. 4.106) below describes the motion of a ball rebounding from a horizontal surface being released from a point above the surface. Assume the ball collides each time with the floor inelastically. The quantity represented on the y-axis is the ball's (take upward direction as positive)

- a. displacement b. velocity
c. acceleration d. momentum

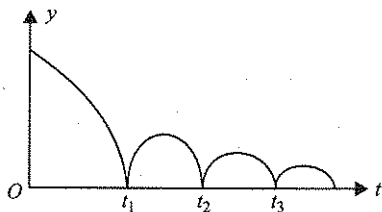


Fig. 4.106

19. The acceleration versus time graph of a particle is shown in Fig. 4.107. The respective v - t graphs of the particle are

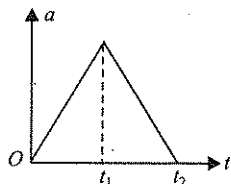
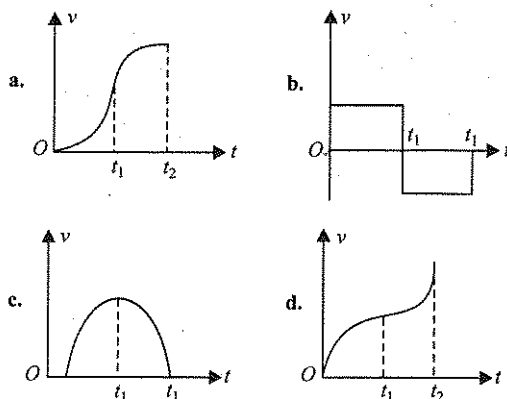


Fig. 4.107



20. The displacement-time graph of a moving particle with constant acceleration is shown in Fig. 4.108. The velocity-time graph is given by

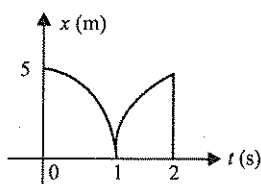
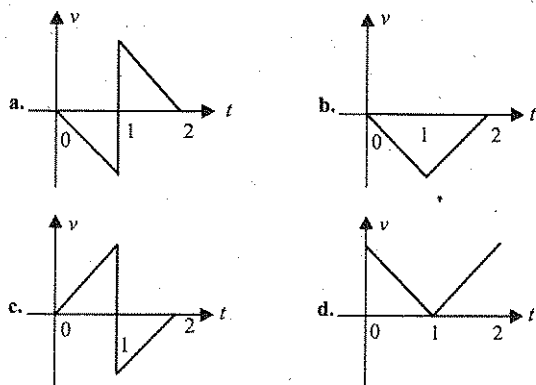
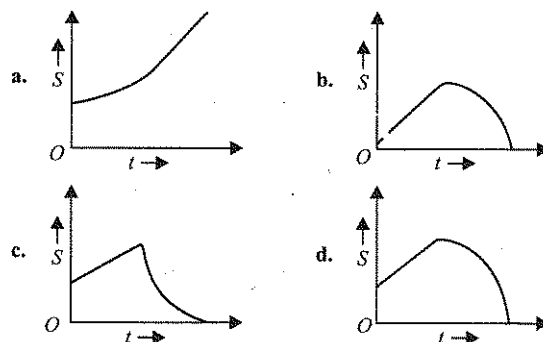


Fig. 4.108



21. Two balls are dropped from the top of a high tower with a time interval of t_0 second, where t_0 is smaller than the time taken by the first ball to reach the floor, which is perfectly inelastic. The distance s between the two balls, plotted against the time lapse t from the instant of dropping the second ball is best represented by



22. The acceleration versus time graph of a particle moving in a straight line is shown in Fig. 4.109. The velocity-time graph of the particle would be

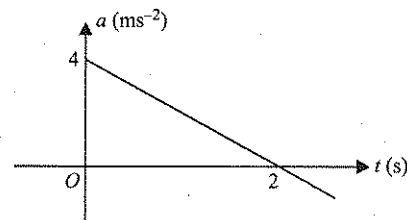


Fig. 4.109

- a. a straight line
b. a parabola
c. a circle
d. an ellipse
23. The acceleration-time graph of a particle moving along a straight line is as shown in Fig. 4.110. At what time the particle acquires its initial velocity?

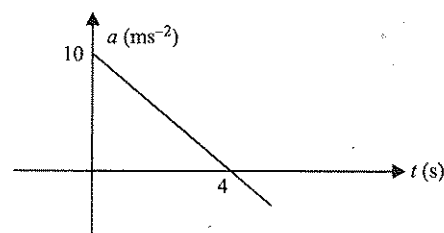


Fig. 4.110

- a. 12 s
b. 5 s
c. 8 s
d. 16 s
24. Plot acceleration-time graph of the velocity-time graph given in Fig. 4.111

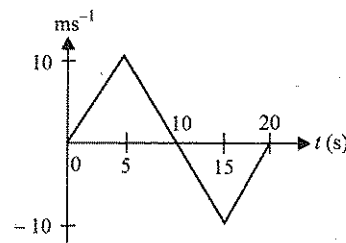
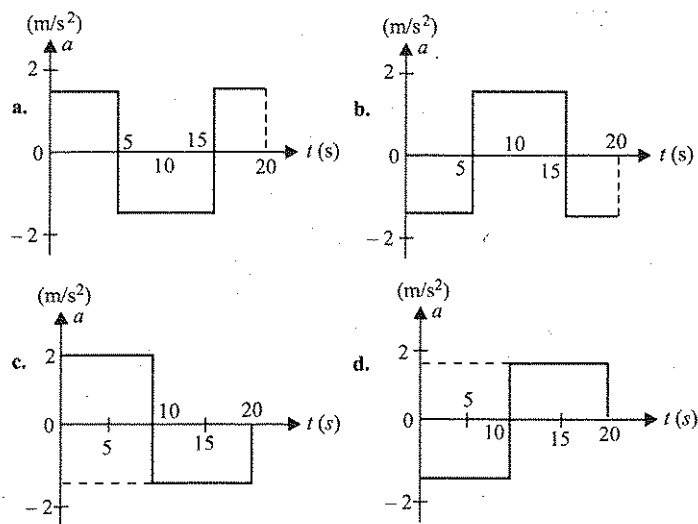


Fig. 4.111



25. For velocity–time graph given in Fig. 4.112.

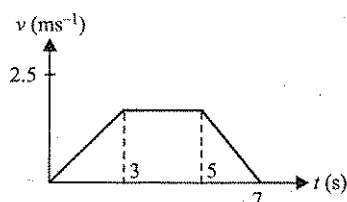
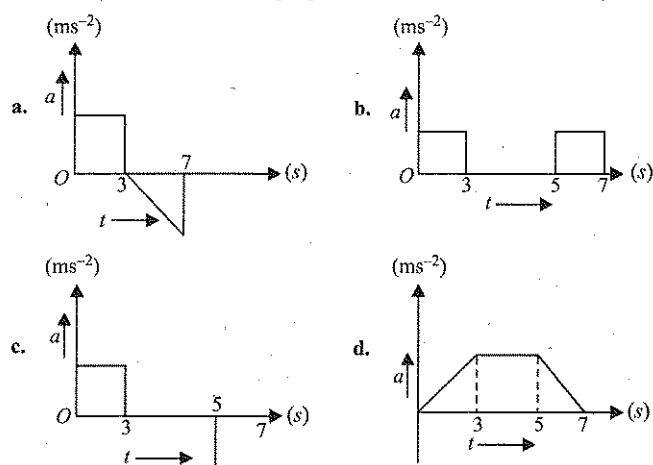


Fig. 4.112

The acceleration–time graph of the motion of the body is



26. The displacement–time graph of a moving particle is shown in Fig. 4.113. The instantaneous velocity of the particle is negative at the point

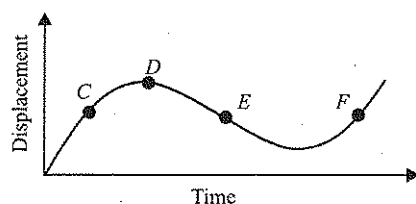
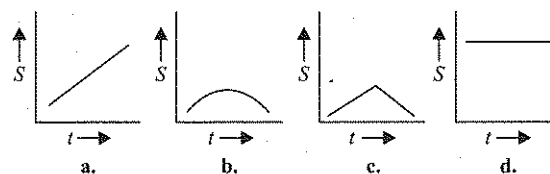


Fig. 4.113

- a. D b. F c. C d. E

27. Which graph represents uniform motion:



Multiple Correct Answers Type

Solutions on page 4.59

- Check up the only correct statement in the following:
 - A body having a constant velocity can have a varying speed.
 - A body having a constant speed can have a varying velocity.
 - A body having constant speed can have an acceleration.
 - If velocity and acceleration are in same direction, then distance is equal to displacement.
- A block slides down a smooth inclined plane when released from the top, while another falls freely from the same point
 - sliding block will reach the ground first
 - freely falling block will reach the ground first
 - both the blocks will reach the ground with different speeds
 - both the blocks will reach the ground with same speed
- A car accelerates from rest at a constant rate of 2 ms^{-2} for some time. Then it retards at a constant rate of 4 ms^{-2} and comes to rest. It remains in motion for 6 s.
 - Its maximum speed is 8 ms^{-1}
 - Its maximum speed is 6 ms^{-1}
 - It travelled a total distance of 24 m
 - It travelled a total distance of 18 m
- At $t = 0$, an arrow is fired vertically upwards with a speed of 100 ms^{-1} . A second arrow is fired vertically upwards with the same speed at $t = 5 \text{ s}$. Then
 - the two arrows will be at the same height above the ground at $t = 12.54 \text{ s}$
 - the two arrows will reach back their starting points at $t = 20 \text{ s}$ and $t = 25 \text{ s}$
 - the ratio of the speeds of the first and second arrows at $t = 20 \text{ s}$ will be 2:1
 - the maximum height attained by either arrow will be 1000 m
- Two bodies of masses m_1 and m_2 are dropped from heights h_1 and h_2 , respectively. They reach the ground after time t_1 and t_2 and strike the ground with v_1 and v_2 , respectively. Choose the correct relations from the following:
 - $\frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$
 - $\frac{t_1}{t_2} = \sqrt{\frac{h_2}{h_1}}$
 - $\frac{v_1}{v_2} = \sqrt{\frac{h_1}{h_2}}$
 - $\frac{v_1}{v_2} = \frac{h_2}{h_1}$
- From the top of a tower of height 200 m, a ball A is projected up with 10 ms^{-1} and 2 s later another ball B is projected

vertically down with the same speed. Then

- both A and B will reach the ground simultaneously
 - ball A will hit the ground 2 s later than B hitting the ground
 - both the balls will hit the ground with the same velocity
 - both the balls will hit the ground with the different velocity
7. A body starts from rest and then moves with uniform acceleration. Then
- its displacement is directly proportional to the square of the time
 - its displacement is inversely proportional to the square of the time
 - it may move along a circle
 - it always moves in a straight line
8. Which is/are correct?
- If velocity of a body changes, it must have some acceleration
 - If speed of a body changes, it must have some acceleration
 - If body has acceleration, its speed must change
 - If body has acceleration, its speed may change
9. The body will speed up if
- velocity and acceleration are in same direction
 - velocity and acceleration are in opposite direction
 - velocity and acceleration are in perpendicular direction
 - velocity and acceleration are acting at acute angle w.r.t. each other
10. Average acceleration is in the direction of
- initial velocity
 - final velocity
 - change in velocity
 - final velocity if initial velocity is zero
11. Which of the following statement is true?
- A body can have varying speed without having varying velocity
 - A body can have constant speed but varying velocity
 - A body can have velocity without having acceleration
 - A body can have acceleration without having velocity
12. A particle is projected vertically upward with velocity u from a point A , when it returns to point of projection
- its average speed is $u/2$
 - its average velocity is zero
 - its displacement is zero
 - its average speed is u
13. A particle moves along a straight line and its velocity depends on time as $v = 4t - t^2$. Then for first 5 s
- Average velocity is $25/3$ m
 - Average speed is 10 m/s
 - Average velocity is $5/3$ m/s
 - Acceleration is 4 m/s^2 at $t = 0$
14. The velocity–time plot for a particle moving on a straight line is shown in the Fig. 4.114.
- The particle has a constant acceleration
 - The particle has never turned around
 - The particle has zero displacement
 - The average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s

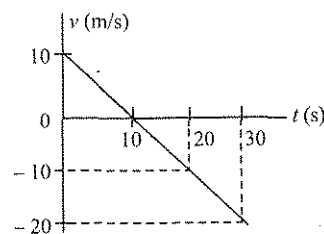


Fig. 4.114

15. Figure 4.115 shows the velocity (v) of a particle plotted against time (t).

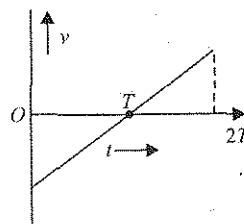


Fig. 4.115

- The particle changes its direction of motion at some point
 - The acceleration of the particle remains constant
 - The displacement of the particle is zero
 - The initial and final speeds of the particle are the same
16. The displacement of a particle as a function of time is shown in the Fig. 4.116. It indicates

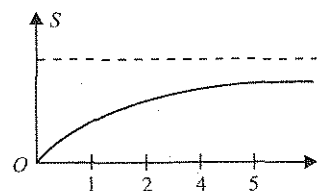


Fig. 4.116

- the particle starts with a certain velocity, but the motion is retarded and finally the particle stops
 - the velocity of the particle decreases
 - the acceleration of the particle is in opposite direction to the velocity
 - the particle starts with a constant velocity, the motion is accelerated and finally the particle moves with another constant velocity
17. A particle moves in a straight line with the velocity as shown in the Fig. 4.117. At $t = 0$, $x = -16$ m

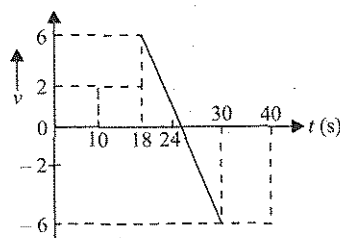


Fig. 4.117

- the maximum value of the position coordinate of the particle is 54 m

- b. the maximum value of the position coordinate of the particle is 36 m
 c. the particle is at the position of 36 m at $t = 18$ s
 d. the particle is at the position of 36 m at $t = 30$ s
18. Velocity variations of an object moving along a straight line are recorded in the enclosed Fig. 4.118.

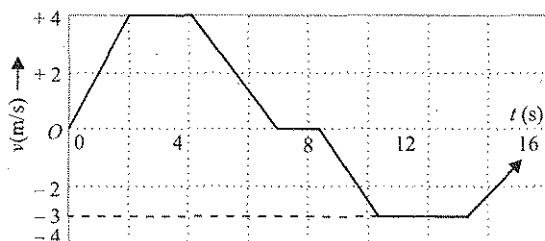


Fig. 4.118

- a. The acceleration is maximum in first two seconds
 b. The acceleration magnitude is maximum during the ninth second
 c. The object is farthest from the starting point after 16 s
 d. The displacement is largest at 7 s

Assertion-Reasoning Type

Solutions on page 4.61

In the following questions, each question contains STATEMENT I (Assertion) and STATEMENT II (Reason). Each question has 4 choices (a), (b), (c), and (d) out of which **only one** is correct.

- (a) Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
 (b) Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
 (c) Statement I is True, Statement II is False.
 (d) Statement I is False, Statement II is True.

1. **Statement I:** The displacement of a body may be zero, though its distance can be finite.

Statement II: If a body moves such that finally it arrives at initial point, then displacement is zero while distance is finite.

2. **Statement I:** The distance and displacement are different physical quantities.

Statement II: Distance and displacement have same dimension.

3. **Statement I:** Average velocity of the body may be equal to its instantaneous velocity.

Statement II: For a given time interval of a given motion, average velocity is single valued while average speed can have many values.

4. **Statement I:** A body can have acceleration even if its velocity is zero at a given instant.

Statement II: A body is momentarily at rest when it reverses its direction of velocity.

5. **Statement I:** An object can possess acceleration even at a time when it has uniform speed

Statement II: It is possible when the direction of motion keeps changing.

6. **Statement I:** Retardation is directed opposite to the velocity.
Statement II: Retardation is equal to the time rate of decrease of velocity.

7. **Statement I:** Magnitude of average velocity is equal to average speed, if velocity is constant.

Statement II: If velocity is constant, then there is no change in the direction of motion.

8. **Statement I:** The velocity of a particle may vary even when its speed is constant.

Statement II: Such a body may move along a circular path.

9. **Statement I:** Two balls of different masses are thrown vertically upward with same speed. They will pass through their point of projection in the downward direction with the same speed.

Statement II: The height and the downward velocity attained at the point of projection are independent of the mass of ball.

10. **Statement I:** At any instant, acceleration of a body can change its direction without any change in direction of velocity.

Statement II: At any instant, direction of acceleration is same as that of direction of change in velocity vector at that instant.

Comprehensive Type

Solutions on page 4.61

For Problems 1–2

The displacement of a body is given by $4s = M + 2Nt^4$, where M and N are constants.

1. The velocity of the body at any instant is

- a. $\frac{M + 2Nt^4}{4}$ b. $2N$
 c. $\frac{M + 2N}{4}$ d. $2Nt^3$

2. The velocity of the body at the end of 1 s from the start is

- a. $2N$ b. $\frac{M + 2N}{4}$
 c. $2(M + N)$ d. $\frac{2M + N}{4}$

For Problems 3–5

A body is dropped from the top of a tower and falls freely.

3. The distance covered by it after n seconds is directly proportional to

- a. n^2 b. n
 c. $2n - 1$ d. $2n^2 - 1$

4. The distance covered in the n^{th} second is proportional to

- a. n^2 b. n
 c. $2n - 1$ d. $2n^2 - 1$

5. The velocity of the body after n seconds is proportional to

- a. n^2 b. n
 c. $2n - 1$ d. $2n^2 - 1$

For Problems 6–7

A body, at rest, is acted upon by a constant force (it means acceleration of the body will be constant).

6. What is the nature of the displacement–time graph?

- Straight line
- Parabola
- Asymmetric parabola
- Rectangular hyperbola

7. What is the nature of velocity–time graph?

- Straight line
- Parabola
- Elliptical
- Hyperbola

For Problems 8–9

A car accelerates from rest at a constant rate α for some time and then decelerates at a constant rate β to come to rest. The total time elapsed is t . (IIT-JEE, 1978)

8. The maximum velocity attained by the car is

- $\frac{\alpha\beta}{2(\alpha + \beta)}t$
- $\frac{\alpha\beta}{\alpha + \beta}t$
- $\frac{2\alpha\beta}{\alpha + \beta}t$
- $\frac{4\alpha\beta}{\alpha + \beta}t$

9. Total distance travelled by the car is

- $\frac{\alpha\beta t^2}{4(\alpha + \beta)}$
- $\frac{\alpha\beta t^2}{2(\alpha + \beta)}$
- $\frac{2\alpha\beta t^2}{(\alpha + \beta)}$
- $\frac{4\alpha\beta t^2}{(\alpha + \beta)}$

For Problems 10–11

A body is moving with uniform velocity of 8 m/s. When the body just crossed another body, the second one starts and moves with uniform acceleration of 4 ms^{-2} .

10. The time after which the two bodies meet, will be

- 2 s
- 4 s
- 6 s
- 8 s

11. The distance covered by the second body when they meet is

- 8 m
- 16 m
- 24 m
- 32 m

For Problems 12–14

A body is allowed to fall from a height of 100 m. If the time taken for the first 50 m is t_1 and for the remaining 50 m is t_2 .

12. Which is correct?

- $t_1 = t_2$
- $t_1 > t_2$
- $t_1 < t_2$
- depends upon the mass

13. The ratio of t_1 and t_2 is nearly

- 5:2
- 3:1
- 3:2
- 5:3

14. The ratio of times to reach the ground and to reach first half of the distance is

- $\sqrt{3}:1$
- $\sqrt{2}:1$
- 5:2
- $1:\sqrt{3}$

For Problems 15–16

A body is dropped from a balloon moving up with a velocity of 4 ms^{-1} when the balloon is at a height of 120.5 m from the ground.

15. The height of the body after 5 s from the ground is ($g = 9.8 \text{ m/s}^2$)

- 8 m
- 12 m
- 18 m
- 24 m

16. The distance of separation between the body and the balloon after 5 s is

- 122.5 m
- 100.5 m
- 132.5 m
- 112.5 m

For Problems 17–18

A bus starts moving with acceleration 2 ms^{-2} . A cyclist 96 m behind the bus starts simultaneously towards the bus at a constant speed of 20 m/s.

17. After what time will he be able to overtake the bus?

- 4 s
- 8 s
- 12 s
- 16 s

18. After some time the bus will be left behind. If the bus continues moving with the same acceleration, after what time from the beginning, the bus will overtake the cyclist?

- 10 s
- 12 s
- 14 s
- 16 s

For Problems 19–21

Distance is a scalar quantity. Displacement is a vector quantity. The magnitude of displacement is always less than or equal to distance. For a moving body displacement can be zero but distance cannot be zero. Same concept is applicable regarding velocity and speed. Acceleration is the rate of change of velocity. If acceleration is constant, then quantities of kinematics are applicable. For one dimensional motion under the gravity in which air resistance is considered, the value of acceleration depends on the density of medium. Each motion is measured with respect to the frame of reference. Relative velocity may be greater/smaller to the individual velocities.

19. A particle moves from A to B. The ratio of distance to displacement is (Fig. 4.119)

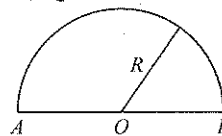


Fig. 4.119

- $\frac{\pi}{2}$
- $\frac{2}{\pi}$
- $\frac{\pi}{4}$
- 1 : 1

20. A person is going 40 m north, then 30 m east and then $30\sqrt{2}$ m southwest. The net displacement will be

- 10 m towards east
- 10 m towards west
- 10 m towards south
- 10 m towards north

21. A particle is moving along the path $y = 4x^2$. The distance and displacement from $x = 1$ to $x = 2$ is (nearly)

- a. 150, 12 b. 160, 20
c. 200, 30 d. 150, 20

For Problems 22–23

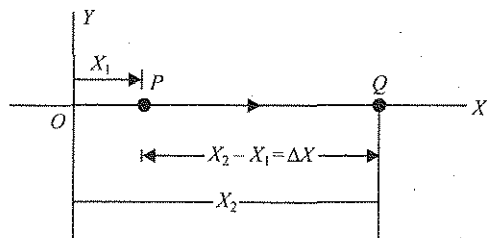
A car is moving towards south with a speed of 20 m/s. A motorcyclist is moving towards east with a speed of 15 m/s. At a certain instant, the motorcyclist is due south of the car and is at a distance of 50 m from the car.

22. The shortest distance between the motorcyclist and the car is
a. 20 m b. 10 m c. 40 m d. 30 m
23. The time after which they are closest to each other
a. 1/3 s b. 8/3 s c. 1/5 s d. 8/5 s

For Problems 24–28

Consider a particle moving along x -axis as shown in Fig. 4.120. Its distance from the origin O is described by the coordinate x , which varies with time. At a time t_1 , the particle is at point P , where its coordinate is x_1 , and at time t_2 it is at point Q , where its coordinate is x_2 . The displacement during the time interval from t_1 to t_2 is the vector from P to Q : the x -component of this vector is $(x_2 - x_1)$ and all other components are zero.

It is convenient to represent the quantity $x_2 - x_1$, the change in x , by means of a notation using the Greek letter Δ (capital delta) to designate a change in any quantity. Thus we write $\Delta x = x_2 - x_1$ in which Δx is not a product but is to be interpreted as a single symbol representing the change in the quantity x . Similarly, we denote the time interval from t_1 to t_2 as $\Delta t = t_2 - t_1$.

**Fig. 4.120**

The average velocity of the particle is defined as the ratio of the displacement Δx to the time interval Δt . We represent average velocity by the letter v with a bar ($\bar{v}$) to signify average value. Thus

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

24. A particle moves half the time of its journey with u . The rest of the half time it moves with two velocities V_1 and V_2 such that half the distance it covers with V_1 and the other half with V_2 . Find the net average velocity. Assume straight line motion.

- a. $\frac{u(V_1 + V_2) + 2V_1V_2}{2(V_1 + V_2)}$ b. $\frac{2u(V_1 + V_2)}{2u + V_1 + V_2}$
c. $\frac{u(V_1 + V_2)}{2V_1}$ d. $\frac{2V_1V_2}{u + V_1 + V_2}$

25. A particle moves according to the equation $x = t^2 + 3t + 4$. The average velocity in the first 5 s is

- a. 8 ms^{-1} b. 7.6 ms^{-1}
c. 6.4 ms^{-1} d. 5.8 ms^{-1}

26. The resistive force suffered by a motor boat is $\propto V^2$ where V is instantaneous velocity. The engine was shut down when the velocity of the boat was V_0 . Find the average velocity at any time t .

- a. $\frac{V_0 + V}{2}$ b. $\frac{VV_0}{2(V_0 + V)}$
c. $\frac{VV_0 \log_e(V_0/V)}{(V_0 - V)}$ d. $\frac{2VV_0 \log_e(V_0/V)}{(V_0 + V)}$

27. A particle moves from A to B such that $x = t^2 + t - 3$. Its average velocity from $t = 2$ s to $t = 5$ s is

- a. 6 ms^{-1} b. 8 ms^{-1}
c. 8.5 ms^{-1} d. 7 ms^{-1}

28. A boy throws a ball to his friend 20 m away. The ball reaches to the friend in 4 s. The friend then throws the ball back to boy and it reaches the boy in 5 s. Assume the ball travels with constant velocity during any throw.

- a. The average velocity is $40/9 \text{ ms}^{-1}$.
b. The average acceleration is zero.
c. The average velocity is zero but average acceleration is nonzero.
d. Average acceleration of the motion cannot be defined.

For Problems 29–30

Two particles A and B are initially 40 m apart, A is behind B . Particle A is moving with uniform velocity of 10 m/s towards B . Particle B starts moving away from A with constant acceleration of 2 m/s^2 .

29. The time at which there is a minimum distance between the two is

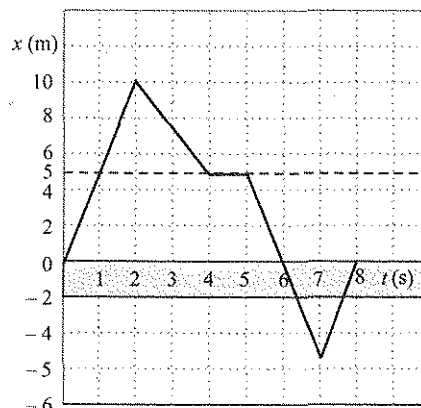
- a. 2 s b. 4 s c. 5 s d. 6 s

30. The minimum distance between the two is

- a. 20 m b. 15 m c. 25 m d. 30 m

For Problems 31–35

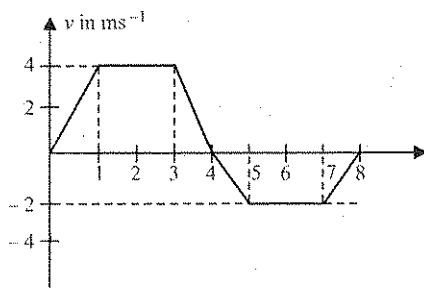
Figure 4.121 shows displacement versus time graph for a particle moving along x -axis. Find the average velocity in the time interval

**Fig. 4.121**

31. from 0 – 2 s
 a. 5 m/s
 b. 0 m/s
 c. 15 m/s
 d. 10 m/s
32. from 0 – 4 s
 a. $3/4$ m/s
 b. $4/5$ m/s
 c. $2/3$ m/s
 d. $5/4$ m/s
33. from 2 s – 4 s
 a. -2.5 m/s
 b. 5 m/s
 c. 3 m/s
 d. 4 m/s
34. from 4 s – 7 s
 a. $2/3$ m/s
 b. $-10/3$ m/s
 c. 0 m/s
 d. 15 m/s
35. from 0 – 8 s
 a. 10 m/s
 b. 20 m/s
 c. 0 m/s
 d. 15 m/s

For Problems 36–38

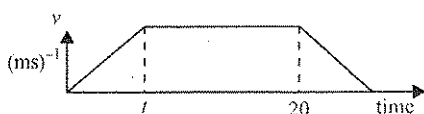
The velocity–time graph of a particle in straight line motion is shown in Fig. 4.122. The particle starts its motion from origin.

**Fig. 4.122**

36. The distance travelled by the particle in 8 s is
 a. 18 m b. 16 m c. 8 m d. 6 m
37. The distance of the particle from the origin after 8 s is
 a. 18 m b. 16 m c. 8 m d. 6 m
38. Find the average acceleration from 2 s to 6 s.
 a. -2 m/s² b. -1 m/s²
 c. 2 m/s² d. 1 m/s²

For Problems 39–41

The velocity–time graph of a particle moving along a straight line is as shown in Fig. 4.123. The rate of acceleration and deceleration is constant and it is equal to 5 ms⁻². If the average velocity during the motion is 20 ms⁻¹, then

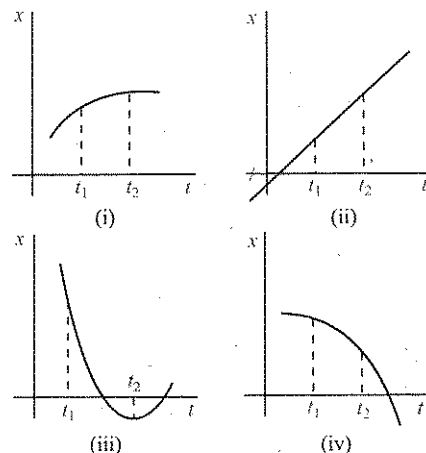
**Fig. 4.123**

39. the value of t is
 a. 5 s b. 10 s c. 20 s d. $5\sqrt{2}$ s

40. the maximum velocity of the particle is
 a. 20 m/s b. 25 m/s
 c. 30 m/s d. 40 m/s
41. the distance travelled with uniform velocity is
 a. 375 m b. 125 m
 c. 300 m d. 450 m

For Problems 42–43

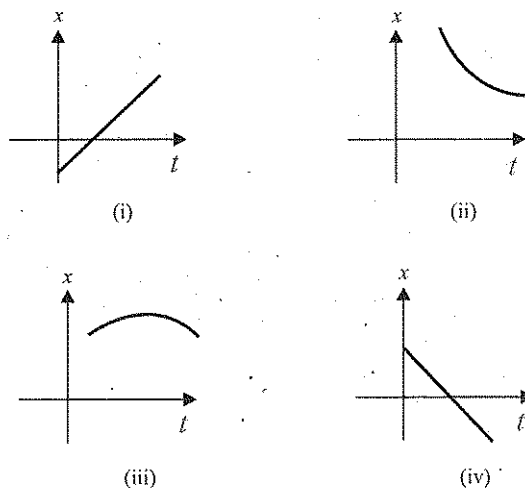
Study the four graphs given below. Answer the following questions on the basis of these graphs.



42. In which of the graphs, the particle has more magnitude of velocity at t_1 than at t_2 .
 a. (i), (iii) and (iv)
 b. (i) and (iii)
 c. (ii) and (iii)
 d. None of the above
43. Acceleration of the particle is positive
 a. in graph (i) b. in graph (ii)
 c. in graph (iii) d. in graph (iv)

For Problems 44–45

Study following graphs



44. The particle is moving with constant speed
 a. in graph (i) and (iii)

- b. in graph (i) and (iv)
 c. in graph (i) and (ii)
 d. in graph (i)

45. The particle has a negative acceleration

- a. in graph (i) b. in graph (ii)
 c. in graph (iii) d. in graph (iv)

Matching Column Type

Solutions on page 4.63

1. A particle moves along a straight line such that its displacement S varies with time t as $S = \alpha + \beta t + \gamma t^2$.

Column I	Column II
i. Acceleration at $t = 2$ s	a. $\beta + 5\gamma$
ii. Average velocity during 3 rd s	b. 2γ
iii. Velocity at $t = 1$ s	c. α
iv. Initial displacement	d. $\beta + 2\gamma$

2. For a body projected vertically up with a velocity $\vec{v}_0$ from the ground, match the following

Column I	Column II
i. $\vec{v}_{av}$	a. Zero for round trip
ii. u_{av}	b. $\frac{\vec{v}_1 + \vec{v}_2}{2}$ over any time interval
iii. T_{ascent}	c. $\frac{v_0}{2}$ over the total time of its flight
iv. $T_{descent}$	d. $\frac{v_0}{g}$

3. A ball is thrown vertically upward from the top of a cliff. Take the starting position of motion as origin and upward direction as positive. **Column I** specifies the position, velocity and/or acceleration of the particle at any instant. **Column II** gives their signs (+) or (−) at that moment. Match the columns

Column I	Column II
i. When the ball is above the point of projection, its displacement is	a. always positive
ii. When the ball is above the point of projection, its velocity is	b. always negative
iii. When the ball is above the point of projection, its acceleration is	c. may be positive or may be negative
iv. When the ball is below the point of projection, its acceleration is	d. may be zero

4. The displacement versus time curve is given (Fig. 4.124)

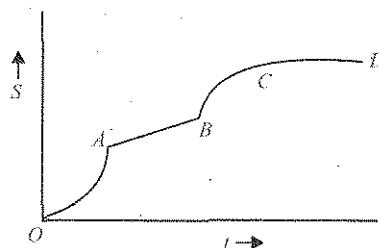


Fig. 4.124

Column I	Column II
i. OA	a. velocity increase with time linearly
ii. AB	b. velocity decreases with time
iii. BC	c. velocity is independent of time
iv. CD	d. velocity is zero

5. Study the following $v-t$ graphs in Column I carefully and match appropriately with the statements given in Column II. Assume that motion takes place from time 0 to T .

Column-I	Column-II
i.	a. net displacement is positive, but not zero
ii.	b. net displacement is negative, but not zero
iii.	c. particle returns to its initial position again
iv.	d. acceleration is positive

ANSWERS AND SOLUTIONS

Subjective Type

$$1. t_1 = \frac{AB}{s} = \frac{\frac{\pi}{2}r}{s}, t_2 = \frac{BC}{2s} = \frac{\frac{2\pi}{3}r}{2s}, t_3 = \frac{CA}{3s} = \frac{\frac{5\pi}{6}r}{3s}$$

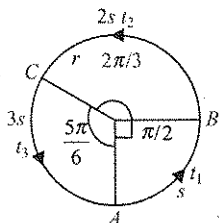


Fig. 4.125

$$\text{av. speed} = \frac{2\pi r}{t_1 + t_2 + t_3} = 1.8 \text{ s}$$

$$2. t_1 = \frac{S/2}{3} = \frac{5}{6}, t_2 = \frac{S_1}{4.5} = \frac{S_2}{7.5}, S_1 + S_2 = S/2$$

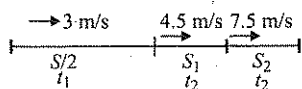


Fig. 4.126

$$\text{av. speed} = \frac{S}{t_1 + 2t_2} = 4 \text{ m/s}$$

$$3. \frac{D_4}{D_5} = \frac{(g/2)(2 \times 4 - 1)}{(g/2)(2 \times 5 - 1)} = \frac{7}{9}$$

4. Both will pass with same speed, because free fall motion is independent of mass.

5. Average velocity on each lap will be zero, because displacement is zero.

$$\text{av. speed} = \frac{2\pi r}{t} = \frac{2\pi \times 200}{62.8} = 20 \text{ m/s}$$

$$6. v_0 = 0 + \alpha t_1 \Rightarrow t_1 = v_0/\alpha, 0 = v_0 - \beta t_2 \Rightarrow t_2 = v_0/\beta$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{\beta}{\alpha}$$

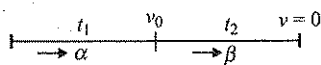


Fig. 4.127

$$7. a. v_{av-1} = \frac{50}{20} = 2.5 \text{ m/s}$$

$$b. v_{av-2} = \frac{50}{22} = 2.27 \text{ m/s}$$

c. v_{av} is zero for round trip because displacement is zero.

$$8. v_0 = \alpha t_1 = \beta t_2, d = \frac{1}{2}\alpha t_1^2 + \frac{1}{2}\beta t_2^2 = \frac{1}{2}(\alpha t_1)t_1 + \frac{1}{2}(\beta t_2)t_2$$

$$= \frac{1}{2}v_0 t_1 + \frac{1}{2}v_0 t_2 = \frac{1}{2}v_0(t_1 + t_2) = \frac{1}{2}v_0 t$$

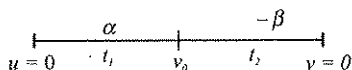


Fig. 4.128

$$\text{L.H.S.} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{t_1}{v_0} + \frac{t_2}{v_0} = \frac{t}{v_0} = \frac{t}{2d/t} = \frac{t^2}{2d}$$

$$= \text{R.H.S.}$$

$$9. v = \alpha t_1 = \beta t_3 \text{ (Fig. 4.129)}$$

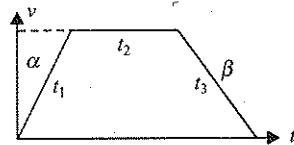


Fig. 4.129

$$l = (t_1 + t_2 + t_3) \frac{v}{2}$$

$$\Rightarrow \frac{2l}{v} = 2t_2 + \frac{v}{\alpha} + \frac{v}{\beta}$$

$$t_2 = \frac{l}{v} - \frac{1}{2} \left[\frac{v}{\alpha} + \frac{v}{\beta} \right]$$

$$t_1 + t_2 + t_3 = \frac{v}{\alpha} + \frac{v}{\beta} + t_2 = \frac{l}{v} + \frac{v}{2} \left[\frac{1}{\alpha} + \frac{1}{\beta} \right]$$

$$10. t = \frac{s_{\text{rel}}}{v_{\text{rel}}} = \frac{150}{10 + 5} = 10 \text{ s}$$

$$11. v^2 - u^2 = 2a \times 180$$

$$\Rightarrow 45^2 - u^2 = 360a$$

$$v = u + at \Rightarrow 45 = u + a \times 6$$

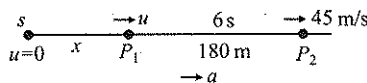


Fig. 4.130

Solve to get $u = 15 \text{ m/s}$, $a = 5 \text{ m/s}^2$

$$u^2 = 0^2 + 2ax \Rightarrow 15^2 = 0^2 + 2 \times 5x \Rightarrow x = 22.5 \text{ m.}$$

12. Taking upward direction as positive, let us work in the frame of lift. Acceleration of ball relative to lift = $(g + a)$ downward, so $a_{\text{rel}} = -(g + a)$, initial velocity: $u_{\text{rel}} = v$, final velocity: $v_{\text{rel}} = -v$ as the ball will reach the man with same speed w.r.t lift

$$\text{Apply } v_{\text{rel}} = u_{\text{rel}} + a_{\text{rel}}t \Rightarrow -v = v + (-g - a)t$$

$$\Rightarrow \frac{2v}{t} = g + a$$

13. Let at any time $t = 0$, balloon is at position A, where its velocity is u (Fig. 4.131). At $t = 2 \text{ s}$ it reaches at B, where its velocity becomes v , then

$$AB = S = u \times 2 + \frac{1}{2}a(2)^2 = 2v + 2$$

$$\text{also } v = u + a \times 2 = u + 2,$$

$$-S_1 = u \times 3.5 + \frac{1}{2}(-g)(3.5)^2,$$

$$-S_2 = v \times 1.5 + \frac{1}{2}(-g)(1.5)^2$$

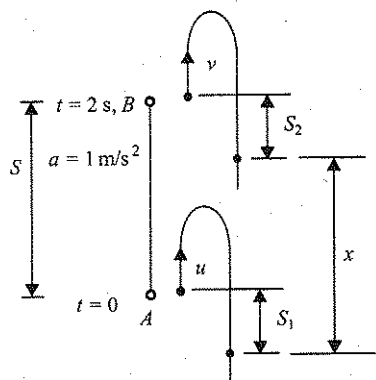


Fig. 4.131

Required distance between the stones:

$$x = S_1 + S - S_2$$

Solve to get $x = 55$ m

Alternatively: If we work from the frame of balloon, then acceleration of each stone w.r.t. balloon will be $g + a$ after releasing from it (Fig. 4.132). Initial velocity of each stone will be zero w.r.t. balloon.

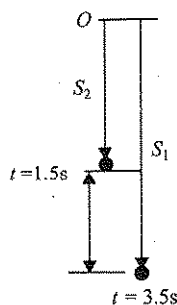


Fig. 4.132

$$S_1 = \frac{1}{2}(g + a)(3.5)^2$$

$$S_2 = \frac{1}{2}(g + a)(1.5)^2$$

$$x = S_1 - S_2 = 55 \text{ m}$$

$$14. t_1 + t_2 = 4 \text{ min}, v = a_1 t_1 = a_2 t_2$$

$$S = \frac{1}{2} \times 4v \Rightarrow 4 = 2v \Rightarrow v = 2$$

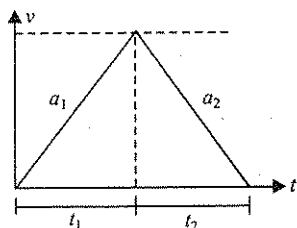


Fig. 4.133

$$t_1 + t_2 = v \left[\frac{1}{a_1} + \frac{1}{a_2} \right] \Rightarrow 4 = 2 \left[\frac{1}{a_1} + \frac{1}{a_2} \right]$$

$$\Rightarrow \frac{1}{a_1} + \frac{1}{a_2} = 2$$

$$15. S_1 = \frac{1}{2}(g + f)(t + t')^2, S_2 = \frac{1}{2}(g + f)t'^2$$

$$S = S_1 - S_2 = \frac{1}{2}(g + f)(t)(t + 2t')$$

16. Time taken by stone 1 to fall through $h - x$ is equal to time taken by stone 2 to fall through $h - y$ (Fig. 4.134)

$$h - y = \frac{1}{2}gt^2, u_1 = \sqrt{2gx}$$

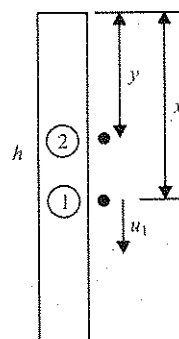


Fig. 4.134

$$h - x = u_1 t + \frac{1}{2}gt^2, \text{ solve to get } h = \frac{(x + y)^2}{4x}$$

$$17. a = g \sin \theta = 10 \times \frac{5}{10} = 5 \text{ m/s}^2$$

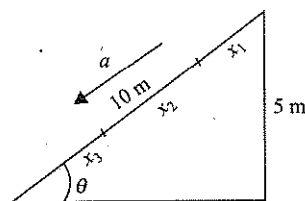


Fig. 4.135

$$x_1 = \frac{1}{2}at^2, x_1 + x_2 = \frac{1}{2}a(2t)^2$$

$$10 = x_1 + x_2 + x_3 = \frac{1}{2}a(3t)^2$$

$$\text{solve to get } x_1 = \frac{10}{9} \text{ m}, x_2 = \frac{30}{9} \text{ m}, x_3 = \frac{50}{9} \text{ m}, t = \frac{2}{3} \text{ s}$$

18. a. Let collision occurs at time t .

$$\text{For car w.r.t. truck: } S_{\text{rel}} = u_{\text{rel}}t + \frac{1}{2}a_{\text{rel}}t^2$$

$$60 = (30 - 10)t + \frac{1}{2}(-5 - 0)t^2$$

$t^2 - 8t + 24 = 0$, from here we will get no real value of t , it means collision does not occur.

b. Relative distance covered in 0.5 s $= (30 - 10)0.5 = 10$ m,

$$50 = (30 - 10)t + \frac{1}{2}(-a - 0)t^2$$

$$\Rightarrow at^2 - 40t + 100 = 0$$

To avoid collision, its discriminant, $D \leq 0$

$$\Rightarrow 40^2 - 4a100 \leq 0 \Rightarrow -a \geq 4 \text{ m/s}^2$$

$$19. \text{ From Fig. 4.136 } h = u \times 1 - \frac{1}{2}g(1)^2$$

$$h + 1 = u \times 1.1 - \frac{1}{2}g(1.1)^2 \Rightarrow u = 20.5 \text{ m/s}$$

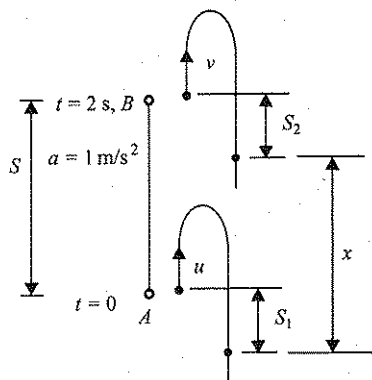


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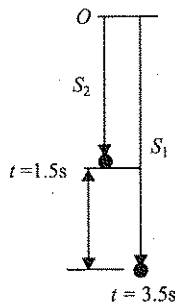


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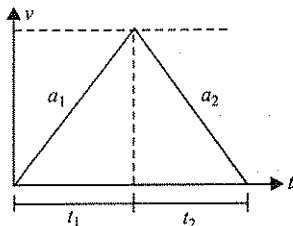


Fig. 4.133

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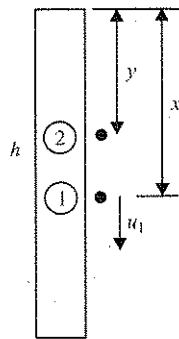


Fig. 4.134

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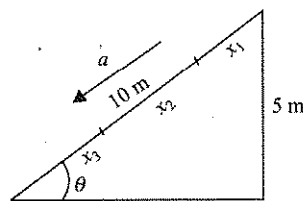


Fig. 4.135

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$$10 = x_1 + x_2 + x_3 = \frac{1}{2}a(3t)^2$$

$$\text{solve to get } x_1 = \frac{10}{9} \text{ m}, x_2 = \frac{30}{9} \text{ m}, x_3 = \frac{50}{9} \text{ m}, t = \frac{2}{3} \text{ s}$$

18. a. Let collision occurs at time t .

$$\text{For car w.r.t. truck: } S_{\text{rel}} = u_{\text{rel}}t + \frac{1}{2}a_{\text{rel}}t^2$$

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$$50 = (30-10)t + \frac{1}{2}(-a-0)t^2$$

$$\Rightarrow at^2 - 40t + 100 = 0$$

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$$\Rightarrow 40^2 - 4a100 \leq 0 \Rightarrow -a \geq 4 \text{ m/s}^2$$

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$$h+1 = u \times 1.1 - \frac{1}{2}g(1.1)^2 \Rightarrow u = 20.5 \text{ m/s}$$

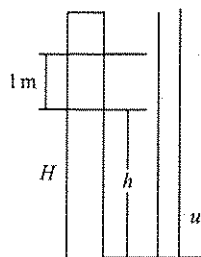


Fig. 4.136

$$H = \frac{u^2}{2g} = \frac{(20.5)^2}{2 \times 10} = 21 \text{ m}$$

$$20. t = \sqrt{\frac{2h}{ng}}, h - \frac{h}{n} = ut - \frac{1}{2}gt^2$$

$$\Rightarrow u = \frac{h}{t} = \sqrt{\frac{nhg}{2}}$$

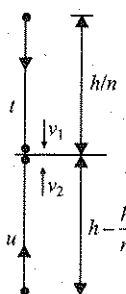


Fig. 4.137

$$v_1 = gt, v_2 = u - gt, \frac{v_2}{v_1} = \frac{u - gt}{gt} = \frac{u}{gt} - 1$$

$$\Rightarrow \frac{v_2}{v_1} = \frac{h}{gt^2} - 1 = \frac{hng}{g^2h} - 1 \Rightarrow \frac{v_1}{v_2} = \frac{2}{n-2}$$

$$21. a. t = \sqrt{\frac{2h}{g+a}} = \sqrt{\frac{2 \times 2.5}{10+1.25}} = \frac{2}{3} \text{ s}$$

b. velocity of lift after 1 s:

$$v = 0 + 1.25 \times 1 = 1.25 \text{ m/s} = 5/4 \text{ m/s}$$

This will be the initial velocity of bolt.

Distance moved up by lift in $\frac{2}{3}$ s:

$$x = \frac{5}{4} \times \frac{2}{3} + \frac{1}{2} \cdot 1.25 \left(\frac{2}{3}\right)^2 = \frac{10}{9} \text{ m}$$

$$\text{Displacement of bolt} = 2.5 - x = 2.5 - \frac{10}{9} = \frac{25}{18} \text{ m}$$

Maximum height attained by bolt above point of dropping

$$= \frac{v^2}{2g} = \frac{(5/4)^2}{2 \times 10} = \frac{5}{64} \text{ m}$$

$$\text{So distance travelled} = 2 \times \frac{5}{64} + \frac{25}{18} = \frac{445}{288} \text{ m}$$

$$22. t_1 = t_2 - t, v_1 = v_2 + v, S = \frac{1}{2}a_1t_1^2, S = \frac{1}{2}a_2t_2^2$$

$$v_1 = a_1t_1, v_2 = a_2t_2 \Rightarrow v_2 + v = a_1t_1$$

$$\Rightarrow a_2t_2 + v = a_1t_1 = a_1t_2 - a_1t$$

$$t_2 = \frac{v + a_1t}{a_1 - a_2}$$

$$\begin{aligned} \sqrt{\frac{a_2}{a_1}} &= \frac{t_1}{t_2} = 1 - \frac{t}{t_2} \Rightarrow \sqrt{\frac{a_2}{a_1}} = 1 - \frac{t(a_1 - a_2)}{(v + a_1t)} \\ \Rightarrow \frac{\sqrt{a_2}}{\sqrt{a_1}} &= \frac{v + a_1t}{v + a_1t} \\ \Rightarrow \sqrt{a_2}v + a_1\sqrt{a_2}t &= v\sqrt{a_1} + a_2\sqrt{a_1}t \\ \Rightarrow v &= (\sqrt{a_1a_2})t \end{aligned}$$

$$23. t_1 = \sqrt{2h/g} \rightarrow \text{for first stone}$$

$$-h = -u[t_1 - n] - \frac{1}{2}g(t_1 - n)^2 \rightarrow \text{for second stone}$$

$$h = u \left(\sqrt{\frac{2h}{g}} - n \right) + \frac{1}{2}g \left(\sqrt{\frac{2h}{g}} - n \right)^2$$

$$\text{simplify to get: } 8h(u - gn)^2 = gn^2(2u - gn)^2$$

$$24. d - a = u(3n) + \frac{1}{2}\alpha(3n)^2 = 3un + 9 \left(\frac{1}{2}\alpha n^2 \right) \quad (i)$$

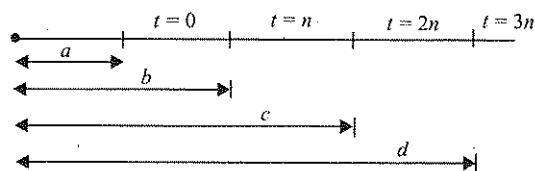


Fig. 4.138

$$c - a = u(2n) + \frac{1}{2}\alpha(2n)^2 \quad (ii)$$

$$b - a = un + \frac{1}{2}\alpha n^2 \quad (iii)$$

$$(ii) - (iii) \Rightarrow c - b = un + \frac{3}{2}\alpha n^2 \quad (iv)$$

$$\text{from (i) and (iv), } d - a = 3(c - b)$$

$$(ii) - 2 \times (iii), c - a - 2(b - a) = \alpha n^2$$

$$\Rightarrow \alpha = \frac{c + a - 2b}{n^2}$$

$$25. t_1 + t_2 = 2 \Rightarrow \frac{S}{30} + \frac{S}{60} = 2 \Rightarrow S = 40 \text{ km, total distance} = 80 \text{ km}$$

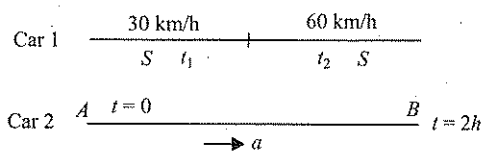


Fig. 4.139

$$80 = \frac{1}{2}a(2)^2, a = 40 \text{ km/h}^2, t_1 = \frac{40}{30} = \frac{4}{3} \text{ h}, t_2 = \frac{40}{60} = \frac{2}{3} \text{ h}$$

for $t < \frac{4}{3} \text{ h}$, h: If their speeds are equal,

$$30 = at \Rightarrow t = \frac{3}{2} \text{ h}$$

$$\text{If one overtakes the other: } 30t = \frac{1}{2}at^2 \Rightarrow t = \frac{3}{2} \text{ h}$$

for $t > \frac{4}{3} \text{ h}$: If their speeds are equal:

$$60 = at \Rightarrow t = \frac{3}{2} \text{ h}$$

$$\text{If one over takes other: } 40 + 60 \left(t - \frac{4}{3} \right) = \frac{1}{2} at^2$$

$$40 + 60t - 80 = 20t^2 \Rightarrow t^2 - 3t + 2 = 0$$

$$(t - 1)(t - 2) = 0 \Rightarrow t = 1 \text{ h (not valid), } 2 \text{ h}$$

$$26. S_1 = \frac{1}{2}(3 \times 10^{-2})(30)^2 = 13.5 \text{ m}$$

$$S_2 = \frac{1}{2}(3 \times 10^{-2})(90)^2 = 121.5 \text{ m}$$

$$d = s_2 - s_1 = 108 \text{ m}$$

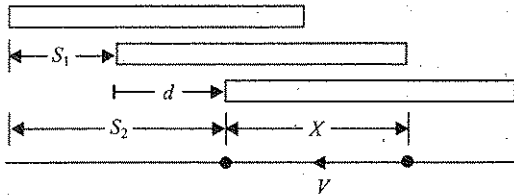


Fig. 4.140

- a. 350 m, with respect to train

$$x = l - d = 350 - 108 = 242 \text{ m, with respect to earth}$$

$$b. v = \frac{242}{60} = 4.03 \text{ m/s}$$

27. a. No, if position-time graph is a straight line parallel to position axis, its slope is infinite which indicates infinite velocity.

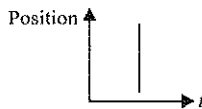


Fig. 4.141

It is not possible practically

- b. Displacement
c. Distance
d.

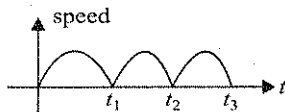


Fig. 4.142

- e. Velocity
f. Speed
g. No, its slope is infinite which indicates infinite velocity. It is not possible practically.
h. (c), its slope is negative

$$i. \text{ Slope of } PQ = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\text{total displacement}}{\text{time taken}}$$

$$= \text{Average velocity}$$

- j. We know that slope of velocity-time graph is equal to acceleration. So
i. as slope is zero, so zero acceleration.
ii. as slope is positive and constant, so constant positive acceleration.
iii. as slope is infinite, so infinite acceleration.
iv. as slope is negative and constant, so constant negative acceleration.

- v. as the slope is positive and increasing, so positive increasing acceleration.
vi. as the slope is positive and decreasing, so positive decreasing acceleration.

28. a. Cannot represent one dimensional motion, because there cannot be two values of displacements at one time. Also we don't consider time to be negative.
b. Cannot represent one dimensional motion, because distance cannot be negative. Also we don't consider time to be negative.
c. represents one dimensional motion, as displacement can be negative.
d. Cannot represent one dimensional motion, because there cannot be two values of displacements at one time.
e. Cannot represent one dimensional motion, because speed cannot be negative.
f. represents one dimensional motion, as velocity can be negative.
g. Cannot represent one dimensional motion, because there cannot be two values of velocity at one time.
h. Cannot represent one dimensional motion, because total path length does not decrease with time.

29. a. True, From graph, we see that for time $t = 0$ to $t = 2$ s, displacement of the particle remains same as 3 m. It means particle was at rest for this time interval.
b. True, Velocity is maximum when the slope is maximum. And slope is maximum for time $t = 5$ s to $t = 7$ s. velocity during this time = slope = $\frac{0 - 5}{7 - 5} = -2.5 \text{ m/s}$

30. a. true, average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

$$= \frac{3 - 8}{4 - 1.5} = -2.5 \text{ m/s}$$

- b. False, slope at $t = 2$ s is,

$$\text{Slope} = -\frac{13}{3.5} = -3.7 \text{ m/s}$$

- c. True, since slope is zero at $t = 4$ s, so velocity is zero at $t = 4$ s.

31. Slope for C is greatest so C will have greater velocity and slope for A is least so A will have least velocity.

32. a. 0 - 20 s, 140 - 180 s, 240 - 280 s, 380 - 440 s: For these time intervals, the graph is parallel to time axis.

- b. 20 - 140 s, 280 - 380 s: For these time intervals, the value of position (x) is increasing.

- c. 180 - 240 s: For this time interval, the value of position (x) is decreasing.

33. a. At point IV velocity is zero because slope is zero at this point.

- b. At point I slope is positive and constant.

- c. At point V slope is negative and constant

- d. At point II slope suddenly increases.

- e. At point III slope is positive but decreasing.

34. a. At $t = 3$ s the graph is horizontal so the acceleration is 0. From $t = 5$ s to $t = 9$ s, the acceleration is constant (from

the graph) and equal to $\frac{45-20}{9-5} = 6.25 \text{ m/s}^2$

From $t = 9 \text{ s}$ to $t = 13 \text{ s}$ the acceleration is constant and equal to $\frac{0-45}{13-9} = -11.25 \text{ m/s}^2$

- b. In the first five seconds, the area under the graph is the area of the rectangle, $20 \times 5 = 100 \text{ m}$.

Between $t = 5 \text{ s}$ and $t = 9 \text{ s}$, the area under the trapezoid is $(1/2)(45+20)(4) = 130 \text{ m}$ and so the total distance in the first 9 s is $100 + 130 = 230 \text{ m}$.

Between $t = 9 \text{ s}$ and $t = 13 \text{ s}$, the area under the triangle is $(1/2)(45)(4) = 90 \text{ m}$, and so the total distance in the first 13 s is $230 + 90 = 320 \text{ m}$.

35. a. Slope of graph is constant, i.e., $\frac{v_2 - v_1}{t_2 - t_1} = \text{constant}$.

To find velocity at $t = 4 \text{ s}$:

$$\text{Slope: } \frac{8-0}{0-6} = \frac{8-v}{0-4} \Rightarrow v = \frac{8}{3} \text{ cm/s}$$

To find velocity at $t = 7 \text{ s}$:

$$\text{Slope: } \frac{8-0}{0-6} = \frac{8-v}{0-7} \Rightarrow v = 4/3 \text{ cm/s}$$

- b. $a = \text{slope of } v-t \text{ graph} = -\frac{8.0}{6.0} = -4/3 \text{ cm/s}$ which is constant.

- c. For first 4.5 s:

Let us first find the velocity at $t = 4.5 \text{ s}$

$$\frac{8-0}{0-6} = \frac{8-v}{0-4.5} \Rightarrow v = 2 \text{ cm/s}$$

Now $\Delta x = \text{area under } v-t \text{ graph for first 4.5 s}$

$$\Rightarrow \Delta x = A_{\text{Rectangle}} + A_{\text{Triangle}}$$

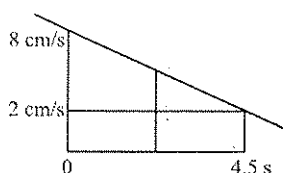


Fig. 4.143

$$= 4.5 \times 2 + \frac{1}{2} \times 4.5 \times 6 = 22.5 \text{ cm}$$

From $t = 0$ to $t = 7.5 \text{ s}$:

Let us first find velocity at $t = 7.5 \text{ s}$

$$\frac{8-0}{0-6} = \frac{8-v}{0-7.5} \Rightarrow v = -2 \text{ cm/s}$$

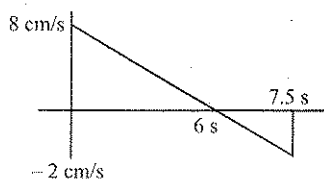


Fig. 4.144

The distance is the sum of the magnitudes of the areas.

$$d = \frac{1}{2} (6\text{s}) \left(\frac{8 \text{ cm}}{\text{s}} \right) + \frac{1}{2} (1.5\text{s}) \left(\frac{2 \text{ cm}}{\text{s}} \right) = 25.5 \text{ cm}$$

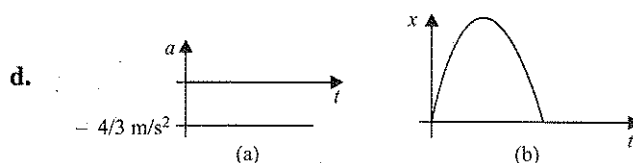


Fig. 4.145

36. To find, a , we apply the relation $a = dv/dt$, which is the same as $a = \Delta v / \Delta t$ over time intervals in which v varies linearly with t . For the time interval between $t_0 = 0$ and $t_1 = 2 \text{ s}$, the slope of the graph is positive and constant, and is equal to

$$a_1 = \frac{\Delta v}{\Delta t} = \frac{(v_1 - v_0)}{(t_1 - t_0)} = \frac{(2 \text{ m/s} - 0)}{2\text{s} - 0} = 1 \text{ m/s}^2$$

Between $t_1 = 2 \text{ s}$ and $t_2 = 4 \text{ s}$, the slope is zero, so the acceleration is zero for this time interval. Between $t_2 = 4 \text{ s}$ and $t_3 = 5 \text{ s}$, the slope is negative and constant, equal to

$$a_2 = \frac{\Delta v}{\Delta t} = \frac{(v_3 - v_2)}{(t_3 - t_2)} = \frac{(0 - 2 \text{ m/s})}{5\text{s} - 4\text{s}} = -2 \text{ m/s}^2$$

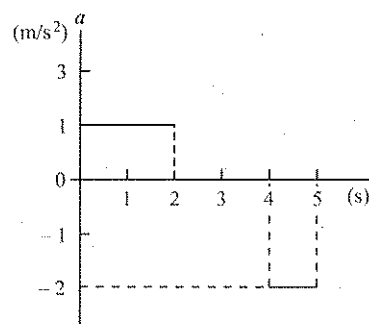


Fig. 4.146

Figure 4.146 is a graph of the instantaneous acceleration for the motion.

37. We know that displacement is area under $v-t$ graph. Area from 0 to 2 s: $4 \times 2 = 8 \text{ m}$, area from 2 to 4 s: $-2 \times 2 = 4 \text{ m}$, area from 4 to 6 s: $2 \times 2 = 4 \text{ m}$

So displacement $= 8 - 4 + 4 = 8 \text{ m}$

When we find distance, we consider all areas to be +ve.

So distance $= 8 + 4 + 4 = 16 \text{ m}$

38. At $t = 5 \text{ s}$ $v_1 = u + at = 0 + 2 \times 5 = 10 \text{ m/s}$ (Fig. 4.147)

at $t = 15 \text{ s}$: $v_1 = 10 \text{ m/s}$

at $t = 25 \text{ s}$: $v_2 = v_1 + at = 10 - 2 \times 10 = -10 \text{ m/s}$

at $t = 35 \text{ s}$, $v_2 = -10 \text{ m/s}$

at $t = 40 \text{ s}$: $v_3 = v_2 + at = -10 + 2 \times 5 = 0 \text{ m/s}$

To find x

$$\text{for } 0 \text{ to } 5 \text{ s, } x = \frac{1}{2} \times 2 \times 5^2 = 25 \text{ m}$$

$$\text{for } 5 \text{ to } 15 \text{ s, } x = 25 + 10 \times 10 = 125 \text{ m}$$

$$\text{for } 15 \text{ to } 20 \text{ s, } x = 125 + v_1 t + \frac{1}{2} at^2 \\ = 125 + 10 \times 5 + \frac{1}{2} (-2) 5^2 = 150 \text{ m}$$

$$\text{for } 20 \text{ to } 25 \text{ s, } x = 150 + 0 \times 5 + \frac{1}{2} (-2) 5^2 = 125 \text{ m}$$

$$\text{for } 25 \text{ to } 35 \text{ s, } x = 125 + v_2 t = 125 - 10 \times 10 = 25 \text{ m}$$

$$\text{for } 35 \text{ to } 40 \text{ s, } x = 25 + v_2 t + \frac{1}{2} at^2$$

$$= 25 - 10 \times 5 + \frac{1}{2} \times 2 \times 5^2 = 0$$

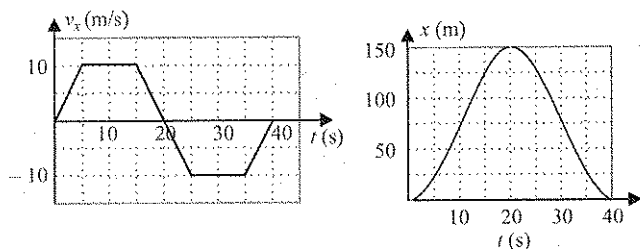


Fig. 4.147

39. Time taken to reach the office $= \frac{2.5}{5} = \frac{1}{2} \text{ h} = 30 \text{ min}$ (Fig. 4.148).

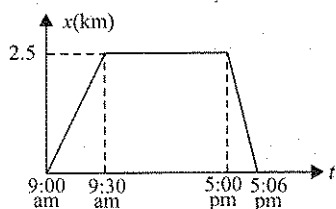


Fig. 4.148

$$\text{Time taken to return home} = \frac{2.5}{25} = \frac{1}{10} \text{ h} = 6 \text{ min}$$

40. We first sketch Fig. 4.149 (a) to depict the motion and indicate our choice of locating the origin of the coordinate system where the runner starts. We also show significant positions of the runner during the motion.

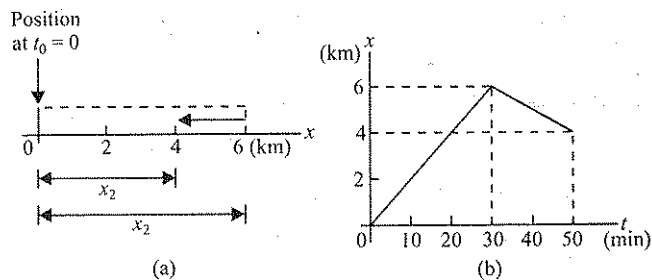


Fig. 4.149

Position	Time
$x_0 = 0$	$t_0 = 0$
Jogging $x_1 = 6 \text{ km}$	$t_1 = 30 \text{ min}$
Walking $x_2 = 4 \text{ km}$	$t_2 = 50 \text{ min}$

- a. True, $\Delta x = (x_2 - x_0) = (4 \text{ km} - 0) = 4 \text{ km}$
 b. True, average speed $= \frac{\text{Total distance}}{\text{Total time}} = \frac{8 \text{ km}}{50 \text{ min}} = 0.160 \text{ km/min}$

- c. False, the average velocity for the entire trip is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{(x_2 - x_0)}{(t_2 - t_0)} = \frac{4 - 0}{50 - 0} = 0.080 \text{ km/min}$$

- d. False, while jogging

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{(x_1 - x_0)}{(t_1 - t_0)} = \frac{6 - 0}{30 - 0} = 0.200 \text{ km/min}$$

- e. True, while walking

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{(x_1 - x_1)}{(t_1 - t_1)} = \frac{(4 \text{ km} - 6 \text{ km})}{(50 \text{ min} - 30 \text{ min})} = \frac{-2 \text{ km}}{20 \text{ min}} = -0.100 \text{ km/min (walking)}$$

The negative value indicates a velocity in the $-x$ direction.

A word of caution: Please note that the runner's average velocity for the entire trip is not simply the average of her jogging and walking velocities, because the times she spends in these two different motions are not equal.

41. a. The truck's position as a function of time is given by $x_T = v_T t$, with v_T being the truck's constant speed, and the car's position is given by $x_C = (1/2) a_C t^2$. Equating the two expressions and dividing by a factor of t (this reflects the fact that the car and the truck are at the same place at $t = 0$) and solving for t yields

$$t = \frac{2v_T}{a_C} = \frac{2(20.0 \text{ m/s})}{3.20 \text{ m/s}^2} = 12.5 \text{ s}$$

and at this time $x_T = x_C = 250 \text{ m}$.

- b. $v_c = a_C t = (3.20 \text{ m/s}^2)(12.5 \text{ s}) = 40.0 \text{ m/s}$

- c.

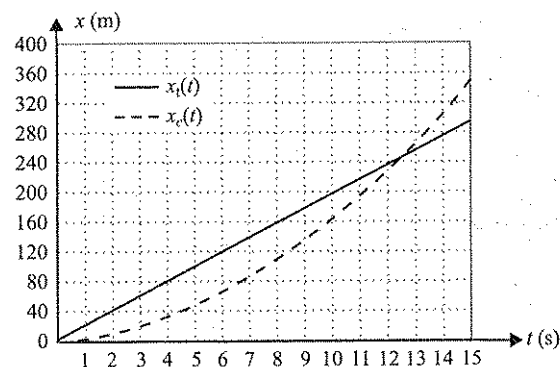


Fig. 4.150

- d. Let us see when the velocities of both become same

$$\text{Fig. 4.151: for this } v_c = v_T \Rightarrow a_C t = v_T \\ 3.2t = 20 \Rightarrow t = 6.25 \text{ s}$$

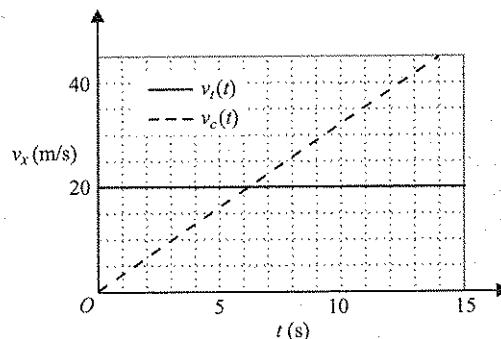


Fig. 4.151

42. a. $\Delta x_{01} = x_f - x_i = -2 - 0 = -2 \text{ m}$
 $\Delta x_{13} = x_f - x_i = 6 - (-2) = 8 \text{ m}$
 b. $\bar{v}_{01} = \frac{\Delta x_{01}}{\Delta t} = \frac{-2 \text{ m}}{1 \text{ s}} = -2 \text{ m/s}$
 $\bar{v}_{13} = \frac{\Delta x_{13}}{\Delta t} = \frac{8 \text{ m}}{2 \text{ s}} = 4 \text{ m/s}$
 c. $v = \frac{dx}{dt} = \frac{d}{dt}(-4t + 2t^2) = 4(-1 + t) \text{ m/s}$
 Therefore, at $t = 2.5 \text{ s}$, $v = 4(-1 + 2.5) = 6 \text{ m/s}$

43. $a = 2t - 2$

a. $\int_0^v dv = \int_0^t a dt \Rightarrow v = (2t^2/2 - 2t)_0^t = t^2 - 2t$
 b. $S = \int_2^4 v dt = \left(\frac{t^3}{3} - 2 \frac{t^2}{2} \right)_2^4 = \frac{20}{3} \text{ m}$

Objective Type

1. b. If the displacement is zero, the distance moved may or may not be zero. For example, if a particle returns to its initial position after moving through a distance, displacement will be zero but distance covered will not be zero.
 2. a.
 3. d. We know that average velocity = $\frac{\text{Displacement}}{\text{Time}}$ and

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}}$$

 Since displacement can be less than or equal to distance, so average velocity can be less than or equal to average speed.
 4. b. At any instant velocity and speed will be equal.
 5. c.
 6. a. If the location of a particle changes, then both distance and displacement must have some value.
 7. a. During retarded motion, acceleration and velocity are in opposite directions.
 8. b.
 9. c. To have distance equal to magnitude of displacement, the particle has to move in the same direction. The velocity may or may not be constant.
 10. d. $s = kt^2$

$$\Rightarrow v = \frac{ds}{dt} = 2kt$$

$$\Rightarrow a = \frac{dv}{dt} = 2k$$

From above, acceleration is independent of time, hence acceleration is constant.

11. b.
 12. c.
 13. b. $s = kt$, differentiating s twice to get acceleration, we see that acceleration comes out to be zero.
 14. c. Since maximum velocity is more than average velocity, therefore ratio of the average velocity to maximum velocity has to be less than one.

15. d. $s = kt^{1/2} \Rightarrow a = \frac{d^2s}{dt^2} = -\frac{1}{4}kt^{-3/2}$
 As t increases, retardation decreases.

16. a.

17. a. The only force acting on both will be gravity which will produce same acceleration g in both. Further, both the balls are dropped simultaneously from same height, hence both will come together on the ground.

18. d. $s = \frac{1}{2}at^2 \Rightarrow t \propto \sqrt{s}$

19. b. When a body possesses constant velocity, then both its magnitude (i.e., speed) and direction must remain constant. On the other hand, if the speed of a body is constant, then its velocity may or may not remain constant. For example, in uniform circular motion, though the speed of body remains constant but velocity changes from point to point due to change in direction.

A body moving with a constant speed along a circular path constantly experiences a centripetal acceleration.

20. c. $x = at^2 - bt^3$
 velocity = $\frac{dx}{dt} = 2at - 3bt^2$
 and acceleration = $\frac{d}{dt} \left(\frac{dx}{dt} \right) = 2a - 6bt$
 Acceleration will be zero if

$$2a - 6bt = 0 \Rightarrow t = \frac{2a}{6b} = \frac{a}{3b}$$

21. a. $V_{av} = \frac{v_1(t/2) + v_2(t/2)}{t} = \frac{v_1 + v_2}{2}$
 22. d. $t_1 = \frac{S/2}{v_1}$, $t_2 = \frac{S/2}{v_2}$, $v_{av} = \frac{S}{t_1 + t_2} = \frac{2v_1v_2}{v_1 + v_2}$
 23. b. $t_1 = \frac{S/3}{v_1}$, $t_2 = \frac{S/3}{v_2}$, $t_3 = \frac{S/3}{v_3}$

$$v_{av} = \frac{S}{t_1 + t_2 + t_3} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

 24. c. $S_1 = \frac{1}{2}at^2 = \frac{1}{2}(at)t = \frac{60t}{2} = 30t$

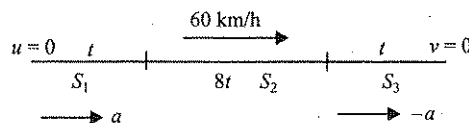


Fig. 4.152

$$S_2 = 60 \times 8t = 480t, S_3 = S_1 = 30t$$

$$v_{av} = \frac{S_1 + S_2 + S_3}{t + 8t + t} = 54 \text{ km/h}$$

25. a. $t_1 = \frac{x/2}{3} = \frac{x}{6}$, $x_1 = 4.5t_2$, $x_2 = 7.5t_2$

$$\text{Also } x_1 + x_2 = \frac{x}{2} = (4.5 + 7.5)t_2 \Rightarrow t_2 = \frac{x}{24}$$

$$\text{Total time: } t = t_1 + 2t_2 = \frac{x}{6} + \frac{2x}{24} = \frac{x}{4}$$

$$v_{av} = \frac{x}{t} = 4 \text{ m/s}$$

26. c. Time of flight: $T = \sqrt{\frac{2H}{g}} \Rightarrow T \propto \sqrt{H}$
27. a. As both balls are dropped from same height, hence both will come together on the earth.
28. b. Distance travelled in 4th second = $16 - 9 = 7$ m
Now $D_n = u + \frac{a}{2}(2n - 1)$, Given $u = 0$
 $\Rightarrow 7 = 0 + \frac{a}{2}(2 \times 4 - 1) \Rightarrow a = 2 \text{ m/s}^2$
29. a. $30 = u + a \times 2$, $60 = u + a \times 4$
Solve to get $u = 0$
30. b. $x = 2 - 5t + 6t^2 \Rightarrow v = \frac{dx}{dt} = -5 + 12t$
 $v_{t=0} = -5 + 12 \times 0 = -5 \text{ m/s}$
31. d. $2ax = (50)^2 - (10)^2$ and $2(-a)(-x) = v^2 - (50)^2$
This gives $v^2 - (50)^2 = (50)^2 - (10)^2$, i.e., $v = 70 \text{ m/s}$
32. a. $v^2 = 108 - 9x^2$ (i)

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{d(\sqrt{108 - 9x^2})}{dx} \cdot \frac{dx}{dt}$$

$$a = \frac{1(-18x)}{2\sqrt{108 - 9x^2}} \cdot \sqrt{108 - 9x^2} = -9x \text{ m/s}^2$$

33. c. We have $h = \frac{1}{2}gT^2$
In $\frac{T}{3}$ s, the distance fallen = $\frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{h}{9}$
So position of the ball from the ground
 $= h - \frac{h}{9} = \frac{8h}{9} \text{ m}$
34. b. $x = a \cos t$, $\frac{dx}{dt} = -a \sin t$; $\frac{d^2x}{dt^2} = -a \cos t$
35. b. Because one taxi leaves every 10 min, hence at any instant there will be 11 taxis on the way towards each station, one will be arriving and another leaving the other station. Figure shows the location of taxis going from X to Y at the instant 2.00 PM. The taxi which left station X at 0.00 PM has just arrived at station Y. Consider the taxi leaving the station Y at 2.00 PM.

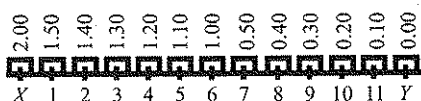


Fig. 4.153

It will meet all the 11 taxis marked 1 to 11 as well as 12 other taxis which would leave the station X from 2.00 PM to 3.50 PM. When it arrives at the station X at 4.00 PM, there will be one more taxi leaving that station. However, it will not be counted among the taxis crossed by taxi under consideration. That is, it will cross 23 taxis leaving the station X from 0.10 PM to 3.50 PM.

36. a. Given that $u = 0$ (the electron starts from rest)
At any time t : $v = kt = 2t$. $a = \frac{dv}{dt} = 2 \text{ m/s}^2$ (constant acceleration)

$$\text{Now } s = ut + \frac{1}{2}at^2 = 0 \times 3 + \frac{1}{2} \times 2 \times (3)^2 = 9 \text{ m}$$

37. d. $v^2 - u^2 = 2as$, $v = 0$
Maximum retardation, $s \propto u^2$
When the initial velocity is made n times, then the distance over which it can be stopped becomes n^2 times.
38. d. Relative velocity of policeman w.r.t. the thief is $10 - 9 = 1 \text{ m/s}$. Since the relative separation between them is 100 m, hence, the time taken will be = relative separation/relative velocity = $100/1 = 100 \text{ s}$.
39. c. We know that slope of displacement-time graph is equal to velocity. So $v_A = \tan 30^\circ = 1/\sqrt{3}$,
 $v_B = \tan 60^\circ = \sqrt{3}$, hence $v_A/v_B = 1/3$
40. d. Distance covered by the object in first 2 s:

$$h_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

Similarly distance covered by the object in next 2 s will also be 20 m, hence the required height = $H - 20 - 20 = H - 40 \text{ m}$

41. c. $h = \frac{1}{2}gt^2$ and $h - 20 = \frac{1}{2}g(t - 1)^2$
Solving them we get, $t = 2.5 \text{ s}$ and $h = 31.25 \text{ m}$.
42. a. Relative velocity of overtaking = $40 - 30 = 10 \text{ ms}^{-1}$.
Total relative distance covered with this relative velocity during overtaking = $100 + 200 = 300 \text{ m}$
So time taken = $300/10 = 30 \text{ s}$
43. a. Time of ascent = $1 \text{ s} \Rightarrow \frac{u}{g} = 1 \Rightarrow u = 10 \text{ m/s}$
Maximum height attained = $\frac{u^2}{2g} = \frac{10^2}{2 \times 10} = 5 \text{ m}$
44. c. Maximum height attained $\propto u^2$
45. b. Given $7x = \frac{g}{2}(2n - 1)$ and $x = \frac{1}{2}g(1)^2$
Solving these two equations: $n = 4$ s
46. c. $v^2 - u^2 = 2as$
Suppose velocity of the middle part = v_m
Then $v_m^2 - u^2 = 2as \times \frac{1}{2} = as$ or
 $v_m^2 = u^2 + as = u^2 + \frac{v^2 - u^2}{2} = \frac{u^2 + v^2}{2}$
 $\Rightarrow v_m = \sqrt{\frac{u^2 + v^2}{2}}$
47. a. $t = \alpha x^2 + \beta x = x(\alpha x + \beta)$
 $1 = 2\alpha \frac{dx}{dt} \cdot x + \beta \frac{dx}{dt}$
 $v = \frac{dx}{dt} = \frac{1}{\beta + 2\alpha x}$; $\frac{dv}{dt} = \frac{-2\alpha v}{(\beta + 2\alpha x)^2} = -2\alpha v^3$
48. a. $t = \sqrt{x} + 3$, differentiating with respect to t , we get, $1 = \frac{1}{2\sqrt{x}} \frac{dx}{dt} + 0$ or $\frac{dx}{dt} = 2\sqrt{x}$
when velocity is zero, then $2\sqrt{x} = 0$ or $x = 0$.

49. d. If speeds are comparable to the velocity of light c , according to theory of relativity, velocity of B relative to A (when both are moving along the same line in opposite directions) is given by: $v_{BA} = \frac{v_B + v_A}{1 + \frac{v_A v_B}{c^2}}$. From this, it is clear that if v_A is equal to c , $v_{BA} = \frac{v + c}{1 + \frac{v}{c}} = c$

Note: Speed of light is independent of relative motion between source and observer.

50. b. $a_x = \frac{d^2x}{dt^2} = 8$ and $a_y = \frac{d^2y}{dt^2} = 0$

Hence, net acceleration $= \sqrt{a_x^2 + a_y^2} = 8 \text{ m/s}^2$

51. b. The required ratio is 1:3:5: ... so on

52. c. Relative acceleration of both will be zero w.r.t. each other. So, $s_{\text{rel}} = u_{\text{rel}}t$ or $100 = 100t$ or $t = 1 \text{ s}$

53. a. Since the last five steps covering 5 m land the drunkard fell into the pit, the displacement prior to this is $(11 - 5) \text{ m} = 6 \text{ m}$.

Time taken for first eight steps (displacement in first eight steps $= 5 - 3 = 2 \text{ m}$) $= 8 \text{ s}$. Then Time taken to cover

first six meters of journey $= \frac{6}{2} \times 8 = 24 \text{ s}$

Time taken to cover last 5 m $= 5 \text{ s}$

Total time $= 24 + 5 = 29 \text{ s}$

54. c. Here $h = \frac{1}{2} \times 10 \times (5)^2 = 125 \text{ m}$

In 3 s it falls through:

$h_1 = \frac{1}{2} \times 10 \times (3)^2 = 45 \text{ m}$

Rest 80 m is covered in 4 s. Hence, total time taken $= 3 \text{ s} + 4 \text{ s} = 7 \text{ sec}$.

55. b. $200 = u \times 2 - (1/2)a(2)^2$

or $u - a = 100$ (i)

$200 + 220 = u(2 + 4) - (1/2)(2 + 4)^2 a$

or $u - 3a = 70$ (ii)

Solving equations (i) and (ii), we get $a = 15 \text{ cm/s}^2$ and $u = 115 \text{ cm/s}$.

Further, $v = u - at = 115 - 15 \times 7 = 10 \text{ cm/s}$.

56. c. Graphically, area of $(v-t)$ curve represents displacement:

$S = \frac{1}{2} v_{\text{max}} t_1$ or $t_1 = \frac{2S}{v_{\text{max}}}$

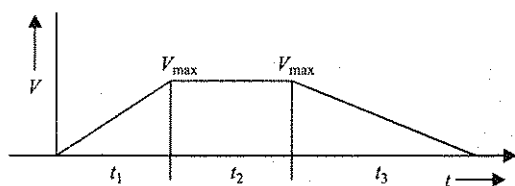


Fig. 4.154

$2S = v_{\text{max}} t_2$ or $t_2 = \frac{2S}{v_{\text{max}}}$

$5S = \frac{1}{2} v_{\text{max}} t_3$ or $t_3 = \frac{10S}{v_{\text{max}}}$

$v_{\text{av}} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{S + 2S + 5S}{\frac{2S}{v_{\text{max}}} + \frac{2S}{v_{\text{max}}} + \frac{10S}{v_{\text{max}}}}$

$\frac{v_{\text{av}}}{v_{\text{max}}} = \frac{8S}{14S} = \frac{4}{7}$

Alternatively:

$\frac{v_{\text{av}}}{v_{\text{max}}} = \frac{\text{Total displacement}}{2 \left(\begin{array}{l} \text{Total displacement} \\ \text{during acceleration} \\ \text{and retardation} \end{array} \right) + \left(\begin{array}{l} \text{Displacement} \\ \text{during uniform} \\ \text{velocity} \end{array} \right)}$

$\frac{v_{\text{av}}}{v_{\text{max}}} = \frac{8S}{2(S + 5S) + 2S} = \frac{8}{14} = \frac{4}{7}$

57. b. When a body slides on an inclined plane, component of weight along the plane produces an acceleration.

$a = \frac{mg \sin \theta}{m} = g \sin \theta = \text{constant}$. If s be the length of the inclined plane, then

$s = 0 + \frac{1}{2} at^2 = \frac{1}{2} g \sin \theta \times t^2$

$\frac{s'}{s} = \frac{t'^2}{t^2}$ or $\frac{s'}{s'} = \frac{t'^2}{t'^2}$

Given, $t = 4 \text{ s}$ and $s' = \frac{s}{4}$

$t' = 1 \sqrt{\frac{s'}{s}} = 4 \sqrt{\frac{s}{4s}} = \frac{4}{2} = 2 \text{ s}$

58. a. Bomb B_1 will have less velocity upward on dropping, so it will reach ground first.

59. c. Time taken to cover first $n \text{ m}$ is given by

$n = \frac{1}{2} g t_n^2$ or $t_n = \sqrt{\frac{2n}{g}}$

Time taken to cover first $(n+1) \text{ m}$ is given by

$t_{n+1} = \sqrt{\frac{2(n+1)}{g}}$

So time taken to cover $(n+1) \text{ m}$ is given by

$t_{n+1} - t_n = \sqrt{\frac{2(n+1)}{g}} - \sqrt{\frac{2n}{g}} = \sqrt{\frac{2}{g}} [\sqrt{n+1} - \sqrt{n}]$

This gives the required ratio as:

$\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), \dots$ etc. (starting from $n = 0$)

60. c. $AB = 30 \text{ m}$, $BC = 20 \text{ m}$, $CD = 30\sqrt{2} \text{ m}$

$\triangle BCE$ is isosceles (Fig. 4.155), so

$BE = BC = 20 \text{ m}$

$AE = AB - BE$

$= 30 - 20 = 10 \text{ m}$

$EC = 20\sqrt{2} \text{ m}$

$ED = CD - EC = 30\sqrt{2} - 20\sqrt{2} = 10\sqrt{2} \text{ m}$

$\triangle ADE$ is isosceles, so $AD = AE = 10 \text{ m}$

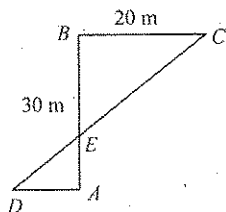


Fig. 4.155

61. a.
- $u = 0$
- ,
- $v = 27.5$
- m/s,
- $t = 10$
- s

$$a = \frac{v - u}{t} = \frac{27.5}{10} = 2.75 \text{ m/s}^2$$

In the first 10 s, distance travelled:

$$s_1 = 0 \times 10 + \frac{1}{2} \times 2.75 \times 10^2 = 137.5 \text{ m}$$

In the first 20 s, distance travelled:

$$s_2 = 0 \times 20 + \frac{1}{2} \times 2.75 \times 20^2 = 550 \text{ m}$$

Required distance = $550 - 137.5 = 412.5$ m

62. b. Distance covered in first three seconds = Distance covered in last second

$$\Rightarrow \frac{1}{2}g(3)^2 = \frac{g}{2}(2t - 1) \Rightarrow t = 5 \text{ s}$$

63. a.
- $S = ut + \frac{1}{2}at^2$

$$-76 = 4 \times 6 - \frac{1}{2} \times 10 \times (6)^2 \Rightarrow u = \frac{52}{3} \text{ m/s}$$

64. a. Suppose
- v
- be the velocity of the body after falling through half the distance. Then

$$s = \frac{39.2}{2} = 19.6 \text{ m}, u = 0 \text{ and } g = 9.8 \text{ m/s}^2$$

$$v^2 = u^2 + 2gh = 0^2 + 2 \times 9.8 \times 19.6$$

$$\Rightarrow v = 19.6 \text{ m/s.}$$

When the acceleration due to gravity ceases to act, the body travels with the uniform velocity of 19.6 m/s. So it hits the ground with velocity 19.6 m/s.

65. c. Distance travelled by first train,

$$s_1 = \frac{u^2}{2a} = \frac{(15)^2}{2 \times 1} = 112.5 \text{ m}$$

Distance travelled by second train,

$$s_2 = \frac{(20)^2}{2 \times 1} = 200 \text{ m}$$

Total distance travelled = $112.5 + 200 = 312.5$ mDistance of separation = $500 - 312.5 = 187.5$ m

66. d.
- $t = \frac{S_{\text{rel}}}{v_{\text{rel}}} \Rightarrow \frac{4}{60} = \frac{5}{30 + v_2} \Rightarrow v_2 = 45 \text{ km/h}$

67. a.
- $3v_A = v_B$
- ,
- $S_{\text{rel}} = v_{\text{rel}}t \Rightarrow 100 + 60 = (v_A + v_B) \times 4$
-
- Solve to get,
- $v_A = 10$
- m/s and
- $v_B = 30$
- m/s

68. d. Relative speed of trains =
- $37.5 + 37.5 = 75$
- km/h

Time taken by the trains to meet = $90/75 = 6/5$ h

Since speed of bird = 60 km/h, So distance travelled by the bird = $60 \times 6/5 = 72$ km.

69. c. Given
- $u = 0$
- . Let during three phases time taken are
- t
- ,
- $8t$
- and
- t
- , respectively.
- $v_{\text{max}} = at = 60$
- km/h

$$v_{\text{av}} = \frac{\frac{1}{2}at^2 + v_{\text{max}}8t + \frac{1}{2}at^2}{t + 8t + t} = \frac{at^2 + 8v_{\text{max}}t}{10t} = \frac{at + 8v_{\text{max}}}{10}$$

$$= \frac{60 + 8 \times 60}{10} = 54 \text{ km/h}$$

70. c. According to 3rd equation of motion.

$$v^2 - u^2 = 2as$$

Given $v = 3u$, $u = v$ and $a = g$

$$\text{Or } (3v)^2 - v^2 = 2gs \text{ or } s = \frac{4v^2}{g}$$

71. a. Time of fall =
- $\sqrt{\frac{2h}{g}}$

Time taken by the sound to come out = $\frac{h}{c}$

$$\text{Total time} = \sqrt{\frac{2h}{g}} + \frac{h}{c} = h \left[\sqrt{\frac{2}{gh}} + \frac{1}{c} \right]$$

72. c.
- $t_1 = \frac{s}{v_1}$
- ,
- $t_2 = \frac{s}{v_2}$
- , ...,
- $t_n = \frac{s}{v_n}$

Average speed = (total distance)/(total time)

$$\frac{s}{v_1} + \frac{s}{v_2} + \dots + \frac{s}{v_n}$$

Fig. 4.156

$$\Rightarrow \bar{V} = \frac{ns}{t_1 + t_2 + \dots + t_n}$$

$$\Rightarrow \bar{V} = \frac{ns}{\frac{s}{v_1} + \frac{s}{v_2} + \dots + \frac{s}{v_n}} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n}}$$

taking reciprocal, we get $\frac{1}{\bar{V}} = \frac{1}{n} \left(\frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n} \right)$

73. c.
- $H = \frac{u^2}{2g}$
- Given
- $v_2 = 2v_1$
- (1)

$$A \text{ to } B: v_1^2 = u^2 - 2gh \quad (2)$$

$$A \text{ to } C: v_2^2 = u^2 - 2g(-h) \quad (3)$$

Solving (1), (2), and (3), find the value of u^2 and then get the value of H by using $H = \frac{u^2}{2g}$ (Fig. 4.157)

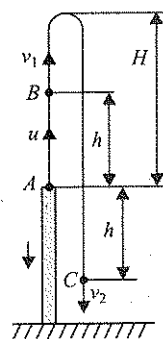


Fig. 4.157

74. b. Time taken by same ball to return to the hands of juggler
 $= \frac{2u}{g} = \frac{2 \times 20}{10} = 4$ s. So he is throwing the balls after each 1 s. Let at some instant he is throwing ball number 4. Before 1 s of it he throws ball 3. So height of ball 3:

$$h_3 = 20 \times 1 - \frac{1}{2} 10(1)^2 = 15 \text{ m}$$

Before 2 s, he throws ball 2. So height of ball 2:

$$h_2 = 20 \times 2 - \frac{1}{2} 10(2)^2 = 20 \text{ m}$$

Before 3 s, he throws ball 1. So height of ball 1:

$$h_1 = 20 \times 3 - \frac{1}{2} 10(3)^2 = 15 \text{ m}$$

75. b. Let initial relative velocity, relative acceleration and the relative displacement of the coin with respect to the floor of the lift be u_r , a_r , and s_r , then

$$s_r = u_r t + (1/2) a_r t^2$$

$$\text{and } u_r = u_c - u_l = 10 - 10 = 0$$

$$a_r = a_c - a_l = (-9.8) - 0 = -9.8 \text{ m/s}^2$$

$$s_r = s_c - s_l = -2.45 \text{ m}$$

$$-2.45 = 0(t) + (1/2)(-9.8)t^2$$

$$\text{or } t^2 = 1/2 \text{ or } t = 1/\sqrt{2} \text{ s}$$

76. d. $a = g \sin \alpha = g \sin(90^\circ - \theta)$ (Fig. 4.158)

$$= g \cos \theta$$

$$l = 2R \cos \theta$$

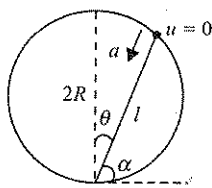


Fig. 4.158

$$\text{Now using } s = ut + \frac{1}{2} at^2$$

$$\Rightarrow l = 0 \times t + \frac{1}{2} g \cos \theta t^2$$

$$\Rightarrow 2R \cos \theta = \frac{1}{2} g \cos \theta t^2$$

$$\Rightarrow t = \sqrt{\frac{2R}{g}}$$

This is independent of θ .

77. c. Suppose the body be projected vertically upwards from A with a speed u_0 .

$$\text{Using equation, } s = ut + \left(\frac{1}{2}\right) at^2$$

$$\text{For first case, } -h = u_0 t_1 - \left(\frac{1}{2}\right) g t_1^2 \quad (i)$$

$$\text{For second case, } -h = -u_0 t_2 - \left(\frac{1}{2}\right) g t_2^2 \quad (ii)$$

$$(i) - (ii) \Rightarrow 0 = u_0(t_2 + t_1) + \left(\frac{1}{2}\right) g(t_2^2 - t_1^2)$$

$$u_0 = \left(\frac{1}{2}\right) g(t_1 - t_2) \quad (iii)$$

Put the value of u_0 in (ii),

$$-h = -\left(\frac{1}{2}\right) g(t_1 - t_2)t_2 - \left(\frac{1}{2}\right) g t_2^2$$

$$\Rightarrow h = \frac{1}{2} g t_1 t_2 \quad (iv)$$

For third case, $u = 0$, $t = ?$

$$-h = 0 \times t - \left(\frac{1}{2}\right) g t^2 \text{ or } h = \left(\frac{1}{2}\right) g t^2 \quad (v)$$

Combining equation (iv) and equation (v), we get

$$\frac{1}{2} g t^2 = \frac{1}{2} g t_1 t_2 \text{ or } t = \sqrt{t_1 t_2}$$

78. c. For no collision, the speed of car A should be reduced to v_B before the cars meet, i.e., final relative velocity of car A with respect to car B is zero, i.e., $V_r = 0$.

$$\text{Here } u_r = \text{initial relative velocity} = V_A - V_B$$

$$\text{Relative acceleration} = a_r = -a - 0 = -a$$

$$\text{Let relative displacement} = s_r$$

$$\text{Then using the equation, } v_r^2 = u_r^2 + 2a_r s_r$$

$$0^2 = (V_A - V_B)^2 - 2a s_r \text{ or } s_r = \frac{(V_A - V_B)^2}{2a}$$

$$\text{For no collision, } s_r \leq s, \text{ i.e., } \frac{(V_A - V_B)^2}{2a} \leq s$$

79. a. From considerations of symmetry, the four persons meet at the centre of the square. The displacement from the corner to the centre of the square for each person is given by

$$S_r = \frac{\sqrt{d^2 + d^2}}{2} = \frac{d}{\sqrt{2}}$$

The speed of each person can be resolved into two components: the radial component and the perpendicular component. Throughout the journey, the radial component of velocity towards the centre is given by

$$V_r = V \cos 45^\circ = \frac{V}{\sqrt{2}}, \text{ Time} = \frac{S_r}{V_r} = \frac{d/\sqrt{2}}{V/\sqrt{2}} = \frac{d}{V}$$

80. d. Here $\frac{dv}{dt} = -kv^3$

$$\text{or } \frac{dv}{v^3} = -k dt \text{ or } \int_{v_0}^v \frac{dv}{v^3} = \int_0^t -k dt$$

$$\text{or } \left[-\frac{1}{2v^2}\right]_{v_0}^v = -kt \text{ or } -\frac{1}{2v^2} + \frac{1}{2v_0^2} = -kt$$

$$\text{or } v^2 = \frac{v_0^2}{1 + 2v_0^2 kt} \text{ or } v = \frac{v_0}{\sqrt{2v_0^2 kt + 1}}$$

81. b. Given $v = 3x^2 - 2x$, differentiating v , we get

$$\frac{dv}{dt} = (6x - 2) \frac{dx}{dt} = (6x - 2)v$$

$$\Rightarrow a = (6x - 2)(3x^2 - 2x). \text{ Now put, } x = 2 \text{ m}$$

$$\Rightarrow a = (6 \times 2 - 2)(3(2)^2 - 2 \times 2) = 80 \text{ m/s}^2$$

82. a. Suppose h be the height of each storey, then

$$25h = 0 + \frac{1}{2} \times 10 \times t^2 = \frac{1}{2} \times 10 \times 5^2 \text{ or } h = 5 \text{ m}$$

In first second, let the stone passes through n storey, so

$$n \times 5 = \frac{1}{2} \times 10 \times (1)^2 \text{ or } n = 1$$

83. b. Suppose v be the velocity attained by the body after time t_1 . Then $v = u - gt_1$ (1)

Let the body reaches the same point at time t_2 . Now velocity will be downward with same magnitude v , then

$$-v = u - gt_2 \quad (2)$$

$$(1) - (2) \Rightarrow 2v = g(t_2 - t_1)$$

$$\text{or } t_2 - t_1 = \frac{2v}{g} = \frac{2}{g}(u - gt_1) = 2\left(\frac{u}{g} - t_1\right)$$

84. a. $-h = 4 \times 4 - \frac{1}{2}10(4)^2 \Rightarrow h = 64 \text{ m}$

85. a. The velocity v acquired by the parachutist after 10 s:

$$v = u + gt = 0 + 10 \times 10 = 100 \text{ m/s}$$

$$\text{Then, } s_1 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 10 \times 10^2 = 500 \text{ m}$$

The distance travelled by the parachutist under retardation, $s_2 = 2495 - 500 = 1995 \text{ m}$

Let v_g be this velocity on reaching the ground.

$$\text{Then } v_g^2 - v^2 = 2as_2$$

$$\text{or } v_g^2 - (100)^2 = 2 \times (-2.5) \times 1995 \text{ or } v_g = 5 \text{ m/s}$$

86. c. If police is able to catch the dacoit after time t , then

$$vt \geq x + \frac{1}{2}\alpha t^2. \text{ This gives } \frac{\alpha}{2}t^2 - vt + x = 0$$

$$\text{or } t = \frac{v \pm \sqrt{v^2 - 2\alpha x}}{\alpha}$$

$$\text{For } t \text{ to be real, } v^2 \geq 2\alpha x$$

87. d. Here relative velocity of the train w.r.t. other train

$$= V - v. \text{ Hence, } 0 - (V - v)^2 = 2ax$$

$$\text{or } a = -\frac{(V - v)^2}{2x}$$

$$\text{Minimum retardation} = \frac{(V - v)^2}{2x}$$

88. b. distance covered $= s = v_{av} \times \text{time}$

$$\text{For first second: } S_1 = 5 \times 1 = 5 \text{ m}$$

$$\text{For second second: } S_2 = 10 \times 1 = 10 \text{ m}$$

$$\text{For third second: } S_3 = 15 \times 1 = 15 \text{ m}$$

Total distance travelled

$$S = S_1 + S_2 + S_3 = 5 + 10 + 15 = 30 \text{ m}$$

89. b. Given $v_{av} = \frac{v + u}{2} = 0.34$ and $v - u = 0.18$

Solving these two equations, we get

$$u = 0.25 \text{ m/s, } v = 0.43 \text{ m/s. Given } s = 3.06 \text{ m}$$

Now use $v^2 - u^2 = 2as$ to find a .

90. d. By the time 5th water drop starts falling, the first water drop reaches the ground.

$$u = 0, h = \frac{1}{2}gt^2 \Rightarrow 5 = \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 1 \text{ s}$$

Hence, the interval of falling of each water drop

$$= \frac{1 \text{ s}}{4} = 0.25 \text{ s}$$

When the 5th drop starts its journey towards ground, the third drop travels in air for

$$0.25 + 0.25 = 0.5 \text{ s}$$

$\therefore$ Height (distance) covered by 3rd drop in air is

$$h_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.5)^2 = 5 \times 0.25 = 1.25 \text{ m}$$

The third water drop will be at a height of

$$= 5 - 1.25 = 3.75 \text{ m.}$$

91. b. $S_1 + S_2 + S_3 + S_4 = 16 \text{ m,}$

$$S_1 : S_2 : S_3 : S_4 = 1 : 3 : 5 : 7$$

Solve to get $S_1 = 1 \text{ m, } S_2 = 3 \text{ m, } S_3 = 5 \text{ m, } S_4 = 7 \text{ m}$

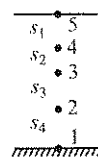


Fig. 4.159

92. c. $x^2 = 1 + t^2$ or $x = (1 + t^2)^{1/2}$

$$\frac{dx}{dt} = \frac{1}{2}(1 + t^2)^{-1/2} 2t = t(1 + t^2)^{-1/2}$$

$$a = \frac{d^2x}{dt^2} = (1 + t^2)^{-1/2} + t \left(-\frac{1}{2}\right) (1 + t^2)^{-3/2} 2t$$

$$= \frac{1}{x} - \frac{t^2}{x^3}$$

93. b. Suppose u be the initial velocity.

Velocity after time t_1 : $v_{11} = u + at_1$

Velocity after time $t_1 + t_2$: $v_{22} = u + a(t_1 + t_2)$

and Velocity after time $t_1 + t_2 + t_3$: $v_{33} = u + a(t_1 + t_2 + t_3)$

$$\text{Now } v_1 = \frac{u + v_{11}}{2} = \frac{u + u + at_1}{2} = u + \frac{1}{2}at_1$$

$$v_2 = \frac{v_{11} + v_{22}}{2} = u + at_1 + \frac{1}{2}at_2$$

$$v_3 = \frac{v_{22} + v_{33}}{2} = u + at_1 + at_2 + \frac{1}{2}at_3$$

$$\text{So } v_1 - v_2 = -\frac{1}{2}a(t_1 + t_2)$$

$$v_2 - v_3 = -\frac{1}{2}a(t_2 + t_3)$$

$$(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$$

94. c. Let the man starts crossing the road at an angle θ with the roadside. For safe crossing, the condition is that the man must cross the road by the time truck describes the distance $(4 + 2 \cot \theta)$

$$\text{So, } \frac{4 + 2 \cot \theta}{8} = \frac{2l \sin \theta}{v} \text{ or } v = \frac{8}{2 \sin \theta + \cos \theta}$$

For minimum v , $\frac{dv}{d\theta} = 0$

$$\text{or } \frac{-8(2 \cos \theta - \sin \theta)}{(2 \sin \theta + \cos \theta)^2} = 0 \text{ or } 2 \cos \theta - \sin \theta = 0$$

$$\text{or } \tan \theta = 2, \text{ so } \sin \theta = \frac{2}{\sqrt{5}}, \cos \theta = \frac{1}{\sqrt{5}}$$

$$v_{\min} = \frac{8}{2\left(\frac{2}{\sqrt{5}}\right) + \frac{1}{\sqrt{5}}} = \frac{8}{\sqrt{5}} = 3.57 \text{ m/s}$$

Graphical Concepts

1. **b.** Because in options (a), (c) and (d), we can find from the graphs that at a single time there can be more than one velocities, which is not possible practically.
2. **b.** We know that area under $v-t$ graph is displacement.
 Area from 0 to 6 s = $\frac{1}{2} \times 6 \times 2 = 6$ m
 Area from 6 to 8 s = $\frac{1}{2} \times 2 \times (-2) = -2$ m
 Area from 8 to 10 s = $2 \times 1 = 2$ m
 So Net displacement = $6 - 2 + 2 = 6$ m
3. **c.** Displacement = area under the graph

$$= 2 \times 2 + \frac{1}{2}(2+6) \times 1 + \frac{1}{2} \times 1 \times 6 - \frac{1}{2} \times 1 \times 6 - 1 \times 6 + 2 \times 4 = 10$$
 m
4. **a.** Area from 0 to 10 s = $\frac{1}{2}[10+4] \times 5 = 35$ m
 Area from 10 to 12 s = $\frac{1}{2} \times 2 \times (-2.5) = -2.5$ m
 Distance travelled = $35 + 2.5 = 37.5$ m
5. **c.** Maximum height will be attained at 110 s. Because after 110 s, velocity becomes negative and rocket will start coming down. Area from 0 to 110 s

$$= \frac{1}{2} \times 110 \times 1000 = 55,000 \text{ m} = 55 \text{ km}$$
6. **d.** Displacement in 12 s = area under $v-t$ graph

$$= \frac{1}{2} \times (12+5) \times 4 = 34 \text{ m}$$

$$V_{av} = \frac{\text{Displacement}}{\text{Time}} = \frac{34}{12} = \frac{17}{6} \text{ m/s}$$

 Hence (a) is incorrect.
 Option b is incorrect because during first 3 s velocity increases from 0 to 4 m/s
 Option c is incorrect, because in part AB, velocity is constant.
7. **c.** Acceleration between 8 to 10 s (or at $t = 9$ s):

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{5 - 15}{10 - 8} = -5 \text{ m/s}^2$$
8. **c.** Near B velocity decreases with time. Hence there is retardation or there is opposing force acting on the body.
9. **a.** Maximum acceleration will be from 30 to 40 s, because slope in this interval is maximum

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{60 - 20}{40 - 30} = 4 \text{ m/s}^2$$
10. **a.** In option a., there is some part of the graph which is $\perp$ to t axis. It indicates infinite acceleration which is not possible practically.
11. **c.** During OA, acceleration = $\tan 30^\circ = \frac{1}{\sqrt{3}} \text{ m/s}^2$
 During AB, acceleration = $-\tan 60^\circ = -\sqrt{3} \text{ m/s}^2$

$$\text{Required ratio} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

12. **d.** Velocity given by OA = $\tan 60^\circ = \sqrt{3} \text{ m/s}$
 Velocity given by AB = $-\tan 30^\circ = -\frac{1}{\sqrt{3}} \text{ m/s}$

$$\text{Required ratio} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3:1$$

13. **d.** From 0 to t_1 , velocity is positive and constant as indicated by positive and constant slope.
 From t_1 to t_3 , slope is zero, hence velocity is zero.
 From t_3 to t_4 , velocity is negative and constant as indicated by negative and constant slope.
 Option d satisfies all these observations.
14. **d.** At $t = 0$, velocity is positive and maximum. As the particle goes up, velocity decreases and becomes zero at the highest point. When the particle starts coming down, velocity increases in negative direction.
15. **a.** At time t , let displacement of first stone is $S_1 = \frac{1}{2}gt^2$ and that of second stone is $S_2 = ut - \frac{1}{2}gt^2$.
 Distance between the two stones at time t :

$$S = S_1 + S_2 = ut \Rightarrow S = ut$$

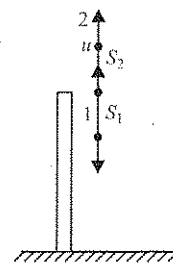


Fig. 4.160

So the graph should be a straight line passing through origin as shown by option a.

16. **b.** Let the particle is thrown up with initial velocity u . Displacement (S) at any time t is $S = ut - \frac{1}{2}gt^2$.
 The graph should be parabolic downward as shown by option b.
17. **b.** In (I), slope is negative and its magnitude is decreasing with time. It means slope is increasing numerically. So velocity is increasing towards right, that means acceleration is positive.
 In (IV), slope is positive and its magnitude is increasing with time. So velocity is increasing towards right, that means acceleration is positive.
18. **a.** We know that for a body thrown up, its displacement is given as $S = ut - \frac{1}{2}gt^2$. So the $s-t$ graph is parabolic downward.
 Also the ball collides inelastically, so it will rebound to less height every time as shown by the graph.
 t_1, t_2, t_3 are the instants when the ball collides with the ground. Here slope of $s-t$ graph is suddenly changing from nega-

tive to positive. It means velocity before collision is negative (downward) and after collision is positive (upward).

19. **a.** From 0 to t_1 , acceleration is increasing linearly with time, hence $v-t$ graph should be parabolic upwards.
From t_1 to t_2 , acceleration is decreasing linearly with time, hence $v-t$ graph should be parabolic downwards.
20. **a.** At $t = 0$, slope of $x-t$ graph is zero, hence velocity is zero at $t = 0$. As time increases, slope increases in negative direction, hence velocity increases in negative direction. At point '1' slope changes suddenly from negative to positive value, hence velocity changes suddenly from negative to positive and then velocity starts decreasing and becomes zero at '2'. Option (a) represents all these clearly.

21. **d.** Before the second ball is dropped, the first ball would have travelled some distance say $S_0 = \frac{1}{2}gt_0^2$. After dropping second ball, relative acceleration of both balls becomes zero. So distances between them increases linearly. After some time, the first ball will collide with the ground and the distance between them will start decreasing and magnitude of relative velocity will be increasing for this time. Option (d) represents all these clearly.

22. **b.** $a = -\frac{4}{2}t = 4 \Rightarrow a = -2t + 4$

$$v = \int a dt + C = -\int (2t + 4) + C = -t^2 + 4t + C$$

Hence graph will be parabolic

23. **c.** Particle will acquire the initial velocity when areas A_1 and A_2 are equal (Fig. 4.161). For this $t_0 = 8$ s

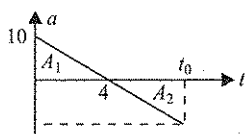


Fig. 4.161

24. **a.** For 0 to 5 s, acceleration is positive; for 5 to 15 s acceleration is negative; for 15 to 20 s, acceleration is positive.
25. **c.** For 0 to 3 s, acceleration is positive; from 3 to 5 s, acceleration is zero; from 5 to 7 s, acceleration is negative.
26. **d.** The slope of the graph is negative at this point.
27. **a.** Uniform motion involves equal distances covered in equal time intervals or the velocity is constant.

Multiple Correct Answers Type

1. **b., c., d.** A body having a constant speed can have a varying velocity due to change in direction of velocity. Thus a body having constant speed can have an acceleration.
If velocity and acceleration are in same direction, then distance is equal to displacement, because then there is no change in direction of motion. The body will continuously travel in one direction only.

2. **b., d.** Freely falling block will reach the ground first, because it has to travel less distance and with greater acceleration in comparison to the other block.

Both the blocks will reach the ground with same speed, because potential energy of both decrease, by same amount which gets converted into KE.

3. **a., c.** $a_1 = 2 \text{ ms}^{-2}$, $a_2 = -4 \text{ ms}^{-2}$

$$v_0 = a_1 t_1 = 2t_1, v_0 = a_2 t_2 = 4t_2$$

$$t_1 + t_2 = 6 \Rightarrow \frac{v_0}{2} + \frac{v_0}{4} = 6$$

$$\Rightarrow v_0 = 8 \text{ m/s}$$

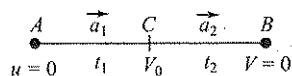


Fig. 4.162

Total distance travelled

$$S = AC + CB = \frac{1}{2}v_0 t_1 + \frac{1}{2} \times v_0 t_2$$

$$\Rightarrow S = \frac{1}{2}v_0[t_1 + t_2] = \frac{1}{2} \times 8 \times 6 = 24 \text{ m}$$

4. **a., b., c.** Let them meet at height h after time t .

$$h = 100t - \frac{1}{2}gt^2 \rightarrow \text{for first arrow}$$

$$= 100(t - 5) - \frac{1}{2}g(t - 5)^2 \rightarrow \text{for second arrow}$$

$$\Rightarrow t = 12.5 \text{ (after solving). So a is correct.}$$

$$\text{Time of flight of first arrow: } T = \frac{24}{g} = \frac{2 \times 100}{10} = 20 \text{ s.}$$

Second arrow will reach after 5 s of reaching first, so (b) is correct

$$v_1 = 100 - 10 \times 20 = -100 \text{ m/s}$$

$$v_2 = 100 - 10 \times 15 = -50 \text{ m/s}$$

$$\text{ratio: } \frac{v_1}{v_2} = 2 : 1, \text{ so c is correct.}$$

Maximum height attained

$$H = \frac{u^2}{2g} = \frac{(100)^2}{2 \times 10} = 500 \text{ m. Hence d is incorrect}$$

5. **a., c.** $t = \sqrt{\frac{2h}{g}}$, $v = \sqrt{2gh}$

6. **a., c.** Ball A will return to the top of tower after

$$T = \frac{2u}{g} = \frac{2 \times 10}{10} = 2 \text{ s}$$

with a speed of 10 m/s downward.

And this time B is also projected downward with 10 m/s. So both reach ground simultaneously. Also they will hit the ground with same speed.

7. **a., d.** From $s = \frac{1}{2}at^2$, $u = 0$

$s \propto t^2$. Since particle starts from rest and acceleration is constant, so there is no change in direction of velocity and particle moves in a straight line always.

8. **a., b., d.** $a = \frac{dv}{dt}$, if velocity changes, definitely there will be acceleration. If speed changes, then velocity also changes, so

definitely there will be acceleration.

Acceleration may be due to change in direction of velocity only and not magnitude.

If body has acceleration, its speed may change if acceleration is due to change in magnitude of velocity.

9. **a., d.** The body will speed up if angle between velocity and acceleration is acute.
10. **c., d.** Since average acceleration = change in velocity/time, so average acceleration is in the direction of change in velocity. Also if initial velocity is zero then final velocity and change in velocity will be in same direction.
11. **b., c., d.** In circular motion speed is constant but velocity is varying. For a body moving with constant velocity, acceleration is zero. At highest point of vertical path, velocity is zero but acceleration is finite. A body cannot have varying speed without having varying velocity.
12. **a., b., c.** If a vertically projected body, returns to the starting point, its displacement and average velocity become zero. As acceleration is constant, so average speed during upward or downward motion is $(u + 0)/2 = u/2$. The same will be the average speed for the whole motion.

$$13. \text{ c., d. Average velocity } \vec{v} = \frac{\int_0^5 v dt}{\int_0^5 dt} = \frac{\int_0^5 (4t - t^2) dt}{\int_0^5 dt}$$

$$= \frac{\left[2t^2 - \frac{t^3}{3} \right]_0^5}{5} = \frac{50 - \frac{125}{3}}{5} = \frac{25}{3 \times 5} = \frac{5}{3}$$

For average speed, let us put $v = 0$, which gives $t = 0$ and $t = 4$ s.

$$\therefore \text{ average speed} = \frac{\left| \int_0^4 v dt \right| + \left| \int_4^5 v dt \right|}{\int_0^5 dt}$$

$$= \frac{\left| \int_0^4 (4t - t^2) dt \right| + \left| \int_4^5 v dt \right|}{5}$$

$$= \frac{\left[2t^2 - \frac{t^3}{3} \right]_0^4 + \left[2t^2 - \frac{t^3}{3} \right]_4^5}{5}$$

$$= \frac{\left(32 - \frac{64}{3} \right) + \left[2(25 - 16) - \frac{1}{3}(125 - 64) \right]}{5} = \frac{5}{3}$$

$$\text{For acceleration: } a = \frac{dv}{dt} = \frac{d}{dt}(4t - t^2) = 4 - 2t$$

$$\text{At } t = 0, a = 4 \text{ m/s}^2$$

$\therefore$ Options c and d are correct and options a and b are wrong.

14. **a., d.** Since the graph is a straight line, its slope is constant, it means acceleration of the particle is constant.

Velocity of the particle changes from positive to negative at $t = 10$ s, so particle changes direction at this time.

The particle has zero displacement up to 20 s, but not for the entire motion.

The average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s because distance covered in both time intervals is same.

15. **a., b., c., d.** Particle changes direction of motion at $t = T$. Acceleration remains constant, because velocity-time graph is a straight line. Displacement is zero, because net area is zero. Initial and final speeds are equal.

16. **a,b,c** Initially at origin, slope is not zero, so the particle has some initial velocity but with time we see that slope is decreasing, and finally the slope becomes zero, so the particle stops finally.

As the magnitude of velocity is decreasing, so velocity and acceleration will be in opposite directions.

17. **a., c., d.** Maximum value of position coordinate = initial co-ordinate + area under the graph upto $t = 24$ s (As upto $t = 24$ s the displacement of the particle will be positive (Fig. 4.163))

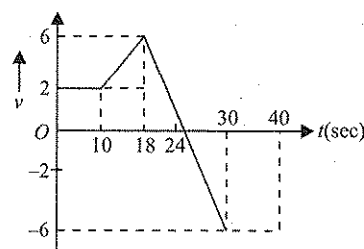


Fig. 4.163

$$= -16 + \left[(2 \times 10) + \left(\frac{2+6}{2} \right) \times (18-10) \right]$$

$$+ \frac{1 \times 6}{2} \times (24-18) \Big]$$

$$= -16 + [20 + 32 + 18] = 54 \text{ m}$$

At time $t = 18$ s

$$\text{Position} = -16 + \text{Area of graph upto } t = 18 \text{ s}$$

$$= -16 + [20 + 32] = 36 \text{ m}$$

At time $t = 30$ s

$$\text{Position} = -16 + \text{Area of graph upto } t = 30 \text{ s}$$

$$= -16 + \left[70 - \frac{1}{2} \times 6 \times 6 \right] = 36 \text{ m}$$

18. **a., b., d.** Four slopes are +2, $-\frac{4}{3}$, -3, 2. For maximum acceleration, option a is correct. For maximum magnitude of acceleration, option b is correct. For displacement, from $t = 0$ to $t = 7$ s, it is increasing then falling, so option d is also correct.

Assertion-Reasoning Type

1. a. When the body returns to its initial point, its displacement is zero, but distance travelled is not zero.
2. b. Two different physical quantities may have same dimensions.
3. c. Average velocity of the body may be equal to its instantaneous velocity, because instantaneous velocity can take any value.

For a given time interval of a given motion, both average velocity and average speed can have only one value as displacement and distance will have single values.

4. a. When body reverses the direction of motion it is momentarily at rest, but it still possesses acceleration. Velocity zero does not mean that acceleration is also zero.
5. a. If direction of velocity changes (magnitude may or may not change), we say that velocity changes. If velocity changes then definitely there will be acceleration.
6. a. When there is retardation, velocity decreases. So retardation is equal to the time rate of decrease of velocity. This retardation will be oppositely directed to velocity.
7. a. If velocity is constant, then displacement and distance will be equal, then magnitude of average velocity is equal to average speed.
8. b. Such a body can move along any curved path including circular path.
9. a. In kinematical equations, mass does not appear.
10. b. Acceleration depends upon the force applied.

Comprehensive Type

For Problems 1–2

1. d., 2. a.

Sol.

$$2. a. \quad s = \frac{M + 2Nt^4}{4} \Rightarrow v = \frac{ds}{dt} = 2Nt^3$$

putting $t = 1$ s, we get $v = 2$ N.

For Problems 3–5

3. a., 4. c., 5. b.

$$\text{Sol. } S = \frac{1}{2}gn^2 \Rightarrow S \propto n^2$$

$$S = \frac{a}{2}(2n-1) \Rightarrow S \propto (2n-1).$$

$$V = gn \Rightarrow v \propto n$$

For Problems 6–7

6. b., 7. a.

Sol.

6. b. Here acceleration is constant. So we can use

$$s = ut + \frac{1}{2}at^2, s-t \text{ graph will be parabolic.}$$

7. a. Using $v = u + at$, we find that velocity–time graph will be straight.

For Problems 8–9

8. b., 9. b.

Sol.

8. b.

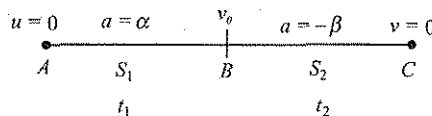


Fig. 4.164

From A to B: apply $v = u + at$

$$\Rightarrow v_0 = 0 + \alpha t_1 \Rightarrow t_1 = v_0/\alpha$$

From B to C: again apply $v = u + at$

$$\Rightarrow 0 = v_0 - \beta t_2 \Rightarrow t_2 = v_0/\beta$$

$$\text{Given } t_1 + t_2 = t \Rightarrow \frac{v_0}{\alpha} + \frac{v_0}{\beta} = t \Rightarrow v_0 = \frac{\alpha\beta t}{\alpha + \beta}$$

v_0 is the maximum velocity attained.

9. b. From A to B: apply $s = ut + \frac{1}{2}at^2$

$$\Rightarrow S_1 = 0 \times t_1 + \frac{1}{2}\alpha t_1^2 = \frac{1}{2}(\alpha t_1)t_1 = \frac{1}{2}v_0 t_1 \quad (1)$$

Similarly from B to C:

$$\Rightarrow S_2 = v_0 t_2 + \frac{1}{2}(-\beta)t_2^2 = v_0 t_2 - \frac{1}{2}(\beta t_2)t_2 = \frac{1}{2}v_0 t_2$$

Now total distance travelled:

$$= S_1 + S_2 = \frac{1}{2}v_0 t_1 + \frac{1}{2}v_0 t_2 = \frac{1}{2}v_0(t_1 + t_2)$$

$$= \frac{1}{2} \frac{\alpha\beta t}{\alpha + \beta} t = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$$

Also, we can find (1) and (2) by using the formula $s = \frac{u+v}{2}t$

For Problems 10–11

10. b., 11. d.

Sol.

10. b. Let they meet after time t , then distance travelled by both in time t should be same

$$s = 8t = \frac{1}{2}4t^2 \Rightarrow t = 4 \text{ s}$$

11. d. $s = 8t = 8 \times 4 = 32$ m

For Problems 12–14

12. b., 13. a., 14. b.

Sol.

$$12. b. \quad s = \frac{1}{2}gt_1^2$$

$$\text{or } t_1^2 = \frac{50 \times 2}{g} = \frac{100}{g} \text{ or } t_1 = \frac{10}{\sqrt{g}}$$

$$\text{and } 100 = \frac{1}{2}gt^2 \text{ or } t = \frac{10\sqrt{2}}{\sqrt{g}}$$

$$t_2 = t - t_1 = \frac{10}{\sqrt{g}}(\sqrt{2} - 1) = 0.4 t_1$$

$$t_1 > t_2$$

$$13. a. \quad t_1 : t_2 = \frac{10}{\sqrt{g}} \times \frac{\sqrt{g}}{10(\sqrt{2} - 1)} = \frac{1}{0.4} = \frac{10}{4} = \frac{5}{2}$$

$$14. \text{ b. } t : t_1 = \frac{10\sqrt{2}}{\sqrt{g}} \times \frac{\sqrt{g}}{10} = \sqrt{2}$$

For Problems 15–16

15. c., 16. a.

Sol.

$$15. \text{ c. } s = ut + \frac{1}{2}at^2 = 4 \times 5 - \frac{1}{2} \times 9.8 \times 5^2 \\ = 20 - 122.5 = -102.5 \text{ m}$$

This shows that the body is 102.5 m below the initial position, i.e., height of the body = $120.5 - 102.5 = 18 \text{ m}$.

$$16. \text{ a. } \text{The distance travelled by the balloon in 5 s upwards} \\ = 4 \times 5 = 20 \text{ m.}$$

$$\text{Distance of separation} = 20 + 102.5 = 122.5 \text{ m}$$

For Problems 17–18

17. b., 18. b.

Sol.

$$17. \text{ b. } \text{Let at time } t, \text{ the cyclist overtakes the bus, then} \\ 96 + (\text{distance travelled by bus in time } t)$$

$$= (\text{distance travelled by cyclist in time } t)$$

$$\Rightarrow \frac{1}{2} \times 2 \times t^2 + 96 = 20 \times t$$

$$\Rightarrow t^2 - 20t + 96 = 0$$

This gives, $t = 8 \text{ s}$ or 12 s . Hence, the cyclist will overtake the bus at 8 s.

$$18. \text{ b. } \text{It is quite clear from above explanation that the bus will overtake the cyclist after a time of 12 s.}$$

For Problems 19–21

19. a., 20. d., 21. a.

Sol.

$$19. \text{ a. } \text{Distance} = \pi R, \text{ Displacement} = 2R$$

$$\text{Ratio is } \frac{\pi R}{2R} = \frac{\pi}{2}$$

$$20. \text{ d. } \text{See Fig. 4.165}$$

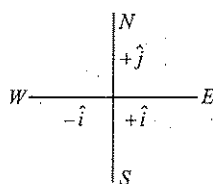


Fig. 4.165

$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$$

$$= 40\hat{j} + 30\hat{i} - 30\sqrt{2}(\hat{i}\cos 45^\circ + \hat{j}\sin 45^\circ) = +10\hat{j}$$

$$21. \text{ a. } \text{Fig. 4.166}$$

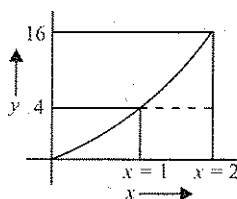


Fig. 4.166

$$\text{Displacement} = \sqrt{1^2 + 12^2} = \sqrt{145} \approx 12$$

$$\int dS = \int_1^2 [1 + (8x)^2] dx = [x]_1^2 + \frac{64}{3}[2^3 - 1^3] \\ = 1 + \frac{64}{3} \times 7 = 1 + \frac{448}{3} \approx 150$$

For Problems 22–23

22. d., 23. d

Sol.

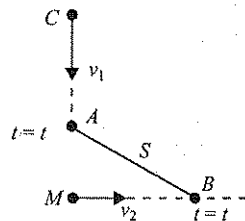


Fig. 4.167

$$CM = 50 \text{ m}$$

$$v_1 = 20 \text{ m/s}, v_2 = 15 \text{ m/s}, AC = v_1 t, MB = v_2 t$$

$$S = \sqrt{MA^2 + MB^2} = \sqrt{(50 - v_1 t)^2 + (v_2 t)^2}$$

$$\text{From } \frac{ds}{dt} = 0 \text{ (for minima), find } t \text{ and put } t \text{ in } s.$$

For Problems 24–28

24. a., 25. a., 26. c., 27. b., 28. c.

Sol.

$$24. \text{ a. } S_1 = \frac{ut}{2}, \frac{S_2}{2} = v_1 t_1 = v_2 t_2 \quad (1)$$

$$\begin{array}{c} \frac{S_2}{2} \quad \frac{S_2}{2} \\ | \quad | \quad | \\ S_1 \quad \frac{t}{2} \quad v_1 t_1 \quad v_2 t_2 \end{array}$$

Fig. 4.168

$$\frac{t}{2} = t_1 + t_2 \quad (2)$$

From (1) and (2)

$$t_1 = \frac{v_2 t}{2(v_1 + v_2)}$$

$$V_{av} = \frac{S}{t} = \frac{S_1 + S_2}{t} = \frac{\frac{ut}{2} + 2v_1 t_1}{t}$$

$$\text{Put the value of } t_1 \text{ and get } v_{av} = \frac{u(v_1 + v_2) + 2v_1 v_2}{2(v_1 + v_2)}$$

$$25. \text{ a. } x_{1(t=0)} = 4, x_{2(t=5)} = 44$$

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{44 - 4}{5 - 0} = 8 \text{ m/s}$$

$$26. \text{ c. } F = -kV^2 \text{ or } ma = -kv^2 \text{ or } m \frac{dv}{dt} = -kv^2$$

$$v = \frac{v_0}{v_0 \alpha t + 1}, \text{ where } \alpha = \frac{k}{m}$$

$$\text{Put } v = \frac{ds}{dt} \text{ and find } s. \text{ Then } v_{av} = \frac{s}{t}$$

27. b.

28. c. Net displacement is zero, so average velocity is zero.

$$v_i = \frac{20}{4} = 5 \text{ m/s}, v_f = \frac{20}{5} = 4 \text{ m/s},$$

$$a_{av} = \frac{v_f - v_i}{t} = \frac{-4 - 5}{9} = -1 \text{ m/s}^2$$

For Problems 29–30

29. c., 30. b.

Sol.

29. c. Fig. 4.169

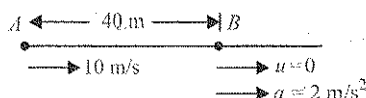


Fig. 4.169

Distance between the particles will be minimum when velocity of B becomes equal to that of A, i.e., 10 m/s.

Apply $v = u + at \Rightarrow 10 = 0 + 2t \Rightarrow t = 5 \text{ s}$

30. b. Distance travelled by A in time 5 s, $S_1 = 10 \times 5 = 50 \text{ m}$
Distance travelled by B in time 5 s:

$$S_2 = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 5^2 = 25 \text{ m}$$

Minimum distance = $40 + S_2 - S_1 = 15 \text{ m}$

For Problems 31–35

31. a., 32. d., 33. a., 34. b., 35. c.

Sol.

31. a. We know that, $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{10 - 0}{2 - 0} = 5 \text{ m/s}$

32. d. $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5 - 0}{4 - 0} = \frac{5}{4} \text{ m/s}$

33. a. $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5 - 10}{4 - 2} = -2.5 \text{ m/s}$

34. b. $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5 - 5}{7 - 4} = -\frac{10}{3} \text{ m/s}$

35. c. $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = 0 \text{ m/s}$

For Problems 36–38

36. a., 37. d., 38. b.

Sol.

36. a. Distance covered = area of the speed–time graph

$$= \frac{1}{2} \times (4 + 2) \times 4 + \frac{1}{2} (4 + 2) \times 2 = 18 \text{ m}$$

37. d. Distance from the origin will be equal to the final displacement of the particle which is equal to the area of velocity–time graph

$$= \frac{1}{2} \times (4 + 2) \times 4 - \frac{1}{2} (4 + 2) \times 2 = 6 \text{ m}$$

38. b. $a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{-2 - 2}{6 - 2} = -1 \text{ m/s}^2$

For Problems 39–41

39. a., 40. b., 41. a.

Sol.

39. a. Average speed = 20 ms^{-1}

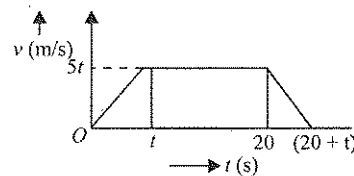


Fig. 4.170

$$\frac{\text{Total distance}}{\text{Total time}} = 20 \Rightarrow \frac{\text{Area of graph}}{20 + t} = 20$$

$$\Rightarrow \frac{1}{2} (20 + t + 20 - t) \times 5t = 20(20 + t) \Rightarrow t = 5 \text{ s}$$

40. b. Maximum velocity = $5t = 5 \times 5 = 25 \text{ m/s}$

41. a. $5t(20 - t) = 100t - 5t^2 = 375 \text{ m}$

For Problems 42–43

42. d., 43. c.

Sol.

42. d. In graph (i) and (iii), magnitude of slope is greater at t_1 than that at t_2 .

43. c. In graph (iii), magnitude of slope of graph is decreasing. Hence magnitude of velocity is decreasing but in -ve direction, hence acceleration is positive.

For Problems 44–45

44. b., 45. c.

Sol.

44. b. For the graphs (i) and (iv), slope is constant hence the velocity becomes negative.

45. c. For the graph (iii) the particles velocity first decreases and then increases in -ve direction. It means negative acceleration is involved in this motion.

Matching Column Type

1. i. $\rightarrow$ b., ii. $\rightarrow$ a., iii. $\rightarrow$ d., iv. $\rightarrow$ c.

$$v = \frac{dS}{dt} = \beta + 2\gamma t, \quad a = \frac{dv}{dt} = 2\gamma, \text{ so, i} \rightarrow \text{b}$$

$$(v) = \frac{\int_2^3 v dt}{\int_2^3 dt} = \frac{\beta(3-2) + \gamma(9-4)}{1} = \beta + 5\gamma,$$

so, ii. $\rightarrow$ a.

Velocity at $t = 1 \text{ s}$: $v = \beta + 2\gamma \times 1 = \beta + 2\gamma$, so iii $\rightarrow$ d
Initial displacement i.e., $t = 0$, $S = \alpha$, so, iv $\rightarrow$ c

2. i $\rightarrow$ a., b., ii. $\rightarrow$ c., iii. $\rightarrow$ d., iv. $\rightarrow$ d.

For a round trip displacement is zero, hence $\vec{v}_{av} = 0$.

Also $\vec{v}_{av} = \frac{\vec{v}_1 + \vec{v}_2}{2}$, when $\vec{v}_1$ is initial, $\vec{v}_2$ is final. Hence, i. $\rightarrow$ a., b.

$$\text{Average speed } (U_{av}) = \frac{\text{total distance}}{\text{time of flight}}$$

$$= \frac{2(v_0^2/2g)}{2v_0/g} = \frac{v_0}{2}. \text{ Hence ii.} \rightarrow \text{c.}$$

$$T_{\text{ascent}} = T_{\text{descent}} = \frac{v_0}{g}. \text{ Hence iii.} \rightarrow \text{d., iv.} \rightarrow \text{d.}$$

3. i. $\rightarrow$ a., ii. $\rightarrow$ c., d., iii. $\rightarrow$ b., iv. $\rightarrow$ b.

When the ball is above the point of projection, its displacement is always positive, but its velocity may be positive (when moving up), zero (at top point) or negative (when coming down).

Acceleration is always directed downward, so it is always negative.

4. i. $\rightarrow$ a., ii. $\rightarrow$ c., iii. $\rightarrow$ b., iv. $\rightarrow$ d.

$$\text{In } OA, S \propto t^2, v = \frac{dS}{dt} \propto 2t, \text{ i.e., } v \propto t.$$

i.e., velocity increases with time.

$$\text{In } AB, S \propto t, v = \frac{dS}{dt} \propto t^2$$

i.e., velocity is uniform, i.e., constant or independent of time.

In BC , body is retarded, i.e., velocity decreases with time.

In CD , $S \propto t^0$, i.e., $v = \text{zero}$ i.e., body is at rest.

5. i. $\rightarrow$ b., d., ii. $\rightarrow$ a., d., iii. $\rightarrow$ c., iv. $\rightarrow$ a.

a. Area of $v-t$ graph lies below time axis, so displacement is negative, but slope is positive, so acceleration is positive.

b. Area of $v-t$ graph lies above time axis, so displacement is positive, and slope is positive, so acceleration is also positive.

c. Displacement is zero, because half area is above time axis and half below. Slope is negative, so acceleration is negative.

d. Area of $v-t$ graph lies above time axis, so displacement is positive, and slope is negative, so acceleration is also negative.

Motion in Two Dimensions

- Relative Velocity
- Motion with Uniform Acceleration in a Plane
- Kinematics of Circular Motion
- Non-uniform Circular Motion
- Centripetal Acceleration
- Relative Angular Velocity

RELATIVE VELOCITY

It is given by the time rate of change of position of one object w.r.t. another. Relative velocity of a body B with respect to some other body A means velocity of B is recorded by an observer sitting on A . Mathematically,
Relative velocity of B w.r.t. A : $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$

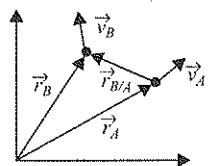


Fig. 5.1

Proof: From Fig. 5.1, $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$. Differentiating this equation w.r.t. time, we get

$$\frac{d(\vec{r}_{B/A})}{dt} = \frac{d\vec{r}_B}{dt} - \frac{d\vec{r}_A}{dt} \quad \text{but} \quad \frac{d\vec{r}_A}{dt} = \vec{v}_A, \quad \frac{d\vec{r}_B}{dt} = \vec{v}_B \quad \text{and} \quad \frac{d(\vec{r}_{B/A})}{dt} = \vec{v}_{B/A}$$

putting these values we get $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$. Hence proved.

Similarly, we can prove that *relative velocity* of A w.r.t. B :
 $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$.

Graphical Method to Find Relative Velocity

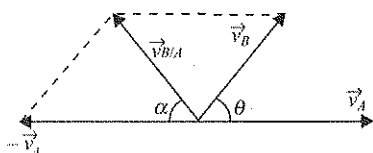


Fig. 5.2

When two bodies move at an angle θ with each other then their relative velocity is given by:

$$\begin{aligned} \text{Magnitude: } |\vec{v}_{B/A}| &= |\vec{v}_B - \vec{v}_A| = \\ &= \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180 - \theta)} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos \theta} \\ \text{Direction: } \tan \alpha &= \frac{v_B \sin(180 - \theta)}{v_A + v_B \cos(180 - \theta)} \\ \Rightarrow \tan \alpha &= \frac{v_B \sin \theta}{v_A - v_B \cos \theta} \end{aligned}$$

Note: If $\theta = 90^\circ$ then $|\vec{v}_{B/A}| = \sqrt{v_A^2 + v_B^2}$ and $\tan \alpha = \frac{v_B}{v_A}$

We can find the velocity of a particle in a frame if we know the particle's velocity in some other frame and the relative velocity of frames w.r.t. each other.

In Fig. 5.3, two observers are watching a moving particle P from the origins of reference frames A and B , while B moves at a constant velocity $\vec{v}_{B/A}$ relative to A . (The corresponding axes of these three frames remain parallel.)

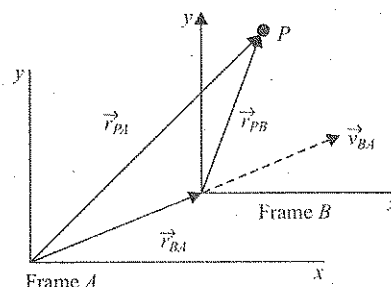


Fig. 5.3

Figure 5.3 shows a certain instant during the motion. At this instant, the position vector of B relative to A is $\vec{r}_{BA}$. Also, the position vectors of particle P are $\vec{r}_{PA}$ relative to A and $\vec{r}_{PB}$ relative to B . From the arrangement of heads and tails of those three position vectors, we can relate the vectors with $\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$.

By taking the time derivative of this equation, we can relate the velocities $\vec{v}_{P/A}$ and $\vec{v}_{P/B}$ of particle P relative to our observers. We get $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$.

We can understand the concept of relative velocity by a simple situation as follows:

Let two cars P and Q move with velocities $\vec{v}_P$ and $\vec{v}_Q$ towards north and east, respectively.

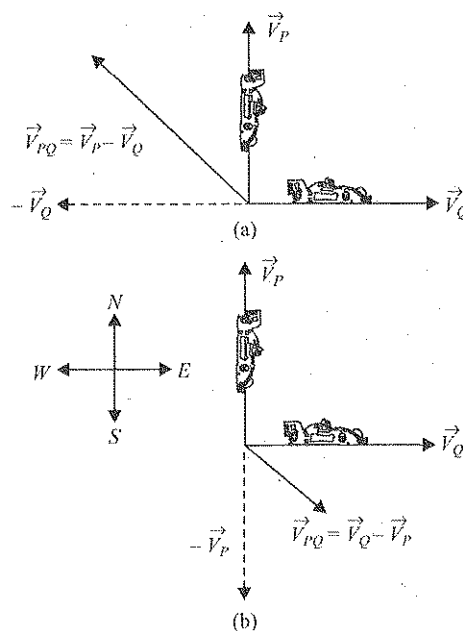


Fig. 5.4

Velocity of car P w.r.t. Q (i.e., velocity of car P for an observer on car Q) is $\vec{v}_{P/Q}$, where $\vec{v}_{P/Q} = \vec{v}_P - \vec{v}_Q$ as shown in Fig. 5.4(a), which implies that for an observer on car Q , car P appears to move along north-west instead of north. Similarly for an observer on P , car Q appears to move along south-east instead of east as shown in Fig. 5.4(b).

Illustration 5.1 A car A moves with a velocity of 15 m/s and B with a velocity of 20 m/s as shown in Fig. 5.5. Find the relative velocity of B w.r.t. A and A w.r.t. B .

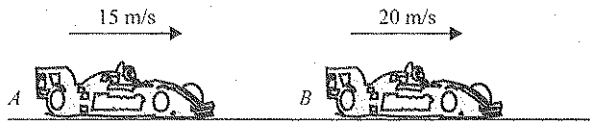


Fig. 5.5

Sol. Given: $v_A = 15 \text{ m/s}$, $v_B = 20 \text{ m/s}$.

$$\text{Relative velocity of B w.r.t A: } v_{B/A} = v_B - v_A \\ = 20 - 15 = 5 \text{ m/s.}$$

$$\text{Relative velocity of A w.r.t B: } v_{A/B} = v_A - v_B \\ = 15 - 20 = -5 \text{ m/s.}$$

(Negative sign indicates that this relative velocity is in backward direction.)

Note: If the cars were moving in opposite directions as shown, then $v_A = -15 \text{ m/s}$, $v_B = 20 \text{ m/s}$,

Relative velocity of B w.r.t A: $v_{B/A} = v_B - v_A = 20 - (-15) = 35 \text{ m/s}$

Relative velocity of A w.r.t B: $v_{A/B} = v_A - v_B = -15 - 20 = -35 \text{ m/s}$.

(Negative sign indicates that this relative velocity is in backward direction.)

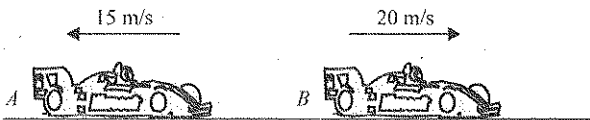


Fig. 5.6

Imp: $\vec{v}_{B/A}$ and $\vec{v}_{A/B}$ have equal magnitude but opposite directions.

Now the following illustration shows how to find relative velocity when two objects move in directions at some angle.

Illustration 5.2 Two objects A and B are moving along the directions as shown in Fig. 5.7. Find the magnitude and the direction of relative velocity of B w.r.t A.



Fig. 5.7

Sol. Analytically:

$$\vec{v}_A = 10\hat{i}, \vec{v}_B = 20 \cos 30^\circ \hat{i} + 20 \sin 30^\circ \hat{j} = 10\sqrt{3}\hat{i} + 10\hat{j}$$

Relative velocity of B w.r.t A:

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = 10(\sqrt{3} - 1)\hat{i} + 10\hat{j}$$

$$\Rightarrow |\vec{v}_{B/A}| = 10\sqrt{(\sqrt{3} - 1)^2 + 1^2} = 10\sqrt{5 - 2\sqrt{3}} \text{ m/s,}$$

$$\tan \alpha = \frac{10}{10(\sqrt{3} - 1)} = \frac{1}{\sqrt{3} - 1}$$

Graphically: $\vec{v}_{B/A} = \vec{v}_B + (-\vec{v}_A)$, from Fig. 5.8

$$NS = MP = 20 \sin 30^\circ = 10$$

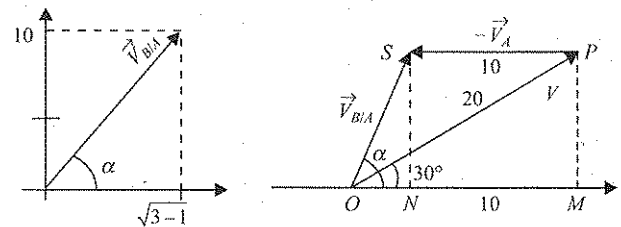


Fig. 5.8

$$\text{and } ON = OM - NM = 20 \cos 30^\circ - 10 = 10(\sqrt{3} - 1)$$

$$|\vec{v}_{B/A}| = \sqrt{ON^2 + NS^2} = 10\sqrt{(\sqrt{3} - 1)^2 + 1^2}$$

$$= 10\sqrt{5 - 2\sqrt{3}} \text{ m/s}$$

$$\tan \alpha = \frac{NS}{ON} = \frac{10}{10(\sqrt{3} - 1)} = \frac{1}{\sqrt{3} - 1}$$

Illustration 5.3 Ram was riding in a car travelling at a speed of 20 m/s towards east. Shyam was standing on the sideway when the car passed him. Just before the car passed close to Shyam (when it was north-west of him) Ram had threw a banana peel with a velocity of 10 m/s relative to him as $\vec{v}_{\text{Banana, Ram}} = (-10)\hat{i} + (-10)\hat{j}$. The banana peel had velocity components 10 m/s towards south and 10 m/s towards west in Ram's reference frame. When Ram met Shyam later, Shyam accused that Ram had thrown the banana peel at him. Explain this with the help of velocity diagram.

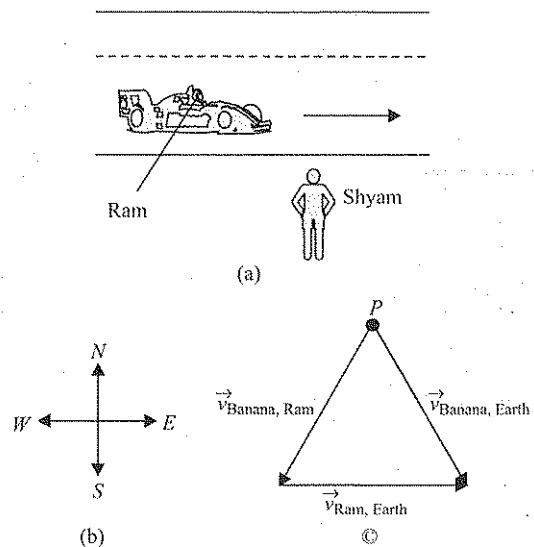


Fig. 5.9

Sol. The banana peel was thrown from a point in south-west direction relative to car (reference frame of Ram). Shyam is on the ground; velocity of banana peel relative to him can be considered to be absolute velocity of banana peel.

$$\vec{v}_{\text{Banana/Ram}} = \vec{v}_{\text{Banana}} - \vec{v}_{\text{Ram}}$$

$$\Rightarrow \vec{v}_{\text{Banana}} = \vec{v}_{\text{Banana/Ram}} + \vec{v}_{\text{Ram}}$$

$$= (-10)\hat{i} + (-10)\hat{j} + (20)\hat{i} = (10)\hat{i} - (10)\hat{j}$$

i.e., the absolute velocity of banana peel is directed towards south-east, or towards Shyam as shown by the third figure. The above example shows that two observers moving at constant velocity relative to one another find that their velocity measurements yield different results.

When One Body Moves on the Surface of Other Body

Illustration 5.4 In Fig. 5.10 a plank is moving on ground with a velocity v_1 and a block is moving on the plank with velocity v_2 relative to the plank. What will be the actual velocity of block?

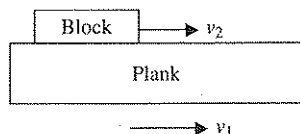


Fig. 5.10

Sol. From Fig. 5.10 we have $v_p = v_1$ and $v_{b/p} = v_2$.

Now apply the formula $v_{b/p} = v_b - v_p$
 $\Rightarrow v_b = v_{b/p} + v_p \Rightarrow v_b = v_2 + v_1$

Illustration 5.5 A block slips along an incline of a wedge. Due to reaction of the block on the wedge it slips backwards. An observer on the wedge will see the block moving straight down the incline. Discuss how to find absolute velocity of block w.r.t the wedge.

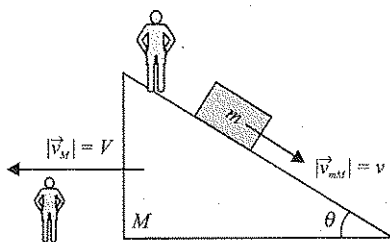


Fig. 5.11

Sol. We know that $\vec{v}_{m/M} = \vec{v}_m - \vec{v}_M$

$\Rightarrow \vec{v}_m = \vec{v}_{m/M} + \vec{v}_M$

Note that a single subscript implies absolute velocity. Absolute velocity of block is vector sum of its velocity relative to the wedge and velocity of wedge relative to the ground.

Absolute velocity of block (ground reference frame) is shown in the vector diagram given in Fig. 5.12.

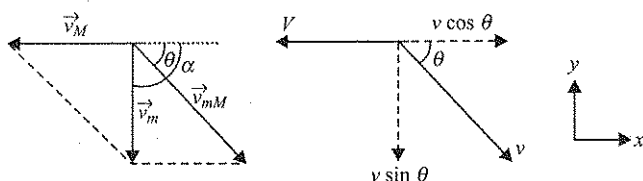


Fig. 5.12

$$|\vec{v}_m| = \sqrt{v^2 + V^2 + 2vV \cos(\pi - \theta)} = \sqrt{v^2 + V^2 - 2vV \cos \theta}$$

We can derive this result by resolving v into its components.

Sum of x-components $V_x = v \cos \theta - V$

Sum of y-components $V_y = v \sin \theta$

Resultant velocity $= \sqrt{V_x^2 + V_y^2}$

$$= \sqrt{(v \cos \theta - V)^2 + (v \sin \theta)^2}$$

$$= \sqrt{v^2 + V^2 - 2vV \cos \theta}; \tan \alpha = \frac{V_y}{V_x} = \frac{v \sin \theta}{v \cos \theta - V}$$

Change in velocity:

Let initial velocity of an object be $\vec{v}_1$ and therefore after some time velocity of the object becomes $\vec{v}_2$.

Then, change in velocity: $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$
 magnitude of change in velocity: $|\Delta \vec{v}| = |\vec{v}_2 - \vec{v}_1|$ and change in magnitude of velocity (or change in speed): $|\vec{v}_2| - |\vec{v}_1|$.

Note: Be careful that average acceleration is change in velocity (not change in speed) divided by time taken.

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t}$$

Illustration 5.6 A particle is moving with a velocity of 5 m/s. After 5 s it is found to be moving with a velocity of 12 m/s in a direction perpendicular to the original direction. During these 5 s find

1. change in speed,
2. change in velocity and magnitude of change in velocity, and
3. average acceleration and its magnitude.

Sol.

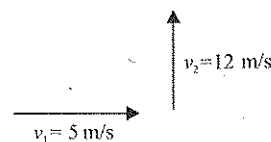


Fig. 5.13

1. Change in speed: $= 12 - 5 = 7$ m/s
2. Change in velocity: $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1 = 12\hat{j} - 5\hat{i}$
 $= -5\hat{i} + 12\hat{j}$

Magnitude of change in velocity $= |\Delta \vec{v}| = \sqrt{5^2 + 12^2} = 13$ m/s

3. $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{-5\hat{i} + 12\hat{j}}{5} = -\hat{i} + \frac{12}{5}\hat{j}$
 Magnitude of average acceleration $= |\vec{a}_{av}|$

$$= \sqrt{1^2 + \left(\frac{12}{5}\right)^2} = \frac{13}{5} \text{ m/s}^2$$

We can also find magnitude of average acceleration from

$$|\vec{a}_{av}| = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{13}{5} \text{ m/s}^2$$

River-Boat or River-Man Problems

Let a boat travel in still water with a velocity v . If water also moves in the river, then net velocity of boat (w.r.t. ground) will be different from v .

Net velocity of the boat: $\vec{v}_b = \vec{v}_{b/w} + \vec{v}_w$, where $\vec{v}_{b/w}$ is velocity of boat in water or w.r.t. water and $\vec{v}_w$ is the velocity of water in river.

Special cases:

1. If a boat travels downstream we have $\vec{v}_{b/w} = v\hat{i}$ and $\vec{v}_w = u\hat{i}$.

$$\text{So, } \vec{v}_b = \vec{v}_{b/w} + \vec{v}_w = v\hat{i} + u\hat{i} = (v + u)\hat{i}$$

Hence, the net velocity of the boat will be $v + u$ downstream.

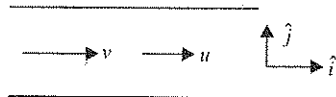


Fig. 5.14

2. If the boat travels upstream,

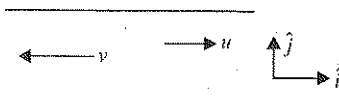


Fig. 5.15

We have $\vec{v}_{b/w} = -v\hat{i}$ and $\vec{v}_w = u\hat{i}$. So

$$\vec{v}_b = \vec{v}_{b/w} + \vec{v}_w = -v\hat{i} + u\hat{i} = -(v - u)\hat{i}$$

(-ve sign indicates backward direction.) Hence, the net velocity of the boat will be $v - u$ upstream.

3. If the boat travels at some angle θ with downstream then,

$$\vec{v}_w = u\hat{i}, \quad \vec{v}_{b/w} = v \cos \theta \hat{i} + v \sin \theta \hat{j}$$

$$\vec{v}_b = \vec{v}_{b/w} + \vec{v}_w = (u + v \cos \theta)\hat{i} + v \sin \theta \hat{j}$$

$$v_b = \sqrt{[u + v \cos \theta]^2 + (v \sin \theta)^2} = \sqrt{u^2 + v^2 + 2uv \cos \theta}$$

$$\text{Time taken to cross the river: } t = \frac{d}{v \sin \theta},$$

$$\text{Drift: } x = (u + v \cos \theta)t \Rightarrow x = \frac{(u + v \cos \theta)d}{v \sin \theta}$$

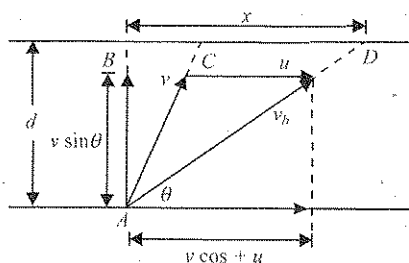


Fig. 5.16

To cross the river in shortest time:

$$\sin \theta = 1 \Rightarrow \theta = 90^\circ, t_{\min} = \frac{d}{v}, x = \frac{du}{v}$$

x can be calculated from Fig. 5.16. Also: $\tan \alpha = \frac{u}{v} = \frac{x}{d}$

$$\Rightarrow x = \frac{du}{v}$$

Net velocity of boat (put $\theta = 90^\circ$): $\vec{v}_b = u\hat{i} + v\hat{j}$,

$$\text{Magnitude: } |\vec{v}_b| = \sqrt{v^2 + u^2}$$

To cross the river by shortest path: For this $x = 0$

$$\Rightarrow u + v \cos \theta = 0 \text{ (Fig. 5.17)}$$

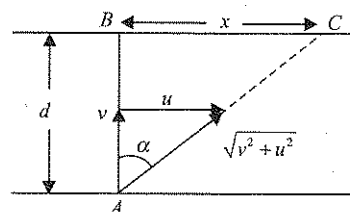


Fig. 5.17

$$\Rightarrow \cos \theta = -\frac{u}{v} \Rightarrow \cos(90 + \beta) = -\frac{u}{v} \Rightarrow \sin \beta = \frac{u}{v}$$

$$t = \frac{d}{v \sin \theta} = \frac{d}{v \sqrt{1 - \cos^2 \theta}} = \frac{d}{v \sqrt{1 - \frac{u^2}{v^2}}}$$

$$\Rightarrow t = \frac{d}{\sqrt{v^2 - u^2}}$$

Net velocity of boat: $\vec{v}_b = v \sin \theta \hat{j}$

$$\text{Magnitude: } |\vec{v}_b| = v \sin \theta = v \sqrt{1 - \frac{u^2}{v^2}} = \sqrt{v^2 - u^2}$$

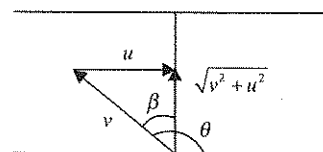


Fig. 5.18

Illustration 5.7 A man wishes to cross a river in a boat.

If he crosses the river in minimum time then he takes 10 min with a drift of 120 m. If he crosses the river taking shortest route, he takes 12.5 min, find the velocity of the boat with respect to water.

$$\text{Sol. } t_1 = 10 = \frac{d}{v} \Rightarrow d = 10v, x = ut_1 \Rightarrow 120 = u \times 10$$

$$\Rightarrow u = 12 \text{ m/min}$$

$$t_2 = 12.5 = \frac{d}{\sqrt{v^2 - u^2}} \Rightarrow 12.5 = \frac{10v}{\sqrt{v^2 - 12^2}}$$

$$\Rightarrow v = 20 \text{ m/min}$$

Rain-Man Problems

Formula to be applied: $\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$, where $\vec{v}_{r/m}$ is velocity of rain w.r.t. man, $\vec{v}_r$ is the velocity of rain (w.r.t. ground), and $\vec{v}_m$ is the velocity of man (w.r.t. ground).

Case (i): Rain is falling vertically downwards with a velocity v_r and a man is running horizontally with a velocity v_m as shown in Fig. 5.19. What is the relative velocity of rain w.r.t. man?

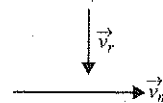


Fig. 5.19

$$\text{Given: } \vec{v}_r = -v_r \hat{j}, \quad \vec{v}_m = v_m \hat{i},$$

$$\text{Now } \vec{v}_{r/m} = \vec{v}_r - \vec{v}_m = -v_r \hat{j} - v_m \hat{i} \text{ or } \vec{v}_{r/m} = -v_m \hat{i} - v_r \hat{j}.$$

Magnitude: $v_{r/m} = \sqrt{v_m^2 + v_r^2}$ and direction: $\tan \alpha = \frac{v_m}{v_r}$

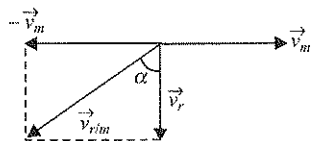


Fig. 5.20

Case (ii): If rain is already falling at some angle θ with horizontal, then with what velocity the man should travel so that the rain appears falling vertically downwards to him?



Fig. 5.21

Here, $\vec{v}_m = v_m \hat{i}$, $\vec{v}_r = v_r \sin \theta \hat{i} - v_r \cos \theta \hat{j}$

Now, $\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m = (v_r \sin \theta - v_m) \hat{i} - v_r \cos \theta \hat{j}$

Now for rain to appear falling vertically, the horizontal component of $\vec{v}_{r/m}$ should be zero, i.e.,

$$v_r \sin \theta - v_m = 0 \Rightarrow \sin \theta = \frac{v_m}{v_r} \text{ and } v_{r/m} = v_r \cos \theta$$

$$= v_r \sqrt{1 - \sin^2 \theta} = v_r \sqrt{1 - \frac{v_m^2}{v_r^2}}$$

$$\text{or } v_{r/m} = \sqrt{v_r^2 - v_m^2}$$

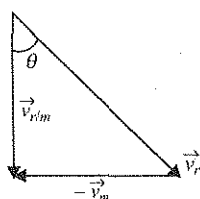


Fig. 5.22

We can illustrate the whole situation by the diagrams given in Fig. 5.23:

It is quite interesting to notice the steady rainfall sitting in a vehicle such as bus, car, etc. While moving on a straight track the direction of rainfall changes when the vehicle changes its velocity. That means, the velocity of the rain you observe is the velocity of the rain relative to you. Therefore, your observed velocity of rainfall (both magnitude and direction of velocity of rainfall) is the velocity of the rain with respect to the vehicle (you) $\vec{v}_{r/m}$. It cannot be equal to the velocity when the vehicle becomes stationary $\vec{v}_m = 0$. If you measure the velocity of the rainfall while the vehicle is stationary, that gives actual velocity of rainfall.

Problem Solving Tips for Relative Velocity

- If the velocity is mentioned without specifying the frame, assume it is with respect to the ground.
- In many cases, a body travels on water or in air. Depending on the context you will have to figure out whether the velocity is with respect to the water/air or with respect to the ground.

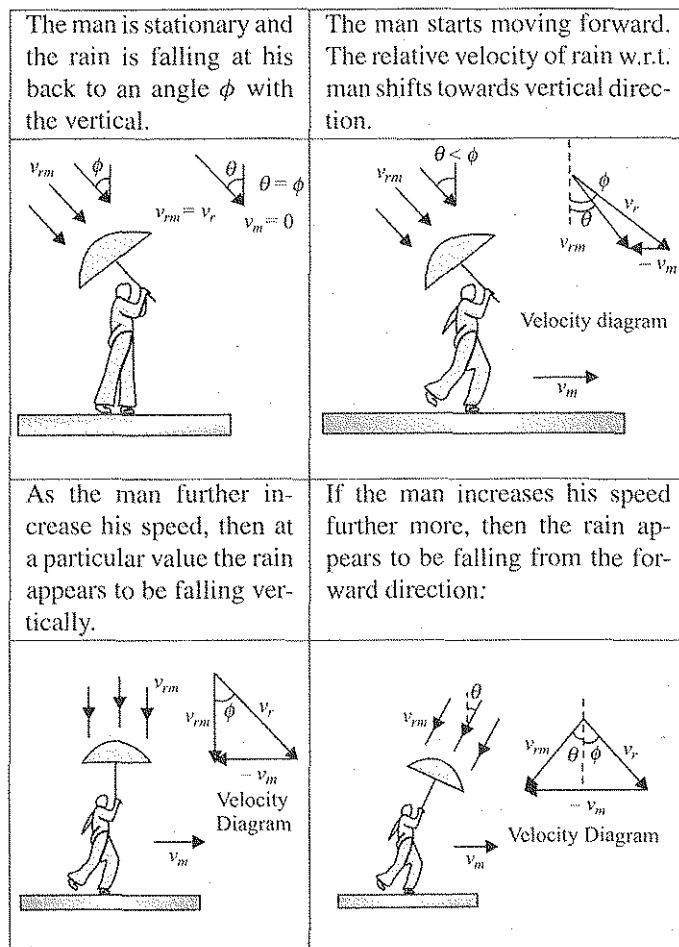


Fig. 5.23

- In some situations you have to presume the velocities. For example, if the problem says that a man can walk at a maximum of 8 kmph and if it asks you to find the velocity on a train, then you have to assume that the velocity of the man with respect to the surface he is on (in this case the train is 8 kmph). Similarly, the velocity of a bullet is always measured respect to the gun. If the gun is mounted on a truck, the bullet will have a different velocity.
- Similarly if I throw a ball, the velocity of the ball is normally given with respect to the man throwing the ball. If the man is having some velocity, then you need to add this up when you want to find out the velocity of the ball with respect to the ground.
- Therefore, depending on the circumstances, the velocity will have to be computed with respect to the ground or the object on which the body is moving.
- You can consider any of the bodies as observers and any of the bodies as observed. However, note that at a minimum you will have 3 bodies, 2 observers on different reference frames and one body, which is being observed. You can change one of the observers as the observed and vice versa and use the relative velocity/relative displacement equations. You will always get the same answer.

Concept Application Exercise 5.1

1. State True or False:

- A ball is dropped from the window of a train moving along horizontal rails. The path that ball travels to reach the ground is parabola.
 - In the above part, the ball appears to move along circular path for the man in train who dropped the ball.
 - In order to cross the river in shortest time a boat needs to move in the perpendicular direction to the flow of river.
 - Similar to above, if the river has to be crossed through the shortest path, the boat must move at an angle $\theta < 90^\circ$ inclined downwards to the flow.
 - A man has to hold the umbrella at an angle $\theta = \tan^{-1} \left(\frac{v_m}{v_r} \right)$ with vertical (where symbols have their usual meaning) while moving in rain falling vertically in an attempt to save himself from the rain.
 - A person aiming to reach exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at an angle of 120° with the direction of flow of water. The speed of water in the stream is 0.25 m/s.
- A train 200 m long is moving with a velocity of 72 km/h. Find the time taken by the train to cross the 1 km long bridge.
 - Two cars A and B are moving on the straight parallel paths with speeds 36 km/h and 72 km/h, respectively, starting from the same point in the same direction. After 20 min how much behind is car A from car B?
 - Two trains, 110 m and 90 m long, respectively, are running in opposite directions with velocities 36 km/h and 54 km/h. Find the time taken by the two trains to completely cross each other.
 - Two persons A and B are walking with a speeds 6 km/h and 10 km/h, respectively, in the same direction. Find the separation between A and B after 3 h.
 - Two parallel roads run east-west. Car A moves east while car B moves west with speeds 54 km/h and 90 km/h, respectively. Find relative velocity of B w.r.t. A and relative velocity of ground w.r.t. B.
 - The relative velocity of A w.r.t. B is 6 ms^{-1} due north. What will be the relative velocity of B w.r.t. A?
 - A man A is going due east with a speed of 5 m/s. Another man B is going due north with a speed of 7 m/s. Find
 - the relative velocity of B w.r.t. A.
 - the relative velocity of A w.r.t. B.
 - A car is going due south with a speed of 15 m/s and a bus is going due north-west with a speed of $10\sqrt{2} \text{ m/s}$. What velocity of car, a passenger in bus will record?
 - A moving sidewalk in an airport terminal building moves at a speed of 1.0 m/s and is 35.0 m long. If a woman steps on at one end and walks at 1.5 m/s relative to the moving sidewalk then find the time that she requires to reach the opposite end when (i) she walks in the same direction the sidewalk is moving; and (ii) when she walks in the opposite direction.

- A man can swim in still water with a velocity 2.5 km/h. He wants to cross the river flowing at a speed of 1.5 km/h. He always swims at an angle 120° with downstream. The width of river is 800 m. Find
 - the time in which he will cross the river.
 - where will he arrive on the opposite bank?

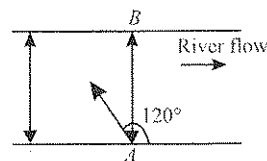


Fig. 5.24

- the time in which he will cross the river.
 - where will he arrive on the opposite bank?
- A car with a vertical wind shield moves in a rain storm at the speed of 40 km/h. The rain drops fall vertically with a terminal speed of 20 m/s. Find the angle with the vertical at which the rain drops strike the wind shield.

MOTION WITH UNIFORM ACCELERATION IN A PLANE

Let a particle move in x - y plane with a constant acceleration $\vec{a} = a_x \hat{i} + a_y \hat{j}$.

Magnitude of acceleration: $a = \sqrt{a_x^2 + a_y^2}$

Let the particle is at point A at $t = t_1$, where its velocity is

$$\vec{u} = u_x \hat{i} + u_y \hat{j}$$

At $t = t_2$, it reaches B where its velocity becomes

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

Now apply, $\vec{v} = \vec{u} + \vec{a}t$

$$\Rightarrow v_x \hat{i} + v_y \hat{j} = u_x \hat{i} + u_y \hat{j} + (a_x \hat{i} + a_y \hat{j})t$$

$$\Rightarrow v_x = u_x + a_x t \text{ and } v_y = u_y + a_y t$$

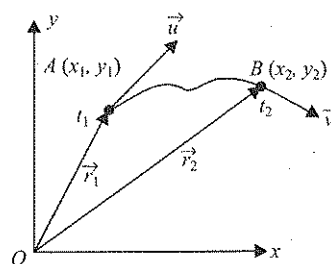


Fig. 5.25

Here we see that the velocity along x -axis is related to acceleration along x -axis only and the velocity along y -axis is related to acceleration along y -axis only.

It means that acceleration in some direction can affect the velocity in that direction only, not in a perpendicular direction.

Displacement

$$\vec{r}_2 - \vec{r}_1 = \vec{v}_{av}t \Rightarrow \vec{r}_2 = \vec{r}_1 + \left(\frac{\vec{u} + \vec{v}}{2} \right)t$$

$$\Rightarrow \vec{r}_2 = \vec{r}_1 + \frac{\vec{u} + \vec{u} + \vec{a}t}{2}t$$

$$\Rightarrow \vec{r}_2 = \vec{r}_1 + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\Rightarrow x_2\hat{i} + y_2\hat{j} = x_1\hat{i} + y_1\hat{j} + u_x t\hat{i} + u_y t\hat{j} + \frac{1}{2}a_x t^2\hat{i} + \frac{1}{2}a_y t^2\hat{j}$$

From here, we get, $x_2 = x_1 + u_x t + \frac{1}{2}a_x t^2$ and

$$y = y_1 + u_y t + \frac{1}{2}a_y t^2$$

Displacement along x-axis: $S_x = x_2 - x_1 = u_x t + \frac{1}{2}a_x t^2$

Displacement along y-axis: $S_y = y_2 - y_1 = u_y t + \frac{1}{2}a_y t^2$

Note: Equation of path can be obtained by eliminating t from both the above equations.

From here, we note that the displacement in a direction is covered by velocity and acceleration in that direction only.

Finally, equations of motion for two dimensional motion under uniform acceleration are as follows:

$$v_x = u_x + a_x t \quad (i)$$

$$v_y = u_y + a_y t \quad (ii)$$

$$S_x = u_x t + \frac{1}{2}a_x t^2 \quad (iii)$$

$$S_y = u_y t + \frac{1}{2}a_y t^2 \quad (iv)$$

$$v_x^2 = u_x^2 + 2a_x S_x \quad (v)$$

$$v_y^2 = u_y^2 + 2a_y S_y \quad (vi)$$

Projectile Motion

A projectile is the name given to a body thrown with some velocity at an angle with horizontal and then allowed to move under the action of gravity alone, without any external force being applied to it.

The path followed by a projectile is known as its **trajectory**.

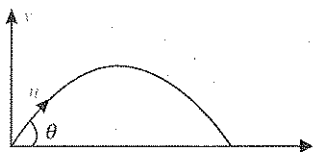


Fig. 5.26

Example:

A stone thrown at some angle

stone $\rightarrow$ projectile

curved path $\rightarrow$ Trajectory

A projectile moves under the effect of two velocities:

1. A uniform velocity along horizontal direction, which would not change provided that there is no air resistance.
2. A uniformly changing velocity (either increase or decrease) in the vertical direction due to gravity (the motion is taking place along horizontal) as well as in vertical direction.

To study the motion of a projectile, we make the following assumptions:

1. No frictional resistance of air.
2. The effect due to rotation of earth and curvature of earth is negligible.
3. g remains constant at all points of the motion of projectile. It does not change with height.

Projectile given angular projection:

Let a particle is projected at $t = 0$ from a point A (origin) with velocity u at an angle θ with horizontal.

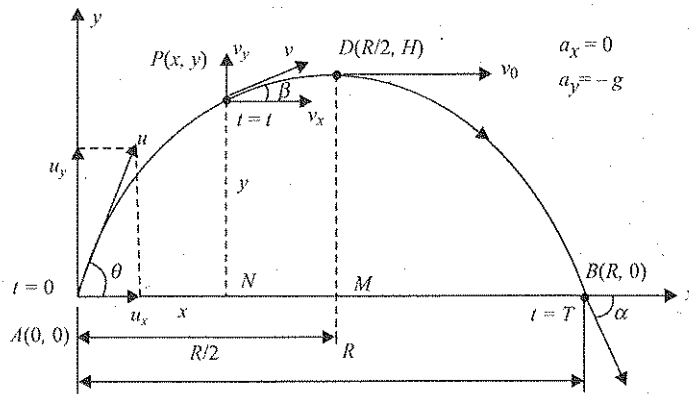


Fig. 5.27

$u_x = u \cos \theta \rightarrow$ horizontal component of velocity,

$u_y = u \sin \theta \rightarrow$ vertical component of velocity

The object covers horizontal displacement due to horizontal velocity and vertical displacement due to vertical velocity.

Let the particle reaches at point $P(x, y)$ at any time $t = t$. Velocity of projectile at this point becomes v , say at an angle β with horizontal. Then $v_x = v \cos \beta$ and $v_y = v \sin \beta$.

For motion from A to P: apply $v_x = u_x + a_x t$, we get

$$v_x = u_x \text{ because } a_x = 0.$$

It means horizontal component of velocity remains constant. It is because there is no force on the object in horizontal direction. So, finally we get:

$$v \cos \beta = u \cos \theta \quad (i)$$

Now apply $v_y = u_y + a_y t$, we get $v_y = u_y - gt$

$$\Rightarrow v \sin \beta = u \sin \theta - gt \quad (ii)$$

It means vertical component of velocity decreases as the projectile goes up. It is because force of gravity is downward.

At highest point D, velocity is purely horizontal, $\beta = 0$

After crossing the highest point, again vertical component starts increasing in downward direction.

Path of Projectile

(Equation of trajectory)

From A to P: apply $S_x = u_x t + \frac{1}{2}a_x t^2$, where,

$$S_x = x - 0 = x \Rightarrow x = u \cos \theta t$$

$$\Rightarrow t = \frac{x}{u \cos \theta}$$

Now apply $S_y = u_y t + \frac{1}{2} a_y t^2$, where $S_y = y - 0 = y$

$$\Rightarrow y = u \sin \theta \left(\frac{x}{u \cos \theta} \right) - \frac{1}{2} g \left(\frac{x}{\cos \theta} \right)^2$$

$$\Rightarrow y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta} \quad (\text{iii})$$

It is known as equation of trajectory. It is an equation of parabola. Hence path of a projectile is parabolic.

Equation (iii) can also be written as

$$\begin{aligned} y &= x \tan \theta - \frac{x^2}{2 u^2 \cos^2 \theta} = x \tan \theta - \frac{x^2}{\left(\frac{2 u^2 \cos \theta \sin \theta}{g} \right) \cos \theta} \\ &= x \tan \theta - \frac{x^2 \tan \theta}{R} \Rightarrow y = x \tan \alpha \left(1 - \frac{x}{R} \right) \end{aligned}$$

This is another form of equation (iii) which has important applications.

Time of Flight (T)

It is the total time for which the object remains in flight (air). Time of flight consists of two parts:

1. Time from A to D $\rightarrow$ known as time of ascent, let it be t_1
2. Time from D to B $\rightarrow$ known as time of descent, let it be t_2

It is found that time of ascent = time of descent $\Rightarrow t_1 = t_2 = \frac{T}{2}$

First method: from A to D, $v_y = u_y + a_y t$

$$\Rightarrow 0 = u \sin \theta - g t_1$$

$$\Rightarrow t_1 = \frac{u \sin \theta}{g}$$

$$\Rightarrow T/2 = \frac{u \sin \theta}{g}$$

$$\Rightarrow T = \frac{2 u \sin \theta}{g} \quad (\text{iv})$$

Second method: from A to B, $s_y = u_y t + \frac{1}{2} a_y t^2$

$$\Rightarrow 0 = u \sin \theta T - \frac{1}{2} g T^2$$

$$\Rightarrow g T^2 = 2 u \sin \theta T \Rightarrow T = \frac{2 u \sin \theta}{g}$$

$$\text{also, } T = \frac{2 u_y}{g} \quad (\text{v})$$

Maximum Height: (H)

From A to D: $S_y = u_y t + \frac{1}{2} a_y t^2$

$$\Rightarrow H = u \sin \theta t_1 - \frac{1}{2} g t_1^2$$

$$\Rightarrow H = u \sin \theta \left(\frac{u \sin \theta}{g} \right) - \frac{1}{2} g \left(\frac{u \sin \theta}{g} \right)^2$$

$$\Rightarrow H = \frac{u^2 \sin^2 \theta}{g} - \frac{1}{2} \frac{u^2 \sin^2 \theta}{g} \Rightarrow H = \frac{u^2 \sin^2 \theta}{2g}$$

Alternatively: From A to D, $v_y^2 = u_y^2 + 2 a_y s_y$

$$\Rightarrow 0^2 = (u \sin \theta)^2 - 2 g H$$

$$\Rightarrow H = \frac{u^2 \sin^2 \theta}{2g} \quad (\text{vi})$$

$$\text{Also } H = \frac{u_y^2}{2g} \quad (\text{vii})$$

- Time of flight and the maximum height attained depend only upon vertical components of velocities.
- $H \rightarrow$ it is the maximum vertical height attained by the object above the point of projection.

Horizontal Range: (R)

It is the horizontal displacement covered between point of projection and point of hitting.

$$\text{From A to B: } S_x = u_x t + \frac{1}{2} a_x t^2 \Rightarrow R = u \cos \theta T$$

$$\Rightarrow R = u \cos \theta \frac{2 u \sin \theta}{g} = \frac{2 u_x u_y}{g} \quad (\text{viii})$$

$$\text{Also } R = \frac{u^2 \sin 2\theta}{g} \quad (\text{ix})$$

Alternatively: Put $x = R$, $y = 0$ in equation of trajectory to get $R = \frac{u^2 \sin 2\theta}{g}$.

Maximum Horizontal Range

We know that $R = \frac{u^2 \sin^2 \theta}{g}$. R will be maximum if $\sin 2\theta = 1$

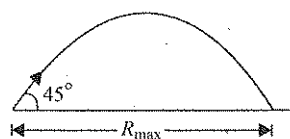


Fig. 5.28

$$\Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}, \text{ then } R_{\max} = \frac{u^2}{g} \quad (\text{x})$$

Two angles of projection for the same horizontal range, provided the two answers are complementary.

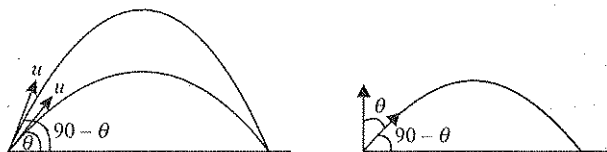


Fig. 5.29

$$R_1 = \frac{u^2 \sin 2\theta}{g}, R_2 = \frac{u^2 \sin [2(90 - \theta)]}{g}$$

$$= \frac{u^2 \sin(180 - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g} \Rightarrow R_2 = R_1$$

- So the horizontal range is the same whether the angle of projection is θ or $90 - \theta$, their sum is $\theta + 90 - \theta = 90^\circ$.
- It means that the horizontal range will be the same for those two angles of projection whose sum is 90° .
- Also, if the angle of projection with the horizontal is $90 - \theta$, then with the vertical it is θ . Hence the range will be the same whether the angle of projection θ is with horizontal or with vertical.

Note: When the range is maximum, the height H reached by the projectile is

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g} = \frac{R_{\max}}{4}$$

i.e., if a person can throw a projectile to a maximum distance $R_{\max}$, the maximum height to which it will rise is $(R_{\max}/4)$ and the angle of projection is 45° .

Velocity of Projection at any Time

We have earlier obtained equations (i) and (ii). These are $v \cos \beta = u \cos \theta$ and $v \sin \beta = u \sin \theta - gt$.

On squaring and adding these equations:

$$v^2 = (u \cos \theta)^2 + (u \sin \theta - gt)^2$$

$$\Rightarrow v = \sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$$

$$\text{Dividing them, we get, } \tan \beta = \frac{u \sin \theta - gt}{u \cos \theta} \quad (\text{xi})$$

$$v^2 = u^2 - 2gh \Rightarrow v = \sqrt{u^2 - 2gh}$$

Note: We can apply the following technique to find the speed.

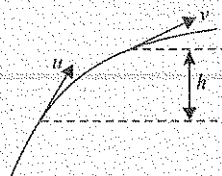


Fig. 5.30

Illustration 5.8 A batsman kicks a ball at an angle of 30° with an initial speed of 30 m/s. Assume that the ball travels in a vertical plane, calculate

1. the time at which the ball reaches the highest point.
2. the maximum height reached.
3. the horizontal range of the ball.
4. the time for which the ball is in the air.

Sol. Here $\theta = 30^\circ$, $u = 30$ m/s

1. The time taken by the ball to reach the highest point is half of the total time of flight. As time of ascending and descending is the same for a projectile without air resistance, then the time to reach the highest point is

$$t_H = \frac{T}{2} = \frac{u \sin \theta}{g} = \frac{30}{10} \times \sin 30^\circ = 1.5 \text{ s}$$

$$2. \text{ The maximum height reached } = \frac{u^2 \sin^2 \theta}{2g}$$

$$= \frac{(30)^2 \times (\sin 30^\circ)^2}{2g} = \frac{900}{2 \times 10 \times 4} = 11.25 \text{ m}$$

$$3. \text{ Horizontal range } = \frac{u^2 \sin 2\theta}{g} = \frac{(30)^2 \times \sin 2(30^\circ)}{10} = \frac{900 \times \sqrt{3}}{20} = 45\sqrt{3} \text{ m}$$

4. The time for which the ball is in air is same as its time of

$$\text{flight} = \frac{2u \sin \theta}{g} = \frac{2 \times 30 \times \sin 30^\circ}{10} = 3 \text{ s.}$$

Illustration 5.9

A footballer kicks the football to check his stamina with velocity $60\sqrt{2} \text{ ms}^{-1}$ at an angle of 45° . Find following after 3 s.

1. Velocity of football.
2. Angle made by the velocity with the horizontal.
3. Horizontal and the vertical displacement.

Sol.

1. Given $u = 60\sqrt{2} \text{ ms}^{-1}$, $\theta = 45^\circ$.

We know that the horizontal component of velocity remains the same during projectile and only vertical component changes (Fig. 5.31). So we need to find the vertical component of velocity after 3 s. Hence, we will have both v_x and v_y . We can use $v = \sqrt{v_x^2 + v_y^2}$ to find out velocity after 3 s.

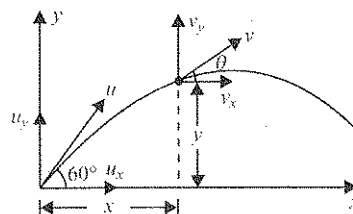


Fig. 5.31

$$\text{So, } u_x = u \cos 45^\circ = 60\sqrt{2} \times \frac{1}{\sqrt{2}} = 60 \text{ ms}^{-1};$$

$$u_y = u \sin 45^\circ = 60\sqrt{2} \times \frac{1}{\sqrt{2}} = 60 \text{ ms}^{-1}$$

The value of the vertical component of velocity after 30 s is

$$v_y = u_y + a_y t = u_y - gt = 60 - 10 \times 3 = 30 \text{ ms}^{-1},$$

$$v_x = 60 \text{ ms}^{-1}$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{3600 + 900} = \sqrt{4500} = 30\sqrt{5} \text{ ms}^{-1}$$

2. The angle made by the direction of the movement of a projectile or its velocity with horizontal at any time during journey is given by

$$\tan \theta = \frac{v_y}{v_x} \Rightarrow \tan \theta = \frac{30}{60} \Rightarrow \theta = \tan^{-1} \left(\frac{1}{2} \right)$$

3. As horizontal component of velocity remains constant. So to calculate the horizontal distance covered we can use the relation—Distance = speed $\times$ time $\Rightarrow x = v_x t$

But vertical component keeps on changing side by side as gravity keeps on acting in downward direction. So we have to use the relation $y = u_y t + \frac{1}{2} a_y t^2$

$$\text{So } x = 60 \times 3 = 180 \text{ m and } y = 60 \times 3 - \frac{1}{2} \times g \times (3)^2 \\ = 180 - 45 = 135 \text{ m}$$

Hence the horizontal displacement is 180 m and vertical displacement is 135 m.

Projectile Fired from Some Height

$$S_x = R, S_y = -h, u_x = u \cos \theta, u_y = u \sin \theta, a_x = 0, a_y = -g$$

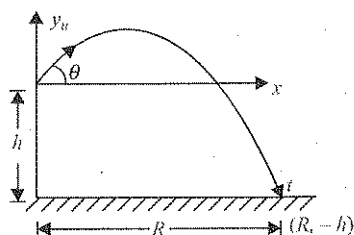


Fig. 5.32

$$s_x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow R = u \cos \theta t, s_y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow -h = u \sin \theta t - \frac{1}{2} g t^2$$

We can use these equations to calculate range and time taken.

Projectile Given Horizontal Projection

Initially: velocity is purely horizontal and vertically it is zero.

(i.) Horizontal velocity remains constant; (ii) vertical velocity increases in downward direction due to gravity.

Particle covers horizontal displacement due to horizontal velocity and vertical displacement due to vertical velocity.

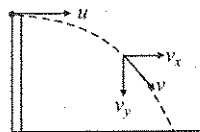


Fig. 5.33

Path of projectile:

$$u_x = u, u_y = 0, a_x = 0, a_y = -g$$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x = ut \Rightarrow t = \frac{x}{u}$$

$$\Rightarrow s_y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow y = -\frac{1}{2} g \left(\frac{x}{u} \right)^2$$

$$\Rightarrow y = -\frac{g}{2u^2} x^2$$

$$\Rightarrow y = -kx^2, \text{ where } k = \frac{g}{2u^2}$$

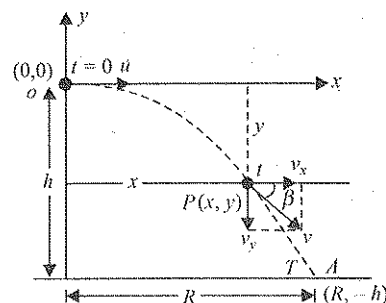


Fig. 5.34

Hence the path of projectile is parabolic.

Time of flight: (T) From O to A: $S_y = u_y t + \frac{1}{2} a_y t^2$

$$\Rightarrow -h = 0 \times T - \frac{1}{2} g T^2$$

$$\Rightarrow T = \sqrt{\frac{2h}{g}}$$

Horizontal Range: (R) $S_x = u_x t + \frac{1}{2} a_x t^2$

$$\Rightarrow R = uT \Rightarrow R = u \sqrt{\frac{2h}{g}}$$

Velocity of any Time: $v_x = u_x + a_x t$

$$\Rightarrow v_x = u, v_y = u_y + a_y t \Rightarrow v_y = -gt$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2 t^2}, \tan \beta = \frac{v_y}{v_x}$$

$$= \frac{-gt}{u} \Rightarrow \beta = \tan^{-1}(-gt/u)$$

Illustration 5.10 A ball is thrown horizontally from the top of a tower with velocity of 30 ms^{-1} . During its motion, at a particular point, horizontal and vertical velocities become equal. Determine the time elapsed to reach this point.

Sol. Given $v = 30 \text{ ms}^{-1}$

By finding out horizontal and vertical velocities separately and by equating them we can find out the required time. Also, remember that the horizontal velocity remains the same and the vertical velocity keeps changing during motion.

So, horizontal velocity $u_x = v_x = 30 \text{ ms}^{-1}$ (i)

Vertical velocity $v_y = u_y + a_y t = 0 + gt = +10t$ (ii)

Comparing equations (i) and (ii); $10t = 30 \Rightarrow t = 3 \text{ s}$.

Projectile from a Moving Body

Let us consider a trolley which is moving on a horizontal plane with a constant velocity v . A boy throws a ball from a mov-

$$v = \omega r \sin \theta$$

and its direction is perpendicular to the plane containing $\vec{r}$ and $\vec{\omega}$.

By right hand screw rule, we can write; $\vec{v} = \vec{\omega} \times \vec{r}$.

Illustration 5.11 A rigid body is spinning with an angular velocity of 4 rad/s about an axis parallel to $3\hat{j} - \hat{k}$ passing through the point $\hat{i} + 3\hat{j} - \hat{k}$. Find the velocity of the particle at the point $4\hat{i} - 2\hat{j} + \hat{k}$.

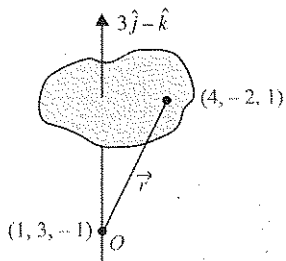


Fig. 5.44

Sol. Let $\hat{n}$ be the unit vector in the direction of $3\hat{j} - \hat{k}$. Then,

$$\hat{n} = \frac{3\hat{j} - \hat{k}}{\sqrt{3^2 + (-1)^2}} = \frac{(3\hat{j} - \hat{k})}{\sqrt{10}}$$

$\therefore$ angular velocity of the particle is

$$\vec{\omega} = \omega \hat{n} = \frac{4}{\sqrt{10}}(3\hat{j} - \hat{k}) \text{ rad/s}$$

The position vector of the point with reference to point $(\hat{i} + 3\hat{j} - \hat{k})$ is

$$\vec{r} = (4\hat{i} - 2\hat{j} + \hat{k}) - (\hat{i} + 3\hat{j} - \hat{k}) = 3\hat{i} - 5\hat{j} + 2\hat{k} \text{ m}$$

Hence, linear velocity

$$\begin{aligned} \vec{v} &= \vec{\omega} \times \vec{r} = \frac{4}{\sqrt{10}}(3\hat{j} - \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k}) \\ &= \frac{4}{\sqrt{10}}(\hat{i} + 3\hat{j} + 9\hat{k}) \text{ m/s} \end{aligned}$$

Illustration 5.12 Two particles A and B are moving as shown in Fig. 5.45. At this moment of time, find the angular speed of A relative to B.

Sol. We know that,

$$[V_{AB}]_y = V_{Ay} - V_{By} = V_A \sin \theta_A - V_B \sin \theta_B$$

$$\text{and } \omega = \frac{[V_{AB}]_y}{AB}$$

$$= \left[\frac{V_A \sin \theta_A - V_B \sin \theta_B}{r} \right] \text{ in clockwise direction.}$$

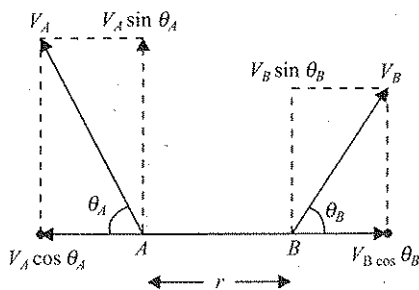


Fig. 5.45

NON-UNIFORM CIRCULAR MOTION

Angular acceleration (α): It is defined as the time rate of change of angular velocity, $\alpha = \frac{d\omega}{dt}$. Unit of α : rad/s².

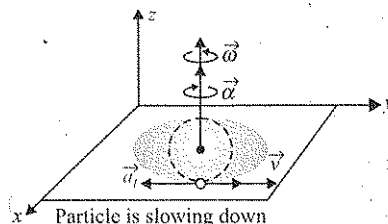
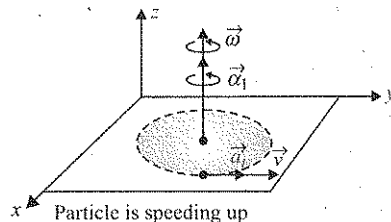


Fig. 5.46

Relation between linear acceleration and angular acceleration:

$$v = \omega r \Rightarrow \frac{dv}{dt} = \left(\frac{d\omega}{dt} \right) r \Rightarrow a_t = \alpha r$$

This acceleration is along tangent, so it is known as **tangential acceleration**.

Centripetal acceleration: There is one more type of acceleration in any kind of circular motion, which is known as centripetal acceleration. This is given by

$$a_c = \omega^2 r = \frac{v^2}{r}$$

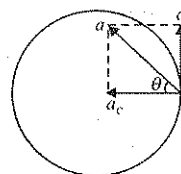


Fig. 5.47

This acceleration is directed always towards the centre of the circle. It is also known as **normal acceleration** or **radial acceleration**.

Net acceleration: In Fig. 5.47 a is the net acceleration.

$$\text{Magnitude of net acceleration: } a = \sqrt{a_c^2 + a_t^2}$$

Direction of net acceleration:

$$\tan \theta = \frac{a_t}{a_c} \Rightarrow \theta = \tan^{-1} \left(\frac{a_t}{a_c} \right)$$

Analysis of Uniform Circular Motion

In the uniform circular motion, the particle moves in a circular path with constant speed.

Let us choose the centre of the circular path as the origin of the reference frame. Point P is an arbitrary point on the path whose position vector is $\vec{r} = x\hat{i} + y\hat{j}$, where r , the radius of the circular path is related to x and y by following equations:

$$x = r \cos \theta \text{ and } y = r \sin \theta$$

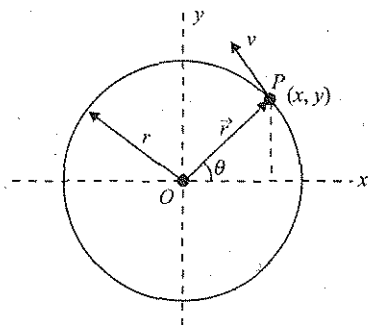


Fig. 5.48

$$\text{Also, } x^2 + y^2 = r^2$$

$$\vec{r} = r \cos \theta \hat{i} + r \sin \theta \hat{j}$$

Now, the velocity of particle P is given as

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} = \frac{d(r \cos \theta)}{dt} \hat{i} + \frac{d(r \sin \theta)}{dt} \hat{j}$$

$$\vec{v} = -r \sin \theta \frac{d\theta}{dt} \hat{i} + r \cos \theta \frac{d\theta}{dt} \hat{j}$$

But $\frac{d\theta}{dt} = \omega = \text{angular velocity [constant, for uniform circular motion]}$

$$\text{Thus } \vec{v} = \omega r (-\sin \theta \hat{i} + \cos \theta \hat{j})$$

Now $\vec{v} \cdot \vec{r} = \omega r^2 (-\sin \theta \cos \theta + \sin \theta \cos \theta) = 0$, i.e., $\vec{v}$ is perpendicular to $\vec{r}$

Now, $\vec{v}$ is along the tangent $|\vec{v}| = \omega r \sqrt{\sin^2 \theta + \cos^2 \theta} = \omega r$

$$\Rightarrow v = \omega r$$

Now, acceleration $\vec{a}$ is given as

$$\vec{a} = \frac{d\vec{v}}{dt} = \omega r \left(-\cos \theta \frac{d\theta}{dt} \hat{i} - \sin \theta \frac{d\theta}{dt} \hat{j} \right)$$

$$\Rightarrow \vec{a} = -\omega^2 r (\cos \theta \hat{i} + \sin \theta \hat{j})$$

$$\vec{a} = -\omega^2 \times \vec{r} = \omega^2 (-\vec{r})$$

The above equation shows that $\vec{a}$ is directed in the opposite direction of $\vec{r}$. Thus, $\vec{a}$ is always directed towards the centre.

Magnitude of $\vec{a}$: $a = \omega^2 r \sqrt{\cos^2 \theta + \sin^2 \theta} = \omega^2 r$

$$\Rightarrow a = \omega^2 r$$

CENTRIPETAL ACCELERATION

In uniform circular motion the velocity is constantly changing, there must be only the centripetal or normal acceleration. The tangential acceleration here is zero as the magnitude of velocity remains constant. Let us consider an instant a particle is at position P shown in Fig. 5.49(a), in uniform circular motion it is moving with velocity v_P . In the short interval of time Δt , the particle moves by an angular displacement $\Delta \theta$ to another point Q . During the time interval Δt , the velocity changes by an amount $\Delta \vec{v} = \vec{v}_Q - \vec{v}_P$. The average acceleration for this small duration is

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \quad (1)$$

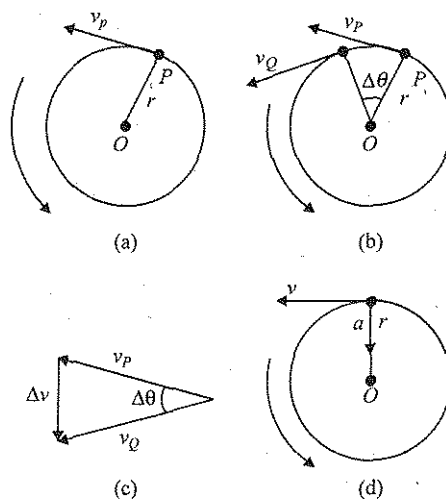


Fig. 5.49

To evaluate the acceleration we translate the vectors v_P and v_Q to a common origin shown in Fig. 5.49(c). The change in velocity Δv is also shown. As the time interval Δt is made smaller, points P and Q are found closer together. We can see that angle is also the angle between the two velocity vectors shown in figure. When becomes so small that v_P and v_Q are almost parallel and their difference Δv is almost perpendicular to both of them. In the limit when Δt tends to 0, Δv is perpendicular to v . Hence the instantaneous acceleration which is in the same direction as Δv , is directed radially towards the centre of the circular path. Therefore, a particle moving with constant speed around a circle is always accelerated toward the centre [Fig. 5.49(d)].

From Fig. 5.49(d), the acceleration can be easily evaluated. The magnitude of change in velocity Δv can be given as

$$\Delta v = v \Delta \theta \quad (2)$$

[arc length = radius $\times$ angle subtended by arc]

Here $|v_P| = |v_Q|$, and from equation (2), centripetal acceleration is

$$a = \frac{v \Delta \theta}{\Delta t} = v \omega \text{ or } a = \frac{v^2}{r} \text{ [as } v = r \omega] \quad (3)$$

whose direction is always towards the centre of the circle.

General Method to find Acceleration in Circular Motion

We know that $\vec{v} = \vec{\omega} \times \vec{r}$. Differentiating it w.r.t. time t , we will get the net acceleration.

$$\begin{aligned} \vec{a} &= \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r} \\ &= \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r} \quad \left(\because \frac{d\vec{r}}{dt} = \vec{v}, \frac{d\vec{\omega}}{dt} = \vec{\alpha} \right) \end{aligned}$$

$$\Rightarrow \vec{a} = \vec{a}_c + \vec{a}_t$$

where, $\vec{a}_c = \vec{\omega} \times \vec{v}$ is called centripetal acceleration because it is directed towards the centre of the circle and its magnitude is $a_c = \omega v = \omega^2 r = v^2/r$. It is responsible for changing the direction of motion and $\vec{a}_t = \vec{\alpha} \times \vec{r}$ is called tangential acceleration because its direction is along the tangent. Its magnitude is $a_t = \alpha r$ and it is responsible for changing the speed of the body.

Case I: When $a_t = 0$, $a = a_c = \omega^2 r$; in this case motion is called uniform circular motion.

Case II: When $a_t \neq 0$, $a_c = \omega^2 r$, in this case motion is called non-uniform circular motion.

Kinematical equations in circular motion for constant α :

1. $\omega_2 = \omega_1 + \alpha t$
2. $\theta = \omega_1 t + \frac{1}{2} \alpha t^2$
3. $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

Illustration 5.13 Tangential acceleration of a particle moving in a circular path of radius 5 cm is 2 m/s^2 . Angular velocity of the particle increases from 10 rad/s to 20 rad/s during some time. Find

1. this duration of time, and
2. the number of revolutions completed during this time.

Sol. $r = 5 \text{ cm} = 0.05 \text{ m}$, $a_t = 2 \text{ m/s}^2$, $\omega_1 = 10 \text{ rad/s}$,
 $\omega_2 = 20 \text{ rad/s}$

1. $\omega_2 = \omega_1 + \alpha t \Rightarrow 20 = 10 + 40t \Rightarrow t = 0.25 \text{ s}$
2. $\theta = \omega_1 t + \frac{1}{2} \alpha t^2 = 10 \times \frac{1}{4} + \frac{1}{2} \times 40 \times \left(\frac{1}{4}\right)^2 = \frac{15}{4} \text{ rad}$

$$\text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{15}{4 \times 2\pi} = 0.6$$

Illustration 5.14 A particle moves on a circle of radius r with centripetal acceleration as function of time as $a_c = k^2 r t^2$, where k is a positive constant. Find the following quantities as function of time at an instant:

1. the speed of the particle,
2. the tangential acceleration of the particle,
3. the resultant acceleration, and
4. angle made by the resultant with tangential direction.

Sol.

1. In the problem it is given that $a_c = k^2 r t^2$

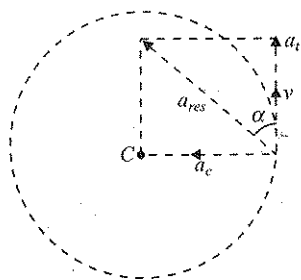


Fig. 5.50

$$\text{Since } a_c = \frac{v^2}{r} \Rightarrow \frac{v^2}{r} = k^2 r t^2$$

$$\Rightarrow v^2 = k^2 r^2 t^2 \text{ taking square root on both sides.}$$

$$\text{We get, } v = k r t$$

2. The tangential acceleration: $a_t = \frac{dv}{dt} = \frac{d}{dt}(k r t) = k r$
3. The resultant acceleration

$$a_{\text{res}} = \sqrt{a_c^2 + a_t^2} = \sqrt{(k^2 r t^2)^2 + (k r)^2} = k r \sqrt{k^2 t^4 + 1}$$
4. From Fig. 5.50: $\tan \alpha = \frac{a_c}{a_t} = \frac{k^2 r t^2}{k r} = k t^2$

$$\Rightarrow \alpha = \tan^{-1}(k t^2)$$

RELATIVE ANGULAR VELOCITY

Angular velocity is defined with respect to the point from which the position vector of the moving particle is drawn. Let a particle P is moving with velocity $\vec{v}$. Its position vector w.r.t a point A is $\vec{r}$, say. Take some fixed line as a reference line and let $\vec{r}$ makes an angle θ with this line as shown in Fig. 5.51(a). Then angular velocity of point P w.r.t. an observer on point A can be defined as $\omega = \frac{d\theta}{dt}$. It is also given as $\omega = \frac{v_{\perp}}{r} = \frac{v \sin \alpha}{r}$.

Angular velocity of a given particle can be different about different points. For example in the Fig. 5.51(b), a particle P is moving on the circumference of a circle.

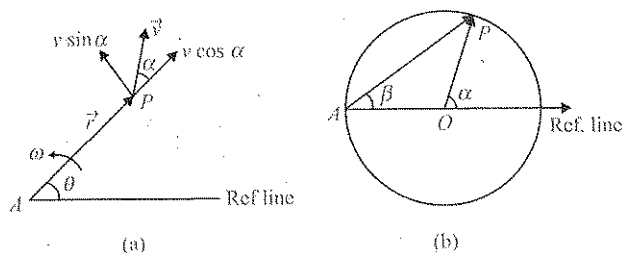


Fig. 5.51

Here the angular velocity of the particle w.r.t. O and A will be different and given as

$$\omega_{P/O} = \frac{d\alpha}{dt} \text{ and } \omega_{P/A} = \frac{d\beta}{dt}$$

Definition

Relative angular velocity of a particle B with respect to another moving particle A is the angular velocity of the position vector of B with respect to A . This means it is the rate at which position vector of B with respect to A rotates at that instant.

$$\omega_{B/A} = \frac{\text{Relative velocity of } B \text{ w.r.t. } A \text{ perpendicular to line } AB}{\text{Separation between } A \text{ and } B}$$

$$= \frac{(v_{B/A})_{\perp}}{r}$$

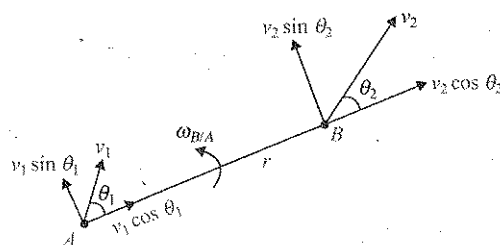


Fig. 5.52

where, $(v_{B/A})_{\perp} = v_2 \sin \theta_2 - v_1 \sin \theta_1$, so we get

$$\omega_{B/A} = \frac{v_2 \sin \theta_2 - v_1 \sin \theta_1}{r}$$

Important Points

1. If two particles are moving on the same circle or different coplanar concentric circles in same direction with different uniform angular speed ω_A and ω_B , respectively, then the angular velocity of B relative to A (for both the Figs 5.53(a) and (b) below) for an observer at the center will be $\omega_{B/A} = \omega_B - \omega_A = \frac{d\theta}{dt}$

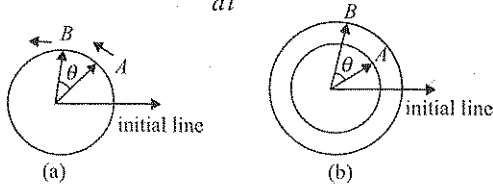


Fig. 5.53

So the time taken by one to complete one revolution around O w.r.t. the other is

$$T = \frac{2\pi}{\omega_{\text{rel}}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2}, \text{ where } \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}$$

2. If two particles are moving on two different concentric circles with different velocities then angular velocity of B relative to A as observed by A will depend on their positions and velocities. Consider the case when A and B are closest to each other moving in same direction as shown in Fig. 5.54. In this situation

$$v_{\text{rel}} = |\vec{v}_B - \vec{v}_A| = v_B - v_A \quad \text{and} \quad r_{\text{rel}} = |\vec{r}_B - \vec{r}_A| = r_B - r_A$$

so, $\omega_{BA} = \frac{(v_{\text{rel}})_{\perp}}{r_{\text{rel}}} = \frac{v_B - v_A}{r_B - r_A}$ here $(v_{\text{rel}})_{\perp}$ = Relative velocity perpendicular to position vector

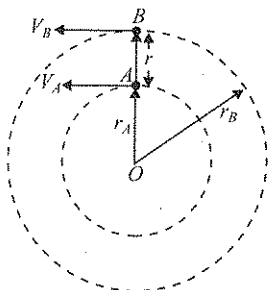


Fig. 5.54

Radius of Curvature

Any curved path can be assumed to be made of infinite circular arcs. Radius of curvature at a point is defined as the radius of

that circular arc which fits at that particular point on the curve as shown in Fig. 5.55.

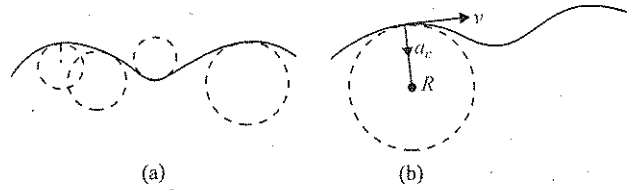


Fig. 5.55

We know that, $a_c = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_c}$. This is the expression for radius of curvature.

Concept Application Exercise 5.3

1. Calculate the angular velocity of
 - a. second's hand of a clock,
 - b. minute's hand of a clock, and
 - c. hour hand of a clock.
2. A stone tied to the end of 2 m long string is whirled in a horizontal circle with constant speed. If the stone makes 10 revolutions in 20 s, calculate the magnitude and the direction of acceleration.
3. A motor car is travelling at 30 m/s on a circular road of radius 500 m. It starts increasing its speed at the rate of 2 ms^{-2} . What is its acceleration at this moment?
4. A body is moving in a circle of radius 100 cm with a time period of 2 s. Find the acceleration.
5. Calculate the centripetal acceleration of the moon moving around the earth in a circular orbit in terms of its time period T and radius of the orbit R .

Solved Examples

Example 5.1

A bird is flying due east with a velocity of 4 ms^{-1} . The wind starts to blow with a velocity of 3 ms^{-1} due north. What is the magnitude of relative velocity of bird w.r.t. wind? Find out its direction also.

Sol. Method 1: Whenever two bodies are moving at right angle to each other, then their relative velocity is obtained by taking square root of the sum of squares of respective velocities.

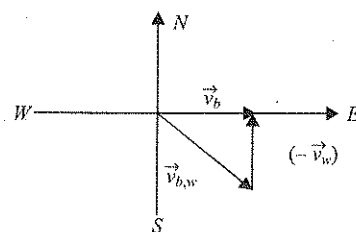


Fig. 5.56

Here velocity of bird = $v_b = 4 \text{ ms}^{-1}$ towards east.
Velocity of wind = $v_w = 3 \text{ ms}^{-1}$ towards north.

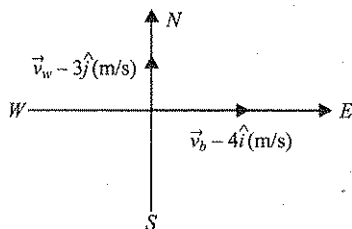


Fig. 5.57

Velocity of bird w.r.t. wind = $v_{bw} = ?$

$$\vec{v}_{b,w} = \vec{v}_b - \vec{v}_w = \vec{v}_b + (-\vec{v}_w)$$

$$\text{So, } v_{bw} = \sqrt{v_b^2 + v_w^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5 \text{ ms}^{-1}.$$

To find the direction we need to find the angle β , the angle between the relative velocity and the velocity of bird.

$$\tan \beta = \frac{3 \sin 90^\circ}{4 + 3 \cos 90^\circ} = \frac{3}{4}$$

$$[\because \theta = 90^\circ, \text{ angle between } v_b \text{ and } v_w.] \quad \beta = \tan^{-1} \left(\frac{3}{4} \right)$$

Method 2: $\vec{v}_{b,w} = \vec{v}_b - \vec{v}_w$

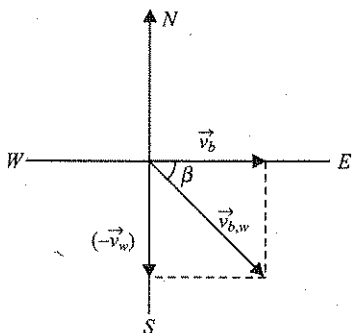


Fig. 5.58

$$= \vec{v}_b + (-\vec{v}_w) = 4\hat{i} + (-3\hat{j}) \text{ (m/s)} = 4\hat{i} - 3\hat{j} \text{ (m/s)}$$

$$|\vec{v}_{b,w}| = \sqrt{(4)^2 + (3)^2} = 5 \text{ m/s}$$

Here, the direction of the relative velocity of the bird is

$$|\tan \beta| = \frac{3}{4} \Rightarrow \beta = \tan^{-1} \left(\frac{3}{4} \right)$$

Hence, the relative velocity of the bird with respect to wind is 5 m/s and the direction is $\tan^{-1} \left(\frac{3}{4} \right)$ from east towards south.

Example 5.2

Assume two cars A and B each 5 m long.

Car A is travelling at 84 km/h and overtakes another car B which is traveling at low speed of 12 km/h. Find out the time taken for overtaking.

Sol. To analyse the motion in case of overtaking we need relative velocity of object which overtakes w.r.t. the other object. Therefore, we need to find relative velocity of car A w.r.t. car B which is

$$84 - 12 = 72 \text{ km/h} = 20 \text{ ms}^{-1}$$

Total relative distance covered with this velocity = sum of lengths of car A and car B = 5 + 5 = 10 m.

$$\therefore \text{the time taken} = \frac{\text{Distance covered}}{\text{Relative velocity}} = \frac{10}{20} = 0.5 \text{ s}$$

Example 5.3

Rain is falling vertically with a speed of 12 ms^{-1} . A cyclist is moving east to west with a speed of $12\sqrt{3} \text{ ms}^{-1}$. In order to protect himself from rain at what angle he should hold his umbrella?

Sol. Method 1: In the case of rain falling vertically with a velocity of $\tan \theta = \frac{v_{re}}{v_{br}}$ and a person (cyclist, bikers, etc.) is moving horizontally with a speed $\vec{v}_m$, the person can protect himself from rain by keeping umbrella in the direction of relative velocity of rain w.r.t. person $\vec{v}_{rp}$. If θ is the angle that $\vec{v}_{rp}$ makes with vertical or rain.

$\therefore$ velocity of rain w.r.t. cyclist

$$\vec{v}_{rp} = \vec{v}_r - \vec{v}_p$$

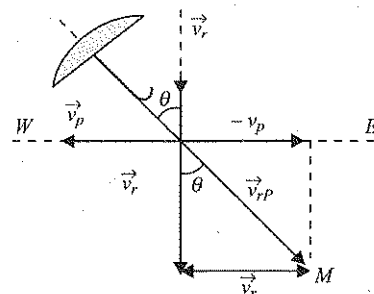


Fig. 5.59

$$\text{Here, } v_r = 12 \text{ ms}^{-1} \text{ and } v_p = 12\sqrt{3}, \tan \theta = \left(\frac{v_p}{v_r} \right)$$

$$\text{and } \theta = \tan^{-1} (\sqrt{3}) = 60^\circ$$

So the cyclist has to hold the umbrella at an angle 60° to the vertical.

Method 2: $\vec{v}_{\text{person}} = -12\sqrt{3}\hat{i} \text{ (m/s)}$ (Fig. 5.60)

$$\vec{v}_{\text{rain}} = -12\hat{j} \text{ (m/s); } \vec{v}_{\text{rain, person}} = \vec{v}_{\text{rain}} - \vec{v}_{\text{person}}$$

$$= [(-12\hat{j}) - (-12\sqrt{3}\hat{i})] \text{ (m/s)} = (12\sqrt{3}\hat{i} - 12\hat{j}) \text{ m/s}$$

Hence, the direction of orientation of umbrella with vertical is

$$\tan \theta = \frac{12\sqrt{3}}{12} = \sqrt{3} \Rightarrow \theta = 60^\circ$$

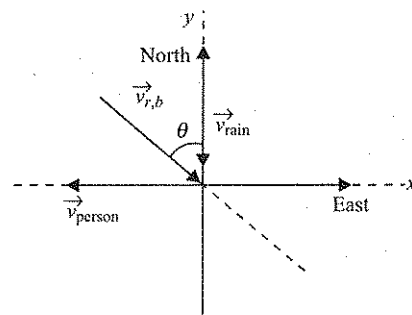


Fig. 5.60

Example 5.4

An aeroplane pilot wishes to fly due west. A wind of 100 km/h is blowing toward the south (Fig. 5.61).

1. If the airspeed of the plane (its speed in still air) is 300 km/h, in which direction should the pilot head?

2. What is the speed of the plane over the ground? Illustrate with a vector form.

Sol. Given,

Velocity of air with respect to ground $\vec{v}_{A/G} = 100 \text{ km/hr}$

Velocity of plane with respect to air $\vec{v}_{P/A} = 300 \text{ km/hr}$

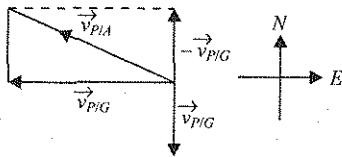


Fig. 5.61

1. As the plane is to move towards west, due to air in south direction, air will try to drift the plane in south direction. Hence, the plane has to make an angle θ towards north-west, south west direction, in order to reach at point on west.

$$\vec{v}_{P/A} = \vec{v}_{P/G} - \vec{v}_{A/G} \text{ and } v_{P/A} \sin \theta = v_{A/G}$$

Example 5.5 A river flows due south with a speed of 2.0 m/s. A man steers a motorboat across the river; his velocity relative to the water is 4 m/s due east. The river is 800 m wide.

1. What is his velocity (magnitude direction) relative to the earth?
2. How much time is required to cross the river?
3. How far south of his starting point will he reach the opposite bank?

Sol. Velocity of river (i.e., speed of river w.r.t. earth) $\vec{v}_{re} = 2 \text{ m/s}$
 Velocity of boat w.r.t. river $\vec{v}_{br} = 4 \text{ m/s}$
 Width of the river = 800 m

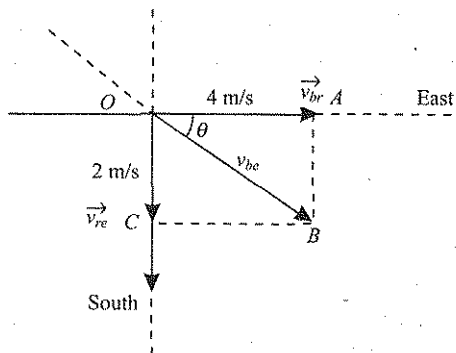


Fig. 5.62

Velocity of boat w.r.t. earth $\vec{v}_{be} = ? \text{ m/s}$

According to the given statement the diagram will be as given in Fig. 5.62.

1. When two vectors are acting at an angle of 90° , their resultant can be obtained by pythagorous theorem,

$$v_{be} = \sqrt{v_{br}^2 + v_{re}^2} = \sqrt{16 + 4} = \sqrt{20} = 4.6 \text{ m/s}$$

To find direction, we have

$$\tan \theta = \frac{v_{re}}{v_{br}} = \frac{2}{4} = \frac{1}{2} \Rightarrow \theta = \tan^{-1} \left(\frac{1}{2} \right)$$

2. Time taken to cross the river = $\frac{\text{Displacement}}{\text{Velocity of boat w.r.t. river}}$
 Velocity of boat w.r.t. river is used since it is the velocity with which the river is crossed. $\Rightarrow \frac{800}{4} = 200 \text{ s}$
 So the boat will cross in 200 s.

3. Desired position on other side is A, but due to current of river boat is drifted to position B. To find out this drift we need time taken in all to cross the river (200 s) and speed of current (2 ms^{-1}).

$$\text{So the distance } AB = \text{Time taken} \times \text{speed of current} = 200 \times 2 = 400 \text{ m}$$

Hence, the boat is drifted by 400 m away from position A.

Example 5.6

1. In which direction should the motorboat given in Example 5.5 head in order to reach a point on the opposite bank directly east from the starting point? The boat's speed relative to the water remains 4 m/s.
2. What is the velocity of the boat relative to the earth?
3. How much time is required to cross the river?

Sol. As the boat has to reach exact opposite end to the point of start, has to start (velocity 4 m/s) at an angle θ aiming somewhat upstream. Taking into count the push given by the current.

Velocity of boat w.r.t. river $\vec{v}_{br} = \vec{OA} = 4 \text{ m/s}$

Velocity of river w.r.t. earth $\vec{v}_{re} = \vec{AB} = 2 \text{ m/s}$

Velocity of boat w.r.t. earth $\vec{v}_{be} \text{ m/s} = \vec{OB} = ?$

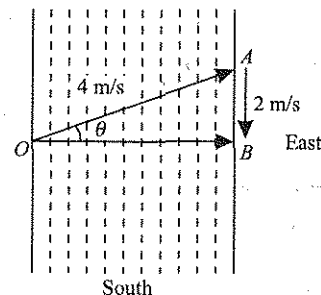


Fig. 5.63

1. To find the direction of boat in which boat has to go we need to find angle θ .

$$\text{From rt } \triangle OBA, \sin \theta = \frac{AB}{OA} = \frac{2}{4} = \frac{1}{2}$$

$$\theta = 30^\circ$$

Hence, the motorboat has to head at 30° north to east.

2. To find the velocity of boat w.r.t. earth, we can use pythagorous theorem again,

Using rt. $\triangle OBA$, we have

$$v_{br}^2 = v_{be}^2 + v_e^2 \Rightarrow v_{be}^2 = v_{br}^2 - v_e^2$$

$$\Rightarrow v_{be} = \sqrt{v_{br}^2 - v_e^2} = \sqrt{4^2 - 2^2} = 2\sqrt{3} \text{ m/s}$$

3. Time taken to cross the river

$$= \frac{\text{Width of river}}{\text{Velocity of boat w.r.t. earth}}$$

v_{be} is taken in this because v_{be} is along OB (the line of movement) = $\frac{800}{2\sqrt{3}} = \frac{400\sqrt{3}}{3}$ s

Example 5.7 A person walks up a stationary escalator in t_1 second. If he remains stationary on the escalator, then it can take him up in t_2 second. If the length of the escalator is L , then

1. Determine the speed of man with respect to the escalator.
2. Determine the speed of the escalator.
3. How much time would it take him to walk up the moving escalator?

Sol.

1. As the escalator is stationary, so the distance covered in t_1 second is L which is the length of the escalator.

$$\text{Speed of the man w.r.t. the escalator } v_{mc} = \frac{L}{t_1}$$

2. When the man is stationary, by taking man as reference point the distance covered by the escalator is L in time t_2 .

$$\text{Speed of escalator } v_c = \frac{L}{t_2}$$

3. Speed of man w.r.t. the ground $v_m = v_{mc} + v_c$

$$= \frac{L}{t_1} + \frac{L}{t_2} = L \left[\frac{1}{t_1} + \frac{1}{t_2} \right] = L \left[\frac{t_1 + t_2}{t_1 t_2} \right]$$

$$\Rightarrow L = v_m \left[\frac{t_1 t_2}{t_1 + t_2} \right]$$

$\left[\frac{t_1 t_2}{t_1 + t_2} \right]$ is the time taken by the man to walk up the moving escalator.

Example 5.8 A person standing on a road has to hold his umbrella at 60° with the vertical to keep the rain away. He throws the umbrella and starts running at 20 ms^{-1} . He finds that rain drops are falling on him vertically. Find the speed of the rain drops with respect to

1. the road, and
2. the moving person.

Sol. Given $\theta = 60^\circ$ and velocity of person

$$\vec{v}_p = \vec{OA} = 20 \text{ ms}^{-1}.$$

This velocity is the same as the velocity of person w.r.t. ground. First of all let's see how the diagram works out.

$$\vec{v}_{rp} = \vec{OB} = \text{velocity of rain w.r.t. the person.}$$

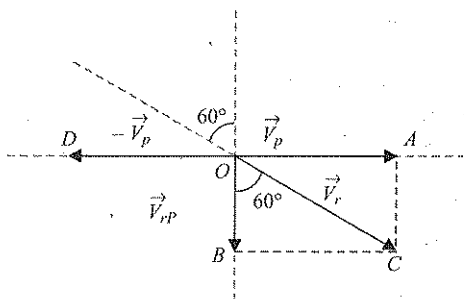


Fig. 5.64

$\vec{v}_r = \vec{OC}$ = velocity of rain w.r.t. earth $\vec{v}_{rp}$ is along $\vec{OB}$ as a person has to hold umbrella at an angle with vertical which is

the angle between velocity of rain and velocity of rain w.r.t. the person.

Values of $\vec{v}_r$ and $\vec{v}_{rp}$ can be obtained by using simple trigonometric relations.

1. Speed of rain drops w.r.t. earth = $\vec{v}_r = \vec{OC}$

$$\text{From rt } \triangle OCM, \frac{CB}{OC} = \sin 60^\circ \Rightarrow OC = \frac{CB}{\sin 60^\circ}$$

$$= \frac{20}{\sqrt{3}/2} = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} \text{ ms}^{-1}$$

2. Speed of rain w.r.t. the person $\vec{v}_{rp} = \vec{OB}$

$$\text{From rt } \triangle OCM, \frac{OB}{CB} = \cot 60^\circ$$

$$\Rightarrow OB = CB \cot 60^\circ = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \text{ ms}^{-1}$$

Example 5.9 A boat heading due north crosses a wide river with a speed of 12 km/h relative to the water. The water in the river has a uniform speed of 5 km/h due east relative to the earth. Determine the velocity of the boat relative to an observer standing on either bank and the direction of boat.

Sol. Imagine a situation in your mind of a boat moving across the river. The boat is heading north, which means it wants to go straight, where the current pushes the boat along the direction of current, i.e., east. We are given

$$\text{Velocity of boat relative to the river } v_{br} = 10 \text{ km/h}$$

$$\text{Velocity of river} = \text{velocity of river relative to earth} = v_{re} = 5 \text{ km/h}$$

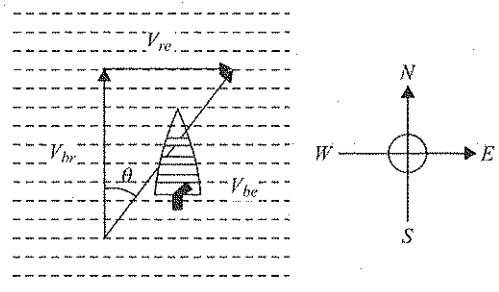


Fig. 5.65

Velocity of the river can be taken as relative to the earth as the velocity measured has only earth as reference. We have to find out the velocity of the boat relative to an observer standing on the bank. Since the observer is stationary with respect to earth, so the velocity of boat relative to observer will be same as the velocity of boat relative to earth.

Let us suppose due to push of current the boat gets drifted by an angle θ from the straight line path (see Fig. 5.65).

As seen from Fig. 5.65, velocities in situation from a right angled triangle and we have the values of two sides. Therefore, the third side can be calculated which represents the desired velocity.

1. From pythagorous theorem, $b_{bE} = \sqrt{v_{br}^2 + v_{re}^2}$
 $= \sqrt{(12)^2 + (5)^2} = \sqrt{144 + 25} = 13 \text{ km/h}$

2. To find out the direction, we need to find the angle θ through which boat has deviated.

$$\tan \theta = \frac{v_{re}}{v_{br}} \Rightarrow \theta = \tan^{-1} \left(\frac{v_{re}}{v_{be}} \right) = \tan^{-1} \left(\frac{5}{12} \right)$$

Hence, the boat is moving with a velocity 13 km/h in the direction $\tan^{-1} \left(\frac{5}{12} \right)$ east of north relative to earth.

Example 5.10 If the boat of the preceding example travels with the speed of 13 km/h relative to the river and is to travel due north as shown in Fig. 5.66, what should its angle of direction be?

Sol. Given $v_{br} = 10$ km/h.

As the boat has to move due north, so it needs to start at an angle θ move upward direction of the river.

This is necessary because the boat during the motion will be drifted downwards due to the push of current.

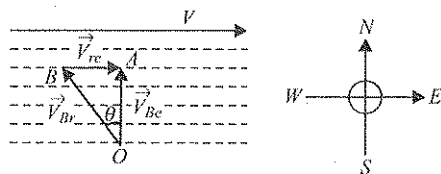


Fig. 5.66

v_{be} = velocity of boat w.r.t. earth is along hypotenuse = 13 km/h

v_{re} = velocity of river w.r.t. earth is along perpendicular = 5 km/h

v_{be} = velocity of boat w.r.t. earth is along base = ?

$$\vec{v}_{Br} = \vec{v}_{Be} - \vec{v}_{re} \Rightarrow \vec{v}_{Br} = 13 \text{ km/hr}$$

$$\vec{v}_{Be} = \vec{v}_{Br} + \vec{v}_{re} \Rightarrow \vec{v}_{re} = 5 \text{ km/hr}$$

Using Pythagoras theorem we have, $v_{br}^2 = v_{be}^2 + v_{re}^2$

$$\Rightarrow v_{be}^2 = v_{br}^2 - v_{re}^2 \Rightarrow v_{be} = \sqrt{v_{br}^2 - v_{re}^2}$$

$$\therefore v_{be} = \sqrt{(13)^2 - (5)^2} = \sqrt{169 - 25} = \sqrt{144} = 12 \text{ m/s}$$

Now to find the right direction of movement of boat so that it goes straight in north direction, the angle θ needs to be obtained

$$\tan \theta = \frac{v_{re}}{v_{be}} \Rightarrow \theta = \tan^{-1} \left(\frac{v_{re}}{v_{be}} \right) = \tan^{-1} \left(\frac{5}{12} \right)$$

Hence the boat has to start at an angle $\tan^{-1} \left(\frac{5}{12} \right)$ in order to move due north.

Example 5.11 A political party has to start its procession in an area where wind is blowing at a speed of 41.4 km/h and party flags on the vehicles are fluttering along north-east direction. If the procession starts with a speed of 40 km/h towards north, find the direction of flags on the vehicles.

Sol. When the procession is stationary, the flags flutter along the north-east direction. It means wind is flowing along the north-east direction. The flags will start fluttering along the direction of relative velocity of wind w.r.t. procession. As soon as procession gets into motion (see Fig. 5.67).

As the car will move along north with velocity v_C , the air will flow in opposite direction to that of car and will influence the direction of fluttering of car.

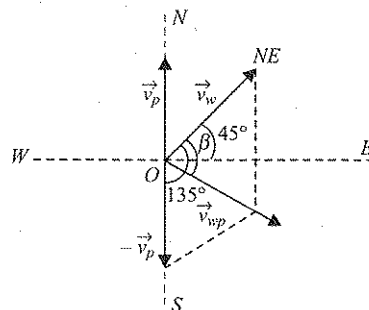


Fig. 5.67

The flags will flutter in the resultant direction of winds coming from two sides (i) along N.E direction (ii) in backward direction due to forward motion. We can say the flag will flutter along $\vec{v}_{wC}$, relative velocity of wind w.r.t. the car.

$$\vec{v}_{wC} = \vec{v}_w + (-\vec{v}_C)$$

[as velocity of air due to car is opposite to motion of car].

Here $|\vec{v}_w| = 41.4$ km/h, $|\vec{v}_C| = 40$ km/h

Angle between $\vec{v}_w$ and $\vec{v}_C$ is 135° , i.e., $\theta = 135^\circ$. Then direction of the resultant is given by

$$\tan \beta = \frac{41.4 \sin 135^\circ}{40 + 41.4 \cos 135^\circ}$$

$$= \frac{41.4 \sin (90^\circ + 45^\circ)}{40 + 41.4 \cos (90^\circ + 45^\circ)}$$

$$= \frac{41.4 \times \frac{1}{\sqrt{2}}}{40 + 41.4 (-\sin 45^\circ)} = \frac{41.4 \times \frac{1}{\sqrt{2}}}{40 - 41.4 \frac{1}{\sqrt{2}}}$$

$$= \frac{10}{40 - 10} = \frac{1}{3}$$

$$\Rightarrow \beta = \tan^{-1} \left(\frac{1}{3} \right)$$

$$\therefore \text{angle w.r.t. east direction } \beta = \tan^{-1} \frac{1}{3} - 45^\circ$$

Example 5.12 Two roads intersect at right angle, one goes along x-axis another along y-axis. At any instant two cars A and B are moving along y and x directions, respectively meets at intersection. Draw the direction of motion of

1. car A as seen from car B, and
2. car B as seen from car A.

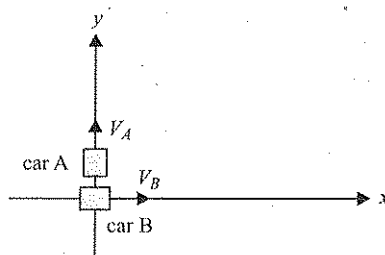


Fig. 5.68

Sol. Direction of motion of car A as seen from car B

$$\vec{v}_{A,B} = \vec{v}_A - \vec{v}_B = \vec{v}_A + (-\vec{v}_B)$$

Here car B will consider as rest and car A will be observed.

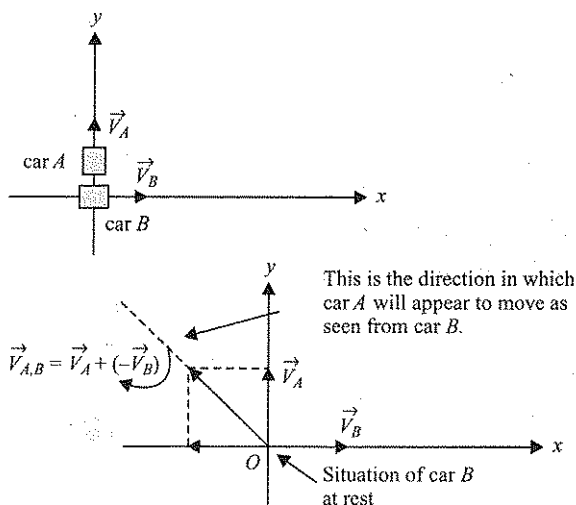


Fig. 5.69

Example 5.13 Two roads one along y-axis and another along a direction at angle θ with x-axis are as shown in Fig. 5.70. Two cars A and B are moving along the roads. Consider the situation of the diagram. Draw the direction of

1. car B as seen from car A, and
2. car A as seen from car B.

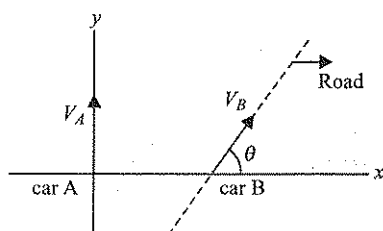


Fig. 5.70

Sol. 1. Direction of motion of B as seen from A.

$$\vec{v}_{B,A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{v}_{B,A} = \vec{v}_B + (-\vec{v}_A)$$

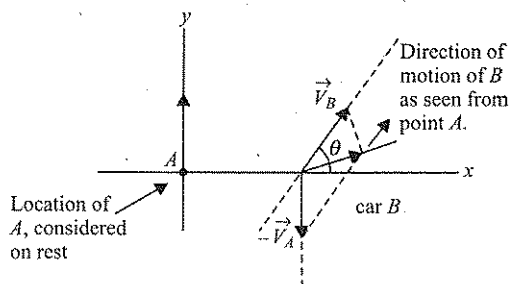


Fig. 5.71

2. Direction of motion of A as seen from B.

$$\vec{v}_{A,B} = \vec{v}_A - \vec{v}_B \Rightarrow \vec{v}_{A,B} = \vec{v}_A + (-\vec{v}_B)$$

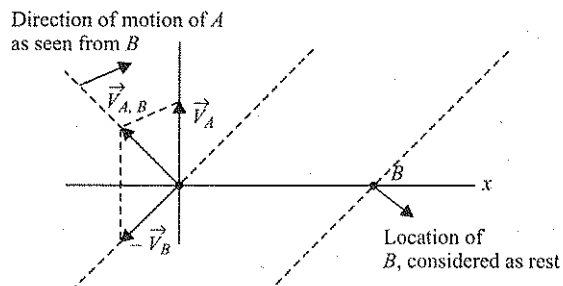


Fig. 5.72

Example 5.14 Consider the situation given in Fig. 5.73, two cars are moving along road 1 and road 2. Draw the direction of the motion of

1. car B as seen from car A, and
2. car A as seen from car B.

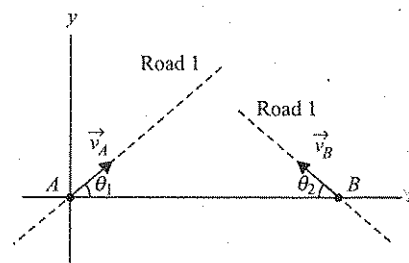


Fig. 5.73

Sol. 1. Direction of motion of B as seen from A Fig. 5.74

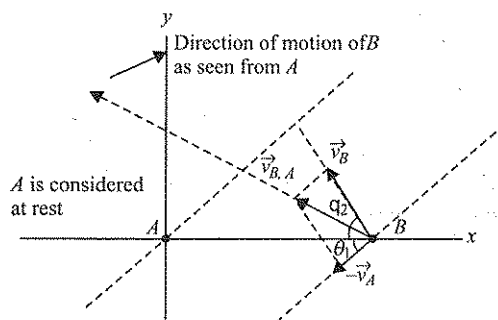


Fig. 5.74

$$\vec{v}_{B,A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{v}_{B,A} = \vec{v}_B + (-\vec{v}_A)$$

2. Direction of motion of A as seen from B (Fig. 5.75)

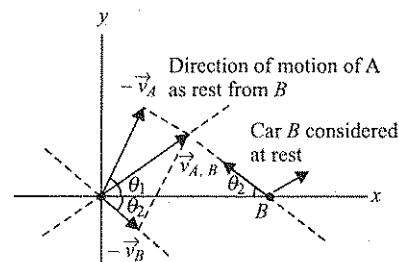


Fig. 5.75

$$\vec{v}_{A,B} = \vec{v}_A - \vec{v}_B \Rightarrow \vec{v}_{A,B} = \vec{v}_A + (-\vec{v}_B)$$

Example 5.15 Consider two cities P and Q between which consistent bus service is available in both directions

every x minutes. A morning jogger is jogging towards Q from P with a speed of 10 km/h. Every 18 mins a bus crosses this jogger in its own direction of motion and every 6 mins another bus crosses in opposite direction. What is the time period between two consecutive buses and also find the speed of buses?

Sol. Analysis of problem: The equations of motion are applicable in any frame of reference. Attaching the frame of reference at jogger. The velocity of bus is now required w.r.t. jogger. Suppose speed of buses $s = v$ km/h

Distance between two buses on road: $s = xv$ The relative



Fig. 5.76

velocity of bus w.r.t. the jogger, for the buses moving from P to $Q = (v - 10)$ km/h

$$\text{So we have } 18 = \frac{vx}{v - 10} \Rightarrow vx = 18v - 180 \quad (i)$$

Similarly, for the buses moving from Q to $P = (v + 10)$ km/h

$$\text{So we have } 6 = \frac{vx}{v + 10} \Rightarrow vx = 6v + 60 \quad (ii)$$

Solving equations (i) and (ii), we find

$$v = 20 \text{ km/h and } x = 9 \text{ min.}$$

Example 5.16 The horizontal range of a projectile is

$2\sqrt{3}$ times its maximum height. Find the angle of projection.

Sol. If u and α be the initial velocity of projection and angle of projection, respectively, then

$$\begin{aligned} \text{The maximum height attained} &= \frac{u^2 \sin^2 \alpha}{2g} \text{ and horizontal} \\ \text{range} &= \frac{2u^2 \sin \alpha \cos \alpha}{g} \end{aligned}$$

$$\begin{aligned} \text{According to the problem, } &= \frac{2u^2 \sin \alpha \cos \alpha}{g} \\ &= 2\sqrt{3} \left(\frac{u^2 \sin^2 \alpha}{2g} \right) \Rightarrow \tan \alpha \left(\frac{2}{\sqrt{3}} \right) \Rightarrow \alpha = \tan^{-1} \left(\frac{2}{\sqrt{3}} \right) \end{aligned}$$

Example 5.17 For a projectile, show that

$$(1) gT^2 = 2R \tan \alpha; (2) H_{\max} = \left(\frac{R}{4} \right) \tan \alpha; \text{ and } (3)$$

$gT^2 = 8H_{\max}$, where the symbols have their usual meanings.

Sol.

$$1. \text{ We know that } T = \frac{2u \sin \alpha}{g}, \text{ and} \quad (i)$$

$$R = \frac{2u^2 \sin \alpha \cos \alpha}{g} \quad (ii)$$

Squaring equation (i) and dividing it by equation (ii) we get

$$\frac{T^2}{R} = \frac{4u^2 \sin^2 \alpha}{g^2} \times \frac{g}{2u^2 \sin \alpha \cos \alpha} = \frac{2}{g} \tan \alpha$$

$$gT^2 = 2R \tan \alpha$$

Hence proved.

$$2. \text{ Again, we know that } H_{\max} = \frac{u^2 \sin^2 \alpha}{2g} \quad (i)$$

$$\text{and } R = \frac{2u^2 \sin \alpha \cos \alpha}{g} \quad (ii)$$

Dividing equations (i) and (ii),

$$\frac{H_{\max}}{R} = \frac{u^2 \sin^2 \alpha}{2g} \times \frac{g}{2u^2 \sin \alpha \cos \alpha} = \frac{1}{4} \tan \alpha$$

$$H_{\max} = \frac{R}{4} \tan \alpha.$$

Hence proved.

$$3. H_{\max} = \frac{u^2 \sin^2 \alpha}{2g}, \text{ and} \quad (i)$$

$$T = \frac{2u \sin \alpha}{g} \quad (ii)$$

$\therefore$ squaring equation (ii) and dividing it by equation (i),

$$\frac{T^2}{H_{\max}} = \frac{4u^2 \sin^2 \alpha}{g^2} \times \frac{2g}{u^2 \sin^2 \alpha} = \frac{8}{g}$$

$$gT^2 = 8H_{\max}$$

Hence proved.

Example 5.18

From a point on the ground at a distance a from the foot of a pole, a ball is thrown, at an angle of 45° , which just touches the top of pole and strikes the ground at a distance of b , on the other side of it. Find the height of the pole.

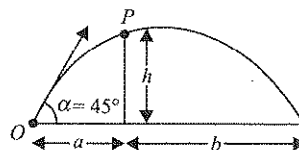


Fig. 5.77

Sol. Let h be the height of the pole.

Using equation, we have $y = x \tan \alpha \left(1 - \frac{x}{R} \right)$

Since top of pole, lie on curve (1),

$$h = a \tan 45^\circ \left(1 - \frac{a}{a+b} \right) = a \left[\frac{a+b-a}{a+b} \right] = \frac{ab}{a+b}$$

Example 5.19

A particle is projected over a triangle from one extremity of its horizontal base. Grazing over the vertex, it falls on the other extremity of the base. If α and β be the base angles of the triangle and θ the angle of projection, prove that $\tan \theta = \tan \alpha + \tan \beta$.

Sol. Let ABC be the triangle with base BC . Let h be the height of the vertex A , above BC . If AM be the perpendicular drawn on base BC from vertex A , then $\tan \alpha = \frac{h}{a}$ and $\tan \beta = \frac{h}{b}$, where $BM = a$ and $CM = b$.

Since $A(a, h)$ lies on the trajectory of the projectile;

$$y = x \tan \theta \left(1 - \frac{x}{R} \right) \quad (i)$$

Therefore it should satisfy equation (i)

$$\text{i.e., } h = a \tan \theta \left(1 - \frac{a}{a+b} \right) [\because \text{range } R = a+b]$$

$$\frac{h}{a} = \tan \theta \left[\frac{b}{a+b} \right]$$

$$\Rightarrow \tan \theta = h \frac{\left(\frac{a+b}{ab} \right) h}{= \frac{h}{b} + \frac{h}{a}}$$

$$\Rightarrow \tan \theta = \tan \alpha + \tan \beta$$

Hence proved.

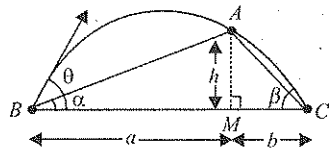


Fig. 5.78

Example 5.20 A particle projected at a definite angle α to the horizontal, passes through points (a, b) and (b, a) , referred to horizontal and vertical axes through the point of projection. Show that:

1. the horizontal range $R = \frac{a^2 + ab + b^2}{a + b}$, and
2. the angle of projection α is given by $\tan^{-1} \left[\frac{a^2 + ab + b^2}{ab} \right]$.

Sol. 1. The equation to the trajectory of the particle:

$$y = x \tan \alpha \left(1 - \frac{x}{R} \right) \quad (i)$$

Since the particle passes through the points with coordinates (a, b) and (b, a) they should satisfy the first equation of curve

$$b = a \tan \alpha \left(1 - \frac{a}{R} \right) \text{ and} \quad (ii)$$

$$a = b \tan \alpha \left(1 - \frac{b}{R} \right) \quad (iii)$$

Solving equation (i) from equations (ii) and (iii)

$$\frac{b}{a} = \left(1 - \frac{a}{R} \right) \tan \alpha \text{ and} \quad (iv)$$

$$\frac{a}{b} = \left(1 - \frac{b}{R} \right) \tan \alpha \quad (v)$$

Dividing equation (iv) by (v) we get (to eliminate $\tan \alpha$)

$$\frac{b^2}{a^2} = \frac{\left(1 - \frac{a}{R} \right)}{\left(1 - \frac{b}{R} \right)} \Rightarrow b^2 - \frac{b^3}{R} = a^2 - \frac{a^3}{R}$$

$$\Rightarrow \frac{1}{R} [a^3 - b^3] = a^2 - b^2$$

$$\Rightarrow R = \frac{a^3 - b^3}{a^2 - b^2} = \frac{(a-b)(a^2 + ab + b^2)}{(a-b)(a+b)} = \frac{a^2 + ab + b^2}{a+b}$$

Hence proved [$\because a \neq b$].

2. Substituting the expression for R in equation (ii)

$$\frac{b}{a} = \tan \alpha \left[1 - \frac{a(a+b)}{a^2 + ab + b^2} \right]$$

$$= \tan \alpha \left[\frac{a^2 + ab + b^2 - a^2 - ab}{a^2 + ab + b^2} \right]$$

$$\tan \alpha = \frac{a^2 + ab + b^2}{ab} \Rightarrow \alpha = \tan^{-1} \left[\frac{a^2 + ab + b^2}{ab} \right]$$

Hence proved.

Example 5.21 A ball is thrown from the top of a building 45 m high with a speed 20 m/s above the horizontal at an angle of 30° . Find

1. the time taken by the ball to reach the ground, and
2. the speed of ball just before it touches the ground.

Sol. Given $v = 20$ m/s, $\theta = 30^\circ$, $H = 45$ m

1. As the ball has been projected at an angle of 30° above the horizontal, so first of all we need to analyse the velocity horizontally and vertically. This will be useful while using distance-time relation in horizontal and vertical directions.

$$v_{xi} = v \cos 30^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ m/s;}$$

$$v_{yi} = v \sin 30^\circ = 20 \times \frac{1}{2} = 10 \text{ m/s}$$

It will be easy for us to use distance-time relation in vertical as it will involve less calculation

$$\text{In } y \text{ direction: } -45 = 10t - \frac{1}{2} \times g t^2 \Rightarrow t^2 - 2t - 9 = 0$$

which on solving gives $t = 1 + \sqrt{10}$ s (positive value), (other value is $1 - \sqrt{10}$ s, a negative value of time which is not acceptable).

2. $v_{yf} = 10 - 10 \times 4 = -30$ m/s,

$$v_f = \sqrt{v_{yf}^2 + v_{xf}^2} = \sqrt{30^2 + (10\sqrt{3})^2} = 20\sqrt{3} \text{ m/s}$$

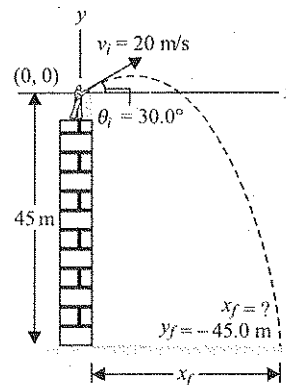


Fig. 5.79

Example 5.22

1. With what velocity v_0 should a ball be projected horizontally from the top of a tower so that the horizontal distance on the ground is ηH , where H is the height of the tower.
2. Also determine the speed of the ball when it reaches the ground.

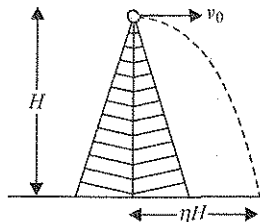


Fig. 5.80

Sol. 1. Given Horizontal distance = ηH , where H = height of the tower.

To find out the sufficient velocity as asked, we have to use the formula. Distance = velocity $\times$ time. Here the time involved will be time of flight as we are considering projectile motion which is given by $T = \sqrt{\frac{2H}{g}}$.

$$\text{So, } v_0 T = \eta H \Rightarrow v_0 \sqrt{\frac{2H}{g}} = \eta H \Rightarrow v_0 = \eta \sqrt{\frac{gH}{2}}$$

2. During projectile motion horizontal component of velocity remains same and its vertical component keeps on changing under the effect of gravity. So horizontal speed $v_x = v_0$, vertical speed $v_y = \sqrt{2gH}$.

$$\begin{aligned} \text{Total speed} &= \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + 2gH} = \sqrt{\eta^2 \frac{gH}{2} + 2gH} \\ &= \sqrt{\left(\frac{\eta^2 + 4}{2}\right) gH} \quad \therefore v_0 = \eta \sqrt{\frac{gH}{2}} \end{aligned}$$

Example 5.23 A particle is projected at angle θ with the horizontal. Calculate the time when it is moving perpendicular to initial direction. Also calculate the velocity at this position.

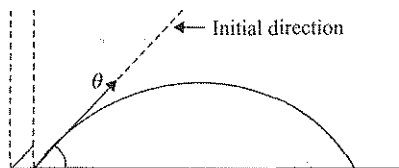


Fig. 5.81

Sol. Method 1 (using properties of projectile motion):

As we have to calculate the time between two positions A and B where final direction of movement is perpendicular to initial direction of movement. So for our own comfortability we can choose initial direction of motion as x -axis. Also let's assume velocity at position B to be v .

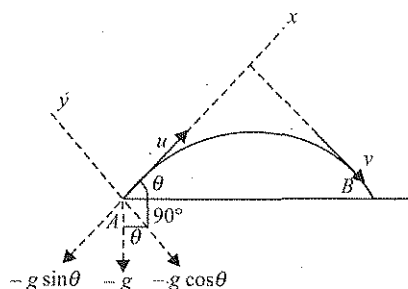


Fig. 5.82

Now analyzing the motion in x and y directions, we have

$$\begin{aligned} u_x &= u & u_y &= 0 \\ a_x &= -g \sin \theta & a_y &= -g \cos \theta \end{aligned}$$

Here we can use the following formula $v = u + at$ in x -direction. As we have values of initial velocity, final velocity, and acceleration we can find t . $\therefore v_x = u_x + a_x t$

At position B, $v_x = 0$, as final velocity is equal to y -component of velocity.

$$0 = u - g \sin \theta t \Rightarrow t = \frac{u}{g \sin \theta} \text{ which is required time to travel.}$$

Method 2 (using vectors)

As u and v both are perpendicular to each other, we can use the orthogonality property of dot product. This means that if two vectors are perpendicular to each other their dot product is zero, to find out time of travel to desired position.

$$\text{So, } \vec{u} \cdot \vec{v} = 0 \Rightarrow \vec{u}(\vec{u} + \vec{a}t) = 0$$

$$\Rightarrow \vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{a}t = 0$$

$$\Rightarrow u^2 + u g \cos(90^\circ + \theta)t = 0$$

[$\therefore$ Angle between u and g is $90^\circ + \theta$ as from figure.]

$$\Rightarrow u^2 + ug(-\sin \theta) \cdot t = 0 \text{ So } t = \frac{u}{g \sin \theta} \text{ is the desired time.}$$

To find out the velocity we can use the same relation as used in this question. But as at final position (considered) only y component of velocity is present. So we need to use the same relation in y -direction.

$$v_y = u_y + a_y t \Rightarrow v = 0 - g \cos \theta t;$$

$$v = -g \cos \theta \frac{u}{g \sin \theta} = -u \cot \theta \text{ is the velocity at position B.}$$

Example 5.24 At what angle should a ball be projected up an inclined plane with a velocity v_0 so that it may hit the incline normally. The angle of the inclined plane with the horizontal is α .

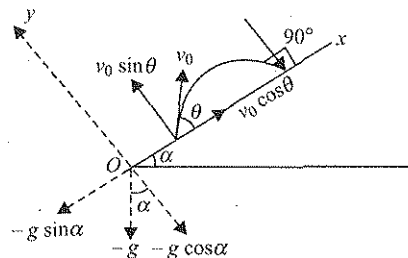


Fig. 5.83

Sol. As the ball has to hit the inclined plane normally, so in that position the x -component of velocity will be zero and the velocity will have y -component only.

The ball will hit the incline normally if its parallel component of velocity reduces to zero during the time of flight.

By analyzing this motion along incline, i.e., x -direction $v_x = u_x + a_x t$.

$$\text{Here } v_x = 0, u_x = v_0 \cos \theta, a_x = -g \sin \alpha$$

$$0 = v_0 \cos \theta - (g \sin \alpha) T \Rightarrow T = \frac{v_0 \cos \theta}{g \sin \alpha} \quad (i)$$

Also the displacement of the particle in y -direction will be zero. Using

$$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 0 = v_0 \sin \theta T - \frac{1}{2} g \cos \alpha T^2$$

This gives $T = \frac{2v_0 \sin \theta}{g \cos \alpha}$

From equations (i) and (ii), we have

$$\frac{v_0 \cos \theta}{g \sin \alpha} = \frac{2v_0 \sin \theta}{g \cos \alpha} \Rightarrow \frac{\cos \theta}{\sin \alpha} = \frac{2 \sin \theta}{\cos \alpha}$$

$$\Rightarrow 2 \tan \theta \tan \alpha = 1 \Rightarrow \tan \theta = \left[\frac{1}{2} \cot \alpha \right]$$

$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{2} \cot \alpha \right)$ which is the required angle of projection.

Example 5.25 Particles P and Q of mass 20 gm and 40 gm, respectively, are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles, respectively, with the horizontal AB as shown in the Fig. 5.84. Each particle has an initial speed of 49 m/s. The separation AB is 245 m. Both particles travel in the same vertical plane and undergo a collision. After the collision, P retraces its path. Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground? Take $g = 9.8 \text{ m/s}^2$.

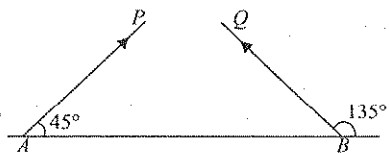


Fig. 5.84

Sol. $m_P = 20 \text{ g}$, $m_Q = 40 \text{ g}$, The horizontal velocities of both the particles are same and since both are projected simultaneously, these particles will meet exactly in the middle of AB (horizontally).

To find the vertical velocity at the time of collision let us consider the motion of P in vertical and horizontal directions.

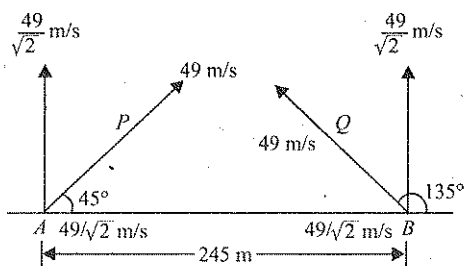


Fig. 5.85

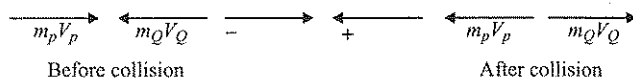
Horizontal direction, $S_x = 122.5$, $t_x = ?$, $v_x = 49/\sqrt{2}$,

$$\text{Vel.} = \frac{\text{disp}}{\text{time}} \Rightarrow \frac{49}{\sqrt{2}} = \frac{122.5}{t_x} \Rightarrow t_x = \frac{(122.5)\sqrt{2}}{49}$$

$$\text{Vertical direction } v_y = ?, u_y = \frac{49}{\sqrt{2}}, t_y = \frac{(122.5)\sqrt{2}}{49},$$

$$a_y = -9.8 \text{ m/s}^2$$

$$v_y = u_y + at_y = \frac{49}{\sqrt{2}} - 9.8 \times \frac{(122.5)\sqrt{2}}{49} = 0$$



$$\text{Also, } v^2 - u^2 = 2as \Rightarrow -\left(\frac{49}{\sqrt{3}}\right)^2 = 2(-9.8) \times s$$

$$\Rightarrow s = 61.25$$

The collision takes place at the maximum height where the velocities of both the particles will be in the horizontal direction.

Applying conservation of linear momentum in the horizontal direction with the information that P retraces its path therefore its momentum will be same in magnitude but different in direction.

Momentum of system before collision = Momentum of system after collision.

$$m_P v_P - m_Q v_Q = -m_P v_P + m_Q v'_Q \Rightarrow v'_Q = \frac{2m_P v_P - m_Q v_Q}{m_Q}$$

$$= \frac{2 \times 0.02 \times (49/\sqrt{2}) - 0.040 (49/\sqrt{2})}{0.040}$$

$$= \frac{49}{\sqrt{2}} \left[\frac{0.04}{0.040} - 1 \right] = \frac{49}{\sqrt{2}} \times 0 = 0$$

New path of P after collision, Considering vertical motion of Q , $u_y = 0$, $s_y = -61.25$, $a_y = -9.8$, $t_y = ?$

$$S = ut + \frac{1}{2} at^2 = \frac{1}{2} \times (-9.8) \times t^2 = (-61.25) \therefore t = 3.53 \text{ s}$$

Considering horizontal motion of Q .

Since the $V'_Q = 0$, therefore the particle Q falls down vertically so it falls down on the mid point of ab .

Example 5.26 A body falling freely from a given height H hits an inclined plane in its path at a height h . As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h/H) the body will take maximum time to reach the ground? (IIT-JEE, 1986)

Sol. For A to B : $u = 0$, $s = -(H - h)$, $a = -g$, $t = ?$

$$S = ut + \frac{1}{2} at^2 \Rightarrow -(H - h) = \frac{1}{2} (-g) t^2$$

$$\Rightarrow t = \left[\frac{2(H - h)}{g} \right]^{1/2}$$

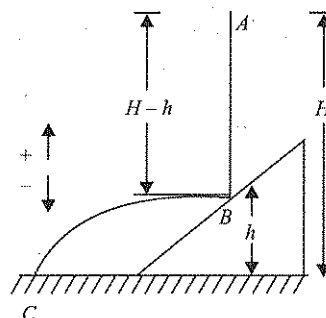


Fig. 5.86

For B to C vertical motion: $u_y = 0$, $s_y = -h$, $a_y = -g$

$$S = ut + \frac{1}{2} at^2 \Rightarrow -h = -\frac{1}{2} at^2 \Rightarrow t' = \sqrt{\frac{2h}{g}}$$

$$\text{Total time of fall } T = t + t' = \left[\frac{2(H-h)}{g} \right]^{1/2} + \left[\frac{2h}{g} \right]^{1/2}$$

For finding the maximum time using the concept of differentiation: $\frac{dT}{dh} = 0$

$$\Rightarrow \frac{d}{dt} \left[\frac{2(H-h)}{g} \right]^{1/2} + \frac{d}{dt} \left[\frac{2h}{g} \right]^{1/2} = 0$$

$$\Rightarrow \frac{1}{2} \left[\frac{2(H-h)}{g} \right]^{1/2} \times \left(\frac{-2}{g} \right) + \frac{1}{2} \left[\frac{2h}{g} \right]^{1/2} \left(\frac{2}{g} \right) = 0$$

$$\Rightarrow \left[\frac{2(H-h)}{g} \right]^{1/2} = \left[\frac{2h}{g} \right]^{1/2} \Rightarrow \frac{2(H-h)}{g} = \frac{2h}{g}$$

$$\Rightarrow H - h = h \Rightarrow \frac{h}{H} = \frac{1}{2}$$

Example 5.27 Two towers AB and CD are situated at a distance d apart as shown in Fig. 5.87. AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD . Simultaneously another object of mass $2m$ is thrown from the top of CD at an angle 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other. (i) Calculate the distance d between the towers. (ii) Find the position where the objects hit the ground.

(IIT-JEE, 1994)

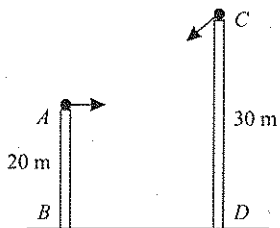


Fig. 5.87

Sol. (i) Let t be the time taken for collision. For mass m thrown horizontally from A for horizontal motion,

$$PM = 10t \quad (1)$$

For vertical motion $u_y = 0$, $s_y = y$, $a_y = g$

$$S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow y = \frac{1}{2} g t^2 \quad (2)$$

$$t_y = t, v_y = u_y + a_y t = g t \quad (3)$$

For mass $2m$ thrown from C , for horizontal motion

$$QM = [10 \cos 60^\circ] t \Rightarrow QM = 5t \quad (4)$$

For vertical motion: $u_y = 10 \sin 60^\circ = 5\sqrt{3}$

$$\Rightarrow v_y = 5\sqrt{3} + g t \quad (5)$$

$$a_y = g, S_y = y + 10, S = ut + \frac{1}{2} at^2$$

$$\Rightarrow y + 10 = 5\sqrt{3} t + \frac{1}{2} g t^2 \quad (6)$$

$t_y = t$. From equations (ii) and (vi)

$$\frac{1}{2} g t^2 + 10 = 5\sqrt{3} t + \frac{1}{2} g t^2$$

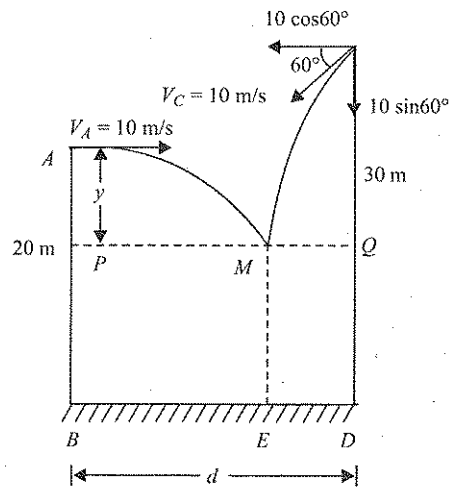


Fig. 5.88

$$\Rightarrow t = \frac{2}{\sqrt{3}} \text{ s}$$

$$BD = PM + MQ = 10t + 5t = 15t = 15 \times \frac{2}{\sqrt{3}} = 10\sqrt{3} = 17.32 \text{ m}$$

(ii) Applying conservation of linear momentum (during collision of the masses at M) in the horizontal direction

$$m \times 10 - 2m \times 10 \cos 60^\circ = 3m \times v_x$$

$$\Rightarrow 10m - 10m = 3m \times v_x \Rightarrow v_x = 0$$

Since the horizontal momentum comes out to be zero, the combination of masses will drop vertically downwards and fall at E $\times BE = PM = 10t = 10 \times \frac{2}{\sqrt{3}} = 11.547 \text{ m}$

Example 5.28 Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed at some interval of time. One gun fires horizontally and other fires upward at an angle of 60° with the horizontal. The shots collide in air at a point P . Find (i) the time-interval between the firings, and (ii) the coordinates of the point P . Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x - y plane.

(IIT-JEE, 1996)

Sol. For bullet A : Let t be the time taken by bullet A to reach P .

$$\text{Vertical motion: } u_y = 0, S = ut + \frac{1}{2} at^2, S_y = 10 - y,$$

$$a_y = 10 \text{ m/s}^2,$$

$$10 - y = 5t^2 \quad (i)$$

$$t_y = t$$

$$\text{Horizontal motion: } x = 5\sqrt{3} t \quad (ii)$$

For bullet B . Let $(t + t')$ to be the time taken by bullet B to reach P .

$$\text{Vertical motion } \uparrow + u_y = +5\sqrt{3} \sin 60^\circ = +7.5$$

$$S = ut + \frac{1}{2} at^2 \quad \downarrow -a_y = -10 \text{ m/s}^2$$

$$y - 10 = 7.5(t + t') - 5(t + t')^2 \quad (iii)$$

$$s_y = -(10 - y) = y - 10 \quad t_y = t + t'$$

$$\text{Horizontal motion } x = (5\sqrt{3} \cos 60^\circ)(t + t')$$

$$\Rightarrow 5\sqrt{3} t + 5\sqrt{3} t' = 2x \quad (iv)$$

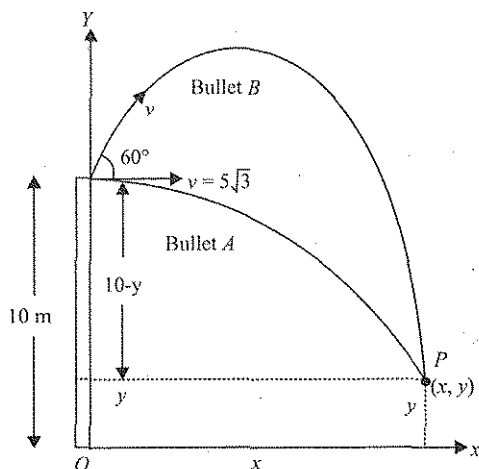


Fig. 5.89

Substituting the value of x from equation (ii) in equation (iv), we get

$$5\sqrt{3}t + 5\sqrt{3}t' = 10\sqrt{3}t \Rightarrow t = t'$$

Putting $t = t'$ in equation (iii) $y - 10 = 15t^2$ (v)

Adding equations (i) and (v) $0 - 15t - 15t^2 \Rightarrow t = 1$ s

(ii) Putting $t = 1$ in equation (ii), we get $x = 5\sqrt{3}$

Putting $t = 1$ in equation (i), we get $y = 5$

Therefore, the coordinates of point P are $(5\sqrt{3}, 5)$ in meters.

Example 5.29 A large, heavy box is sliding without friction down a smooth plane of inclination θ . From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u , and the direction of projection makes an angle α with the bottom as shown in Fig. 5.90. (IIT-JEE, 1998)

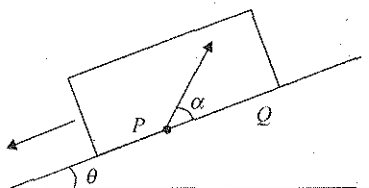


Fig. 5.90

1. Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).
2. If the horizontal displacement of the particle as seen by an observer on the ground is 0, find the speed of the box with respect to the ground at the instant when particle was projected.

Sol. 1. u is the relative velocity of the particle with respect to the box. Resolve u .

u_x is the relative velocity of particle with respect to the box in x -direction.

u_y is the relative velocity with respect to the box in y -direction.

Since there is no velocity of the box in the y -direction, therefore this is the vertical velocity of the particle with respect to

ground also. Y -direction motion (taking relative terms w.r.t. box)

$u_y = +u \sin \alpha$, $a_y = -g \cos \theta$, $S_y = 0$ (activity is taken till the time the particle comes back to the box) $t_y = t$.

$$S = ut + \frac{1}{2}at^2 \Rightarrow 0 = (u \sin \alpha)t - \frac{1}{2}g \cos \theta \times t^2$$

$$\Rightarrow t = 0 \text{ or } t = \frac{2u \sin \alpha}{g \cos \theta}$$

X -direction motion (Taking relative terms w.r.t. box)

$$u_x = +u \cos \alpha$$

$$S = ut + \frac{1}{2}at^2$$

$$a_x = 0 \Rightarrow S_x = u \cos \alpha \times \frac{2u \sin \alpha}{g \cos \theta} = \frac{u^2 \sin 2\alpha}{g \cos \theta} t_x = \frac{2u \sin \alpha}{g \cos \theta}$$

2. For the observer (on ground) to see the horizontal displacement to be zero, the distance travelled by the box in time $\left(\frac{2u \sin \alpha}{g \cos \theta}\right)$ should be equal to the range of the particle. Let the speed of the box at the time of projection of particle be U . Then for the motion of box with respect to ground (Fig. 5.91).

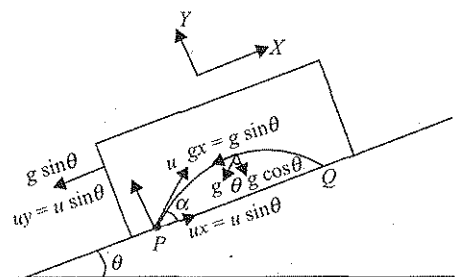


Fig. 5.91

$$u_x = u, S = ut + \frac{1}{2}at^2, a_x = -g \sin \theta$$

$$S_x = \frac{-u^2 \sin^2 2\alpha}{g \cos \theta} - \frac{u^2 \sin 2\alpha}{g \cos \theta} = -U \left(\frac{2u \sin \alpha}{g \cos \theta} \right) - \frac{1}{2}g \sin \theta \left(\frac{2u \sin \alpha}{g \cos \theta} \right)^2$$

$$t_y = \frac{2u \sin \alpha}{g \cos \theta} \text{ on solving, we get } U = \frac{u \cos(\alpha + \theta)}{\cos \theta}$$

Example 5.30 An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$ the plank starts moving along the $+x$ direction with an acceleration 1.5 m/s^2 . At the same instant a stone is projected from the origin with a velocity u as shown in Fig. 5.92. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the x - y plane. Find and the time after which the stone hits the object. (IIT-JEE, 2000)

Sol. Let t be the time after which the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with the horizontal. According to question, we have following three conditions.

- (i) Vertical displacement of stone is 1.25 m. Therefore,

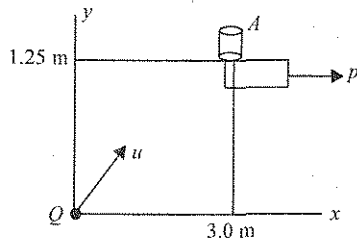


Fig. 5.92

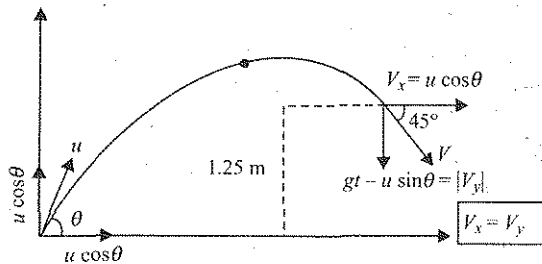


Fig. 5.93

$$1.25 = (u \sin \theta)t - \frac{1}{2}gt^2, \text{ where } g = 10 \text{ m/s}^2$$

$$\text{or } (u \sin \theta)t = 1.25 + 5t^2 \quad (i)$$

(ii) Horizontal displacement of stone = 3 + displacement of object A. Therefore,

$$(u \cos \theta)t = 3 + \frac{1}{2}at^2 \text{ where } a = 1.5 \text{ m/s}^2$$

$$\text{or } (u \cos \theta)t = 3 + 0.75t^2 \quad (ii)$$

(iii) Horizontal component of velocity (of stone) = vertical component (because velocity vector is inclined) at 45° with horizontal). Therefore,

$$(u \cos \theta) = gt - (u \sin \theta) \quad (iii)$$

(The right-hand side is written $gt - u \sin \theta$ because the stone is in its downward motion. Therefore, $gt > u \sin \theta$. In upward motion $u \sin \theta > gt$). Multiplying equation (iii) with t we can write,

$$(u \cos \theta)t + (u \sin \theta)t = 10t^2 \quad (iv)$$

Now, equations (i) - (ii) - (i) gives $4.25t^2 - 4.25 = 0$ or $t = 1$ s

Substituting $t = 1$ s in equations (i) and (ii) we get, $u \sin \theta = 6.25$ m/s or $u_y = 6.25$ m/s and $u \cos \theta = 3.75$ m/s.

or $u_x = 3.75$ m/s, therefore, $\vec{u} = u_x \hat{i} + u_y \hat{j}$ or

$$\vec{u} = (3.75 \hat{i} + 6.25 \hat{j}) \text{ m/s}$$

Example 5.31 On a frictionless horizontal surface, assumed to be the x - y plane, a small trolley A is moving along a straight line parallel to the y -axis (see Fig. 5.94) with a constant velocity of $(\sqrt{3} - 1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x -axis, a ball is thrown along the surface from the origin O . Its velocity makes an angle ϕ with the x -axis and it hits the trolley.

1. The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x -axis in this frame.

2. Find the speed of the ball with respect to the surface, if $\phi = 40/3$. (IIT-JEE, 2002)

Sol. 1. From the Fig. 5.95 $\vec{V}_{BT}$ makes an angle of 45° with the x -axis.

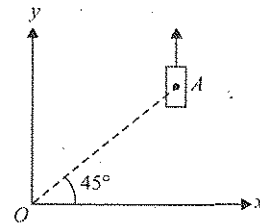


Fig. 5.94

$$2. \text{ Using sine rule } \frac{V_B}{\sin 135^\circ} = \frac{V_T}{\sin 15^\circ} \Rightarrow V_B = 2 \text{ m/s}$$

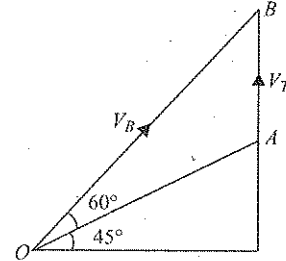


Fig. 5.95

Example 5.32 A particle A moves along a circle of radius $R = 50$ cm so that its radius vector r relative to point O rotates with the constant angular velocity $\omega = 0.40$ rad/s (Fig. 5.96).

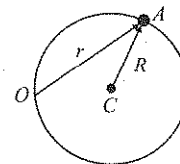


Fig. 5.96

Find the modulus of the velocity of the particle and direction of its total acceleration.

Sol. Consider X and Y axes as shown in Fig. 5.97. Using sine law in triangle CAO , we get

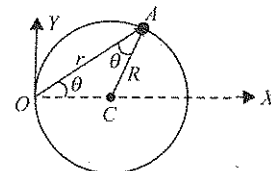


Fig. 5.97

$$\frac{r}{\sin(\pi - 2\theta)} = \frac{R}{\sin \theta} \text{ or } \frac{r}{2 \sin \theta \cos \theta} = \frac{R}{\sin \theta} \therefore r = 2R \cos \theta$$

$$\text{Now } \vec{r} = r \cos \theta \hat{i} + r \sin \theta \hat{j} = 2R \cos^2 \theta \hat{i} + 2R \cos \theta \sin \theta \hat{j}$$

$$\begin{aligned} \text{Now } \vec{v} &= \frac{d\vec{r}}{dt} = -4R \cos \theta \sin \theta \frac{d\theta}{dt} \hat{i} + 2R \cos 2\theta \frac{d\theta}{dt} \hat{j} \\ &= -2R \sin 2\theta \omega \hat{i} + 2R \cos 2\theta \omega \hat{j} \end{aligned}$$

$$\therefore |v| = 2\omega R$$

$$\begin{aligned} \text{Further } \vec{a} &= \frac{d\vec{v}}{dt} = -4R \cos 2\theta \omega \frac{d\theta}{dt} \hat{i} - 4R \omega \sin 2\theta \frac{d\theta}{dt} \hat{j} \\ &= -4R \omega^2 \cos 2\theta \hat{i} - 4R \omega^2 \sin 2\theta \hat{j} \end{aligned}$$

$$\therefore |a| = 4R \omega^2$$

EXERCISES

Subjective Type

Solutions on page 5.47

- Two piers, A and B are located on a river. B is 1,500 m downward from A . Two friends C and D must make round trips from pier A to pier B and return. D rows a boat at a constant speed of 4.00 km/h relative to the water and C walks on the shore at a constant speed of 4.00 km/h. The velocity of the river is 3.5 km/h in the direction from A to B . Find the time taken by C and D to make the round trips.
- A boat takes 2 h to travel 8 km and back in still water. If the velocity of the water is 4 km/h, then what is the time taken for going upstream of 8 km and then coming back.
- A railroad flatcar is traveling to the right at a speed of 13.0 m/s relative to an observer standing on the ground (see Fig. 5.98). Someone is riding a motor scooter on the flatcar. Corresponding to the relative velocities 18 m/s to the right, 3 m/s to the left, and 0 m/s of motor scooter relative to ground, find the relative velocities (magnitude and direction) of motor scooter relative to the flatcar.

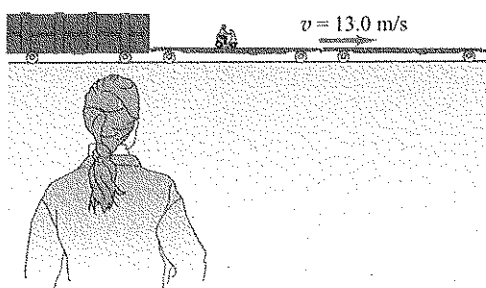


Fig. 5.98

- A boy is cycling with a speed of 20 km/h in a direction making an angle 30° north of east (see Fig. 5.99).

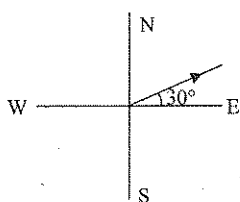


Fig. 5.99

Find the velocity of the second boy moving towards north so that to him the first boy appears to be moving towards east.

- A boat wants to cross a river as soon as possible. In doing so it takes 4 s less than if it travels by shortest path. Find the width of the river if velocity of water in river is 8 ms^{-1} and boat can travel in still water with a velocity of 17 ms^{-1} .
- To a man going with a speed of 5 m/s rain appears to be fall vertically. If the actual speed of the rain is 10 m/s, then what is the angle made by rain with the vertical?

- A ship is sailing due north at a speed of 1.25 m/s. The current is taking it towards east at the rate of 2 m/s and a sailor is climbing a vertical pole in the ship at the rate of 0.25 m/s. Find the magnitude of the velocity of the sailor with respect to the ground.
- A bomber plane moves due east at 100 km/h over a town T at a certain instant of time. Six minutes later an interceptor plane sets off flying due north east from the station S which is 40 km south of T . If both maintain their courses, find the velocity with which the interceptor plane must fly in order to just overtake the bomber.
- Velocity of a swimmer v in still water is less than the velocity of water u in a river. Show that the swimmer must aim himself at an angle $\cos^{-1} \frac{v}{u}$ with upstream in order to cross the river along the shortest possible path. Find the drifting (distance moved along the direction of stream in crossing the river) of the swimmer along this shortest possible path.
- A man wants to reach point B on the opposite bank of a river flowing at a speed as shown in the Fig. 5.100.

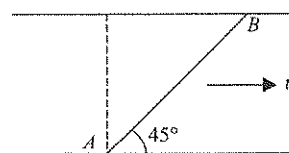


Fig. 5.100

What minimum speed relative to water should the man have so that he can reach directly to point B ? In which direction should he swim?

- A steamer plies between two stations A and B on opposite banks of a river, always following the path AB . The distance between stations A and B is 1,200 m. The velocity of the current is $\sqrt{17} \text{ ms}^{-1}$ and is constant over the entire width of the river. The line AB makes an angle 60° with the direction of the flow. The steamer takes 5 min to cover the distance AB and back. Then find
 - the velocity of steamer with respect to water, and
 - in what direction the steamer should move with respect to line AB .
- A boat is rowed with constant velocity u starting from point A on the bank of a river of width d . Water in the river flows at a constant speed nu . The boat always points towards a point O on the other bank which is directly opposite to A (Fig. 5.101). Find the path equation $r = f(\theta)$ for the boat.

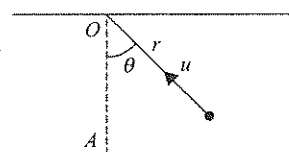


Fig. 5.101

13. A ship *A* streams due north at 16 km/hr and a ship *B* due west at 12 km/h. At a certain instant *B* is 10 km north east of *A*.
- Find the magnitude of velocity of *A* relative to *B*.
 - Find the nearest distance of approach of ships.
14. Two particles start moving simultaneously with constant velocities u_1 and u_2 as shown in Fig. 5.102. First particle starts from *A* along *AO* and second starts from *O* along *OM*. Find the shortest distance between them during their motion.

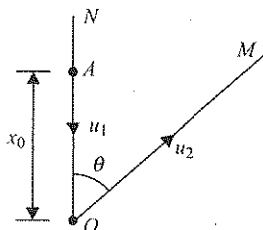


Fig. 5.102

15. The front windscreen of a car is inclined at an angle 60° with the vertical. Hailstones fall vertically downwards with a speed of $5\sqrt{3}$ m/s. Find the speed of the car so that hailstones are bounced back by the screen in vertically upward direction with respect to car. Assume elastic collision of hailstones with wind screen.
16. **State True or False:**
- The speed of the projectile is minimum at its highest position.
 - The normal acceleration of the projectile at the highest position is equal to g .
 - For a given speed, the time of flight does not depend on the angles of projection.
 - The greatest height to which a man can throw a stone is H . The greatest distance upto which he can throw the stone is $H/2$.
 - A body feels weightlessness during projectile motion.
 - The instantaneous velocity of a particle is always tangential to the trajectory of the particle.
 - The instantaneous magnitude of velocity is equal to the slope of the tangent drawn at the trajectory of the particle at that instant.
17. A hit baseball leaves the bat with a velocity of $100\sqrt{2}$ m/s at 45° above the horizontal. The ball hits the top of a screen at the 300 m mark and bounces into the crowd for a home run. How high above the ground is the top of the screen? (Neglect air resistance.)
18. A ball is projected from ground with a velocity of 40 ms^{-1} at an angle of 30° with the horizontal. Determine
- the time of flight,
 - the horizontal range, and
 - the maximum height.
 - At what time is the maximum height attained?
 - What is the magnitude and direction of velocity at the maximum height?
 - What is the magnitude and direction of velocity after 1 s of throwing the ball?

19. A ball is thrown upwards with a velocity of 30 ms^{-1} at an angle of 60° to the horizontal. Find its velocity after 2 s.
20. **a.** A ball is thrown with a velocity of 10 m/s at an angle with horizontal. Find its maximum range.
b. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high can he throw the same ball?
c. Find the minimum velocity with which the horizontal range is 160 m.
21. Prove that a gun will shoot three times as high when its angle of elevation is 60° as when it is 30° , but cover the same horizontal range. Take $g = 10 \text{ ms}^{-2}$.
22. A football is kicked by a player with a speed of 20 ms^{-1} at an angle of projection 45° . A receiver on the goal line 25 m away from the player in the direction of the kick runs the same instant to meet the ball. What must be his speed, if he is to catch the ball before it hits the ground? Ignore the height of the receiver.
23. A stone is thrown from the top of tower at an angle of 30° up with the horizontal with a velocity of 16 ms^{-1} . After 4 s of flight it strikes the ground. Calculate the height of the tower from the ground and the horizontal range of the stone.
24. A ball is thrown downwards at an angle of 30° to the horizontal with a velocity of 10 ms^{-1} from the top of a tower which 30 m high. How long will it take before striking the ground?
25. A bomb is fired horizontally with a velocity of 20 ms^{-1} from the top of a tower which 40 m high. After how much time and at what horizontal distance from the tower will the bomb strike the ground? Take $g = 9.8 \text{ ms}^{-2}$.
26. **a.** An aircraft is flying horizontally with a velocity of 540 kmh^{-1} at a height of 2,000 m from ground. When it is vertically above a point *A* on the ground, a body is dropped from it. The body reaches the ground at a point *B*. Calculate distance *AB*. Take $g = 10 \text{ ms}^{-2}$.
b. A stone is dropped from a window of a bus moving at with a velocity 54 kmh^{-1} . The height of the window is 245 cm. Find the distance along the track which the stone covers before striking the ground. Take $g = 10 \text{ ms}^{-2}$.
27. A ball is thrown horizontally from the top of a tower and strikes the ground in 3 s at an angle of 30° with the vertical.
- Find the height of the tower.
 - Find the speed with which the body was projected.
28. A particle is projected from point *A* to hit an apple as shown in Fig. 5.103. The particle is directly aimed at the apple. Show that particle will not hit the apple. Now show that if the string with which the apple is hung is cut at the time of firing the particle, then particle will hit the apple.

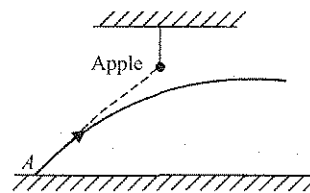


Fig. 5.103

29. A fighter plane flying horizontally at an altitude of 1.5 km with a speed of 720 kmh^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 400 ms^{-1} to hit the plane?
30. A target is fixed on the top of a tower which is 13 m high. A person standing at a distance of 50 m from the pole is capable of projecting a stone with a velocity $10\sqrt{g} \text{ ms}^{-1}$. If he wants to strike the target in the shortest possible time, at what angle should he project the stone?
31. A stone is projected from the point on the ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain the maximum height $3h/2$ above the ground. If at the instant of projection the bird were to fly away horizontally with uniform speed, find the ratio between horizontal velocities of the bird and stone if the stone still hits the bird while descending.
32. A ball rolls to the top of a stairway with a horizontal velocity of magnitude 1.8 ms^{-1} . The steps are 0.20 m high and 0.20 m wide. Which step will the ball hit first? ($g = 10 \text{ ms}^{-2}$)
33. A machine gun is mounted on the top of a tower which is 100 m high. At what angle should the gun be inclined to cover a maximum range of firing on the ground below? The muzzle speed of bullet is 150 ms^{-1} . Take $g = 10 \text{ ms}^{-2}$.
34. Figure 5.104 shows an elevator cabin, which is moving downwards with constant acceleration a . A particle is projected from the corner A, directly towards diagonally opposite corner C. Then prove that

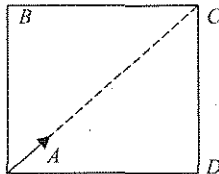


Fig. 5.104

- particle will hit C only when $a = g$
 - particle will hit the wall CD if $a < g$
 - particle will hit the roof BC if $a > g$
35. A ball is thrown with a velocity whose horizontal component is 12 ms^{-1} from a point 15 m above the ground and 6 m away from a vertical wall 18.75 m high in such a way to just clear the wall. At what time will the ball reach the ground? (take $g = 10 \text{ ms}^{-2}$)
36. A particle is projected up an inclined plane of inclination β at an elevation α to the horizon. Show that
- $\tan \alpha = \cot \beta + 2 \tan \beta$, if the particle strikes the plane at right angles.
 - $\tan \alpha = 2 \tan \beta$ if the particle strikes the plane horizontally.
37. Two parallel straight lines are inclined to the horizon at an angle α . A particle is projected from a point mid way between them so as to graze one of the lines and strike the other at the right angle. Show that if θ is the angle between the direction of projection and either of the lines, then $\tan \theta = (\sqrt{2} - 1) \cot \alpha$.
38. Two guns situated on top of a hill of height 10 m fire one shot each with the same speed $5\sqrt{3} \text{ m/s}$ at some interval of

time. One gun fires horizontally and the other fires upwards at an angle of 60° with the horizontal (Fig. 5.105). The shots collide in air at a point P. Find

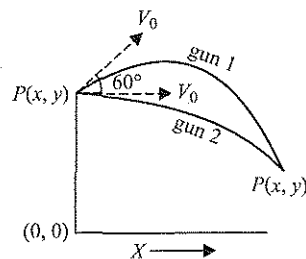


Fig. 5.105

- the time interval between the firings, and
 - the coordinates of point P. Take the origin of coordinate system at the foot of the hill right below the muzzle and trajectories in the xy -plane.
39. A small sphere is projected with a velocity of 3 m/s in a direction 60° from the horizontal y -axis, on the smooth inclined plane (Fig. 5.106). The motion of sphere takes place in the x - y plane.

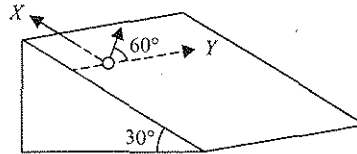


Fig. 5.106

Calculate the magnitude v of its velocity after 2 s.

40. A gun is fired from a moving platform and ranges of the shot are observed to be R_1 and R_2 when the platform is moving forwards and backwards, respectively, with velocity v_p . Find the elevation of the gun α in terms of the given quantities.
41. Tangential acceleration changes the speed of the particles whereas the normal acceleration changes its direction. (True or False)
42. Calculate the centripetal acceleration of a point on the equator of earth due to the rotation of earth about its own axis. Radius of earth = 6,400 km.
43. A cyclist is riding with a speed of 27 kmh^{-1} . As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.5 ms^{-2} . What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?
44. An electric fan has blades of length 30 cm as measured from the axis of rotation. If the fan is rotating at 1,200 rpm, find the acceleration of a point on the tip of the blade.
45. A particle starts from rest and moves in a circular motion with constant angular acceleration of 2 rad/s^2 . Find
- angular velocity, and
 - angular displacement of the particle after 4 s.
 - The number of revolutions completed by the particle during these 4 s.
 - If the radius of the circle is 10 cm, find the magnitude and direction of net acceleration of the particle at the end of 4 s.

c. $6\hat{i} + 8\hat{j}$

d. $5\sqrt{2}$

14. A car is moving towards east with a speed of 25 km/h. To the driver of the car, a bus appears to move towards north with a speed of $25\sqrt{3}$ km/h. What is the actual velocity of the bus?
- 50 km/h, 30° east of north
 - 50 km/h, 30° north of east
 - 25 km/h, 30° east of north
 - 25 km/h, 30° north of east
15. A swimmer wishes to cross a 500 m river flowing at 5 km/hr. His speed with respect to water is 3 km/hr. The shortest possible time to cross the river is
- 10 min
 - 20 min
 - 6 min
 - 7.5 min
16. A train of 150 m length is going towards north direction at a speed of 10 m/s. A parrot flies at a speed of 5 m/s towards south direction parallel to the railway track. The time taken by the parrot to cross the train is equal to
- 12 s
 - 8 s
 - 15 s
 - 10 s
17. A boy can swim in still water at 1 m/s. He swims across a river flowing at 0.6 m/s which is 336 m wide. If he travels in shortest possible time, then what time he takes to cross the river.
- 250 s
 - 420 s
 - 340 s
 - 336 s
18. A man can swim in still water with a speed of 2 m/s. If he wants to cross a river of water current speed $\sqrt{3}$ m/s along the shortest possible path, then in which direction should he swim?
- At an angle 120° to the water current.
 - At an angle 150° to the water current.
 - At an angle 90° to the water current.
 - None of these
19. A plank is moving on a ground with a velocity v and a block is moving on the plank with a velocity u as shown in figure. What is the velocity of block with respect to ground?
- $v - u$ towards right
 - $v - u$ towards left
 - u towards right
 - None of these

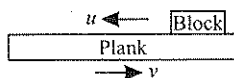


Fig. 5.109

20. A car is going in south with a speed of 5 m/s. To a man sitting in car a bus appears to move towards west with a speed of $2\sqrt{6}$ m/s. What is the actual speed of the bus?
- 4 ms^{-1}
 - 3 ms^{-1}

c. 7 ms^{-1}

d. None of these

21. A truck is moving with a constant velocity of 54 km/h. In which direction (angle with the direction of motion of truck) should a stone be projected up with a velocity of 20 m/s, from the floor of the truck, so as to appear at right angles to the truck, for a person standing on the earth?
- $\cos^{-1}\left(-\frac{3}{4}\right)$
 - $\cos^{-1}\left(-\frac{1}{4}\right)$
 - $\cos^{-1}\left(\frac{2}{3}\right)$
 - $\cos^{-1}\left(\frac{3}{4}\right)$
22. A river flows with a speed more than the maximum speed with which a person can swim in the still water. He intends to cross the river by shortest possible path (i.e., he wants to reach the point on the opposite bank which directly opposite to the starting point). Which of the following is correct?
- He should start normal to the river bank.
 - He should start in such a way that, he moves normal to the bank, relative to the bank.
 - He should start in a particular (calculated) direction making an obtuse angle with the direction of water current.
 - The man cannot cross the river, in that way.
23. Rain, driven by the wind, falls on a railway compartment with a velocity of 20 m/s, at an angle of 30° to the vertical. The train moves, along the direction of wind flow, at a speed of 108 km/hr. Determine the apparent velocity of rain for a person sitting in the train.
- $20\sqrt{7}$ m/s
 - $10\sqrt{7}$ m/s
 - $15\sqrt{7}$ m/s
 - $10\sqrt{7}$ km/h
24. The ratio of the distance carried away by the water current, downstream, in crossing a river, by a person, making same angle with downstream and upstream are, respectively, as 2 : 1. The ratio of the speed of person to the water current cannot be less than
- 1/3
 - 4/5
 - 1/5
 - 4/3
25. A particle is moving in a circle of radius r centred at O with a constant speed v . What is the change in the velocity in moving from A to B ($\angle AOB = 40^\circ$)?
- $2v\sin 20^\circ$
 - $4v\sin 40^\circ$
 - $2v\sin 40^\circ$
 - $v\sin 20^\circ$
26. i. If air resistance is not considered in a projectile motion, the horizontal motion takes place with
- constant velocity
 - constant acceleration
 - constant retardation
 - variable velocity
- ii. When a projectile is fired at an angle θ with horizontal with velocity u , then its vertical component
- remains the same

- b. goes on increasing with the height
 c. first decreases and then increases with the height
 d. first increases then decreases with the height
- iii. Range of a projectile is R , when the angle of projection is 30° . Then, the value of the other angle of projection for the same range is
 a. 45° b. 60° c. 50° d. 40°
- iv. A ball is thrown upwards and it returns to ground describing a parabolic path. Which of the following quantities remains constant throughout the motion?
 a. kinetic energy of the ball
 b. speed of the ball
 c. horizontal component of velocity
 d. vertical component of velocity
27. i. If R is the maximum horizontal range of a projectile, then the greatest height attained by it is
 c. R b. $2R$ c. $R/2$ d. $R/4$
 ii. If the time of flight of a projectile is doubled, what happens to the maximum height attained?
 a. halved
 b. remains unchanged
 c. doubled
 d. become four times
- iii. A person can throw a stone to a maximum horizontal distance of h . The greatest height to which he can throw the stone is
 a. h b. $h/2$ c. $2h$ d. $3h$
- iv. The path of one projectile as seen by an observer on another projectile is a/an
 a. straight line b. parabola c. ellipse d. circle
- v. A body is projected at 30° with the horizontal. The air offers resistance in proportion to the velocity of the body. Which of the following statements is correct?
 a. The trajectory is a symmetrical parabola.
 b. The time of rise to the maximum height is equal to the time of return to the ground.
 c. The velocity at the highest point is directed along the horizontal.
 d. The sum of the kinetic and potential energies remains constant.
28. A particle is projected with a velocity v so that its range on a horizontal plane is twice the greatest height attained. If g is acceleration due to gravity, then its range is
 a. $\frac{4v^2}{5g}$ b. $\frac{4g}{5v^2}$ c. $\frac{4v^3}{5g^2}$ d. $\frac{4v}{5g^2}$
29. During a projectile motion if the maximum height equals the horizontal range, then the angle of projection with the horizontal is
 a. $\tan^{-1}(1)$ b. $\tan^{-1}(2)$ c. $\tan^{-1}(3)$ d. $\tan^{-1}(4)$
30. A particle is projected from ground at some angle with the horizontal. Let P be the point at maximum height H . At what height above the point P the particle should be aimed to have range equal to maximum height?
 a. H b. $2H$ c. $H/2$ d. $3H$
31. The point from where a ball is projected is taken as the origin of the coordinate axes. The x and y components of its displacement are given by $x = 6t$ and $y = 8t - 5t^2$. What is the velocity of projection?
 a. 6 m/s b. 8 m/s c. 10 m/s d. 14 m/s
32. In the above problem, what is the angle of projection with horizontal?
 a. $\tan^{-1}(1/4)$ b. $\tan^{-1}(4/3)$
 c. $\tan^{-1}(3/4)$ d. $\tan^{-1}(1/2)$
33. i. A particle is fired with velocity u making angle θ with the horizontal. What is the change in velocity when it is at the highest point?
 a. $u \cos \theta$ b. u c. $u \sin \theta$ d. $(u \cos \theta - u)$
 ii. In first part of the problem 33, change in speed is
 a. $u \cos \theta$ b. u c. $u \sin \theta$ d. $(u \cos \theta - u)$
34. A ball is thrown upwards at an angle of 60° to the horizontal. It falls on the ground at a distance of 90 m. If the ball is thrown with the same initial velocity at an angle 30° , it will fall on the ground at a distance of
 a. 120 m b. 90 m
 c. 60 m d. 30 m
35. Range of a projectile is R , when the angle of projection is 30° . Then, the value of the other angle of projection for the same range is
 a. 45° b. 60°
 c. 50° d. 50°
36. A projectile will cover same horizontal distance when the initial angles of projection are
 a. $20^\circ, 60^\circ$ b. $20^\circ, 50^\circ$
 c. $20^\circ, 40^\circ$ d. $20^\circ, 70^\circ$
37. javelin is thrown at an angle θ with the horizontal and the range is maximum. The value of $\tan \theta$ is
 a. 1 b. $\sqrt{3}$
 c. $\frac{1}{\sqrt{3}}$ d. 2
38. A shot is fired from a point at a distance of 200 m from the foot of a tower 100 m high so that it just passes over it horizontally. The direction of shot with horizontal is
 a. 30° b. 45° c. 60° d. 70°
39. Two bullets are fired horizontally with different velocities from the same height. Which will reach the ground first?
 a. slower one
 b. faster one
 c. both will reach simultaneously
 d. it cannot be predicted
40. A person can throw a stone to a maximum distance of h metre. The maximum distance to which he can throw the stone is
 a. h b. $h/2$
 c. $2h$ d. $3h$

- a. 2 m/s b. 6 m/s
c. 4 m/s d. 8 m/s
59. A shell fired from the ground is just able to cross horizontally the top of a wall 90 m away and 45 m high. The direction of projection of the shell will be
a. 25° b. 30°
c. 60° d. 45°
60. Two paper screens A and B are separated by 150 m. A bullet pierces A and B. The hole in B is 15 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is ($g = 10 \text{ ms}^{-2}$)
a. $100\sqrt{3} \text{ m/s}$ b. $200\sqrt{3} \text{ m/s}$
c. $300\sqrt{3} \text{ m/s}$ d. $500\sqrt{3} \text{ m/s}$
61. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flight in the two cases, then what is the product of two times of flight?
a. $t_1 t_2 \propto R^2$
b. $t_1 t_2 \propto R$
c. $t_1 t_2 \propto \frac{1}{R}$
d. $t_1 t_2 \propto \frac{1}{R^2}$
62. A ball is thrown at different angles with the same speed u and from the same point and it has the same range in both the cases. If y_1 and y_2 be the heights attained in the two cases, then $y_1 + y_2$ is equal to
a. $\frac{u^2}{g}$ b. $\frac{2u^2}{g}$
c. $\frac{u^2}{2g}$ d. $\frac{u^2}{4g}$
63. The equation of motion of a projectile is
$$y = 12x - \frac{3}{4}x^2$$

The horizontal component of velocity is 3 ms^{-1} . What is the range of the projectile?
a. 18 m b. 16 m
c. 12 m d. 21.6 m
64. The range R of projectile is same when its maximum heights are h_1 and h_2 . What is the relation between R , h_1 , and h_2 ?
a. $R = \sqrt{h_1 h_2}$
b. $R = \sqrt{2h_1 h_2}$
c. $R = 2\sqrt{h_1 h_2}$
d. $R = 4\sqrt{h_1 h_2}$
65. i. The friction of the air causes a vertical retardation equal to 10 % of the acceleration due to gravity (take $g = 10 \text{ ms}^{-2}$). The maximum height will be decreased by
a. 8 % b. 9 % c. 10 % d. 11 %
- ii. In the first part of question 65, the time taken to reach the maximum height will be decreased by
a. 19 % b. 5 %
c. 10 % d. none of these
- iii. In the second part of question 65, the time taken to return to the ground from the maximum height
a. is almost same as in the absence of friction
b. decreases by 1 %
c. increases by 1 %
d. increases by 11 %
66. At what angle with the horizontal should a ball be thrown so that the range R is related to the time of flight as $R = 5T^2$. (Take $g = 10 \text{ ms}^{-2}$)
a. 30° b. 45°
c. 60° d. 90°
67. A ball thrown by one player reaches the other in 2 s. The maximum height attained by the ball above the point of projection will be about
a. 2.5 m b. 5 m
c. 7.5 m d. 10 m
68. A ball rolls off to the top of a staircase with a horizontal velocity $u \text{ m/s}$. If the steps are h metre high and b metre wide, the ball will hit the edge of the n th step, if
a. $n = \frac{2hu}{gb^2}$ b. $n = \frac{2hu^2}{gb}$
c. $n = \frac{2hu^2}{gb^2}$ d. $n = \frac{hu^2}{gb^2}$
69. A projectile is projected with initial velocity $(6\hat{i} + 8\hat{j}) \text{ m/s}$. If $g = 10 \text{ ms}^{-2}$, then horizontal range is
a. 4.8 m b. 9.6 m
c. 19.2 m d. 14.0 m
70. At a height 0.4 m from the ground, the velocity of a projectile in vector form is $\vec{v} = (6\hat{i} + 2\hat{j}) \text{ m/s}$. The angle of projection is
a. 45° b. 60°
c. 30° d. $\tan^{-1}(3/4)$
71. A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 ms^{-1} . Then the time after which its inclination with the horizontal is 45° , is
a. $15(\sqrt{3} - 1) \text{ s}$ b. $15(\sqrt{3} + 1) \text{ s}$
c. $7.5(\sqrt{3} - 1) \text{ s}$ d. $7.5(\sqrt{3} + 1) \text{ s}$
72. A number of bullets are fired in all possible directions with the same initial velocity u . The maximum area of ground covered by bullets is
a. $\pi \left(\frac{u^2}{g} \right)^2$ b. $\pi \left(\frac{u^2}{2g} \right)^2$
c. $\pi \left(\frac{u}{g} \right)^2$ d. $\pi \left(\frac{u}{2g} \right)^2$

73. A person takes an aim at a monkey sitting on a tree and fires a bullet. Seeking the smoke the monkey begins to fall freely; then the bullet will
- always hit the monkey
 - go above the monkey
 - go below the monkey
 - hit the monkey if the initial velocity of the bullet is more than a certain velocity
74. A person sitting at the rear end of the compartment throws a ball towards the front end. The ball follows a parabolic path. The train is moving with uniform velocity of 20 ms^{-1} . A person standing outside on the ground also observes the ball. How will the maximum heights (h_m) attained and the ranges (R) seen by the thrower and the outside observer compare each other?
- same h_m different R
 - same h_m and R
 - different h_m same R
 - different h_m and R
75. Two stones are projected with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one is $\pi/3$ and its maximum height is h_1 then the maximum height of the other will be
- $3h_1$
 - $2h_1$
 - $h_1/2$
 - $h_1/3$
76. A ball is projected from the ground at angle θ with the horizontal. After 1 s it is moving at angle 45° with the horizontal and after 2 s it is moving horizontally. What is the velocity of projection of the ball?
- $10\sqrt{3} \text{ m/s}$
 - $20\sqrt{3} \text{ m/s}$
 - $10\sqrt{5} \text{ m/s}$
 - $20\sqrt{2} \text{ m/s}$
77. A body is projected horizontally from the top of a tower with an initial velocity of 18 ms^{-1} . It hits the ground at an angle of 45° . What is the vertical component of velocity when the body strikes the ground?
- 9 m/s
 - $9\sqrt{2} \text{ m/s}$
 - 18 m/s
 - $18\sqrt{2} \text{ m/s}$
78. A plane flying horizontally at 100 m/s releases an object which reaches the ground in 10 s. At what angle with horizontal it hits the ground?
- 55°
 - 45°
 - 60°
 - 75°
79. A hose lying on the ground shoots a stream of water upward at an angle of 60° to the horizontal with the velocity of 16 ms^{-1} . The height at which the water strikes the wall 8 m away is
- 8.9 m
 - 10.9 m
 - 12.9 m
 - 6.9 m
80. Three particles A, B, and C are projected from the same point with the same initial speeds making angles 30° , 45° , and 60° , respectively, with the horizontal. Which of the following statements is correct?
- A, B, and C have unequal ranges.
 - Ranges of A, and C are equal and less than that of B.
 - Ranges of A, and C are equal and greater than that of B.
 - A, B, and C have equal ranges.
81. There are two values of time for which a projectile is to the same height. The sum of these two times is equal to (T = time of flight of the projectile)
- $3T/2$
 - $4T/3$
 - $3T/4$
 - T
82. A golfer standing on level ground hits a ball with a velocity of $u = 52 \text{ m/s}$ at an angle a above the horizontal. If $\tan a = 5/12$, then the time for which the ball is at least 15 m above the ground will be (take $g = 10 \text{ m/s}^2$)
- 1 s
 - 2 s
 - 3 s
 - 4 s
83. i. The ceiling of a hall is 40 m high. For maximum horizontal distance, the angle at which the ball may be thrown with a speed of 56 ms^{-1} without hitting the ceiling of the hall is
- 25°
 - 30°
 - 45°
 - 60°
- ii. In question 83's part (i), the maximum horizontal distance will be:
- $160\sqrt{3} \text{ m}$
 - $140\sqrt{3} \text{ m}$
 - $120\sqrt{3} \text{ m}$
 - $100\sqrt{3} \text{ m}$
84. A body is projected up a smooth inclined plane with a velocity u from the point A as shown in the Fig. 5.110. The angle of inclination is 45° and the top is connected to a well of diameter 40 m. If the body just manages to cross the well, what is the value of u ? Length of inclined plane is $20\sqrt{2} \text{ m}$.
- 40 m/s
 - $40\sqrt{2} \text{ m/s}$
 - 20 m/s
 - $20\sqrt{2} \text{ m/s}$

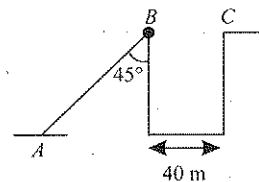


Fig. 5.110

85. A rifle shoots a bullet with a muzzle velocity of 400 m/s at a small target 400 m away. The height above the target at which the bullet must be aimed to hit the target is ($g = 10 \text{ ms}^{-2}$).
- 1 m
 - 5 m
 - 10 m
 - 0.5 m
86. A projectile is fired from the level ground at an angle θ above the horizontal. The elevation angle θ of the highest point as seen from the launch point is related to θ by the relation

- a. $\tan \phi = 2 \tan \theta$
 b. $\tan \phi = \tan \theta$
 c. $\tan \phi = \frac{1}{2} \tan \theta$
 d. $\tan \phi = \frac{1}{4} \tan \theta$

87. A projectile has initially the same horizontal velocity as it would acquire if it had moved from rest with uniform acceleration of 3 ms^{-2} for 0.5 min. If the maximum height reached by it is 80 m, then the angle of projection is ($g = 10 \text{ ms}^{-2}$).

- a. $\tan^{-1} 3$ b. $\tan^{-1}(3/2)$
 c. $\tan^{-1}(4/9)$ d. $\sin^{-1}(4/9)$

88. If a stone is to hit at a point which is at a distance d away and at a height h (see Fig. 5.111) above the point from where the stone starts, then what is the value of initial speed u if the stone is launched at an angle θ ?

- a. $\frac{g}{\cos \theta} \sqrt{\frac{d}{2(d \tan \theta - h)}}$
 b. $\frac{d}{\cos \theta} \sqrt{\frac{d}{2(d \tan \theta - h)}}$
 c. $\sqrt{\frac{gd^2}{h \cos^2 \theta}}$
 d. $\sqrt{\frac{gd^2}{(d - h)}}$

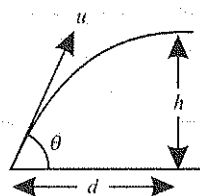


Fig. 5.111

89. The speed of a projectile at its maximum height is $\sqrt{3}/2$ times its initial speed. If the range of the projectile is P times the maximum height attained by it, P is equal to

- a. $4/3$ b. $2\sqrt{3}$ c. $4\sqrt{3}$ d. $3/4$

90. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

- a. $\frac{b^2}{2a'} \tan^{-1}(b)$ b. $\frac{a^2}{b'} \tan^{-1}(2b)$
 c. $\frac{a^2}{4b'} \tan^{-1}(a)$ d. $\frac{2a^2}{b'} \tan^{-1}(a)$

91. A projectile is given an initial velocity of $\hat{i} + 2\hat{j}$. The cartesian equation of its path is ($g = 10 \text{ m/s}^2$).

- a. $y = 2x - 5x^2$
 b. $y = x - 5x^2$
 c. $4y = 2x - 5x^2$
 d. $y = 2x - 25x^2$

92. A particle is projected from the ground with an initial speed of v at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

- a. $\frac{v}{2} \sqrt{1 + 2 \cos^2 \theta}$
 b. $\frac{v}{2} \sqrt{1 + 2 \cos^2 \theta}$
 c. $\frac{v}{2} \sqrt{1 + 3 \cos^2 \theta}$
 d. $v \cos \theta$

93. Two balls A and B are thrown with speeds u and $u/2$, respectively. Both the balls cover the same horizontal distance before returning to the plane of projection. If the angle of projection of ball B is 15° with the horizontal, then the angle of projection of A is

- a. $\sin^{-1} \left(\frac{1}{8} \right)$ b. $\frac{1}{2} \sin^{-1} \left(\frac{1}{8} \right)$
 c. $\frac{1}{3} \sin^{-1} \left(\frac{1}{8} \right)$ d. $\frac{1}{4} \sin^{-1} \left(\frac{1}{8} \right)$

94. A body of mass m is projected horizontally with a velocity v from the top of a tower of height h and it reaches the ground at a distance x from the foot of the tower. If a second body of mass $2m$ is projected horizontally from the top of a tower of height $2h$, it reaches the ground at a distance $2x$ from the foot of the tower. The horizontal velocity of the second body is

- a. v b. $2v$ c. $\sqrt{2}v$ d. $v/2$

95. A car is moving horizontally along a straight line with a uniform velocity of 25 ms^{-1} . A projectile is to be fired from this car in such a way that it will return to it after it has moved 100 m. The speed of the projection must be

- a. 10 ms^{-1} b. 20 ms^{-1}
 c. 15 ms^{-1} d. 25 ms^{-1}

96. The horizontal range and maximum height attained by a projectile are R and H , respectively. If a constant horizontal acceleration $a = g/4$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be

- a. $(R + H), \frac{H}{2}$
 b. $\left(R + \frac{H}{2} \right), 2H$
 c. $(R + 2H), H$
 d. $(R + H), H$

97. A particle is projected with a certain velocity at an angle α above the horizontal from the foot of an inclined plane of inclination 30° . If the particle strikes the plane normally then α is equal to

- a. $30^\circ + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$
 b. 45°
 c. 60°
 d. $30^\circ + \tan^{-1}\left(2\sqrt{3}\right)$

98. In Fig. 5.112, the time taken by the projectile to reach from A to B is t . Then the distance AB is equal to

- a. $\frac{ut}{\sqrt{3}}$
 b. $\frac{\sqrt{3}ut}{2}$
 c. $\sqrt{3}ut$
 d. $2ut$

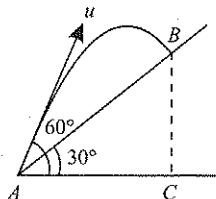


Fig. 5.112

99. A motor cyclist is trying to jump across a path as shown in Fig. 5.113 by driving horizontally off a cliff A at a speed of 5 ms^{-1} . Ignore air resistance and take $g = 10 \text{ ms}^{-2}$. The speed with which he touches the peak B is

- a. 2.0 ms^{-1}
 b. 12 ms^{-1}
 c. 25 ms^{-1}
 d. 15 ms^{-1}

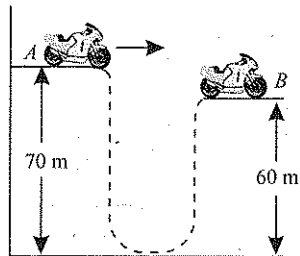


Fig. 5.113

100. An aeroplane is flying horizontally with a velocity of 600 km/hr and at a height of 2,000 m. When it is vertically above a point A on the ground, a bomb is released from it. The bomb strikes the ground at point B. The distance AB is (neglecting air resistance)

- a. 1,200 m
 b. 0.33 km
 c. 3.33 km
 d. 33 km

101. The range of a projectile which is launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile if it is projected at an angle 45° to the horizontal?

- a. 1.5 km
 b. 3 km
 c. 6 km
 d. 0.75 km

102. The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = (8t - 5t^2) \text{ m}$ and $x = 6t \text{ m}$, where t is in seconds. The velocity with which the projectile is projected at $t = 0$, is

- a. 8 m/s
 b. 6 m/s
 c. 10 m/s
 d. not obtainable from the data

103. In the above problem, the angle with the horizontal at which the projectile is projected is

- a. $\tan^{-1}\left(\frac{3}{4}\right)$
 b. $\tan^{-1}\left(\frac{4}{3}\right)$
 c. $\sin^{-1}\left(\frac{3}{4}\right)$
 d. not obtainable from the given data.

104. A body is projected with velocity v_1 from the point A as shown in the Fig. 5.114. At the same time another body is projected vertically upwards from B with velocity v_2 . The point B lies vertically below the highest point. For both the bodies to collide, $\frac{v_2}{v_1}$ should be

- a. 2
 b. $\sqrt{\frac{3}{2}}$
 c. 0.5
 d. 1

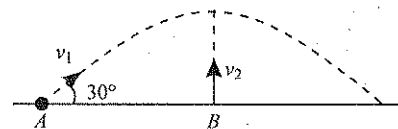


Fig. 5.114

105. A ball is projected from a point A with some velocity at an angle 30° with the horizontal as shown in the Fig. 5.115. Consider a target at point B. The ball will hit the target if it is thrown with a velocity v_0 equal to

- a. 5 ms^{-1}
 b. 6 ms^{-1}
 c. 7 ms^{-1}
 d. none of these

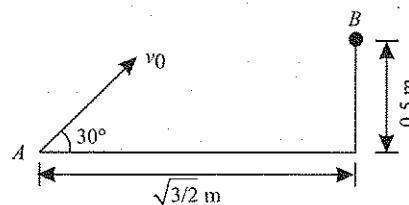


Fig. 5.115

106. A body is moving with a constant speed of 5 m/s in a radius of 2 m. The acceleration of the body is

- a. 0
 b. 12.5 ms^{-2}
 c. 25 ms^{-2}
 d. None of these

107. The velocity of a body moving in a circle in the radial direction is

- a. 0
 b. speed of the body

- ### Multiple Correct Answers Type

Solutions on page 5.62

1. A river is flowing towards east with a velocity of 5 ms^{-1} . The boat velocity is 10 ms^{-1} . The boat crosses the river by shortest path, hence,
 - a. the direction of boat's velocity is 30° west of north.
 - b. the direction of boat's velocity is north-west.
 - c. resultant velocity is $5\sqrt{3} \text{ ms}^{-1}$.
 - d. resultant velocity of boat is $5\sqrt{2} \text{ ms}^{-1}$.
2. A stationary person observes that rain is falling vertically down at 30 km/h . A cyclist is moving up on an inclined plane making an angle 30° with horizontal at 10 km/h . In which direction should the cyclist hold his umbrella to prevent himself from the rain?
 - a. At an angle $\tan^{-1} \left(\frac{3\sqrt{3}}{5} \right)$ with the inclined plane.
 - b. At an angle $\tan^{-1} \left(\frac{3\sqrt{3}}{5} \right)$ with the horizontal.
 - c. At an angle $\tan^{-1} \left(\frac{\sqrt{3}}{7} \right)$ with the inclined plane.
 - d. At an angle $\tan^{-1} \left(\frac{\sqrt{3}}{7} \right)$ with the vertical.
3. Two cities A and B are connected by a regular bus services with buses plying in either direction every T seconds. The speed of each bus is uniform and equal to V_b . A cyclist cycles from A to B with a uniform speed of V_c . A bus goes past the cyclist in T_1 second in the direction A to B and every T_2 second in the direction B to A . Then

a. $T_1 = \frac{V_b T}{V_b + V_c}$ c. $T_1 = \frac{V_b T}{V_b - V_c}$	b. $T_2 = \frac{V_b T}{V_b - V_c}$ d. $T_2 = \frac{V_b T}{V_b + V_c}$
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4. Suppose two particles, 1 and 2, are projected in vertical plane simultaneously.

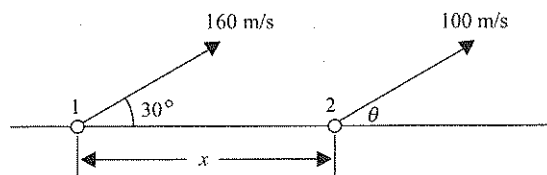


Fig. 5.116

Their angles of projection are 30° and θ , respectively, with the horizontal. Let them collide after a time t in air. Then

- $\theta = \sin^{-1}(4/5)$ and they will have same speed just before the collision.
 - $\theta = \sin^{-1}(4/5)$ and they will have different speed just before the collision.
 - $x < 1280\sqrt{3} - 960$ m.
 - It is possible that the particles collide when both of them are at their highest point.
5. All the particles thrown with the same initial velocity would strike the ground,

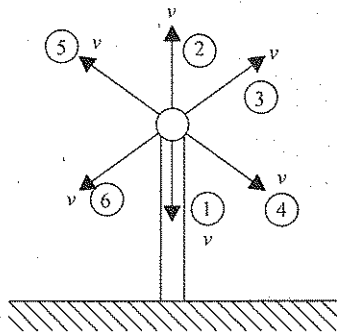
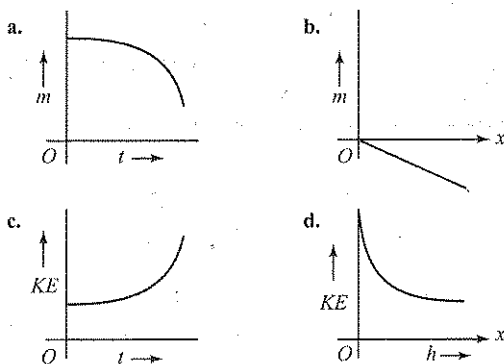
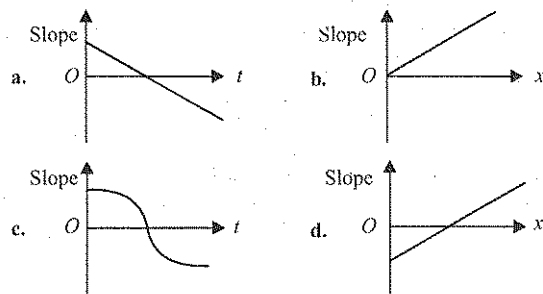


Fig. 5.117

- with the same speed
 - simultaneously.
 - time would be least for the particle thrown with velocity v downward, i.e., particle (1).
 - time would be maximum for the particle (2).
6. A particle is moving in xy -plane with $y = x/2$ and $y = 4 - 2t$. The correct options are
- Initial velocities of x and y directions are negative.
 - Initial velocities of x and y directions are positive.
 - Motion is first retarded, then accelerated.
 - Motion is first accelerated, then retarded.
7. A particle is dropped from a tower in a uniform gravitational field at $t = 0$. The particle is blown over by a horizontal wind with constant velocity. Slope (m) of trajectory of the particle with horizontal and its kinetic energy vary according to the curves.



8. A car moves on a circular road, describing equal angles about the centre in equal intervals of time. Which of the following statements about the velocity of car are not true?
- Velocity is constant.
 - Magnitude of velocity is constant but the direction changes.
 - Both magnitude and direction of velocity change.
 - Velocity is directed towards the centre of circle.
9. A heavy particle is projected with a velocity at an angle with the horizontal into a uniform gravitational field. The slope of the trajectory of the particle varies not according to which of the following curves?



10. The coordinates of a particle moving in a plane are given by $x = a \cos(pt)$ and $y = b \sin(pt)$, where $a, b (< a)$ and p are positive constants of appropriate dimensions. then
- The path of the particle is an ellipse.
 - The velocity and acceleration of the particle are normal to each other at $t = \pi/2p$.
 - The acceleration of the particle is always directed towards a focus.
 - The distance travelled by the particle in time interval, $t = 0$ to $t = \pi/2p$ is a .
11. A body is projected with a velocity u at an angle of projection θ with the horizontal. The direction of velocity of the body makes angle 30° with the horizontal at $t = 2$ s and then after 1 s it reaches the maximum height. Then
- $u = 20\sqrt{3}$ m/s,
 - $\theta = 60^\circ$
 - $\theta = 30^\circ$
 - $u = 10\sqrt{3}$ m/s
12. A particle moves in a circle of radius 20 cm. Its linear speed is given by $v = 2t$ where t is in s and v in m/s. Then
- The radial acceleration at $t = 2$ s is 80 ms^{-2} .
 - Tangential acceleration at $t = 2$ s is 2 ms^{-2} .
 - Net acceleration at $t = 2$ s is greater than 80 ms^{-2} .
 - Tangential acceleration remains constant in magnitude

Assertion-Reasoning Type

Solutions on page 5.63

In the following questions, each question contains STATEMENT I (Assertion) and STATEMENT II (Reason). Each question has 4 choices (a), (b), (c), and (d) out of which **ONLY ONE** is correct.

- Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
- Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
- Statement I is True, Statement II is False.
- Statement I is False, Statement II is True.

- Statement I:** The projectile has only vertical component of velocity at the highest point of its trajectory.
Statement II: At the highest point only one component of velocity is present.
- Statement I:** The time of flight of a body becomes n times of original value, if its speed is made n times.

- Statement II:** This due to the range of the projectile which becomes n times.
3. **Statement I:** If the string of an oscillating simple pendulum is cut, when the bob is at the mean position, the bob falls along a parabolic path.
Statement II: The bob possesses horizontal velocity, at the mean position.
4. **Statement I:** In a plane projectile motion, the angle between instantaneous velocity vector and acceleration vector can be anything between 0 or π (excluding the limiting case).
Statement II: In plane to plane projectile motion, acceleration vector is always pointing vertical downwards. (Neglect air friction).
5. **Statement I:** A body with constant acceleration always moves along a straight line.
Statement II: A body with constant magnitude of acceleration may not speed up.
6. **Statement I:** In a non-uniform circular motion tangential acceleration arises due to change in magnitude of velocity.
Statement II: In a non-uniform circular motion centripetal acceleration is produced due to change in direction of velocity.
7. **Statement I:** In a uniform circular motion angle between velocity vector and acceleration vector is always $\frac{\pi}{2}$.
Statement II: For any type of motion, angle between acceleration and velocity is always $\frac{\pi}{2}$.
8. **Statement I:** Two particles start from the rest simultaneously and proceed with the same acceleration. The relative velocity of these particles will be zero throughout the motion.
Statement II: At every moment the two particles will have the same velocity.
9. **Statement I:** A river is flowing from east to west at a speed of 5 m/min. A man on south bank of river, capable of swimming 10 m/min in still water, wants to swim across the river in shortest time. He should swim due north.
Statement II: For the shortest time the man needs to swim perpendicular to the bank.
10. **Statement I:** Rain is falling vertically downwards with velocity 6 km/h. A man walks with a velocity of 8 km/h. Relative velocity of rain w.r.t. the man is 10 km/h.
Statement II: Relative velocity is the ratio of two velocities.

Comprehensive Type

Solutions on page 5.64

For Problems 1–2

A man is walking due east at the speed of 3 kmh^{-1} . Rain appears to fall down vertically at the rate of 3 kmh^{-1} .

1. The actual velocity of the rainfall is
- $3\sqrt{3} \text{ km/h}$
 - $2\sqrt{2} \text{ km/h}$
 - $4\sqrt{3} \text{ km/h}$
 - $3\sqrt{2} \text{ km/h}$

2. The direction of rainfall with vertical is
- 45°
 - 30°
 - 60°
 - 90°

For Problems 3–4

A person can swim in still water at the rate of 1 km/hr. He tries to cross a river by swimming perpendicular to the river flowing at the rate of 2 km/hr. If the width of the river is 10 m, then

3. The location of the point where he lands on the other side of the river ahead of the point exactly opposite to his start point is
- 20 m
 - 10 m
 - 15 m
 - 25 m
4. The time taken by him to cross the river is
- 30 s
 - 36 s
 - 54 s
 - 45 s

For Problems 5–6

A river 400 m wide is flowing at a rate of 2.0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.

5. Find the time taken by the boat to reach the opposite bank.
- 10 s
 - 30 s
 - 40 s
 - 20 s
6. How far from the point directly opposite to the starting point does the boat reach the opposite bank?
- 100 m
 - 80 m
 - 60 m
 - 40 m

For Problems 7–10

A man can row a boat 4 km/hr in still water. If he is crossing a river where the current is 2 km/hr, then answer the followings

7. In what direction should his boat be headed if he wants to reach a point on the other bank, directly opposite to starting point?
- 120° downstream
 - 120° upstream
 - 60° downstream
 - Perpendicular to the flow of river
8. If width of the river is 4 km how long will it take him to cross the river, with the condition in question 7?
- $\frac{5\sqrt{7}}{4} \text{ hr}$
 - $\frac{2\sqrt{3}}{3} \text{ hr}$
 - $\frac{4}{2\sqrt{3}} \text{ hr}$
 - $\frac{3\sqrt{2}}{4} \text{ hr}$
9. In what direction should he head the boat if he wants to cross river in shortest time?
- 120° downstream
 - 120° upstream
 - 60° downstream
 - Perpendicular to the flow of river

c. $gr = u^2 \cos 2\theta$

d. $gr = u^2 \cos \theta$

25. At what horizontal distance x from the foot of the tower does the ball hit the ground?

a. $\frac{u \cos \theta}{g} \{(u^2 \sin^2 \theta + 2gh)^{1/2} - u \sin \theta\}$

b. $\frac{u \sin \theta}{g} \{(u^2 \cos^2 \theta + 2gh)^{1/2} - u \cos \theta\}$

c. $\frac{u \sin \theta}{g} \{(u^2 \cos^2 \theta + gh)^{1/2} - u \cos \theta\}$

d. $\frac{u \cos \theta}{g} \{(u^2 \sin^2 \theta + gh)^{1/2} - u \sin \theta\}$

For Problems 26–29

A 0.098 kg block slides down a frictionless track as shown in Fig. 5.120.

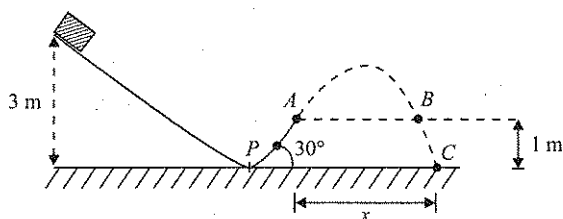


Fig. 5.120

26. The vertical component of the velocity of block at A is

a. $\sqrt{g}$

b. $2\sqrt{g}$

c. $3\sqrt{g}$

d. $4\sqrt{g}$

27. The time taken by the block to move from A to B is

a. $\frac{1}{\sqrt{g}}$

b. $\frac{2}{\sqrt{g}}$

c. $\frac{3}{\sqrt{g}}$

d. $\frac{4}{\sqrt{g}}$

28. The time taken by the block to move from A to C is

a. $\sqrt{\frac{3}{g}}$

b. $\sqrt{\frac{2}{g}}$

c. $\frac{1}{\sqrt{g}}$

d. $\frac{1 + \sqrt{3}}{\sqrt{g}}$

29. The horizontal distance x traveled, by the block in moving from A to C is

a. $(1 + \sqrt{3})\text{ m}$

b. $(1 - \sqrt{3})\text{ m}$

c. $(\sqrt{3} + 3)\text{ m}$

d. $g\text{ m}$

For Problems 30–32

A projectile is thrown with velocity v making an angle θ with the horizontal. It just crosses the tops of two poles, each of height h , after 1 s and 3 s, respectively.

30. The time of flight of the projectile is

a. 1 s

b. 3 s

c. 4 s

d. 8 s

31. Maximum height attained by the projectile is

a. g

b. $2g$

c. $3g$

d. $4g$

32. h is

a. $g/2$

b. g

c. $3g/2$

d. $2g$

For Problems 33–35

A projectile is thrown with velocity v at an angle θ with the horizontal. When the projectile is at a height equal to half of the maximum height, then

33. The vertical component of the velocity of projectile is

a. $v \sin \theta \times 3$

b. $v \sin \theta + 3$

c. $\frac{v \sin \theta}{\sqrt{2}}$

d. $\frac{v \sin \theta}{\sqrt{3}}$

34. The velocity of the projectile when it is at a height equal to half of the maximum height is

a. $v \sqrt{\cos^2 \theta + \frac{\sin^2 \theta}{2}}$

b. $\sqrt{2} v \cos \theta$

c. $\sqrt{2} v \sin \theta$

d. $v \tan \theta \sec \theta$

35. What is the angle of projectile with the vertical if the velocity at the highest point is $\sqrt{\frac{2}{5}}$ times the velocity when it is at a height equal to half of the maximum height?

a. 15°

b. 30°

c. 45°

d. 60°

For Problems 36–39

A stone is thrown with a velocity of 20 m/s at angle of 30° above horizontal from the top of a building 15 m high. Find

36. The time after which the stone strikes the ground.

a. 6 s

b. 3 s

c. 1.5 s

d. 0.75 s

37. The distance of the landing point of the stone from the building.

a. $20\sqrt{2}\text{ m}$

b. $10\sqrt{2}\text{ m}$

c. $20\sqrt{3}\text{ m}$

d. $30\sqrt{3}\text{ m}$

38. The velocity with which the stone hits the ground.

a. $10\sqrt{7}\text{ m/s}$

b. $20\sqrt{7}\text{ m/s}$

c. $10\sqrt{3}\text{ m/s}$

d. $20\sqrt{3}\text{ m/s}$

39. The maximum height attained by the stone above the ground.

a. 5 m

b. 10 m

c. 20 m

d. 40 m

For Problems 40–41

A block slides off a horizontal table top 1 m high with a speed of 3 m/s. Find

40. The horizontal distance from the edge of the table at which the block strikes the floor.

a. $\frac{10}{\sqrt{3}}\text{ m}$

b. $\frac{3}{\sqrt{10}}\text{ m}$

c. $\frac{3}{\sqrt{5}}\text{ m}$

d. $\frac{5}{\sqrt{3}}\text{ m}$

41. The speed of the block when it reaches the floor.
- $\sqrt{37}$ m/s
 - $\sqrt{24}$ m/s
 - $\sqrt{29}$ m/s
 - $\sqrt{18}$ m/s

For Problems 42–43

A Projectile shot at an angle of 45° above the horizontal strikes a building 30m away at a point 15m above the point of projection. Find

42. The speed of projection
- $8\sqrt{5}$ m/s
 - $6\sqrt{10}$ m/s
 - $5\sqrt{6}$ m/s
 - $10\sqrt{6}$ m/s
43. The magnitude and direction of velocity of projectile when it strikes the building.
- $10\sqrt{3}$ m/s, horizontal
 - 15 m/s, horizontal
 - 8 m/s, horizontal
 - 10 m/s, vertical

For Problems 44–46

A particle is projected horizontally with a speed $v = 5$ m/s from the top of a plane inclined at an angle $\theta = 37^\circ$ to the horizontal as shown in the Fig. 5.121.

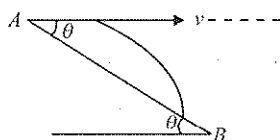


Fig. 5.121

44. How far from the point of projection will the particle strike the plane?
- 75 m
 - $\frac{65}{16}$ m
 - $\frac{75}{16}$ m
 - $\frac{85}{9}$ m
45. Find the time taken by the particle to hit the plane.
- $3/4$ s
 - 3 s
 - 4 s
 - $\frac{4}{3}$ s
46. What is the velocity of the particle just before it hits the plane?
- $5\sqrt{13}$ m/s
 - $\frac{5}{2}\sqrt{13}$ m/s
 - $10\sqrt{13}$ m/s
 - $5\sqrt{26}$ m/s

For Problems 47–49

A particle is projected with a speed u at an angle θ to the horizontal. Find the radius of curvature

47. At the highest point of its trajectory,

- $\frac{u^2 \cos^2 \theta}{2g}$
- $\frac{\sqrt{3} u^2 \cos^2 \theta}{2g}$
- $\frac{u^2 \cos^2 \theta}{g}$
- $\frac{\sqrt{3} u^2 \cos^2 \theta}{g}$

48. At the point where the particle's velocity makes an angle $\theta/2$ with the horizontal.

- $\frac{u^2 \cos^2 \theta \sec^3 \frac{\theta}{2}}{g}$
- $\frac{u^2 \cos^2 \theta \sec^3 \frac{\theta}{2}}{2g}$
- $\frac{2u^2 \cos^2 \theta \sec^3 \frac{\theta}{2}}{g}$
- $\frac{u^2 \cos^2 \theta \sec^3 \frac{\theta}{2}}{\sqrt{3}g}$

49. At the point where the particle is at a height of half the maximum height H attained by it.

- $\frac{2u^2(1 + \cos^2 \theta)^{3/2}}{g 2\sqrt{2} \cos \theta}$
- $\frac{u^2(1 + \cos^2 \theta)^{3/2}}{g 2\sqrt{2} \cos \theta}$
- $\frac{u^2(1 - \sin^2 \theta)^{3/2}}{g 2\sqrt{2} \cos \theta}$
- $\frac{u^2(1 - \tan^2 \theta)^{3/2}}{g \sqrt{2} \cos \theta}$

Matching Column Type

Solutions on page 5.69

1. If $\vec{v}_{mw}$ = velocity of a man relative to water, $\vec{v}_w$ = velocity of water, $\vec{v}_m$ = velocity of man relative to ground, match the following

Column I	Column II
i. Minimum distance for $v_{mw} > v_w$	a. $\theta = \sin^{-1} \left(\frac{v_{mw}}{v_w} \right)$
ii. Minimum time for $v_{mw} \geq v_w$	b. $\vec{v}_m \perp \vec{v}_w$
iii. Minimum distance for $v_{mw} < v_w$	c. $\vec{v}_{mw} \perp \vec{v}_w$
iv. Minimum time for $v_{mw} < v_w$	d. $\theta = \sin^{-1} \frac{v_w}{v_{mw}}$

where θ = angle between $\vec{v}_{mw}$ and the width of the river.

2. A ball is projected from the ground with velocity v such that its range is maximum.

Column I	Column II
i. Velocity at half of the maximum height	a. $\frac{v}{2}$
ii. Velocity at the maximum height	b. $\frac{v}{\sqrt{2}}$
iii. Change in its velocity when it returns to the ground	c. $v\sqrt{2}$
iv. Average velocity when it reaches the maximum height	d. $\frac{v}{2} \sqrt{\frac{5}{2}}$

Column I	Column II
i. For a particle moving in a circle	a. The acceleration may be perpendicular to its velocity
ii. For a particle moving in a straight line	b. The acceleration may be in the direction of velocity
iii. For a particle undergoing projectile motion with angle of projection α ; $0 \leq \alpha \leq \frac{\pi}{2}$	c. The acceleration may be at some angle θ ($0 < \theta < \frac{\pi}{2}$) with the velocity
iv. For a particle is moving in space	d. The acceleration may be opposite to its velocity

ANSWERS AND SOLUTIONS

Subjective Type

1. For D, time taken

$$t_1 = \frac{1.5}{4 + 3.5} + \frac{1.5}{4 - 3.5} = 3.2 \text{ h} = 192 \text{ min}$$

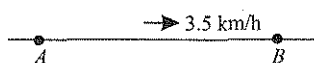


Fig. 5.122

For C; time taken: $t_2 = \frac{1.5}{4} + \frac{1.5}{4} = \frac{3}{4} \text{ h} = 45 \text{ min}$

2. Let velocity of boat is v , then

$$v = \frac{8 + 8}{2} = 8 \text{ km/h}$$

Time taken when the water is flowing;

$$t = \frac{8}{8 + 4} + \frac{8}{8 - 4} = 2 \text{ h } 40 \text{ min}$$

3. a. $v_s = 18 \text{ m/s}$, $v_c = 13 \text{ m/s}$

b. $v_{s/c} = v_s - v_c = 18 - 13 = 5 \text{ m/s}$

c. $v_s = -3 \text{ m/s}$, $v_{s/c} = v_s - v_c = -3 - 13 = -16 \text{ m/s}$

$v_s = 0$, $v_{s/c} = v_s - v_c = 0 - 13 = -13 \text{ m/s}$

Negative sign indicates left direction.

4. $\vec{v}_1 = 20 \cos 30^\circ \hat{i} + 20 \sin 30^\circ \hat{j} = 10\sqrt{3} \hat{i} + 10 \hat{j}$

$\vec{v}_2 = v \hat{j}$, $\vec{v}_{1/2} = \vec{v}_1 - \vec{v}_2 = 10\sqrt{3} \hat{i} + (10 - v) \hat{j}$

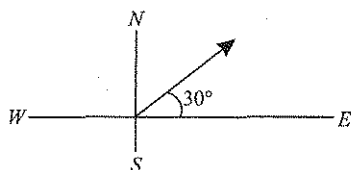


Fig. 5.123

For $\vec{v}_{1/2}$ to be towards east, its $\hat{j}$ comp should be zero.

$$\Rightarrow 10 - v = 0 \Rightarrow v = 10 \text{ km/h}$$

5. $t_{\text{short-path}} - t_{\text{short-time}} = 4 \Rightarrow \frac{d}{\sqrt{v^2 - u^2}} - \frac{d}{v} = 4$

where $v = 17 \text{ m/s}$, $u = 8 \text{ m/s}$

solve to get, $d = 510 \text{ m}$

6. $\vec{v}_m = 5\hat{i}$, $\vec{v}_{r/m} = -v\hat{j}$, $\vec{v}_r = \vec{v}_{r/m} + \vec{v}_m = 5\hat{i} - v\hat{j}$

$$v_r = \sqrt{5^2 + v^2} = 10 \Rightarrow v = 5\sqrt{3}$$

So $\vec{v}_r = 5\hat{i} - 5\sqrt{3}\hat{j}$

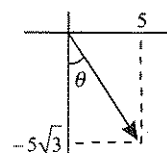


Fig. 5.124

$$\tan \theta = \frac{5}{5\sqrt{3}} \Rightarrow \theta = 30^\circ$$

7. $\vec{v}_s = 2\hat{i} + 1.25\hat{j} + 0.25\hat{k}$

Magnitude: $|\vec{v}_s| = \sqrt{2^2 + 1.25^2 + 0.25^2} = \frac{3}{2}\sqrt{5} \text{ m/s}$

8. Both should reach simultaneously at A.

$$TS = 40 \text{ km} \Rightarrow TA = 40 \text{ km}$$

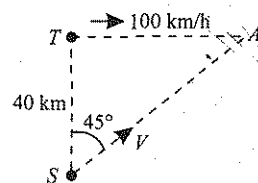


Fig. 5.125

Time taken by bomber to reach A:

$$t = \frac{40}{100} = \frac{2}{5} \text{ h} = 24 \text{ min}$$

So time taken by interceptor plane to reach A:

$$24 - 16 = 8 \text{ min}, v = \frac{40\sqrt{2}}{(8/60)} = \frac{400\sqrt{2}}{3} \text{ km/h}$$

$$9. x = \left(\frac{u - v \sin \theta}{\cos \theta} \right) \frac{d}{v} \Rightarrow \frac{dx}{d\theta} = 0 \Rightarrow \sin \theta = \frac{v}{u}$$

$$\cos \alpha = \frac{v}{u} = \alpha = \cos^{-1} \left(\frac{v}{u} \right)$$

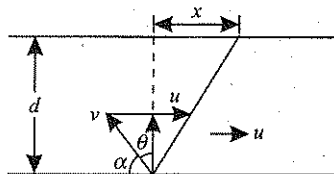


Fig. 5.126

$$\text{we can get } x = \frac{\left(u - \frac{v^2}{u} \right) \frac{d}{v}}{\sqrt{1 - \frac{v^2}{u^2}}} = \frac{(\sqrt{u^2 - v^2}) d}{v}$$

$$10. \vec{v}_m = \vec{v}_{m/w} + \vec{v}_w$$

$$\vec{v}_m = v \cos(\theta + 45^\circ) \hat{i} + v \sin(\theta + 45^\circ) \hat{j}$$

$$+ u \cos 45^\circ \hat{i} - u \sin 45^\circ \hat{j}$$

$$\vec{v}_m = [v \cos(\theta + 45^\circ) + u \cos 45^\circ] \hat{i} + [v \sin(\theta + 45^\circ) - u \sin 45^\circ] \hat{j}$$

To reach the point B, $\hat{j}$ component should be zero.

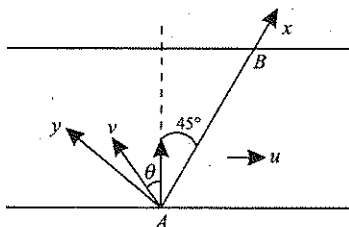


Fig. 5.127

$$v \sin(\theta + 45^\circ) = u \sin 45^\circ \Rightarrow v = \frac{u \sin 45^\circ}{\sin(\theta + 45^\circ)}$$

$$\text{For } v_{\min}, \theta + 45^\circ = 90^\circ \Rightarrow \theta = 45^\circ$$

$$v_{\min} = \frac{u}{\sqrt{2}}, 45^\circ \text{ with upstream.}$$

$$11. \text{ From A to B: } AB = (v \cos \beta + u \cos \alpha) t_1 \text{ from B to A:}$$

$$AB = (u \cos \beta - v \cos \alpha) t_2 \quad (i)$$

$$\alpha = 60^\circ, v \sin \beta = u \sin \alpha$$

$$\text{now } t = t_1 + t_2$$

$$5 \times 60 = \frac{1200}{v \cos \beta + u \cos \alpha} + \frac{1200}{(v \cos \beta - u \cos \alpha)}$$

$$\Rightarrow \text{put } u = \sqrt{17}, \alpha = 60^\circ, \text{ we get}$$

$$\Rightarrow v^2 \cos^2 \beta - 8v \cos \beta - \frac{17}{4} = 0 \Rightarrow v \cos \beta = \frac{17}{2} \quad (ii)$$

$$\text{From equations (i) and (ii) } \tan \beta = \sqrt{\frac{3}{17}}$$

$$\Rightarrow \beta = \tan^{-1} \sqrt{\frac{3}{17}}$$

$$\text{and } v = \sqrt{85} \text{ m/s}$$

$$12. dr = (-u + nu \sin \theta) dt, d\theta = (nu \cos \theta) dt$$

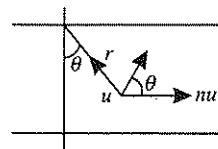


Fig. 5.128

$$\Rightarrow \int_d^r \frac{dr}{r} = \int_0^\theta \frac{nu \sin \theta - u}{nu \cos \theta} d\theta$$

$$\Rightarrow r = \frac{1}{d \sec \theta} \left[\tan \left[\frac{\theta}{2} + \frac{\pi}{4} \right] \right] x$$

$$13. \vec{v}_{A/B} = 12\hat{i} + 16\hat{j}, \tan \alpha = \frac{16}{12} = \frac{4}{3}$$

$$|\vec{v}_{A/B}| = \sqrt{(12)^2 + (16)^2} = 20 \text{ km/h}$$

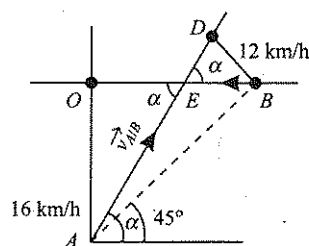


Fig. 5.129

$$DB = EB \sin \alpha = (OB - OE) \sin \alpha$$

$$= (AB \cos 45^\circ - \frac{OA}{\tan \alpha}) \sin \alpha$$

$$= \left[\frac{10}{\sqrt{2}} - AB \cos 45^\circ \times \frac{B}{4} \right] \times \frac{4}{5}$$

$$= \frac{10}{\sqrt{2}} \left[\frac{1}{4} \right] \times \frac{4}{5} = \sqrt{2} \text{ km}$$

$$14. \vec{u}_1 = -u_1 \hat{j}, \quad \vec{u}_2 = u_2 \cos \theta \hat{j} + u_2 \sin \theta \hat{i}$$

$$\Rightarrow \vec{u}_2 - \vec{u}_1 = u_2 \sin \theta \hat{i} + (u_2 \cos \theta + u_1) \hat{j}$$

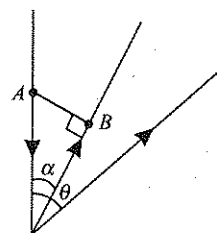


Fig. 5.130

$$\tan \alpha = \frac{u_2 \sin \theta}{u_2 \cos \theta + u_1}$$

$$AB = x_0 \sin \alpha = \frac{x_0 u_2 \sin \theta}{\sqrt{u_1^2 + u_2^2 + 2u_1 u_2 \cos \theta}}$$

$$15. \text{ Let } \vec{v}_c = v_0 \hat{i}, \vec{v}_{h/c} = -\frac{v_1}{2} \hat{j} - v_1 \frac{\sqrt{3}}{2} \hat{i}$$

$$\vec{v}_h = -\frac{v_1}{2}\hat{j} + \left(v_0 - v_1 \frac{\sqrt{3}}{2}\right)\hat{i}$$

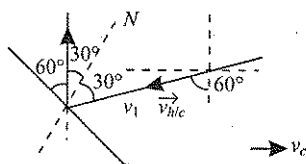


Fig. 5.131

$$\text{Now: } v_0 - v_1 \frac{\sqrt{3}}{2} = 0 \Rightarrow v_1 = \frac{2v_0}{\sqrt{3}}$$

$$\text{also: } \left(\frac{v_1}{2}\right)^2 + \left(v_0 - v_1 \frac{\sqrt{3}}{2}\right)^2 = (5\sqrt{3})^2$$

$$\Rightarrow v_1^2 = 75 \times 4 \Rightarrow \frac{4v_0^2}{3} = 75 \times 4$$

$$\Rightarrow v_0 = 15 \text{ m/s}$$

16. a. True, the speed of the projectile is minimum at its highest position, because as the projectile goes up its speed first decreases and then increases while coming down, becoming minimum at the highest point.

- b. True, at highest point acceleration is perpendicular to velocity.

- c. False, $T = \frac{2u \sin \theta}{g}$, so time of flight depends upon angle of projection.

- d. False, $H = \frac{u^2}{2g} \Rightarrow u^2 = 2gH$

$$R_{\max} = \frac{u^2}{g} = 2H$$

- e. True, a body feels weightlessness during projectile motion because it is a free fall motion.

- f. True, direction of velocity is always along tangent to the path of a particle.

- g. False, the instantaneous magnitude of velocity is not equal to the slope of the tangent drawn at the trajectory of the particle at that instant, but it is equal to the slope of position-time graph.

$$17. y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$y = 300 \tan 45^\circ - \frac{-10(300)^2}{2(100\sqrt{2})^2 \cos^2 45^\circ} = 255 \text{ m}$$

18. Given $\theta = 30^\circ$, $u = 40 \text{ m/s}$

$$\text{a. } T = \frac{2u \sin \theta}{g} = \frac{2 \times 40 \sin 30^\circ}{10} = 4 \text{ s}$$

$$\text{b. } R = \frac{u^2 \sin 2\theta}{g} = \frac{(40)^2 \sin (60^\circ)}{10} = 80\sqrt{3} \text{ m}$$

$$\text{c. } H = \frac{u^2 \sin^2 \theta}{2g} = \frac{40^2 \sin^2 30^\circ}{2 \times 10} = 20 \text{ m}$$

$$\text{d. time of ascent} = \frac{T}{2} = \frac{4}{2} = 2 \text{ s}$$

- e. $v_0 = u \cos \theta = 40 \cos 30^\circ = 20\sqrt{3} \text{ m/s}$ it is in horizontal direction.

$$\text{f. } v = \sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$$

$$= \sqrt{40^2 + 10^2 \times 1^2 - 2 \times 40 \times 10 \times 1 \times \sin 30^\circ}$$

$$= 36 \text{ m/s}$$

$$\tan \beta = \frac{40 \sin 30^\circ - 10 \times 1}{40 \cos 30^\circ} = \frac{1}{2\sqrt{3}} \Rightarrow \beta = 16^\circ$$

19. $u = 30 \text{ m/s}$, $\theta = 60^\circ$

$$v = \sqrt{30^2 + 10^2 \times 2^2 - 2 \times 30 \times 10 \times 2 \sin 60^\circ}$$

$$= 16.14 \text{ m/s}$$

$$\tan \beta = \frac{30 \sin 60^\circ - 10 \times 2}{30 \cos 60^\circ} = \frac{3\sqrt{3} - 4}{3} = 0.4$$

$$\Rightarrow \beta = \tan^{-1}(0.4) = 21.8^\circ$$

$$20. \text{ a. } R_{\max} = \frac{u^2}{g} = \frac{10^2}{10} = 10 \text{ m}$$

$$\text{b. } 100 = \frac{u^2}{10} \Rightarrow u = 10\sqrt{10}$$

$$H_{\max} = \frac{u^2}{2g} = \frac{(10\sqrt{10})^2}{2 \times 10} = 50 \text{ m}$$

$$\text{c. } 160 = \frac{u^2}{10} \Rightarrow u = 40 \text{ m/s}$$

$$21. H_1 = \frac{u^2 \sin^2 30^\circ}{2g} = \frac{u^2}{8g}, H_2 = \frac{u^2 \sin^2 60^\circ}{2g} = \frac{3u^2}{8g}$$

$$\text{Clearly } H_2 = 3H_1$$

Sum of angles = $60 + 30 = 90^\circ$, the horizontal range is same

$$22. R = \frac{20^2 \sin(2 \times 45^\circ)}{10} = 40 \text{ m}, T = \frac{2 \times 20 \sin 45^\circ}{10} = 2\sqrt{2} \text{ s}$$

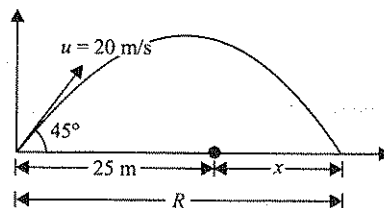


Fig. 5.132

$$x = 40 - 25 = 15 \text{ m},$$

$$\text{Required speed} = \frac{15}{2\sqrt{2}} = 5.3 \text{ m/s}$$

23. Given $u = 16 \text{ m/s}$, $\theta = 30^\circ$

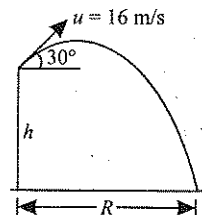


Fig. 5.133

$$R = u_x t = 16 \cos 30^\circ \times 4 = \frac{16\sqrt{3}}{2} \times 4 = 55.4 \text{ m}$$

$$-h = 16 \sin 30^\circ \times 4 - \frac{1}{2}(10)(4)^2 \Rightarrow h = 48 \text{ m}$$

$$24. -30 = -10 \times \sin 30^\circ t - \frac{1}{2} 10 t^2$$

$$\Rightarrow 5t^2 + 5t - 30 = 0 \Rightarrow t^2 + t - 6 = 0$$

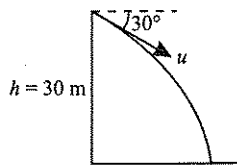


Fig. 5.134

$$\Rightarrow (t+3)(t-2) = 0 \Rightarrow t = 2 \text{ s}$$

$$25. T = \sqrt{\frac{2 \times 40}{9.8}} = 2.86 \text{ s}, R = uT = 20 \times 2.86 = 57.2 \text{ m}$$

$$26. \text{ a. } T = \sqrt{\frac{2 \times 2000}{10}} = 20 \text{ s},$$

$$AB = uT = 540 \times \frac{5}{18} \times 20 = 3000 \text{ m}$$

$$\text{ b. } T = \sqrt{\frac{2 \times 245}{100 \times 10}} = 0.7 \text{ s, Distance} = uT$$

$$= 54 \times \frac{5}{18} \times 0.7 = 10.5 \text{ m}$$

$$27. h = \frac{1}{2} 10 (3)^2 = 45 \text{ m}$$

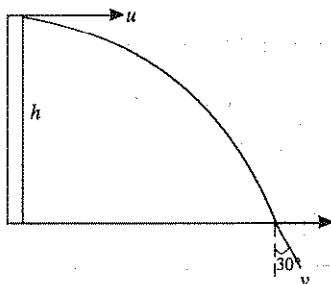


Fig. 5.135

$$v \cos 30^\circ = 10 \times 3 \Rightarrow v = 20\sqrt{3} \text{ m/s}$$

$$u = v \sin 30^\circ = 20\sqrt{3} \times \frac{1}{2} = 10\sqrt{3} \text{ m/s}$$

$$28. \frac{H}{x} = \tan \theta \Rightarrow H = x \tan \theta \text{ (see Fig. 5.136)}$$

Let the particle reaches at P at time t, then

$$h_1 = u \sin \theta t - \frac{1}{2} g t^2$$

$$\text{Height descended by apple in time } t: h_2 = \frac{1}{2} g t^2$$

$$h_1 + h_2 = u \sin \theta t = u \sin \theta \frac{x}{u \cos \theta} = x \tan \theta = H$$

It means particle will hit the apple.

$$29. u_0 \sin \theta = u \Rightarrow 400 \sin \theta = 200 \text{ (see Fig. 5.137)}$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Verify that maximum height attained by the shell is greater than or equal to 1.5 km.

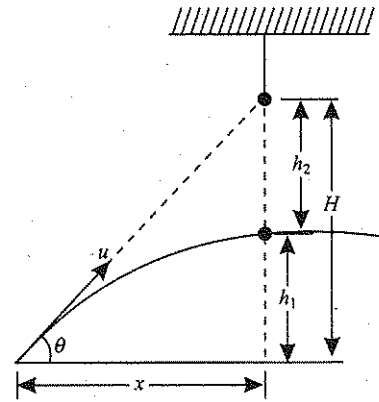


Fig. 5.136

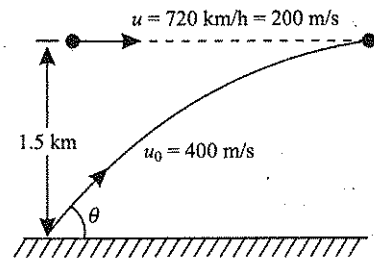


Fig. 5.137

$$30. 13 = 50 \tan \theta - \frac{g (50)^2}{2(10\sqrt{g})^2 \cos^2 \theta}$$

$$25 \tan^2 \theta - 100 \tan \theta + 51 = 0 \Rightarrow \tan \theta = \frac{17}{5}, \frac{3}{5}$$

time taken will be less for $\tan \theta = 3/5$.

$$31. x_2 = u \cos \theta t, x_2 - x_1 = v - t$$

$$\Rightarrow x_1 + vt = u \cos \theta t$$

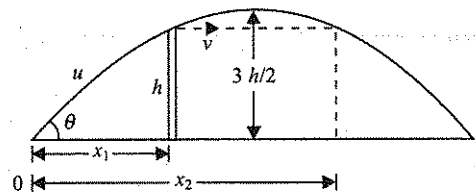


Fig. 5.138

$$h = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta}, \quad \frac{3h}{2} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{Simplify to get } x^2 - 6h \cot \theta x + 6h^2 \cot^2 \theta = 0$$

$$\Rightarrow x = \frac{6h \cot \theta \pm \sqrt{12} h \cot \theta}{2}$$

$$= 3h \cot \theta \pm \sqrt{3} h \cot \theta$$

$$v = \frac{x_2 - x_1}{t} = \frac{2\sqrt{3} h \cot \theta}{t}$$

$$u \cos \theta = \frac{x_2}{t} = \frac{(3 + \sqrt{3}) h \cot \theta}{t}$$

$$\frac{v}{u \cos \theta} = \frac{2\sqrt{3}}{3 + \sqrt{3}} = \frac{2\sqrt{3}(3 - \sqrt{3})}{9 - 3} = \sqrt{3} - 1$$

$$\begin{aligned}
 32. \quad x &= 0.2 \text{ m}, nx = \frac{1}{2}gt^2, nx > ut \Rightarrow n^2x^2 > u^2t^2 \\
 &\Rightarrow n^2x^2 > \frac{u^2 2nx}{g} \\
 &\Rightarrow n > \frac{2u^2}{gx} \Rightarrow n > \frac{2(1.8)^2}{2 \times 0.2} \\
 &\Rightarrow n > 3.24
 \end{aligned}$$

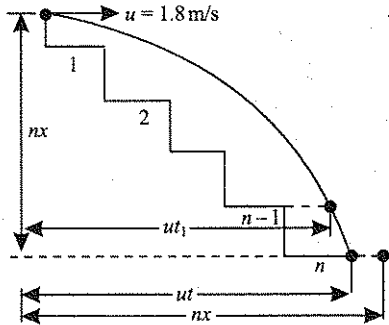


Fig. 5.139

$$\begin{aligned}
 (n-1)x &= \frac{1}{2}gt_1^2 \Rightarrow ut_1 > (n-1)x \\
 &\Rightarrow u^2t_1^2 > (n-1)^2x^2 \\
 &\Rightarrow \frac{u^2 2(n-1)x}{g} > (n-1)^2x^2 \\
 &\Rightarrow \frac{2u^2}{gx} > n-1 \Rightarrow n-1 < 3.24 \Rightarrow n < 4.24
 \end{aligned}$$

$$\begin{aligned}
 33. \quad -h &= R \tan \theta - \frac{gR^2}{2u^2 \cos^2 \theta} \\
 &\Rightarrow h = -R \tan \theta + \frac{gR^2}{2u^2} \sec^2 \theta \quad (i) \\
 &\text{Differentiating w.r.t. } \theta \\
 &\Rightarrow 0 = -R \sec^2 \theta + \tan \theta \frac{dR}{d\theta} \\
 &\quad + \frac{gR}{2u^2} 2 \sec^2 \theta + \frac{g \sec^2 \theta}{2u^2} 2R \frac{dR}{d\theta} \\
 &\text{Put } \frac{dR}{d\theta} = 0, \text{ we get } R = \frac{u^2}{g \tan \theta}, \text{ put } R \text{ in} \quad (i) \\
 &\Rightarrow h = -\frac{u^2}{g} + \frac{g \sec^2 \theta}{2u^2} \frac{u^4}{g^2 \tan^2 \theta} \\
 &\Rightarrow \sin \theta = \sqrt{\frac{u^2}{2(u^2 + gh)}} \\
 &\Rightarrow \theta = 43.78^\circ
 \end{aligned}$$

34. Superimpose an upward acceleration a on the system. The box becomes stationary. The particle has an upward acceleration a and a downward acceleration g . If $a = g$, the particle has no acceleration and will hit C . If $a > g$, the particle has a net upward acceleration, and if $a < g$, the particle has a net downward acceleration.

$$35. u \cos \theta = 12, 6 = u \cos \theta t \Rightarrow t = 0.5 \text{ s (Fig. 5.140)}$$

$$3.75 = u \sin \theta t - \frac{1}{2}gt^2$$

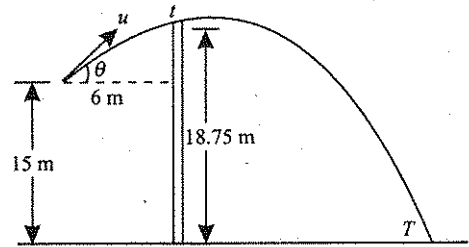


Fig. 5.140

$$\begin{aligned}
 \Rightarrow \frac{15}{4} &= u \sin \theta \times \frac{1}{2} - \frac{1}{2} 10 \left(\frac{1}{2} \right)^2 \\
 &\Rightarrow u \sin \theta = 10 \\
 -15 &= 10T - \frac{1}{2} 10T^2 \Rightarrow T^2 - 2T - 3 = 0 \Rightarrow T = 3 \text{ s}
 \end{aligned}$$

$$\begin{aligned}
 36. \quad a. \quad 0 &= u \sin(\alpha - \beta)t - \frac{1}{2}g \cos \beta t^2 \\
 t &= 2u \sin(\alpha - \beta)/g \cos \beta \quad (i) \\
 0 &= u \cos(\alpha - \beta) - g \sin \beta t \Rightarrow t = \frac{u \cos(\alpha - \beta)}{g \sin \beta} \quad (ii)
 \end{aligned}$$

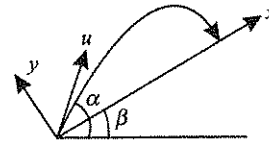


Fig. 5.141

$$\begin{aligned}
 \text{From equations (i) and (ii)} \\
 \frac{2 \sin(\alpha - \beta)}{\cos \beta} &= \frac{\cos(\alpha - \beta)}{\sin \beta}
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow 2 \tan(\alpha - \beta) = \cot \beta \\
 &\Rightarrow \frac{2(\tan \alpha - \tan \beta)}{1 + \tan \alpha \tan \beta} = \cot \beta \\
 &\Rightarrow 2 \tan \alpha - 2 \tan \beta = \cot \beta + \tan \alpha \\
 &\Rightarrow \tan \alpha = \cot \beta + 2 \tan \beta
 \end{aligned}$$

$$b. t = 2u \sin(\alpha - \beta)/g \cos \beta$$

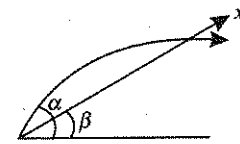


Fig. 5.142

$$\begin{aligned}
 \text{Also, } t &= \frac{u \sin \alpha}{g} \Rightarrow \frac{2 \sin(\alpha - \beta)}{\cos \beta} = \sin \alpha \\
 &\Rightarrow 2 \sin \alpha \cos \beta - 2 \cos \alpha \sin \beta = \sin \alpha \cos \beta \\
 &\Rightarrow \sin \alpha \cos \beta = 2 \cos \alpha \sin \beta \\
 &\Rightarrow \tan \beta = 2 \tan \alpha
 \end{aligned}$$

$$37. A \text{ to } B: 0^2 = u^2 \sin^2 \theta - 2(g \cos \alpha)h$$

$$A \text{ to } C: -h = u \sin \theta t - \frac{1}{2}g \cos \alpha t^2$$

$$\text{And } 0 = u \sin \theta - g \sin \alpha t \Rightarrow t = \frac{u \cos \theta}{g \sin \alpha}$$

$$-h = \frac{u^2 \sin \theta \cos \theta}{g \sin \alpha} - \frac{1}{2} \frac{g \cos \alpha u^2 \cos^2 \theta}{g^2 \sin^2 \alpha}$$

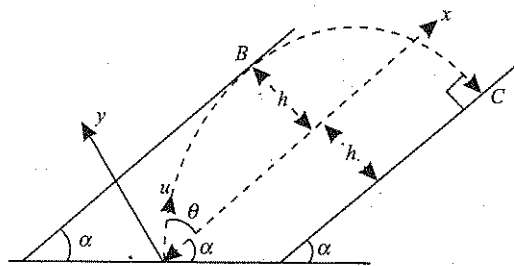


Fig. 5.143

$$\frac{-u^2 \sin^2 \theta}{2g \sin \alpha} = \frac{u^2 \sin \theta \cos \theta}{g \sin \alpha} - \frac{u^2 \cos^2 \alpha \cos^2 \theta}{2g \sin^2 \alpha}$$

$$\frac{-\sin^2 \theta}{2 \cos \alpha} = \frac{\sin \theta \cos \theta}{\sin \alpha} - \frac{\cos \alpha \cos^2 \theta}{2 \sin^2 \alpha}$$

Multiplying both side by $\frac{2 \cos \alpha}{\cos^2 \theta}$, we get

$$-\tan^2 \theta = 2 \cot \alpha \tan \theta - \cot^2 \alpha = 0$$

$$\tan^2 \theta + 2 \cot \alpha \tan \theta - \cot^2 \alpha = 0$$

$$\tan \theta = \frac{-2 \cot \alpha + \sqrt{4 \cot^2 \alpha + 4 \cot^2 \alpha}}{2}$$

$$\tan \theta = (\sqrt{2} - 1) \cot \alpha$$

38. Let gun 1 and gun 2 be fired at an interval Δt , such that,

$$t_1 = t_2 + \Delta t \quad (i)$$

where t_1 and t_2 are the respective times taken by the two shots to reach point P.

For gun 1: $x - x_i = v_i \cos 60^\circ t_1$

$$y - y_i = v_i \sin 60^\circ t_1 - \frac{1}{2} g t_1^2$$

$$\text{or, } x = x_i + \frac{1}{2} v_i t_1 \quad y = y_i + \frac{\sqrt{3}}{2} v_i t_1 - \frac{1}{2} g t_1^2$$

For gun 2: $x - x_i = v_i \cos 0^\circ t_2$

$$y - y_i = v_i \sin 0^\circ t_2 - \frac{1}{2} g t_2^2$$

$$\text{or } x = x_i + v_i t_2 \quad y = y_i - \frac{1}{2} g t_2^2$$

a. Now we can equate x- and y-coordinates of shots, i.e.,

$$\frac{1}{2} v_i t_1 = v_i t_2 \text{ or } t_1 = 2t_2 \quad (ii)$$

$$\text{and } \frac{\sqrt{3}}{2} v_i t_1 - \frac{1}{2} g t_1^2 = -\frac{1}{2} g t_2^2$$

$$\text{or, } \frac{\sqrt{3}}{2} v_i t_1 + \frac{1}{2} g (t_2^2 - t_1^2) = 0 \quad (iii)$$

On substituting t_1 from eqn. (ii) into eqn. (iii), we get,

$$\frac{\sqrt{3}}{2} v_i (2t_2) + \frac{1}{2} g (-3t_2^2) = 0$$

$$\text{or, } t_2 \left(\sqrt{3} v_i - \frac{3}{2} g t_2 \right) = 0$$

$$\text{or, } t_2 = 0 \text{ and } t_2 = \frac{2}{\sqrt{3}} \frac{v_i}{g} = \frac{2}{\sqrt{3}} \times \left(\frac{5\sqrt{3}}{10} \right) = 1 \text{ s}$$

Therefore, $t_1 = 2t_2 = 2(1) = 2 \text{ s}$;

$$\Delta t = t_1 - t_2 = 2 - 1 = 1 \text{ s}$$

b. The coordinates of P at which the two shots collide are

$$x = x_i + v_i t_2 = 0 + (5\sqrt{3})(1) = 5\sqrt{3} \text{ m}$$

$$\text{and } y = y_i - \frac{1}{2} g t_2^2 = 10 - \frac{1}{2} (10)(1)^2 = 5 \text{ m}$$

39. The motion of the sphere takes place in a plane; the x- and y-components of its acceleration are $A_x = g \sin 30^\circ$, $a_y = 0$

The x- and y- component of sphere's velocity at time $t = 2 \text{ s}$ are

$$\begin{aligned} V_x &= v_{0x} - a_x t = 3 \sin 60^\circ - g \sin 30^\circ \times 2 \\ &= -7.40 \text{ m/s} \end{aligned}$$

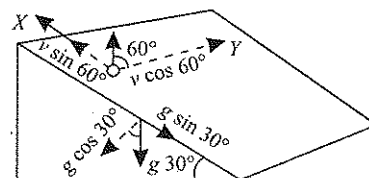


Fig. 5.144

$$V_y = v \cos 60^\circ = \text{constant} = 1.5 \text{ m/s}$$

So the magnitude of sphere's velocity is

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(-7.40)^2 + (1.5)^2} = 7.55 \text{ m/s}$$

$$40. \text{ Horizontal Range} = \frac{2u_x u_y}{g}$$

Let the initial horizontal and vertical components of the velocity of the shot be u and v , respectively.

$$R_1 = \frac{2(u + v_p)v}{g} \text{ platform moving forward}$$

$$R_2 = \frac{2(u - v_p)v}{g} \text{ platform moving backward}$$

$$R_1 + R_2 = \frac{4uv}{g} \text{ and } R_1 - R_2 = \frac{4v_p v}{g}$$

$$\text{Now } (R_1 - R_2)^2 = \frac{16v_p^2 v^2}{g^2}$$

$$\text{and } \frac{(R_1 - R_2)^2}{R_1 + R_2} = \frac{4v_p^2}{g} \times \frac{v}{u}$$

$$\text{or } \frac{v}{u} = \frac{g}{4v_p^2} \frac{(R_1 - R_2)^2}{(R_1 + R_2)}$$

Elevation of gun

$$\alpha = \tan^{-1} \left(\frac{v}{u} \right) = \tan^{-1} \left[\frac{g}{4v_p^2} \times \frac{(R_1 - R_2)^2}{(R_1 + R_2)} \right]$$

41. True, tangential acceleration is along or opposite to the direction of velocity so it changes the magnitude of velocity. Centripetal acceleration is perpendicular to the velocity so it changes only direction of the velocity.

$$42. a_c = \omega^2 R = \left(\frac{2\pi}{24 \times 3600} \right)^2 \times 6400 \times 10^3 = 0.0338 \text{ m/s}^2$$

$$43. v = 27 \times \frac{5}{18} = \frac{15}{2} \text{ m/s}$$

$$44. \omega = 1200 \text{ rpm} = 1200 \times \frac{2\pi}{60} = 40\pi \text{ rad/s}$$

$$a_c = \omega^2 r = (40\pi)^2 \times \frac{30}{100} = 480\pi^2 \text{ m/s}^2$$

$$45. \alpha = 2 \text{ rad/s}^2, \omega_1 = 0, t = 4 \text{ s}$$

$$a. \omega_2 = \omega_1 + \alpha t \Rightarrow \omega_2 = 0 + 2 \times 4 = 8 \text{ rad/s}$$

$$b. \theta = \omega_1 t + \frac{1}{2} \alpha t^2 \Rightarrow \theta = 0 \times 4 + \frac{1}{2} \times 2 \times 4^2 = 16 \text{ rad}$$

$$c. 1 \text{ revolution} = 2\pi \text{ rad} \Rightarrow 1 \text{ rad} = \frac{1}{2\pi} \text{ rev} \Rightarrow 16 \text{ rad} \\ = \frac{16}{2\pi} \text{ rev} = \frac{8}{\pi} \text{ rev} = \frac{8 \times 7}{22} = \frac{28}{11} \text{ rev.}$$

$$d. \text{ after } 4 \text{ s: } v = \omega_2 r = 8 \times 10/100 = 0.8 \text{ m/s,}$$

$$a_c = \frac{v^2}{r} = \frac{(0.8)^2}{10/100} = 6.4 \text{ m/s}^2,$$

$$a_t = \alpha r = 2 \times \frac{10}{100} = 0.2 \text{ m/s}^2$$

Now magnitude of net acceleration:

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{6.4^2 + 0.2^2} = \sqrt{41} \text{ m/s}^2,$$

$$\text{Direction of acceleration: } \tan \theta = \frac{a_t}{a_c} = \frac{0.2}{6.4} = \frac{1}{32}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{32} \right)$$

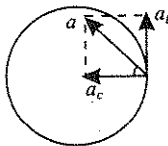


Fig. 5.145

Objective Type

$$1. c. \vec{v}_{r/g} = \vec{v}_r + (-\vec{v}_g)$$

$$v_{rg} = \sqrt{v_r^2 + v_g^2} = \sqrt{16 + 9} \text{ km h}^{-1} = 5 \text{ km h}^{-1}$$

2. a. For B always to be north of A, the velocity components of both along east should be same (see Fig. 5.146).

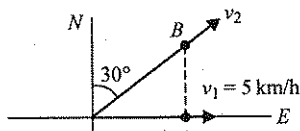


Fig. 5.146

$$v_2 \cos 60^\circ = v_1 \Rightarrow v_2 = 10 \text{ km/h}$$

3. d. Refer to Fig. 5.147(a),

$$\tan \theta = \frac{v_w}{v_m} = \frac{50}{100} = \frac{1}{2} \text{ or } v_m = 2 v_w$$

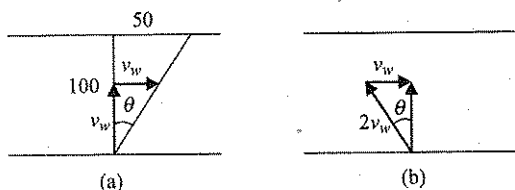


Fig. 5.147

Refer to Fig. 5.147(b),

$$\sin \theta = \frac{v_w}{v_m} = \frac{v_w}{2v_w} = \frac{1}{2} \text{ or } \theta = 30^\circ$$

So, it is 60° upstream.

$$4. a. 10^2 = v^2 + 8^2 \text{ (Fig. 5.148)}$$

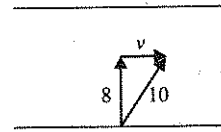


Fig. 5.148

$$\text{or } v^2 = 100 - 64 = 36 \text{ or } v = 6 \text{ kmh}^{-1}$$

$$5. d. v_r = \sqrt{10^2 + 3^2} = \sqrt{109} \text{ km h}^{-1} \text{ (Fig. 5.149)}$$

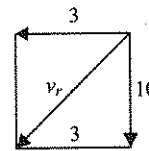


Fig. 5.149

6. b. The bird keeps on flying with a constant speed till the time of crash. So, let us first find the time of crash. If the two trains crash each other after t hrs, then the total distance travelled by the two trains in the same time of t hrs should be 60 km.

$$\therefore 40t + 60t = 60 \Rightarrow t = \frac{60}{100} = 0.6 \text{ hrs}$$

Now, the distance travelled by the bird in 0.6 h is $0.6 \times 30 = 18 \text{ km}$

7. a. Here, $v_R = 25 \text{ ms}^{-1}$, $v_W = 10 \text{ ms}^{-1}$

$\therefore$ Velocity of rain w.r.t. woman : $v_{R/W} = v_R - v_W$

Let $v_{R/W}$ makes an angle θ with vertical then

$$\tan \theta = \frac{v_W}{v_R} = \frac{10}{25} = 0.4$$

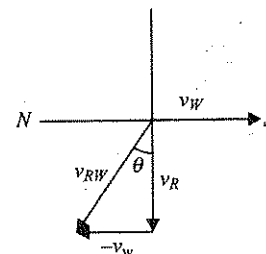


Fig. 5.150

She should hold her umbrella at an angle of $\theta = \tan^{-1}(0.4)$ with vertical towards south.

8. d. When the body is dropped from the balloon, it also acquires the upward velocity of balloon. So w.r.t. a person on the ground, the ball appears to be going up. But a person in the balloon is also going up. So w.r.t. him the velocity of the body will be zero and he will then see the body to be coming down.

9. d. Velocity of police van $= 30 \times \frac{5}{18} = \frac{25}{3}$ m/s
 Muzzle speed of the bullet $= 150 \text{ ms}^{-1}$
 Speed of the bullet w.r.t ground $= (150 + 25/3) \text{ ms}^{-1}$
 Velocity of thief's car

$$= 192 \times \frac{5}{18} = \frac{32 \times 5}{3} = \frac{160}{3} \text{ m/s}$$

$\therefore$ relative velocity of the bullet w.r.t. the thief's car

$$= 150 + \frac{25}{3} - \frac{160}{3} = 150 - \frac{135}{3} = 105 \text{ m/s}$$

10. c. To find the relative velocity of bird w.r.t. train, superimpose velocity $-\vec{V}_T$ on both the objects. Now as a result of it, the train is at rest, while bird possesses two velocities $-\vec{V}_B$ towards N and $-\vec{V}_T$ along west.

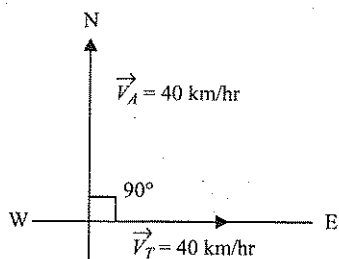


Fig. 5.151

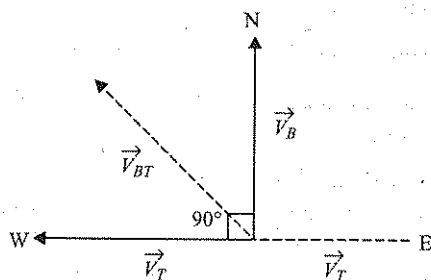


Fig. 5.152

where, $-\vec{V}_B$ = Velocity of bird

$-\vec{V}_T$ = Velocity of train.

$$|\vec{V}_{BT}| = \sqrt{|\vec{V}_B|^2 + |-\vec{V}_T|^2}$$

[By formula, $\therefore \theta = 90^\circ$]

$$= \sqrt{40^2 + 40^2} = 40\sqrt{2} \text{ km/hr North-West}$$

11. b. Finally he will swim along B. $\tan \theta = \frac{v}{u} = \frac{10}{5} = 2$

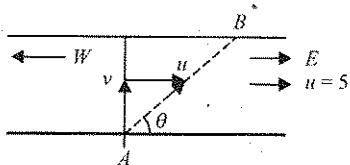


Fig. 5.153

$$\Rightarrow \theta = \tan^{-1}(2) \text{ N of E}$$

12. b. Net velocity of boat in river $= \sqrt{5^2 - u^2}$

$$t = \frac{\text{Distance}}{\text{Velocity}} \Rightarrow \frac{1}{4} = \frac{1}{\sqrt{5^2 - u^2}} \Rightarrow u = 3 \text{ kmh}^{-1}$$

13. c. Relative velocity of boat with respect to water is

$$\vec{v}_b - \vec{v}_w = 3\hat{i} + 4\hat{j} - (-3\hat{i} - 4\hat{j}) = 6\hat{i} + 8\hat{j}$$

14. a. $\vec{v}_c = 25\hat{i}$, $\vec{v}_{b/c} = 25\sqrt{3}\hat{j}$

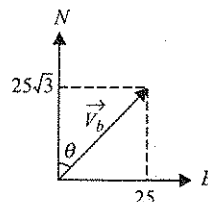


Fig. 5.154

$$\vec{v}_{b/c} = \vec{v}_b - \vec{v}_c \Rightarrow \vec{v}_b = \vec{v}_{b/c} + \vec{v}_c$$

$$\Rightarrow \vec{v}_b = 25\hat{i} + 25\sqrt{3}\hat{j}$$

$$|\vec{v}_b| = \sqrt{25^2 + (25\sqrt{3})^2} = 50 \text{ km/h}$$

$$\tan \theta = \frac{25}{25\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

15. a. Shortest time $= \frac{d}{v} = \frac{1/2}{3} = \frac{1}{6} \text{ hr} = 10 \text{ min}$

16. d. Relative velocity of the bird with respect to the train is:

$$V_{BT} = V_B + V_T = 5 + 10 = 15 \text{ m/s}$$

[because they are going in opposite direction]

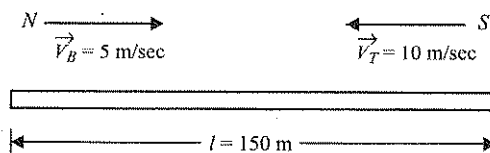


Fig. 5.155

$$\therefore \text{time taken by the bird to cross the train} = \frac{150}{5} = 10 \text{ s}$$

17. d. Shortest possible time: $t = \frac{d}{v} = \frac{336}{1} = 336 \text{ s}$

18. b. $\sin \alpha = \frac{u}{v} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 60^\circ$
 $\Rightarrow \theta = 90^\circ + \alpha = 150^\circ$

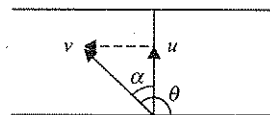


Fig. 5.156

19. a. $\vec{v}_p = v\hat{i}$, $\vec{v}_{b/p} = -u\hat{i}$
 $\vec{v}_b = \vec{v}_p + \vec{v}_{b/p} = (v - u)\hat{i} = v - u$ towards right.

20. c. $\vec{v}_c = -5\hat{j}$, $\vec{v}_{b/c} = -2\sqrt{6}\hat{i}$

$$\vec{v}_b = \vec{v}_{b/c} + \vec{v}_c = -2\sqrt{6}\hat{i} - 5\hat{j}$$

$$v_b = \sqrt{(2\sqrt{6})^2 + 5^2} = 7 \text{ m/s}$$

21. a. Let the stone be projected at an angle α to the direction of motion of truck with a speed of $v = 20 \text{ m/s}$. Since the resultant displacement along horizontal is zero.

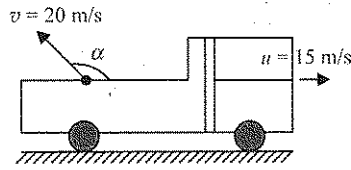


Fig. 5.157

The velocity along horizontal = 0

$$15 + 20 \cos \alpha = 0 \Rightarrow \cos \alpha = -\frac{3}{4}$$

$$\Rightarrow \alpha = \cos^{-1}\left(-\frac{3}{4}\right) = 138^\circ 35'$$

22. d. We know that to cross the river by shortest path: $\sin \alpha = \frac{u}{v}$

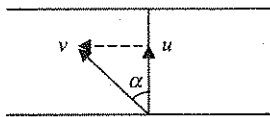


Fig. 5.158

But $u > v$, $\Rightarrow \sin \alpha > 1$, which is not possible.

23. b. Speed of train = $108 \times \frac{5}{18} = 30$ m/s

Let $\vec{v}_R$ and $\vec{v}_T$ represent the respective velocities of rain and train.

Now, the relative velocity of rain w.r.t. person (train) is given by $\vec{v}_{R,T} = \vec{v}_R - \vec{v}_T \Rightarrow \vec{v}_R + (-\vec{v}_T)$

Let $\vec{OR}$ and $\vec{RT}$ represent the vectors and, respectively, in magnitude and direction.

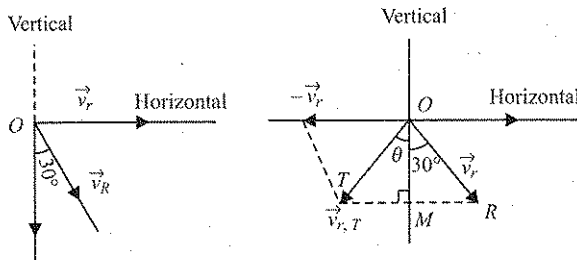


Fig. 5.159

$$\begin{aligned} OT^2 &= OR^2 + RT^2 + 2OR \times RT \cos 120^\circ \\ &= 20^2 + 30^2 - 2 \times 20 \times 30 \times \frac{1}{2} \\ &= 400 + 900 - 600 = 700 = \sqrt{700} \text{ m/s} \end{aligned}$$

From $\triangle OMR$ [where M is the foot of perpendicular drawn from point O on RT]

$$\begin{aligned} OM &= 20 \cos 30^\circ = 10\sqrt{3} \\ MR &= 20 \sin 30^\circ = 10, \quad MT = 30 - 10 = 20 \end{aligned}$$

If θ be the angle which the apparent velocity makes with vertical, then $\tan \theta = \frac{TM}{OM} = \frac{20}{10\sqrt{3}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

24. a. Motion of the person, making an angle (say α) with the downstream.

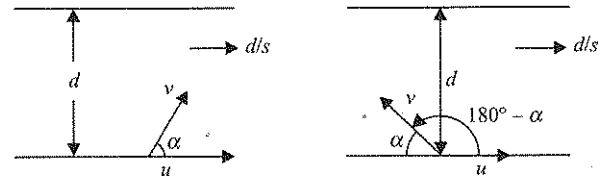


Fig. 5.160

The time taken to cross the river = $\frac{d}{v \sin \alpha}$

The distance carried away down stream in the same time is equal to speed $\times$ time.

$$x = (u + v \cos \alpha) \frac{d}{v \sin \alpha} \quad (i)$$

Motion of the person, making α angle with upstream.

The time taken to cross the river is equal to $\frac{d}{v \sin \alpha}$.

Distance carried away downstream in the same time

$$\begin{aligned} x &= [u + v \cos(180^\circ - \alpha)] \frac{d}{v \sin \alpha} \\ x &= (u - v \cos \alpha) \frac{d}{v \sin \alpha} \quad (ii) \end{aligned}$$

$$\text{Given, } \frac{(u + v \cos \alpha) \frac{d}{v \sin \alpha}}{(u - v \cos \alpha) \frac{d}{v \sin \alpha}} = \frac{2}{1}$$

$$\begin{aligned} \frac{(u + v \cos \alpha)}{(u - v \cos \alpha)} &= \frac{2}{1} \Rightarrow 3v \cos \alpha = u \\ \Rightarrow \frac{v}{u} &= \frac{\sec \alpha}{3} \quad (iii) \end{aligned}$$

$$\sec \alpha \geq 1 \Rightarrow \frac{\sec \alpha}{3} \geq \frac{1}{3}$$

From equation (iii), $\frac{v}{u} \geq \frac{1}{3}$ so, $\frac{v}{u}$ cannot be less than $\frac{1}{3}$.

25. a. Change in velocity = $\vec{v}_f - \vec{v}_i$

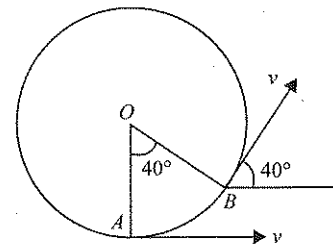


Fig. 5.161

Its magnitude is $\sqrt{v^2 + v^2 - 2vv \cos 40^\circ} = 2v \sin 20^\circ$

26. i. a. In the absence of air resistance, the projectile moves with constant horizontal velocity because acceleration due to gravity is totally vertical.

ii. c. The vertical component goes on decreasing and eventually becomes zero.

iii. b.

iv. c. Horizontal component of velocity remains constant throughout the motion, as it is not affected by acceleration due to gravity which is directed vertically downwards

27. i. d. $R = \frac{u^2}{g}$ and $H = \frac{u^2 \sin^2 \theta}{2g}$

For the maximum range, $\theta = 45^\circ$

$$H = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g} = \frac{R}{4}$$

ii. d. We know that $H = \frac{u^2 \sin^2 \theta}{2g}$ and $T = \frac{2u \sin \theta}{g}$. From these two equations,

we get $H = \frac{gT^2}{8}$. So if T is doubled, H becomes four times.

iii b. Here $R_{\max} = h = \frac{u^2}{g}$

Height H is given by: $H = \frac{u^2 \sin^2 \theta}{2g}$

H is max when $\theta = 90^\circ$ (for a given velocity)

$$\text{Hence } H_{\max} = \frac{u^2}{2g} = \frac{h}{2}$$

iv. a. We know that

$$x = (u \cos \theta)t \text{ and } y = (u \sin \theta)t - \frac{1}{2}gt^2$$

$$\text{Let } x_2 - x_1 = (u_1 \cos \theta_1 - u_2 \cos \theta_2)t = X$$

$$y_2 - y_1 = (u_1 \sin \theta_1)t - \frac{1}{2}gt^2 - (u_2 \sin \theta_2)t + \frac{1}{2}gt^2$$

$$= (u_1 \sin \theta_1 - u_2 \sin \theta_2)t = Y$$

$$\frac{Y}{X} = \frac{u_1 \sin \theta_1 - u_2 \sin \theta_2}{u_1 \cos \theta_1 - u_2 \cos \theta_2} = \text{constant, } m(\text{say})$$

$$Y = mX$$

It is the equation of a straight line passing through the origin.

Alternatively: We can think in this way: Relative acceleration of one projectile w.r.t. another projectile will be zero. Hence relative velocity of one projectile w.r.t. another will be constant. If velocity is constant, it indicates straight line motion.

v. c. The upward motion is with higher retardation while the downward motion is with lesser acceleration. Further, the time of rise is less than the time of return. A part of the kinetic energy is used against friction.

28. a. Since $R = 2H$ or $\frac{v^2 \sin 2\theta}{g} = 2 \times \frac{v^2 \sin^2 \theta}{2g}$
or $2 \sin \theta \cos \theta = \sin^2 \theta$ or $\tan \theta = 2$

$$R = \frac{v^2 \sin 2\theta}{g} = \frac{v^2 2 \sin \theta \cos \theta}{g} = \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$

29. d. As in Fig. 5.162, $H = R$ or $\frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 2 \sin \theta \cos \theta}{g}$

$$\text{or } \tan \theta = 4 \text{ or } \theta = \tan^{-1}(4)$$

30. a. $AC = R/2$, $PC = H$

We have to find $h = MP$

From Problem 29 (Fig. 5.162), we know that if $H = R$, then $\tan \theta = 4$,

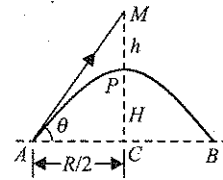


Fig. 5.162

$$\text{Now } \tan \theta = \frac{MC}{AC} = \frac{MP + PC}{AC}$$

$$4 = \frac{h + H}{R/2} \Rightarrow 4 = \frac{(h + H)2}{H}$$

$$\Rightarrow h = H$$

31. c. $v_x = \frac{dx}{dt} = 6$ and $v_y = \frac{dy}{dt} = 8 - 10t$

put $t = 0$ ($\because$ we have to find initial velocity)

$$v_y = 8 - 10 \times 0 = 8$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10 \text{ m/s}$$

32. b. $\tan \theta = \frac{v_y}{v_x} = \frac{8}{6} = \frac{4}{3}$ or $\theta = \tan^{-1} \frac{4}{3}$

33. i. c. Velocity at the highest point,

$$\vec{v}_h = \hat{i}(u \cos \theta)$$

Velocity at the starting point

$$\vec{v}_s = \hat{i}(u \cos \theta) + \hat{j}(u \sin \theta)$$

$$|\Delta \vec{v}| = |\vec{v}_h - \vec{v}_s| = |-\hat{j}(u \sin \theta)| = u \sin \theta$$

ii. d. Speed at the highest point $= u \cos \theta$

Speed at the starting point $= u$.

Hence, change in speed $= (u \cos \theta - u)$

34. b. Range will be the same, because the sum of two angles is 90° .

35. b. The other angle is $90^\circ - 30^\circ = 60^\circ$

36. d. The sum of these two angles is 90° .

37. a. For maximum range: $\theta = 45^\circ$, hence $\tan \theta = \tan 45^\circ = 1$

38. b. $H_{\max} = 100 \text{ m}$, $R_{\max} = 2 \times 200 = 400 \text{ m}$

$$\tan \theta = \frac{4 \times 100}{400} = 1 \Rightarrow \theta = 45^\circ \left[\because \frac{H}{R} = \frac{\tan \theta}{4} \right]$$

39. c. The time taken to reach the ground depends on the height from which the bullets are fired when the bullets are fired horizontally. Here height is same for both the bullets and hence will reach the ground simultaneously.

40. b. $R = h = \frac{u^2 \sin 2\theta}{g}$ When $2\theta = 90^\circ$

$$\Rightarrow \frac{u^2}{g} = h$$

$$\text{Height } H \text{ is given by: } H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{When } \theta = 90^\circ, H = H_{\max} = \frac{u^2}{2g} = \frac{h}{2}$$

41. a. We know that

$$x = (u \cos \theta)t \text{ and } y = (u \sin \theta)t - \frac{1}{2}gt^2$$

$$\text{Then } x_2 - x_1 = (u_2 \cos \theta_2 - u_1 \cos \theta_1)t = x$$

$$\text{And } y_2 - y_1 = (u_2 \sin \theta_2)t - \frac{1}{2}gt^2 - (u_1 \sin \theta_1)t + \frac{1}{2}gt^2$$

$$= (u_1 \sin \theta_1 - u_2 \sin \theta_2)t = Y$$

$$\frac{Y}{X} = \frac{(u_1 \sin \theta_1 - u_2 \sin \theta_2)t}{(u_1 \cos \theta_1 - u_2 \cos \theta_2)t}$$

$$= \frac{u_1 \sin \theta_1 - u_2 \sin \theta_2}{u_1 \cos \theta_1 - u_2 \cos \theta_2}$$

$$= \text{constant, } m \text{ (say)}$$

$\Rightarrow Y = mX$, It is the equation of a straight line passing through the origin.

$$42. \text{ a. } \tan \theta = \frac{4H}{R} = \frac{4 \times 4}{12} = \frac{4}{3} \Rightarrow \sin \theta = \frac{4}{5}$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\Rightarrow u = \frac{\sqrt{2gH}}{\sin \theta} = \frac{\sqrt{2 \times g \times 4}}{4/5} = 5\sqrt{\frac{g}{2}}$$

$$43. \text{ b. } t = \frac{2u \sin \theta}{g} \text{ or } 2 = \frac{2u \sin \theta}{g} \Rightarrow u \sin \theta = g$$

$$h_m = \frac{u^2 \sin^2 \theta}{2g} = \frac{g^2}{2g} = \frac{g}{2} = 5 \text{ m}$$

44. c. Range will be the same, because the sum of two angles is 90° . Hence the ratio 1:1.

45. d. Acceleration remains constant, and equal to g always.

46. a. R is same for both θ and $(90 - \theta)$. If angle w.r.t. vertical is 40° then w.r.t. horizontal direction it will be $90^\circ - 40^\circ = 50^\circ$.

$$47. \text{ d. } \frac{R}{T^2} = \frac{u^2 \sin 2\theta / g}{4u^2 \sin^2 \theta / g^2} = \frac{g}{2} \cot \theta$$

$$\text{i.e., } gT^2 = 2R \tan \theta$$

If T is doubled, then R becomes 4 times.

48. c. We know that at the uppermost point of a projectile the vertical component of the velocity becomes zero, while the horizontal component remains constant. The acceleration due to gravity is always vertically downwards. Therefore, at the uppermost point of a projectile, its velocity and the acceleration are at an angle of 90° .

49. a. For the person to be able to catch the ball, the horizontal component of velocity of the ball should be same as the speed of the person, i.e., $v_0 \cos \theta = \frac{v_0}{2}$ or $\cos \theta = \frac{1}{2}$ or $\theta = 60^\circ$

50. c. $u = 180 \text{ km/h} = 50 \text{ m/s}$

$$\text{Horizontal range} = u \sqrt{\frac{2h}{g}} = 50 \sqrt{\frac{2 \times 490}{9.8}} = 500 \text{ m}$$

51. b. The bullets are fired at the same initial speed. Hence

$$\frac{h}{h'} = \frac{u^2 \sin^2 60^\circ}{2g} \times \frac{2g}{u^2 \sin^2 30^\circ} = \frac{\sin^2 60^\circ}{\sin^2 30^\circ} = \frac{3}{1}$$

52. a. The time of flight is given by

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 30 \times 1/2}{10} = 3 \text{ s}$$

Thus, after 1.5 s the body will be at the highest point. So the direction of motion will be horizontal after 1.5 s, the angle with the horizontal is 0° .

53. c. Here angle of projection $= 45^\circ$

$$R_{\max} = 1.6 = \frac{u^2}{g} = \frac{u^2}{10} \text{ or } u = 4 \text{ ms}^{-1}$$

Hence, the distance covered in 10 s

$$= \text{horizontal speed} \times \text{time} = u \cos 45^\circ \times \text{time}$$

$$= 4 \times \frac{1}{\sqrt{2}} \times 10 = 20\sqrt{2} \text{ m}$$

54. i. a. If h be the maximum height attained by the projectile

$$\text{then } h = \frac{u^2 \sin^2 \theta}{2g} \text{ or } R = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{R}{h} = \frac{2 \sin \theta \cos \theta}{(\sin^2 \theta)/2} = 4 \cot \theta, \text{ therefore } \frac{\Delta R}{R} = \frac{\Delta h}{h}$$

$\therefore$ percentage increase in R = percentage increase in h = 5%

$$\text{ii d. } \frac{h}{T^2} = \frac{u^2 \sin^2 \theta}{2g} \times \frac{g^2}{4u^2 \sin^2 \theta} = \frac{g}{8}$$

$$\text{Thus } \frac{\Delta h}{h} = 2 \frac{\Delta T}{T}, \text{ i.e., } \frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta h}{h}$$

$$\text{or } 100 \times \frac{\Delta T}{T} = \frac{1}{2} \left[\frac{\Delta h}{h} \times 100 \right] = \frac{10}{2} = 5\%$$

55. d. Let $u_x = 3 \text{ m/s}$, $a_x = 0$

$$v_y = u_y + a_y t = 0 + 1 \times 4 = 4 \text{ ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = 5 \text{ m/s}$$

Angle made by the resultant velocity w.r.t. direction of initial velocity, i.e., x -axis, is

$$\beta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{4}{3}$$

56. c. $u_x = u$, $u_y = 0$, $v_x = u_x = u$

$$v_y = u_y + a_y t = 0 + gt = gt$$

$$\text{Resultant vel: } v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2 t^2}$$

57. i. a. Given $h = 490 \text{ m}$, $u = 15 \text{ m/s}$, Apply $T = \sqrt{\frac{2h}{g}}$

ii. c. $v_y = gT$

iii. c. Resultant velocity $= \sqrt{u^2 + g^2 T^2}$

58. b. $h = 150 - 27.5 = 122.5 \text{ m}$

$$\text{Time taken, } T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 122.5}{9.8}} = 5 \text{ s}$$

$$\text{Now } s = uT \text{ or } 30 = 5u \text{ or } u = 6 \text{ m/s}$$

$$59. \text{ d. } h = \frac{u^2 \sin^2 \theta}{2g} \text{ and } R = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{h}{R} = \frac{45}{180} = \frac{\tan \theta}{4} \text{ or } \theta = 45^\circ$$

$$60. \text{ d. Range} = 150 = ut \text{ and } h = \frac{15}{100} = \frac{1}{2} \times gt^2$$

$$\text{or } t^2 = \frac{2 \times 15}{100 \times g} = \frac{30}{1000} \text{ or } t = \frac{\sqrt{3}}{10}$$

$$u = \frac{150}{t} = \frac{150 \times 10}{\sqrt{3}} = 500\sqrt{3} \text{ ms}^{-1}$$

61. b. The horizontal range is the same for the angles of projection θ and $(90^\circ - \theta)$

$$t_1 = \frac{2u \sin \theta}{g}, \quad t_2 = \frac{2u \sin(90^\circ - \theta)}{g} = \frac{2u \cos \theta}{g}$$

$$t_1 t_2 = \frac{2u \sin \theta}{g} \times \frac{2u \cos \theta}{g} = \frac{2}{g} \left[\frac{u^2 \sin 2\theta}{g} \right] = \frac{2}{g} R$$

$$\text{where } R = \frac{u^2 \sin 2\theta}{g}$$

Hence, $t_1 t_2 \propto R$ (as R is constant).

62. c. $y_1 = \frac{u^2 \sin^2 \theta}{2g}, y_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$

$$\Rightarrow y_1 + y_2 = \frac{u^2}{2g}$$

63. b. Given $y = 12x - \frac{3}{4}x^2, u_x = 3 \text{ m/s}$

$$v_y = \frac{dy}{dt} = 12 \frac{dx}{dt} - \frac{3}{2} x \frac{dx}{dt}$$

At $x = 0, v_y = u_y = 12 \frac{dx}{dt} = 12u_x = 12 \times 3 = 36 \text{ m/s}$

$$a_y = \frac{d}{dt} \left(\frac{dy}{dt} \right) = 12 \frac{d^2x}{dt^2} - \frac{3}{2} \left(\frac{dx}{dt} + x \frac{d^2x}{dt^2} \right)$$

but $\frac{d^2x}{dt^2} = a_x = 0$, hence

$$a_y = -\frac{3}{2} \frac{dx}{dt} = -\frac{3}{2} u_x = -\frac{3}{2} \times 3 = -\frac{9}{2} \text{ m/s}^2$$

$$\text{Range } R = \frac{2u_x u_y}{a_y} = \frac{2 \times 3 \times 36}{-9/2} = 16 \text{ m}$$

Alternatively: We have $y = 12x - \frac{3}{4}x^2$. When projectile again comes to ground, then $y = 0$ and $x = R$.

$$0 = 12R - \frac{3}{4}R^2 \Rightarrow R = 16 \text{ m}$$

64. d. Range is same for angles of projection θ and $90 - \theta$.

$$R = \frac{u^2 \sin 2\theta}{g}, h_1 = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{and } h_2 = \frac{u^2 \sin^2(90 - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}, \text{ Hence}$$

$$\sqrt{h_1 h_2} = \frac{u^2 \sin \theta \cos \theta}{2g} = \frac{1}{4} \left[\frac{u^2 \sin 2\theta}{g} \right] = \frac{R}{4}$$

65. i. b. Retardation due to the friction of air = $\frac{g}{10}$. Hence, in upward motion:

$$\text{total retardation } g_1 = g + \frac{g}{10} = \frac{11g}{10}$$

$$H_m = \frac{u^2 \sin^2 \theta}{2g} \text{ and}$$

$$H'_m = \frac{u^2 \sin^2 \theta}{2 \times \frac{11g}{10}} = \frac{10}{11} \times \frac{u^2 \sin^2 \theta}{2g} = \frac{10}{11} H_m$$

$$\% \text{ decreases in } H_m = \frac{H_m - H'_m}{H_m} \times 100$$

$$= \left(1 - \frac{10}{11} \right) \times 100 = 9\%$$

ii. d. $t = \frac{u \sin \theta}{g}, t' = \frac{u \sin \theta}{g'} = \frac{u \sin \theta}{\frac{11g}{10}} = \frac{10}{11} t$

$$\% \text{ decrease in } t = \frac{t - t'}{t} \times 100 = \left(1 - \frac{10}{11} \right) \times 100 = 9\%$$

iii. a. Here $H'_m = \frac{u^2 \sin^2 \theta}{2 \times \frac{11g}{10}} = \frac{10u^2 \sin^2 \theta}{22g}$

Using $x = ut + \frac{1}{2}at^2$ where $x = H'_m, u = 0$ and

$$a = g - \frac{g}{10} = \frac{9g}{10}; \text{ we find } t = \sqrt{2H'_m/a}$$

$$= \sqrt{\frac{200}{198}} \times \frac{u \sin \theta}{g}$$

It is almost equal to the time of fall in the absence of friction.

66. b. $\frac{R}{T^2} = g \frac{\sin 2\theta}{4 \sin^2 \theta} = \frac{g}{2} \cot \theta = 5 \cot \theta$

Given $\frac{R}{T^2} = 5$; hence $5 = 5 \cot \theta$ or $\theta = 45^\circ$

67. b. $t = \frac{2u \sin \theta}{g}$ or $2 = \frac{2u \sin \theta}{g}$ or $u \sin \theta = g$

$$h_m = \frac{u^2 \sin^2 \theta}{2g} = \frac{g^2}{2g} = \frac{g}{2} = 5 \text{ m}$$

68. c. If the ball hits the n th step, then the horizontal distance traversed = nh . Here, the velocity along the horizontal direction = u . Initial velocity along the vertical direction = 0. So

$$nb = ut \quad (i)$$

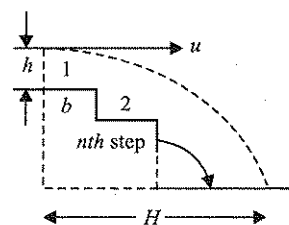


Fig. 5.163

$$nh = 0 + \frac{1}{2}gt^2 \quad (ii)$$

From $t = \frac{nb}{u}$, putting t in equation (ii)

$$nh = \frac{1}{2}g \times \left(\frac{nb}{u} \right)^2 \text{ or } n = \frac{2hu^2}{gb^2}$$

69. b. Here,
- $u_x = 6$
- and
- $u_y = 8$

$$R = \frac{2u_x u_y}{g} = \frac{2 \times 6 \times 8}{10} = 9.6 \text{ m}$$

70. c.
- $v^2 = u^2 - 2gh$
- or
- $u^2 = v^2 + 2gh$

$$\text{or } u_x^2 + u_y^2 = v_x^2 + v_y^2 + 2gh$$

$$u_y^2 = v_y^2 + 2gh \text{ or } u_y^2 = (2)^2 + 2 \times 10 \times 0.4 = 12$$

$$u_y = \sqrt{12} = 2\sqrt{3} \text{ m/s, } u_x = v_x = 6 \text{ m/s}$$

$$\tan \theta = \frac{u_y}{u_x} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

71. c. At the two points of the trajectory during projectile motion, the horizontal component of the velocity is same. Then
- $u \cos 60^\circ = v \cos 45^\circ$

$$\Rightarrow 150 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}} \text{ or } v = \frac{150\sqrt{2}}{\sqrt{2}} \text{ m/s}$$

$$\text{Initially : } u_y = u \sin 60^\circ = \frac{150\sqrt{3}}{2} \text{ m/s}$$

$$\text{Finally : } v_y = v \sin 45^\circ = \frac{150}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{150}{2} \text{ m/s}$$

$$\text{But } v_y = u_y + a_y t \text{ or } \frac{150}{2} = \frac{150\sqrt{3}}{2} - 10t$$

$$10t = \frac{150}{2}(\sqrt{3} - 1) \text{ or } t = 7.5(\sqrt{3} - 1) \text{ s}$$

72. a. A bullet fired at angle
- 45°
- will fall maximum away, and all other bullets will fall with this bullet fired at
- 45°
- .
- $R_{\max} = \frac{u^2}{g}$

$$\text{Maximum area covered} = \pi (R_{\max})^2 = \pi \left(\frac{u^2}{g} \right)^2$$

73. d. If there were no gravity the bullet would reach height
- H
- in the time
- t
- taken by it to travel the horizontal distance
- X
- , i.e.,

$$H = u \sin \theta t \text{ and } x = u \cos \theta t \text{ or } t = \frac{x}{u \cos \theta}$$

However, because of gravity the bullet has an acceleration g vertically downwards, so in time t the bullet will reach a height

$$y = u \sin \theta \times t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2$$

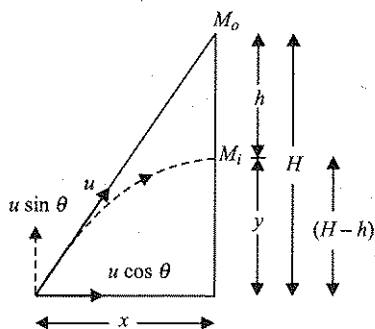


Fig. 5.164

This is lower than H by $\frac{1}{2}gt^2$ which is exactly the amount the monkey falls in this time. So the bullet will hit the monkey regardless of the initial velocity of the bullet so long as it

is great enough to travel the horizontal distance to the tree before hitting the ground. (For large u lesser will be the time of motion; so the monkey is hit near its initial position and for smaller u it is hit just before it reaches the floor.) Bullet will hit the monkey only and only if

$$y > 0, \text{ i.e., } H - \frac{1}{2}gt^2 > 0$$

$$\text{or } H > \frac{1}{2}gt^2 \text{ or } H > \frac{1}{2}g \frac{x^2}{u^2 \cos^2 \theta}$$

$$\text{or } u > \frac{x}{\cos \theta} \sqrt{\frac{g}{2H}} \text{ or } u > \sqrt{\frac{g}{2H}} (x^2 + H^2) = u_0$$

If $u < u_0$, the bullet will hit the ground before reaching the monkey.

74. a. The motion of the train will affect only the horizontal component of the velocity of the ball. Since, vertical component is same for both observers, the
- h_m
- will be same, but
- R
- will be different.

$$\begin{aligned} 75. \text{ d. } \frac{h_2}{h_1} &= \frac{u^2 \sin^2 \theta_2}{2g} \times \frac{2g}{u^2 \sin^2 \theta_1} = \frac{\sin^2 \theta_2}{\sin^2 \theta_1} \\ &= \frac{\sin^2 \pi/6}{\sin^2 \pi/3} = \frac{1}{3} \end{aligned}$$

$$\Rightarrow h_2 = \frac{h_1}{3}$$

76. c. Suppose the angle made by the instantaneous velocity with the horizontal be
- α
- . Then

$$\tan \alpha = \frac{v_y}{v_x} = \frac{u \sin \theta - gt}{u \cos \theta}$$

Given that: $\alpha = 45^\circ$, when $t = 1$ s; $\alpha = 0^\circ$, when $t = 2$ s

$$\text{This gives: } u \cos \theta = u \sin \theta - g \quad (1)$$

$$u \sin \theta g - 2g = 0 \quad (2)$$

Solving equations (1) and (2), we find

$$\text{using } \theta = 2g \text{ and } u \cos \theta = g$$

$$\text{Squaring and adding: } u = \sqrt{5}g = 10\sqrt{5} \text{ m/s}$$

77. c.
- $v \cos 45^\circ = u = 18 \text{ m/s} \Rightarrow v = 18\sqrt{2} \text{ m/s}$

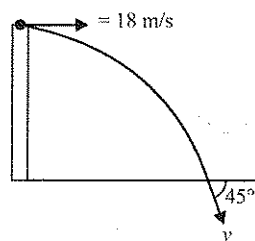


Fig. 5.165

$$\text{Vertical component: } v \sin 45^\circ = 18\sqrt{2} \times \frac{1}{\sqrt{2}} = 18 \text{ m/s}$$

78. b.
- $v_x = u_x = 100 \text{ m/s, } v_y = u_y + a_y t = 0 + 10 \times 10$

$$\tan \theta = \frac{v_y}{v_x} = \frac{100}{100} = 1 \Rightarrow \theta = 45^\circ$$

79. a. $u_x = 16 \cos 60^\circ = 8 \text{ m/s}$

Time taken to reach the wall = $8/8 = 1 \text{ s}$

Now $u_y = 16 \sin 60^\circ = 8\sqrt{3} \text{ m/s}$

$$h = 8\sqrt{3} \times 1 - \frac{1}{2} \times 10 \times 1 = 13.86 - 5 = 8.9 \text{ m}$$

80. b. When a projectile is projected at an angle θ or at an angle $(90^\circ - \theta)$ with the horizontal, the horizontal range remains the same. The horizontal range is maximum when the angle of projection is 45° .

81. d. $t_{OA} = t_{BC}$ (Fig. 5.166)

To find: $t_{OA} + t_{OB}$

$$t_{OA} + t_{OB} = t_{BC} + (t_{OB}) = T$$

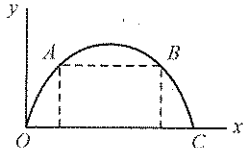


Fig. 5.166

82. b. Let at any time t , the ball is at height of 15 m.

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow 15 = 4 \sin \theta t - \frac{1}{2} g t^2$$

$$\Rightarrow 15 = 52 \times \frac{5}{13} t - \frac{1}{2} \times 10 t^2$$

$$\Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t-3) = 0$$

$$\Rightarrow t = 1 \text{ s}, t = 3 \text{ s, required time: } 3 - 1 = 2 \text{ s.}$$

83. i. b. $h = \frac{u^2 \sin^2 \theta}{2g} = \frac{(56)^2 \sin^2 \theta}{19.6}$

$$\sin^2 \theta = \frac{40 \times 19.6}{(56)^2} = \frac{1}{4} \text{ or } \sin \theta = \frac{1}{2} \text{ or } \theta = 30^\circ \text{ s}$$

ii. a. $R = \frac{u^2 \sin 2\theta}{g} = \frac{(56)^2 \sin 60^\circ}{9.8}$

$$= \frac{56 \times 56 \times \sqrt{3}}{19.6} = 160\sqrt{3} \text{ m}$$

84. d. Angle of projection from B is 45° . As the body is able to cross the well of diameter 40 m, hence $R = \frac{v^2}{g}$ or

$$v = \sqrt{gR} = \sqrt{10 \times 40} = 20 \text{ m/s}$$

On the inclined plane, the retardation is:

$$g \sin \alpha = g \sin 45^\circ = 10\sqrt{2} \text{ m/s}^2$$

$$\text{Using } v^2 - u^2 = 2ax$$

$$(20)^2 - u^2 = 2 \times \left(-\frac{10}{\sqrt{2}}\right) \times 20\sqrt{2}$$

$$u = 20\sqrt{2} \text{ ms}^{-1}, \text{ i.e., } V = 20\sqrt{2} \text{ m/s}$$

85. b. Time taken by bullet to reach the target

$$= \frac{\text{distance}}{\text{velocity}} = \frac{\text{distance}}{u \cos \theta}, \text{ as } \theta \text{ is very small, } \cos \theta = 1$$

$$\text{Time} = \frac{\text{distance}}{u} = \frac{400}{400} = 1 \text{ s}$$

Vertical deflection of bullet

$$= \frac{1}{2} g t^2 = \frac{1}{2} \times 10 \times (1)^2 = 5 \text{ m}$$

86. c. From Fig. 5.167

$$\tan \phi = \frac{H}{R/2} = \frac{u^2 \sin^2 \theta / 2g}{u^2 \sin 2\theta / 2g} = \frac{\sin^2 \theta}{\sin 2\theta} = \frac{1}{2} \tan \theta$$

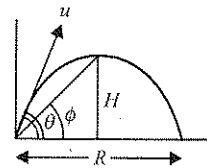


Fig. 5.167

87. c. $H = \frac{u^2 \sin^2 \theta}{2g}$ or $80 = \frac{u^2 \sin^2 \theta}{2 \times 10}$

$$\text{or } u^2 \sin^2 \theta = 1600 \text{ or } u \sin \theta = 40 \text{ ms}^{-1}$$

$$\text{Horizontal velocity} = u \cos \theta \text{ at } 3 \times 30 = 90 \text{ ms}^{-1}$$

$$\frac{u \sin \theta}{u \cos \theta} = \frac{40}{90} \text{ or } \tan \theta = \frac{4}{9} \text{ or } \theta = \tan^{-1} \left(\frac{4}{9} \right)$$

88. b. $h = (u \sin \theta)t - \frac{1}{2} g t^2$

$$d = (u \cos \theta)t \text{ or } t = \frac{d}{u \cos \theta}$$

$$h = u \sin \theta \times \frac{d}{u \cos \theta} - \frac{1}{2} g \times \frac{d^2}{u^2 \cos^2 \theta}$$

$$u = \frac{d}{\cos \theta} \sqrt{\frac{g}{2(d \tan \theta - h)}}$$

89. c. Given $\frac{\sqrt{3}u}{2} = u \cos \theta = \text{speed at maximum height or}$

$$\cos \theta = \frac{\sqrt{3}}{2} \text{ or } \theta = 30^\circ \quad (i)$$

$$\text{Given that } PH_{\max} = R \quad (ii)$$

$$\text{We know } H_{\max} = \frac{R \tan \theta}{4}$$

$$P = \frac{R}{H_{\max}} = \frac{4}{\tan \theta} = \frac{4}{\tan 30^\circ} = 4\sqrt{3}$$

90. c. $y = ax - bx^2$, for height or y to be maximum: $\frac{dy}{dx} = 0$ or

$$a - 2bx = 0 \text{ or } x = \frac{a}{2b}$$

i. $y_{\max} = a \left(\frac{a}{2b} \right) - b \left(\frac{a}{2b} \right)^2 = \frac{a^2}{4b}$

ii. $\left(\frac{dy}{dx} \right)_{x=0} = a = \tan \theta_0$ where $\theta_0 = \text{angle of projection}$
 $\theta_0 = \tan^{-1} \frac{a}{b}$

91. a. $\tan \theta = \frac{u \sin \theta}{u \cos \theta} = \frac{2}{1}$, The desired equation is

$$y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta} = x \times 2 - \frac{10 x^2}{2 (\sqrt{2^2 + 1^2})^2 \left(\frac{1}{\sqrt{5}} \right)^2}$$

$$\text{or } y = 2x - 5x^2$$

$$92. \text{ c. Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{\sqrt{H^2 + R^2/4}}{T/2}$$

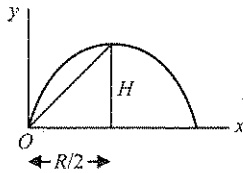


Fig. 5.168

Here, H = max height

$$= \frac{v^2 \sin^2 \theta}{2g}$$

$$R = \text{range} = \frac{v^2 \sin 2\theta}{g}$$

$$\text{and } T = \text{time of flight} = \frac{2v \sin \theta}{g}$$

$$v_{av} = \frac{v}{2} \sqrt{1 + 3 \cos^2 \theta}$$

$$93. \text{ b. } \frac{u^2 \sin 2\theta}{g} = \frac{(u/2)^2 \sin 30^\circ}{g} = \frac{u^2}{8g}$$

$$\therefore \sin 2\theta = \frac{1}{8} \text{ or } \theta = \frac{1}{2} \sin^{-1} \left(\frac{1}{8} \right)$$

$$94. \text{ c. } x = v \sqrt{\frac{2h}{g}}, 2x = v' \sqrt{\frac{2(2h)}{g}} \text{ Solve to get: } v' = \sqrt{2}v$$

$$95. \text{ b. } T = \frac{100}{25} = 4 \text{ s} \Rightarrow \frac{2u \sin \theta}{g} = 4 \Rightarrow u \sin \theta = 20 \text{ m/s}$$

$$96. \text{ d. } H = \frac{u^2 \sin^2 \theta}{2g}, R = \frac{u^2 \sin 2\theta}{g}$$

maximum height will be same because acceleration $a = g/4$ is in horizontal direction

$$R' = u \cos \theta T + \frac{1}{2} a T^2 = R + \frac{1}{2} g \left(\frac{2u \sin \theta}{g} \right)^2 = R + H$$

$$97. \text{ a. } t_{AB} = \text{time of flight of projectile} = \frac{2u \sin(\alpha - 30^\circ)}{g \cos 30^\circ}$$

Now component of velocity along the plane becomes zero at point B.

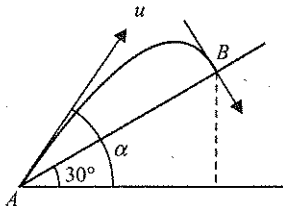


Fig. 5.169

$$0 = u \cos(\alpha - 30^\circ) - g \sin 30^\circ \times T$$

$$\text{or } u \cos(\alpha - 30^\circ) = g \sin 30^\circ \times \frac{2u \sin(\alpha - 30^\circ)}{g \cos 30^\circ}$$

$$\text{or } \tan(\alpha - 30^\circ) = \frac{\cot 30^\circ}{2} = \frac{\sqrt{3}}{2}$$

$$\text{or } \alpha = 30^\circ + \tan^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

$$98. \text{ a. Horizontal component of velocity, } u_H = u \cos 60^\circ = \frac{u}{2}$$

$$AC = u_H \times t = \frac{ut}{2} \text{ and}$$

$$AB = AC \sec 30^\circ = \left(\frac{ut}{2} \right) \left(\frac{2}{\sqrt{3}} \right) = \frac{ut}{\sqrt{3}}$$

99. d. Speed in horizontal direction remains constant during whole journey because there is no acceleration in this direction. So, $v_h = 5 \text{ ms}^{-1}$

In vertical direction: Loss of gravitation potential energy

$$= \text{gain in } KE \text{ i.e., } mgh = \frac{1}{2} m v_v^2$$

$$v_v^2 = 2gh = 2 \times 10 \times (70 - 60) = 200$$

Hence, the speed with which he touches the cliff B is:

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{25 + 200} = \sqrt{225} = 15 \text{ ms}^{-1}$$

$$100. \text{ c. } AB = u \sqrt{\frac{2h}{g}}$$

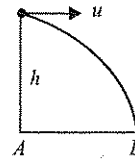


Fig. 5.170

$$= 600 \times \frac{5}{18} \sqrt{\frac{2 \times 2000}{10}} = 3333 \text{ m} = 3.33 \text{ km}$$

$$101. \text{ b. } 1.5 = \frac{u^2 \sin(2 \times 15^\circ)}{g} \Rightarrow \frac{u^2}{g} = 3$$

$$R = \frac{u^2}{g} \sin(2 \times 45^\circ) = 3 \text{ km}$$

$$102. \text{ c. } x = 6t, u_x = \frac{dx}{dt} = 6$$

$$y = 8t - 5t^2, v_y = \frac{dy}{dt} = 8 - 10t$$

$$V_y(t=0) = 8 \text{ m/s}$$

$$u = \sqrt{u_x^2 + v_y^2} = \sqrt{6^2 + 10^2} = 10 \text{ m/s}$$

$$103. \text{ b. } \tan \theta = \frac{u_y}{u_x} = \frac{8}{6} = \frac{4}{3}$$

104. c. vertical component of both should be same:

$$v_2 = v_1 \sin 30^\circ \Rightarrow \frac{v_2}{v_1} = \frac{1}{2} = 0.5$$

105. d. Apply equation of trajectory:

$$0.5 = \frac{\sqrt{3}}{2} \tan 30^\circ - \frac{g \left(\frac{\sqrt{3}}{2} \right)}{2v_0^2 \cos^2 30^\circ} \Rightarrow v_0 = \infty$$

$$106. \text{ b. } a_c = \frac{v^2}{r} = \frac{5^2}{2} = 12.5 \text{ m/s}^2$$

107. a. Since velocity is in tangent direction so its component along radial direction is zero.

108. c. $a_t = 2 \text{ m/s}^2$, $v = u + a_t t = 1 + 2 \times 2 = 5 \text{ m/s}$

$$a_c = \frac{v^2}{r} = \frac{5^2}{25} = 1 \text{ m/s}^2$$

$$\text{net acc} = \sqrt{a_c^2 + a_t^2} = \sqrt{1^2 + 2^2} = \sqrt{5} \text{ m/s}^2$$

109. c. $a_c = \frac{v^2}{r} \rightarrow$ constant in magnitude if v is constant.

110. c. After releasing the string, centripetal acceleration will become zero, due to which direction of velocity cannot change now and stone flies tangentially.

111. a. Angular velocity is always directed perpendicular to the plane of the circular path. Hence, required change in angle = 0° .

112. b. Angular acceleration and angular velocity are along the axis of circular path. So they cannot be perpendicular to each other.

113. c. $\omega = 300 \text{ rpm} = 300 \times 2\pi/60 = 10\pi \text{ rad/s}$

114. c. $r = 1 \text{ km} = 1000 \text{ m}$, $v = 900 \text{ km/h} = 900 \times 5/18 = 250 \text{ m/s}$

$$a_c = \frac{v^2}{r} = \frac{(250)^2}{1000} = 62.5 \text{ m/s}^2$$

115. d. $\alpha = (\omega_2 - \omega_1)/t = (400 - 100)/5 = 60 \text{ rev/min}^2$

$$= \frac{60 \times 2\pi}{(60)^2} = \frac{2\pi}{60} \text{ rad/s}^2$$

$$a_t = \alpha r = \frac{2\pi}{60} \times \frac{50}{100} = \frac{\pi}{60} \text{ m/s}^2$$

Multiple Correct Answers Type

1. a., c. For angle θ in west of north,

$$\sin \theta = \frac{5}{10} = 0.5, \text{ i.e., } \theta = 30^\circ$$

$$V_r = \text{Resultant velocity} = \sqrt{(10)^2 - (5)^2} = 5\sqrt{3} \text{ ms}^{-1}$$

So, options (a) and (c) are correct

2. a., d. $\vec{v}_r = -30\hat{j}$, $\vec{v}_m = \frac{10\sqrt{3}}{2}\hat{i} + \frac{10}{2}\hat{j}$

$$\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m = -5\sqrt{3}\hat{i} - 35\hat{j}$$

$$\tan \theta = \frac{5\sqrt{3}}{35} = \frac{\sqrt{3}}{7} \Rightarrow \theta = \tan^{-1} \left(\frac{\sqrt{3}}{7} \right) \text{ with vertical}$$

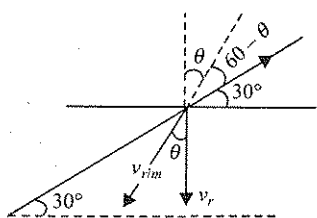


Fig. 5.171

Angle of $v_{r/m}$ with inclined plane is $60 - \theta = \beta$

$$\begin{aligned} \tan \beta &= \tan(60 - \theta) = \frac{\tan 60^\circ - \tan \theta}{1 + \tan 60^\circ \times \tan \theta} \\ &= \frac{\sqrt{3} - \sqrt{3}/7}{1 + \sqrt{3} \times \sqrt{3}/7} = \frac{3\sqrt{3}}{5} \end{aligned}$$

$$\Rightarrow \beta = \tan^{-1} \left(\frac{3\sqrt{3}}{5} \right)$$

3. c., d. Distance between two buses on road = $v_b T$

For A to B direction: distance = relative velocity $\times$ time,

$$v_b T = (v_b - v_c) T_1 \Rightarrow T_1 = \frac{v_b T}{v_b - v_c}$$

For B to A direction:

$$v_b T = (v_b + v_c) T_2 \Rightarrow T_2 = \frac{v_b T}{v_b + v_c}$$

4. b., c., d. If they collide, their vertical component of velocities should be same, i.e.

$$100 \sin \theta = 160 \sin 30^\circ \Rightarrow \sin \theta = 4/5$$

Their vertical components will always be same. Horizontal components:

$$160 \cos 30 = 80\sqrt{3} \text{ m/s}$$

and

$$100 \cos \theta = 100 \times 3/5 = 60 \text{ m/s}$$

They are not same, hence their velocities will not be same at any time. So (b) is correct.

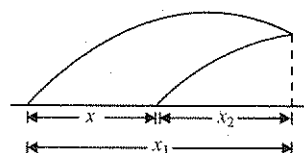


Fig. 5.172

$$x = x_1 - x_2 = 160 \cos 30^\circ t - 100 \cos \theta t$$

$$\Rightarrow x = (80\sqrt{3} - 60)t$$

$$\text{Time of flight: } T = \frac{2 \times 160 \times \sin 30}{g} = 16 \text{ s}$$

Now $t < T_x \rightarrow$ to collide in air

$$\Rightarrow \frac{x}{80\sqrt{3} - 60} < 16 \Rightarrow x < 1280\sqrt{3} - 960$$

Since their times of flight are the same, they will simultaneously reach their maximum height. So it is possible to collide at highest point for certain values of x .

5. a., c., d. [K.E. + P.E.] = [P.E.] projection point with ground

In all situation K.E. (i.e., speed) at the ground are equal. i.e., option a. is correct.

$$h = vt_1 + \frac{1}{2}gt_1^2 \text{ [for (first) particle]} \quad (i)$$

$$h = vt_2 - \frac{1}{2}gt_2^2 \text{ [for (second) particle]} \quad (ii)$$

From equation (i) and equation (ii) $t_2 > t_1$

(t_2) = maximum, (t_1) = minimum

i.e., option (c) and (d) are correct.

6. **b., c.** $y = \frac{x}{2}$ implies that the particle is moving in a straight line passing through origin.

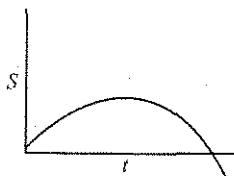


Fig. 5.173

$$v_x = 4 - 2t, \quad v_x = u_x + a_x t, \quad u_x = 4, \quad a_x = -2$$

$$\text{Now, } y = \frac{x}{2} \Rightarrow \frac{dy}{dt} = \frac{1}{2} \frac{dx}{dt}$$

$$v_y = \frac{1}{2} v_x = 2 - t, \quad v_y = u_y + a_y t,$$

$$u_y = 2 \text{ and } a_y = -1$$

7. **b., c.** Since the particle is dropped, it means that the initial velocity of the particle is equal to zero. But the particle is blown over by a wind with a constant velocity along horizontal direction, therefore, the particle has a horizontal component of velocity. Let this component be v_0 . Then it may be assumed that the particle is projected horizontally from the top of the tower with velocity v_0 .

Hence, for the particle, initial velocity $u = v_0$ and angle of projection $\theta = 0^\circ$.

We know equation of trajectory is:

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\text{Here, } y = \frac{gx^2}{2v_0^2}$$

The slope of the trajectory of the particle

$$\frac{dy}{dx} = -\frac{2gx}{2v_0^2} = -\frac{g}{v_0^2} x$$

Hence, the curve between the slope and x will be a straight line passing through the origin and will have a negative slope. It means that option (b) is correct.

Since, horizontal velocity of the particle remains constant, therefore $x = v_0 t$. We get $\frac{dy}{dx} = -\frac{gt}{v_0}$

So the graph between m and time t will have the same shape as the graph between m and x . Hence, option (a) is wrong.

The vertical component of velocity of the particle at time t is equal to gt . Hence, at time t , KE of the particle, $KE = \frac{1}{2} m [(gt)^2 + (v_0)^2]$

It means, the graph between KE and time t should be a parabola having value $\frac{1}{2} m v_0^2$ at $t = 0$. Therefore, option (c) is correct.

As the particle falls, its height decreases and KE increases. The KE increases linearly with height of its fall or the graph between KE and height of the particle will be

a straight line having negative slope. Hence, option (d) is wrong.

8. **a., c., d.** Angular speed is constant. Hence, linear speed is also constant, i.e., magnitude of velocity is constant, but direction is changing.
9. **b., c., d.** If the particle is projected with velocity u at angle θ , then equation of its trajectory will be:

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

We know slope is given by $m = \frac{dy}{dx}$

$$\text{Therefore, slope } m = \tan \theta - \frac{gx}{u^2 \cos^2 \theta}$$

It implies that the graph between slope and x will be straight line having negative slope and a non-zero positive intercept on y -axis.

But x is directly proportional to the time t , therefore, the shape of graph between slope and time will be same as that of the graph between slope and x . Hence, only option (a) is correct, i.e., option (b), (c), and (d) are incorrect.

10. **a., b.** $x = a \cos(pt)$, $y = b \sin(pt)$

$$\text{Equation of path in } x\text{-}y \text{ plane: } \left[\frac{x}{a}\right]^2 + \left[\frac{y}{b}\right]^2 = 1$$

i.e., the path of the particle is an ellipse.

Position vector of a point P is:

$$\vec{r} = a \cos pt \hat{i} + b \sin pt \hat{j}$$

$$\Rightarrow \vec{v} = p(-a \sin pt \hat{i} + b \cos pt \hat{j})$$

$$\text{and, } \vec{a} = -p^2(a \cos pt \hat{i} + b \sin pt \hat{j}) = -p^2 \vec{r}$$

$$\text{also, } \vec{v} \cdot \vec{a} = 0 \text{ at } t = \pi/2p$$

11. **a., b.** Time of ascent $= 2 + 1 = 3$ s

$$\Rightarrow \frac{u \sin \theta}{g} = 3 \Rightarrow u \sin \theta = 30$$

$$\text{And } \tan \theta = \frac{u \sin \theta - gt}{u \cos \theta} \Rightarrow \tan 30^\circ = \frac{30 - 10 \times 2}{u \cos \theta}$$

$$\Rightarrow u \cos \theta = 10\sqrt{3}$$

$$\text{From here } u = \sqrt{(10\sqrt{3})^2 + 10^2} = 20\sqrt{3} \text{ m/s}$$

$$\text{And } \tan \theta = \frac{30}{10\sqrt{3}} = \sqrt{3} \Rightarrow \theta = 60^\circ$$

$$\begin{aligned} 12. \text{ a., b., c., d. } v &= 2t, a_c = \frac{v^2}{r} = \frac{(2t)^2}{0.2} \\ &= 20t^2 = 20 \times 2^2 = 80 \text{ m/s}^2 \end{aligned}$$

$$a_t = \frac{dv}{dt} = 2 \text{ m/s}^2$$

$$\text{Net acceleration: } a = \sqrt{a_c^2 + a_t^2} > 80 \text{ m/s}^2$$

Assertion-Reasoning Type

1. **d.** At the highest point only horizontal component of velocity is present, and vertical component is zero.
2. **c.** $T = \frac{2u \sin \theta}{g} \Rightarrow T \propto u$, $R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R \propto u^2$

3. b. If we cut the string anywhere the bob will follow the parabolic path.
4. b. Before attaining the maximum height, angle is acute and after this the angle is obtuse. At the highest point it is perpendicular.
5. d. Projectile motion is a motion with constant acceleration but it is not a straight line motion.
A body with a constant magnitude of acceleration may not speed up, this is possible in uniform circular motion.
6. b. In a non-uniform circular motion, due to change in magnitude of velocity, tangential acceleration arises and due to change in direction of velocity centripetal acceleration is produced.
7. c. In a uniform circular motion, velocity is along tangential direction and acceleration is always towards centre, so angle between velocity vector and acceleration vector is always $\frac{\pi}{2}$. But in general, the angle between the velocity and the acceleration can be acute or obtuse also.
8. a. Since the relative acceleration is zero and initial relative velocity is also zero, so relative velocity at any moment will be zero.
9. a. Time taken is shortest when one aims perpendicular to the flow.
10. c. $v_{r/m} = \sqrt{v_r^2 + v_m^2}$

Comprehensive Type

For Problems 1–2

1. b., 2. a.

Sol. OA represents velocity of man due east.

OQ represents relative velocity of rain w.r.t. man.

Actual velocity of rain is represented by OR (Fig. 5.174).

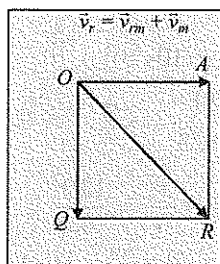


Fig. 5.174

$$OR = \sqrt{OQ^2 + RQ^2} = \sqrt{3^2 + 3^2} = 3\sqrt{2} \text{ km/hr}$$

$$\text{Also, } \tan \theta = \frac{QR}{OQ} = 1 \Rightarrow \theta = 45^\circ$$

The rain is falling at 45° east of vertical with a velocity $3\sqrt{2} \text{ km h}^{-1}$. $\vec{v}_r = \vec{v}_{rm} + \vec{v}_m$ (Fig. 5.175)

For Problems 3–4

3. a., 4. b.

Sol. Assume that the person starts from point A and tries to swim

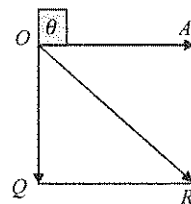


Fig. 5.175

towards B (AB is perpendicular to river flow) (Fig. 5.176)

$$\vec{v}_S = \vec{v}_{SR} + \vec{v}_R$$

$\vec{v}_{SR}$ = velocity of swimmer relative to river

$\vec{v}_R$ = velocity of river

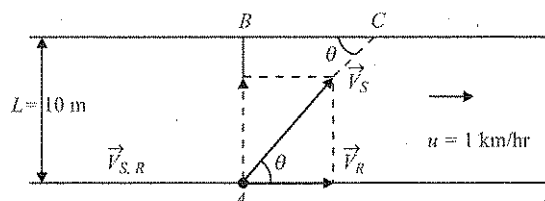


Fig. 5.176

Hence the net velocity of the swimmer is directed from A towards C at an angle θ with the river.

$$\theta = \tan^{-1} \frac{1}{2}$$

From triangle ABC, $BC = AB \cot \theta = (10)(2) = 20 \text{ m}$
 $\therefore$ the swimmer lands on the other side of the river at a point C which is 20 m from the point B, towards which he was trying to swim.

$$\text{time} = S_y / V_y = 10 \text{ m} / 1 \text{ kmhr}^{-1} = 36 \text{ s}$$

Method 2:

$$\vec{v}_S = \vec{v}_{s,R} + \vec{v}_R \quad \vec{v}_{s,R} = 1\hat{j} \text{ km/hr} \quad \vec{v}_R = 2\hat{i} \text{ km/hr}$$

$$\vec{v}_S = 1\hat{j} + 2\hat{i} \text{ (km/hr)} = 2\hat{i} + \hat{j} \text{ (km/hr)}$$

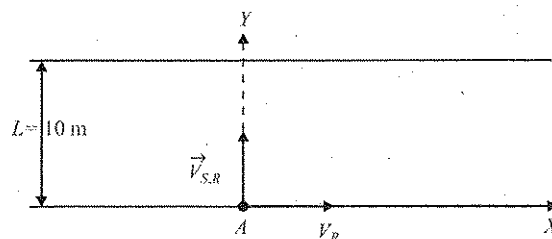


Fig. 5.177

Time to cross river

$$t = \frac{\text{Displacement in y-direction}}{\text{Velocity in y-direction}} = \frac{10}{5/18} = 36 \text{ sec}$$

For Problems 5–6

5. c., 6. b.

Sol. Time taken to cross the river

$$t = \frac{L}{V_{b,w}} = \frac{400}{10}$$

$$t = 40 \text{ s.}$$

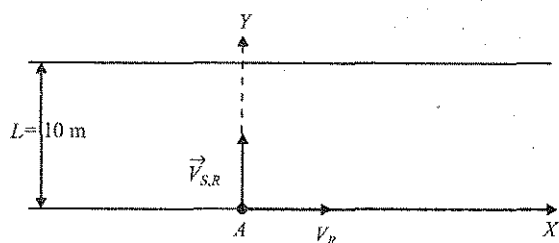


Fig. 5.178

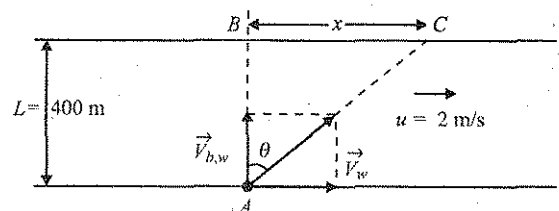


Fig. 5.179

$$\text{Drifting } BC = U_w \times t$$

$$BC = 2 \times 40 = 80 \text{ m.}$$

$$\text{Method 2: } \vec{v}_b = \vec{v}_{b,w} + \vec{v}_w = 10\hat{j} + 2\hat{i}$$

$$\vec{v}_b = 2\hat{i} + 10\hat{j} \text{ m/s}$$

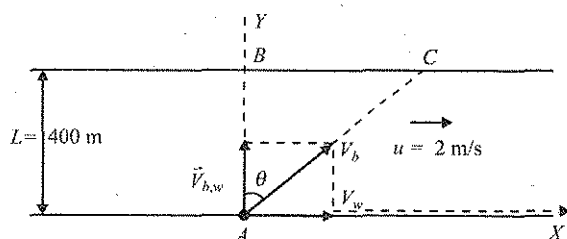


Fig. 5.180

Time to cross the river (Fig. 5.180)

$$t = \frac{L}{V_{b,w}} = \frac{400}{10} = 40 \text{ s}$$

$$\text{Drifting } BC = u \times t = 2 \times 40 = 80 \text{ s.}$$

For Problems 7–10

7. a., 8. c., 9. d., 10. d.

Sol. B is a point directly opposite to the starting point A.

Let the man heads the boat in a direction making an angle θ with the line AB. Here, $\vec{v}_w = 2\hat{i}$

$$\vec{v}_{bw} = -4 \sin \theta \hat{i} + 4 \cos \theta \hat{j}$$

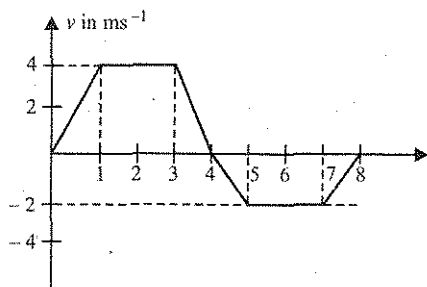


Fig. 5.181

$$\vec{v}_{b(\text{absolute})} = \vec{v}_{bw} + \vec{v}_w = (2 - 4 \sin \theta)\hat{i} + 4 \cos \theta \hat{j}$$

$$v_{bx} = 2 - 4 \sin \theta, v_{by} = 4 \cos \theta$$

7. a. For directly opposite point $v_{bx} = 0$

$$\sin \theta = \frac{1}{2} = \sin 30^\circ \Rightarrow \theta = 30^\circ$$

Hence to reach the point directly opposite to the starting point he should head the boat at an angle

$$\beta = (90 + 30) = 120^\circ \text{ with the river flow.}$$

$$8. \text{ c. } t = \frac{y}{v_{by}} \Rightarrow t = \frac{d}{4 \cos \theta} = \frac{4}{4 \cos 30^\circ} = \frac{2}{\sqrt{3}} \text{ hr.}$$

9. d. For t to be minimum $\cos \theta = 1 \Rightarrow \theta = 0^\circ$

$$t_{\min} = \frac{4}{4 \cos 0} = 1 \text{ hr.}$$

$$10. \text{ d. } T_1 = \frac{2}{(4 - 2)} = 1 \text{ hr, } T_2 = \frac{2}{(4 + 2)} = \frac{1}{3} \text{ hr}$$

$$T = \frac{2}{(4 - 2)} \text{ hr} + \frac{2}{(4 + 2)} \text{ hr} = \left(1 + \frac{1}{3}\right) \text{ hr} = \frac{4}{3} \text{ hr}$$

For Problems 11–12

11. c., 12. d.

Sol. Taking N as +Y-axis and E as +X-axis.

Imagine yourself as an observer sitting inside the car. You will regard the car as being at rest (at C). Relative to you, the speed of the motorcyclist is obtained by imposing the reversed velocity of the car on motorcyclist as shown in the Fig. 5.182.

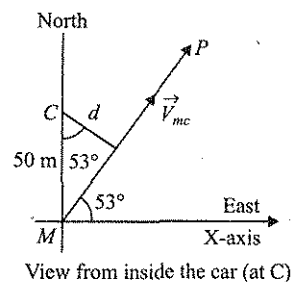


Fig. 5.182

$$v_{mC} = \sqrt{15^2 + 20^2} = 25 \text{ m/s}$$

$$\theta = \tan^{-1} \left(\frac{20}{15} \right) = 53^\circ \text{ with X-axis}$$

The motorcyclist appears to move along the line MP with speed 25 m/s

The shortest distance = perpendicular distance of MP from C = $d \Rightarrow d = 50 \cos 53^\circ \Rightarrow d = 30 \text{ m}$

Time taken to come closest = time taken by motorcyclist to reach B

$$t = \frac{MB}{v_{mc}} = \frac{50 \sin 53^\circ}{25} \Rightarrow t = 1.6 \text{ s.}$$

For Problems 13–14

13. c., 14. a.

Sol.

$$13. \text{ c. } t = \frac{d}{v \sin \theta} = \frac{0.5 \text{ km}}{3 \sin 120^\circ \text{ km/h}} = \frac{1}{3\sqrt{3}} \text{ h}$$

$$14. \text{ a. } x = (u + v \cos \theta)t = (2 + 3 \cos 120^\circ) \frac{1}{3\sqrt{3}}$$

$$= \frac{1}{6\sqrt{3}} \text{ km}$$

For Problems 15–16

15. b., 16. b.

Sol.

$$15. \text{ b. } \sin 30^\circ = \frac{v_m}{v_r} \Rightarrow v_r = 1 \text{ m/s}$$

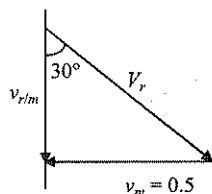


Fig. 5.183

$$16. \text{ c. } v_{r/m} = v_r \cos 30^\circ = 1 \times \frac{\sqrt{3}}{2} = 0.5\sqrt{3} \text{ m/s}$$

For Problems 17–19

17. c. d., 18. d., 19. d.

Sol.

$$17. \text{ c. Velocity of first body at any instant } t, \vec{v}_1 = 2\hat{i} - gt\hat{j}$$

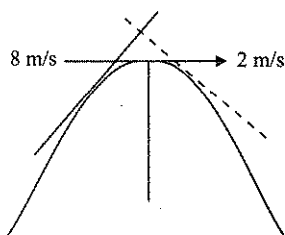


Fig. 5.184

Velocity of second body at any instant

$$\vec{v}_2 = -8\hat{i} - gt\hat{j}$$

Since $\vec{v}_1 \perp \vec{v}_2$ i.e., $\vec{v}_1 \times \vec{v}_2 = 0$

$$(2\hat{i} - gt\hat{j}) \cdot (-8\hat{i} - gt\hat{j}) = 0 \Rightarrow -16 + g^2 t^2 = 0$$

$$\Rightarrow t = \sqrt{\frac{16}{g^2}} \Rightarrow t = \frac{4}{10} = 0.4 \text{ s}$$

$$18. \text{ d. } \vec{S}_1 = (2 \times 0.4)\hat{i}, \vec{S}_2 = (8 \times 0.4)(-\hat{i})$$

$$\vec{S}_1 = 0.8\hat{i} \text{ and } \vec{S}_2 = -3.2\hat{i}$$

$$\text{Separation} = \vec{S}_1 - (-\vec{S}_2) = 0.8\hat{i} + (3.2\hat{i}) = 4\hat{i}$$

$$19. \text{ d. } \vec{S}_1 = 2t\hat{i} - \frac{1}{2}gt^2\hat{j}, \vec{S}_2 = -8t\hat{i} - \frac{1}{2}gt^2\hat{j}$$

as $\vec{S}_1 \perp \vec{S}_2$ i.e., $\vec{S}_1 \cdot \vec{S}_2 = 0$

$$-16t + \frac{1}{2} \times \frac{1}{2} g^2 t^4 = 0 \Rightarrow g^2 t^2 = 16 \times 4$$

$$\Rightarrow gt = 4 \times 2 \Rightarrow t = \frac{8}{10} = 0.8 \text{ s}$$

For Problems 20–21

20. d., 21. c.

Sol.

$$20. \text{ d. } \vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$x\hat{i} + y\hat{j} = (5\hat{i})t + \frac{1}{2}(3\hat{i} + 2\hat{j})t^2$$

$$x = 5t + \frac{3}{2}t^2 \Rightarrow 84 = 5t + \frac{3}{2}t^2$$

$$3t^2 + 10t - 168 = 0 \Rightarrow 3t^2 + 28t - 18t - 168 = 0$$

$$t[3t + 28] - 6[3t + 28] = 0 \quad t = 0$$

$$y = t^2 \quad y = 6^2 = 36 \text{ m}$$

$$21. \text{ c. } y = t^2, x = 5t + \frac{3}{2}t^2$$

$$\frac{dx}{dt} = v_x = 5t + 3t, \frac{dy}{dt} = v_y = 2t$$

$$v_x = 5t + 3(6) = 23, v_y = 12$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(23)^2 + (12)^2} \simeq 26 \text{ m/s}$$

For Problems 22–23

22. c., 23. d.

Sol.

$$22. \text{ c. Maximum height: } h = \frac{u^2 \sin^2 \theta}{2g} \quad (i)$$

$$2h = \frac{u^2 2 \sin \theta \cos \theta}{g} \quad (ii)$$

Dividing equations (i) and (ii); $\tan \theta = 2 \Rightarrow \theta = \tan^{-1}(2)$

$$23. \text{ d. } \tan \theta = \frac{2}{1}$$

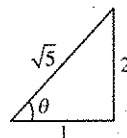


Fig. 5.185

$$\text{From equation (i) } u^2 = \frac{2gh}{\sin^2 \theta}$$

$$= \frac{2gh}{(2/\sqrt{5})^2} = \frac{5gh}{2} \Rightarrow u = \sqrt{5gh/2}$$

For Problems 24–25

24. c., 25. a.

Sol.

24. a. Here r will become range

$$r = \frac{u^2 \sin 2\theta}{g} \Rightarrow gr = u^2 \sin 2\theta$$

$$25. \text{ a. } h = u \sin \theta t + \frac{1}{2}gt^2 \text{ (Fig. 5.188)}$$

$$x = u \cos \theta t$$

$$t = \frac{x}{u \cos \theta}$$

$$h = u \sin \theta \frac{x}{u \cos \theta} + \frac{1}{2}g \frac{x^2}{u^2 \cos^2 \theta}$$

$$+ 2u^2 \sin \theta \cos \theta x - 2hu^2 \cos^2 \theta = 0$$

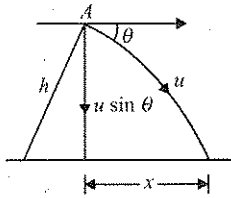


Fig. 5.186

$$\begin{aligned}
 x &= -2u^2 \sin \theta \cos \theta \\
 &+ \sqrt{4u^2 \sin^2 \theta \cos^2 \theta + 8ghu^2 \cos^2 \theta} \\
 &= \frac{u \cos \theta}{g} \left[\sqrt{u^2 \sin^2 \theta + 2gh} - u \sin \theta \right]
 \end{aligned}$$

For Problems 26–29

26. a., 27. b., 28. d., 29. c.

Sol.

$$\begin{aligned}
 26. \text{ a. } \frac{1}{2}mv^2 &= mg(3-1) = 2mg \\
 &\quad \text{[from conservation of energy]}
 \end{aligned}$$

$$\text{or } v = \sqrt{4g} = 2\sqrt{g}$$

$$\text{Vertical component at A} = 2\sqrt{g} \sin 30^\circ = \sqrt{g}$$

27. b. This is equal to time of flight

$$T = \frac{2v \sin \theta}{g} = \frac{2\sqrt{g}}{g} = \frac{2}{\sqrt{g}}$$

28. d. Using $S = ut + \frac{1}{2}at^2$, we get

$$-1 = \sqrt{g}t - \frac{1}{2}gt^2 \text{ or } \frac{1}{2}gt^2 - \sqrt{g}t - t = 0$$

$$t = \frac{\sqrt{g} \pm \sqrt{g + 4 \times \frac{1}{2}g}}{2 \times \frac{g}{2}} \text{ or } t = \frac{1 \pm \sqrt{3}}{\sqrt{g}}$$

$$\text{Neglecting -ve time. } t = \frac{1 + \sqrt{3}}{\sqrt{g}}$$

$$29. \text{ c. } x = 2\sqrt{g} \cos 30^\circ \left[\frac{1 + \sqrt{3}}{\sqrt{g}} \right] \text{ or } x = (\sqrt{3} + 3) \text{ m}$$

For Problems 30–32

30. c., 31. b., 32. c.

Sol.

$$30. \text{ c. } h = v \sin \theta t - \frac{1}{2}gt^2 \text{ or } \frac{1}{2}gt^2 - v \sin \theta t + h = 0$$

$$t_1 + t_2 = -\frac{-v \sin \theta}{\frac{1}{2}g} \text{ or } t_1 + t_2 = \frac{2v \sin \theta}{g} = T$$

$$\text{or } T(1+3) \text{ s} = 4 \text{ s}$$

$$31. \text{ b. } h_{\max} = \frac{v^2 \sin^2 \theta}{2g}$$

$$= \frac{g^2 T^2}{8g} = \frac{1}{8}gT^2 = \frac{1}{8}g \times 4 \times 4 = 2g \left[\because v \sin \theta = \frac{gT}{2} \right]$$

$$32. \text{ c. } h = v \sin \theta t - \frac{1}{2}gt^2$$

$$\text{when } t = 1 \text{ s, then } h = v \sin \theta - \frac{1}{2}g$$

$$\text{or } h = \frac{g \times 4}{2} - \frac{g}{2} = 2g - \frac{g}{2} \text{ or } h = \frac{3g}{2}$$

$$\text{Aliter: } \frac{1}{2}gt^2 - v \sin \theta t + h = 0$$

$$t_1 t_2 = \frac{h}{\frac{1}{2}g} \text{ or } 1 \times 3 = \frac{h}{g/2} \text{ or } h = \frac{3g}{2}$$

For Problems 33–35

33. c., 34. a., 35. d.

Sol.

$$33. \text{ c. } v_y^2 - v^2 \sin^2 \theta = -2g \left[\frac{1}{2} \frac{v^2 \sin^2 \theta}{2g} \right]$$

$$\text{or } v_y^2 = v^2 \sin^2 \theta - \frac{v^2 \sin^2 \theta}{2}$$

$$\text{or } v_y^2 = \frac{v^2 \sin^2 \theta}{2} \text{ or } v_y = \frac{v \sin \theta}{\sqrt{2}}$$

$$34. \text{ a. } v'^2 = (v \cos \theta)^2 + \left(\frac{v \sin \theta}{\sqrt{2}} \right)^2$$

$$\text{or } v'^2 = v^2 \cos^2 \theta + \frac{v^2 \sin^2 \theta}{2}$$

$$\text{or } v'^2 = v^2 \left(\cos^2 \theta + \frac{\sin^2 \theta}{2} \right)$$

$$\text{or } v' = v \sqrt{\cos^2 \theta + \frac{\sin^2 \theta}{2}}$$

$$35. \text{ d. } v^2 \cos^2 \theta = \frac{2}{5}v^2 \left[\cos^2 \theta + \frac{\sin^2 \theta}{2} \right]$$

$$\text{or } 5 \cos^2 \theta = 2 \cos^2 \theta + \sin^2 \theta$$

$$\text{or } 3 \cos^2 \theta = \sin^2 \theta$$

$$\text{or } \tan^2 \theta = 3 \text{ or } \tan \theta = \sqrt{3} \text{ or } \theta = 60^\circ$$

$$\text{Angle of projection with vertical} = 90^\circ - 60^\circ = 30^\circ.$$

For Problems 36–39

36. b., 37. d., 38. a., 39. c.

Sol.

36. b. As shown in Fig. 5.187

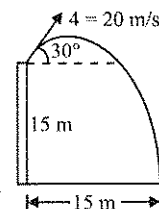


Fig. 5.187

$$-15 = 20 \sin 30^\circ t - \frac{1}{2}10t^2$$

$$\Rightarrow 5t^2 - 10t - 15 = 0$$

$$\Rightarrow t = 3 \text{ s}$$

37. d. $R = 20 \cos 30^\circ t = 30\sqrt{3} \text{ m}$

38. a. $v_x = u_x = u \cos 30^\circ = 10\sqrt{3} \text{ m/s}$
 $u_y = u_y + a_y t = 20 \sin 30^\circ - 10 \times 3 = -20 \text{ m/s}$

$$v = \sqrt{v_x^2 + v_y^2} = 10\sqrt{7} \text{ m/s}$$

39. c. Maximum height above top of tower:

$$H = \frac{(20)^2 \sin^2 30^\circ}{5g} = 5 \text{ m}$$

Maximum height attained above ground
 $= 15 + H = 20 \text{ m}$

For Problems 40–41

40. c., 41. c.

Sol.

40. c. $R = ut = u \sqrt{\frac{2h}{g}} \Rightarrow R = 3 \sqrt{\frac{2 \times 1}{10}} = \frac{3}{\sqrt{5}} \text{ m}$

41. c. $v_x = u_x = 3 \text{ m/s}$
 $v_y^2 = u_y^2 + 2gh \Rightarrow v_y^2 = 0^2 + 2 \times 10 \times 1 = 20$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{29} \text{ m/s}$$

For Problems 42–43

42. d., 43. a.

Sol.

42. d. $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$
 $\Rightarrow 15 = 30 \tan 45^\circ - \frac{10(3)^2}{2u^2 \cos^2 45^\circ} \Rightarrow u = 10\sqrt{6} \text{ m/s}$

43. a. $v_x = u_x = 10\sqrt{6} \cos 45^\circ = 10\sqrt{3} \text{ m/s}$

$$v_y^2 = u_y^2 + 2a_y s_y = (10\sqrt{6} \sin 45^\circ)^2 + 2(-10)15 = 0$$

So the velocity is $10\sqrt{3} \text{ m/s}$ horizontally.

For Problems 44–46

44. c., 45. a., 46. b.

Sol.

44. c.

45. a. Fig. 5.188

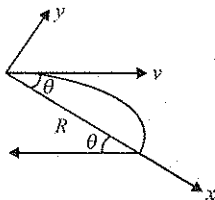


Fig. 5.188

$$a_x = g \sin \theta, a_y = -g \cos \theta$$

$$S_y = 0 \Rightarrow u_y t + \frac{1}{2} a_y t^2 = 0$$

$$\Rightarrow v \sin \theta t - \frac{1}{2} g \cos \theta t^2 = 0$$

$$\Rightarrow t = \frac{2v \sin \theta}{g \cos \theta} = \frac{2 \times 5}{10} \tan \theta = \frac{3}{4} \text{ s}$$

$$R = u_x t + \frac{1}{2} a_x t^2 = v \cos \theta t + \frac{1}{2} g \sin \theta t^2$$

$$5 \cos 37^\circ \times \frac{3}{4} + \frac{1}{2} \times 10 \times \sin 37^\circ \times \left(\frac{3}{4}\right)^2 = \frac{75}{16} \text{ m}$$

46. b. $v_x = u_x + a_x t = 5 \cos 37^\circ + g \sin 37^\circ \times \frac{3}{4}$

$$v_y = u_y + a_y t = 5 \sin 37^\circ - g \cos 37^\circ \times \frac{3}{4} = -3 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(\frac{17}{2}\right)^2 + 3^2} = \frac{5}{2} \sqrt{13} \text{ m/s}$$

For Problems 47–49

47. c., 48. a., 49. b.

Sol.

47. c. We know that $a_c = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a_c}$, where r is known as radius of curvature. At the highest point $v = u \cos \theta$,

$$a_c = g \Rightarrow r = \frac{u^2 \cos^2 \theta}{g}$$

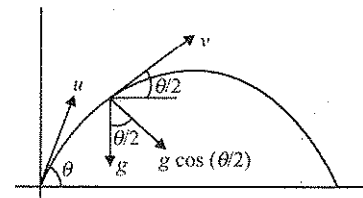
48. a. Similarly find the velocity and a_c at the given point and then find r . a_c will be a component of $g \perp$ to velocity at that point (Fig. 5.189).

Fig. 5.189

$$v \cos(\theta/2) = u \cos \theta \Rightarrow v = u \cos \theta \sec(\theta/2)$$

$$a_c = g \cos(\theta/2). \text{ Now find } r = \frac{v^2}{a_c}$$

49. b. $h = H/2, v \cos \phi = u \cos \theta$ (i)

$$v^2 \sin^2 \phi = u^2 \sin^2 \theta - 2g \frac{H}{2} \quad \text{(ii)}$$

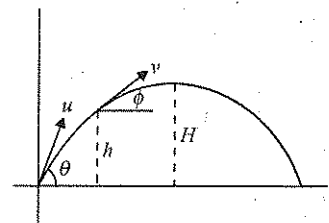


Fig. 5.190

Squaring equation (i) and adding in equation (ii)

$$\Rightarrow v^2 = u^2 - gH$$

$$\text{where } H = \frac{u^2 \sin^2 \theta}{2g}$$

$$a_c = g \cos \phi = \frac{gu \cos \theta}{v}, \text{ Now } r = \frac{v^2}{a_c}$$

Matching Column Type

1. i. $\rightarrow$ b., d., ii. $\rightarrow$ c., iii. $\rightarrow$ a., iv. $\rightarrow$ c.

* Distance will be minimum because the man will reach from point A to point B directly.

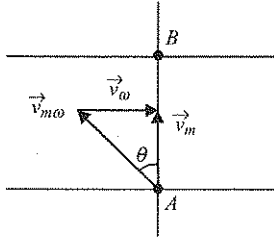


Fig. 5.191

$$\sin \theta = \frac{v_{\omega}}{v_{m\omega}} \text{ and } \vec{v}_m \perp \vec{v}_{\omega}$$

Hence i. $\rightarrow$ b., d.

Time taken is minimum if $\vec{v}_{m\omega}$ is perpendicular to $\vec{v}_{\omega}$.

Hence ii. $\rightarrow$ c., iv. $\rightarrow$ c.

(ii) If $\vec{v}_{m\omega} < \vec{v}_{\omega}$, then drift or distance is shortest if $\sin \theta = \frac{v_{m\omega}}{v_{\omega}}$. Hence (iii) $\rightarrow$ a.

2. i. $\rightarrow$ a., ii. $\rightarrow$ b., iii. $\rightarrow$ c., iv. $\rightarrow$ d.

i. $\rightarrow$ a. Range is maximum, when the angle of projection is 45°

$$H = \frac{v^2}{2g} \sin^2 45^\circ; H = \frac{v^2}{4g} \quad (i)$$

Velocity, at half of the maximum height is v'

$$v'^2 = v^2 \sin^2 45^\circ - 2g \frac{H}{2}; v'^2 = \frac{v^2}{2} - \frac{v^2}{4}$$

$$v'^2 = \frac{v^2}{4} \Rightarrow v' = \frac{v}{2}$$

ii $\rightarrow$ b. Velocity at the maximum height

$$v' = v \cos 45^\circ \Rightarrow v' = \frac{v}{\sqrt{2}}$$

[because vertical component of velocity is zero at the highest point]

iii $\rightarrow$ c. Projection velocity

At projection point, $v_i = v \cos 45^\circ \hat{i} + v \sin 45^\circ \hat{j}$

At the point, when the body strike the ground

$$\vec{v}_f = v \cos 45^\circ \hat{i} - v \sin 45^\circ \hat{j}$$

$$\Delta v = \vec{v}_f + (-\vec{v}_i) = 2v \sin 45^\circ (-\hat{j})$$

$$|\Delta \vec{v}| = 2v \sin 45^\circ = v\sqrt{2}$$

$$\text{iv} \rightarrow \text{d. Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

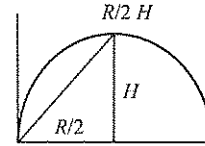


Fig. 5.192

$$\text{Displacement} = \sqrt{\left(\frac{R}{2}\right)^2 + H^2}$$

$$\vec{v}_{av} = \frac{\sqrt{\frac{R^2}{4} + H^2}}{\frac{v \sin \theta}{g}} = \frac{\sqrt{R^2 + 4H^2}}{2v \sin \theta}$$

$$v_{av} = \frac{\sqrt{\left(\frac{v^2}{g}\right)^2 + 4\left(\frac{v^2}{4g}\right)^2}}{\frac{\sqrt{2}v}{g}}$$

$$= \frac{\frac{v^2}{g} \sqrt{1 + \frac{1}{4}}}{\frac{v\sqrt{2}}{g}} = \frac{v^2 \sqrt{5}}{2v\sqrt{2}} = \frac{v}{2} \sqrt{\frac{5}{2}}$$

3. i. $\rightarrow$ a., c., ii. $\rightarrow$ b., d., iii. $\rightarrow$ a., c., iv. $\rightarrow$ a., b., c., d.

i. In the uniform circular motion, the acceleration and the velocity are perpendicular to each other, but in non uniform circular motion the angle between the velocity and the acceleration lies between 0 to $\pi/2$.

ii. In a straight line motion, the acceleration vector and the velocity vector are collinear to each other, i.e., the angle between them is either 0 or 180° .

iii. In a projectile motion, the angle (θ) between the velocity and the acceleration can vary from $0 < \theta < \pi$.

iv. In space angle between velocity and acceleration may be $0 \leq \theta \leq \pi$.

Miscellaneous Assignments and Archives on Chapters 1–5

9. A body with zero initial velocity moves down an inclined plane from a height h and then ascends along the same plane with an initial velocity, such that it stops at the same height h . Considering friction to be present, in which case is the time of motion longer?
- Ascent
 - Descent
 - Same in both
 - Information insufficient
10. A car travelling at a constant speed of 20 m/s overtakes another car which is moving at a constant acceleration of 2 m/s² and it was initially at rest. Assume the length of each car to be 5 m. The total distance covered in overtaking is
- 394.74 m
 - 15.26 m
 - 200.00 m
 - 186.04 m
11. A particle is moving along X-axis whose acceleration is given by $a = 3x - 4$, where x is the location of the particle. At $t = 0$, the particle is at rest at $x = \frac{4}{3}$. The distance travelled by the particle in 5 s is
- zero
 - $\simeq 42$ m
 - infinite
 - None of these
12. A particle has been projected with a speed of 20 m/s at an angle of 30° with the horizontal. The time taken when the velocity vector becomes perpendicular to the initial velocity vector is
- 4 s
 - 2 s
 - 3 s
 - Not possible in this case
13. At a distance of 500 m from the traffic light, brakes are applied to an automobile moving at a velocity of 20 m/s. The position of the automobile relative to the traffic light 50 s after applying the brakes, if its acceleration is -0.5 m/s², is
- 125 m
 - 375 m
 - 400 m
 - 100 m
14. A parachutist jumps off a plane. He falls freely for sometime, and then opens his parachute. Shortly after his parachute inflates, the parachutist
- keeps falling but quickly slows down
 - momentarily stops, then starts falling again, but more slowly
 - suddenly shoots upwards, and then starts falling again but more slowly
 - suddenly shoots upward, and then starts falling again, eventually acquiring the same speed as before the parachute opened
15. An object has velocity $\vec{v}_1$ w.r.t. ground. An observer, moving with a constant velocity $\vec{v}_0$ w.r.t. ground, measures the velocity of the object as $\vec{v}_2$. The magnitudes of three velocities are related by
- $v_0 \geq v_1 + v_2$
 - $v_1 \leq v_2 + v_0$
 - $v_2 \geq v_1 + v_0$
 - All of the above
16. Two particles are projected simultaneously from the same point, with the same speed, in the same vertical plane, and at different angles with the horizontal in a uniform gravitational field acting vertically downwards. A frame of reference is fixed to one particle. The position vector of the other particle, as observed from this frame, is $\vec{r}$. Which of the following statements is correct?
- $\vec{r}$ is a constant vector
 - $\vec{r}$ changes in magnitude as well as direction with time
 - The magnitude of $\vec{r}$ increases linearly with time; its direction does not change

d. The direction of $\vec{r}$ changes with time; its magnitude may or may not change, depending on the angles of projection

17. A point moves such that its displacement as a function of time is given by $x^3 = t^3 + 1$. Its acceleration as a function of time t will be

- $\frac{2}{x^5}$
- $\frac{2t}{x^5}$
- $\frac{2t}{x^4}$
- $\frac{2t^2}{x^5}$

18. Two particles are thrown horizontally in the opposite direction with velocities u and $2u$ from the top of a high tower. The time after which their radius of curvature will be mutually perpendicular is

- $\sqrt{2} \frac{u}{g}$
- $2 \frac{u}{g}$
- $\frac{1}{\sqrt{2}} \frac{u}{g}$
- $\frac{1}{2} \frac{u}{g}$

19. In Fig. 6.7, the angle of inclination of the inclined plane is 30°. The horizontal velocity V_0 so that the particle hits the inclined plane perpendicularly is

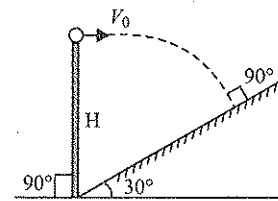


Fig. 6.7

- $V_0 = \sqrt{\frac{2gH}{5}}$
- $V_0 = \sqrt{\frac{2gH}{7}}$
- $V_0 = \sqrt{\frac{gH}{5}}$
- $V_0 = \sqrt{\frac{gH}{7}}$

20. A particle reaches its highest point when it has covered exactly one half of its horizontal range. The corresponding point on the vertical displacement-time graph is characterised by
- zero slope and zero curvature
 - zero slope and non-zero curvature
 - positive slope and zero curvature
 - none of these

21. Two particles A and B are placed as shown in Fig. 6.8. The particle A, on the top of a tower, is projected horizontally with a velocity u and the particle B is projected along the surface towards the tower, simultaneously. If both the particles meet each other. Then the speed of projection of particle B is [Ignore any friction]

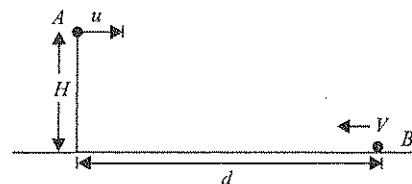


Fig. 6.8

- $d\sqrt{\frac{g}{2H}} - u$
- $d\sqrt{\frac{g}{2H}}$
- $d\sqrt{\frac{g}{2H}} + u$
- u

22. Twelve persons are initially at the twelve corners of a regular polygon of twelve sides of side a . Each person now moves with a uniform speed v in such a manner that 1 is always

directed towards 2, 2 towards 3, 3 towards 4 and so on. The time after which they meet is

- a. $\frac{v}{a}$ b. $\frac{2a}{v}$ c. $\frac{2a}{v(2+\sqrt{3})}$ d. $\frac{2a}{v(2-\sqrt{3})}$

23. An object is subjected to the acceleration $a = 4 + 3v$. It is given that the displacement $S = 0$, when $v = 0$. The value of displacement when $v = 2$ m/s is
a. 0.52 m b. 0.26 m c. 0.39 m d. 0.65 m
24. Two boys P and Q are playing on a river bank. P plans to swim across the river directly and comes back. Q plans to swim downstream by a length equal to the width of the river and comes back. Both of them bet each other, claiming that the boy succeeding in less time will win. Assuming the swimming rate of both P and Q to be the same, it can be concluded that
a. P wins
b. Q wins
c. A draw takes place
d. Nothing certain can be stated
25. A projectile is fired with a velocity v at right angle to the slope which is inclined at an angle θ with the horizontal. The range of the projectile along the inclined plane is

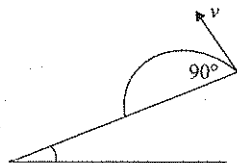


Fig. 6.9

- a. $\frac{2v^2 \tan \theta}{g}$ b. $\frac{v^2 \sec \theta}{g}$
c. $\frac{2v^2 \tan \theta \sec \theta}{g}$ d. $\frac{v^2 \sin \theta}{g}$
26. A ball rolls off the top of a stairway horizontally with a velocity of 4.5 ms^{-1} . Each step is 0.2 m high and 0.3 m wide. If g is 10 ms^{-2} , then the ball will strike the edge of n th step where n is equal to
a. 9 b. 10 c. 11 d. 12
27. A man holds an umbrella at 30° with the vertical to keep himself dry. He, then, runs at a speed of 10 m/s and finds the raindrops to be hitting vertically. Study the following statements and find the correct options.
i. Velocity of rain w.r.t. earth is 20 ms^{-1}
ii. Velocity of rain w.r.t. man is $10\sqrt{3} \text{ ms}^{-1}$
iii. Velocity of rain w.r.t. earth is 30 ms^{-1}
iv. Velocity of rain w.r.t. man is $10\sqrt{2} \text{ ms}^{-1}$
a. Statements (i) and (ii) are correct
b. Statements (i) and (iii) are correct
c. Statements (iii) and (iv) are correct
d. Statements (ii) and (iv) are correct
28. Rain appears to fall vertically on a man walking at 3 km/h , but when he changes his speed to double, the rain appears to fall at 45° with vertical. Study the following statements and find which of them are correct
i. Velocity of rain is $2\sqrt{3} \text{ kmh}^{-1}$

- ii. The angle of fall of rain is $\theta = \tan^{-1} \left(\frac{1}{\sqrt{5}} \right)$
iii. The angle of fall of rain is $\theta = \tan^{-1} \left(\frac{1}{\sqrt{2}} \right)$
iv. Velocity of rain is $3\sqrt{2} \text{ kmh}^{-1}$ given above is with vertical
a. Statements (i) and (ii) are correct
b. Statements (i) and (iii) are correct
c. Statements (iii) and (iv) are correct
d. Statements (ii) and (iv) are correct
29. The maximum range of a projectile is 500 m . If the particle is thrown up a plane which is inclined at an angle of 30° with the same speed, the distance covered by it along the inclined plane will be
a. 250 m b. 500 m c. 750 m d. 100 m

30. Velocity versus displacement graph of a particle moving in a straight line is as shown in Fig. 6.10.

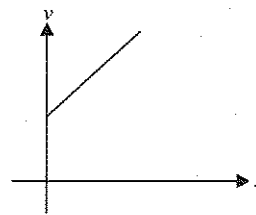


Fig. 6.10

The acceleration of the particle is

- a. constant
b. increases linearly with x
c. increases parabolically with x
d. none of these
31. The acceleration–time graph of a particle moving in a straight line is as shown in Fig. 6.11. The velocity of the particle at time $t = 0$ is 2 m/s . The velocity after 2 s will be

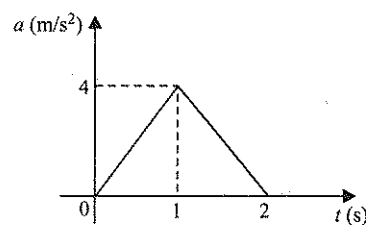


Fig. 6.11

- a. 6 m/s b. 4 m/s c. 2 m/s d. 8 m/s
32. Velocity versus displacement graph of a particle moving in a straight line is shown in Fig. 6.12. Corresponding acceleration versus velocity graph will be

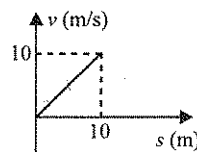
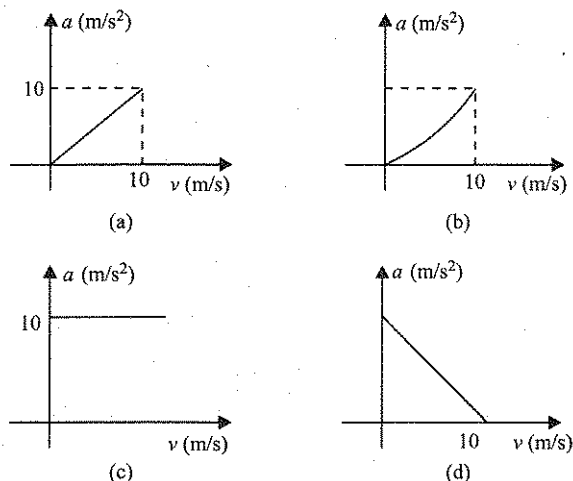


Fig. 6.12



33. Acceleration-velocity graph of a particle moving in a straight line is as shown in Fig. 6.13. Then, the slope of velocity-displacement graph

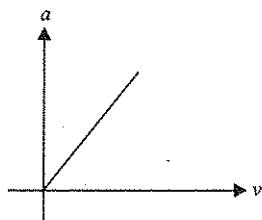


Fig. 6.13

- a. increases linearly
b. decreases linearly
c. is constant
d. increases parabolically
34. The acceleration of a particle travelling along a straight line is shown in Fig. 6.14. The maximum speed of the particle is

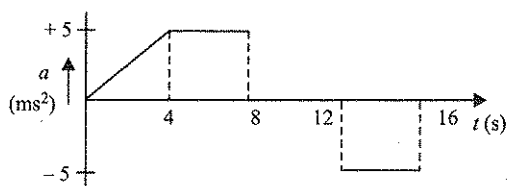


Fig. 6.14

- a. 20 ms^{-1} b. 30 ms^{-1} c. 40 ms^{-1} d. 60 ms^{-1}
35. A block is dragged on smooth plane with the help of a rope which is pulled by velocity v as shown in Fig. 6.15. The horizontal velocity of the block is

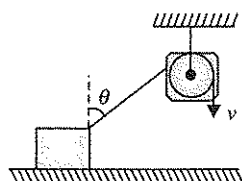


Fig. 6.15

- a. v b. $v/\sin \theta$ c. $v \sin \theta$ d. $v/\cos \theta$
36. An object is moving in the X - Y plane with the position as a function of time given by $\vec{r} = x(t)\hat{i} + y(t)\hat{j}$. Point O is at

$x = 0, y = 0$. The object is definitely moving towards O when

a. $v_x > 0, v_y > 0$ b. $v_x < 0, v_y < 0$
c. $xv_x + yv_y < 0$ d. $xv_x + yv_y > 0$

37. Two particles A and B are moving along a straight line, whose position-time graph is as shown in Fig. 6.16 below. Determine the instant (approx) when both particles are moving with the same velocity.

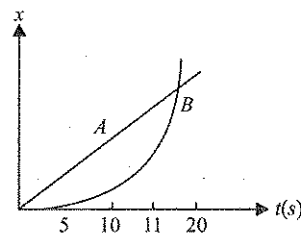


Fig. 6.16

- a. 17 s b. 12 s c. 6 s d. No where
38. An object moves along the x -axis. Its x -coordinates is given as a function of time as $X = 7t - 3t^2$ m, where X is in metre and t is in second. Its average speed over the interval $t = 0$ to $t = 4$ s is
- a. 5 m/s b. -5 m/s c. $-\frac{169}{24}$ m/s d. $\frac{169}{24}$ m/s
39. A cannon fires a projectile as shown in Fig. 6.17. The dashed line shows the trajectory in the absence of gravity. The points M, N, O and P correspond to time at $t = 0, 1.3$ s, 2 s and 3 s, respectively. The lengths of X, Y and Z are respectively

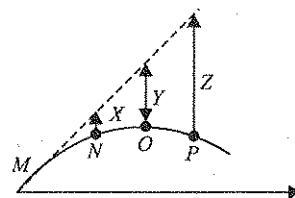


Fig. 6.17

- a. 5 m, 10 m, 15 m b. 10 m, 40 m, 90 m
c. 5 m, 20 m, 45 m d. 10 m, 20 m, 30 m
40. The speed of a projectile at its highest point is v_1 and at the point half the maximum height is v_2 . If $\frac{v_1}{v_2} = \sqrt{\frac{2}{5}}$, then the angle of projection is
- a. 45° b. 30° c. 37° d. 60°
41. A particle is projected at an angle of elevation α and after t seconds it appears to have an angle of elevation β as seen from the point of projection. The initial velocity will be
- a. $\frac{gt}{2 \sin(\alpha - \beta)}$ b. $\frac{gt \cos \beta}{2 \sin(\alpha - \beta)}$
c. $\frac{2gt}{\sin(\alpha - \beta)}$ d. $\frac{gt \cos \beta}{2 \sin(\alpha - \beta)}$
42. A velley shots are fired simultaneously from the top and bottom of a vertical cliff with the elevation $\alpha = 30^\circ, \beta = 60^\circ$, respectively Fig. 6.18. The shots strike an object simultaneously at the same point. If $a = 30\sqrt{3}$ m is the horizontal distance of the object from the cliff, then the height h of the cliff is
- a. 30 m b. 45 m c. 60 m d. 90 m

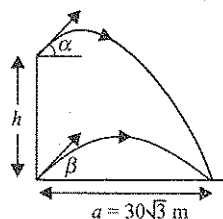


Fig. 6.18

43. Figure 6.19 show that particle A is projected from point P with velocity u along the plane and simultaneously another particle B with velocity v at an angle α with vertical. The particles collide at point Q on the plane. Then

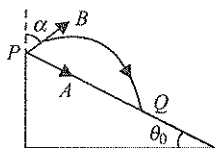


Fig. 6.19

- a. $v \sin(\alpha - \theta_0) = u$ b. $v \cos(\alpha - \theta_0) = u$
 c. $v = u$ d. None of these
44. A platform is moving upwards with an acceleration of 5 m/s^2 . At the moment when its velocity is $u = 3 \text{ m/s}$, a ball is thrown from it with a speed of 30 m/s w.r.t. platform at an angle of $\theta = 30^\circ$ with horizontal w.r.t. platform. The time taken by the ball to return to the platform is
- a. 2 s b. 3 s c. 1 s d. 2.5 s
45. Two balls are projected from points A and B in a vertical plane as shown in Fig. 6.20. AB is a straight vertical line. The balls can collide in mid-air if v_1/v_2 is equal to

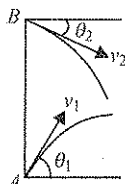


Fig. 6.20

- a. $\frac{\sin \theta_1}{\sin \theta_2}$ b. $\frac{\sin \theta_2}{\sin \theta_1}$ c. $\frac{\cos \theta_1}{\cos \theta_2}$ d. $\frac{\cos \theta_2}{\cos \theta_1}$

Multiple Correct Answers Type

Solutions on page 6.19

- For a particle moving along X-axis, mark the correct statement(s).
 - If x is +ve and is increasing with time, then average velocity of the particle is +ve.
 - If x is -ve and becoming +ve after sometime, then velocity of the particle is always +ve.
 - If x is -ve and becoming less -ve as the time passes, then average velocity of the particle is +ve.
 - If x is +ve and is increasing with time, then velocity of the particle is always +ve.

- For a particle moving along X-axis, $x-t$ graph is as given in Fig. 6.21. Mark the correct statement(s).

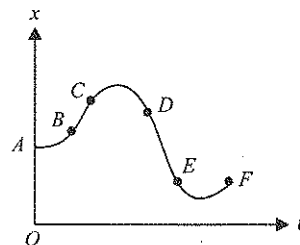


Fig. 6.21

- Initial velocity of the particle is zero.
 - For BC, the acceleration is +ve and for DE, the acceleration is -ve.
 - For EF, the acceleration is +ve.
 - Velocity is getting zero, three times in the motion.
- For a particle moving along X-axis, a scaled $x-t$ graph is shown in Fig. 6.22. Mark the correct statement(s).

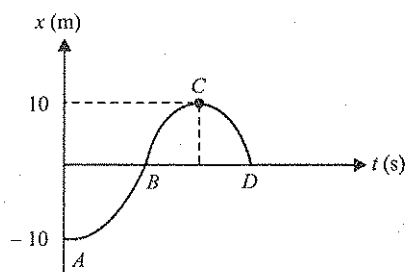


Fig. 6.22

- Speed of the particle is greatest at C.
 - Speed of the particle is greatest at B.
 - Particle is speeding up in region marked CD.
 - Average velocity is greatest for region AB among marked regions.
- Mark the correct statement(s).
 - A particle can have zero displacement and non-zero average velocity.
 - A particle can have zero displacement and non-zero velocity.
 - A particle can have zero acceleration and non-zero velocity.
 - A particle can have zero velocity and non-zero acceleration.
 - At time $t = 0$, a car moving along a straight line has a velocity of 16 m/s . It slows down with an acceleration of $-0.5t \text{ m/s}^2$, where t is in second. Mark the correct statement(s).
 - The direction of velocity changes at $t = 8 \text{ s}$.
 - The distance travelled in 4 s is approximately 59 m.
 - The distance travelled by the particle in 10 s is 94 m.
 - The velocity at $t = 10 \text{ s}$ is 9 m/s.
 - A ball is thrown upwards into the air with a speed that is greater than its terminal speed. It lands at the same place from where it was thrown. Mark the correct statement(s).
 - It acquires terminal speed before it gets to the highest point of the trajectory.
 - Before reaching the highest point of the trajectory, its speed is continuously decreasing.

- c. During the entire flight, the force of air resistance is greatest just after it is thrown.
- d. The magnitude of net force experienced by the ball is maximum just after it is thrown.
7. A particle is moving along X-axis whose position is given by $x = 4 - 9t + \frac{t^3}{3}$. Mark the correct statement(s) in relation to its motion.
- Direction of motion is not changing at any of the instants.
 - Direction of the motion is changing at $t = 3$ s.
 - For $0 < t < 3$ s, the particle is slowing down.
 - For $0 < t < 3$ s, the particle is speeding up.
8. A particle is thrown vertically in upward direction and passes three equally spaced windows of equal heights then

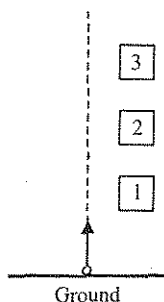


Fig. 6.23

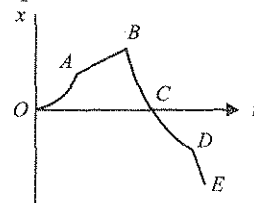
- average speed of the particle while passing the windows satisfies the relation $v_{av1} > v_{av2} > v_{av3}$
 - the time taken by the particle to cross the windows satisfies the relation $t_1 < t_2 < t_3$
 - the magnitude of the acceleration of the particle while crossing the windows satisfies the relation $a_1 = a_2 \neq a_3$
 - the change in the speed of the particle while crossing the windows would satisfy the relation $\Delta v_1 < \Delta v_2 < \Delta v_3$
9. Ship A is located 4 km north and 3 km east of ship B. Ship A has a velocity of 20 km/h towards the south and ship B is moving at 40 km/h in a direction 37° north of east. Take X- and Y-axes along east and north directions, respectively.
- Velocity of A relative to B is $-32\hat{i} - 44\hat{j}$.
 - Position of A relative to B as a function of time is given by

$$\vec{r}_{AB} = (3 - 32t)\hat{i} + (4 - 44t)\hat{j}$$

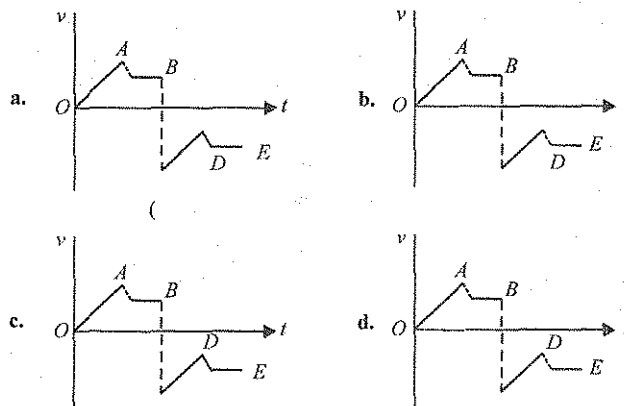
where $t = 0$ when the ships are in position described above.

- The instant at which their separation is minimum is at $t = 65$ s.
 - The least separation between the ships is 4.12 km.
10. Mark the correct statement(s).
- Average speed of a particle between two instants t_1 and t_2 depends only on the position vectors at t_1 and t_2 .
 - Average velocity of a particle between two instants t_1 and t_2 depends only on position vectors at t_1 and t_2 .
 - Average acceleration of a particle between two instants t_1 and t_2 depends only on velocities at t_1 and t_2 .
 - Average speed depends on mid-instants also.

11. A particle is moving along a straight line, whose $x-t$ plot is as shown in Fig. 6.24. Which one of the following can represent $v-t$ graph for the particle?



6.24



12. An object moves in the XY plane with an acceleration that has +ve Y component. At time $t = 0$, the object has a velocity given by $\vec{v} = 3\hat{i} + 4\hat{j}$. Mark the correct statement(s).
- Magnitude of the velocity of the particle is continuously increasing
 - X component of the velocity is continuously increasing
 - Y component of the velocity is continuously increasing
 - Y component of the velocity is constant
13. An object moves with the constant acceleration $\vec{a}$. Which of the following expressions is/are also constant?

- $\frac{d|\vec{v}|}{dt}$
- $\left| \frac{d\vec{v}}{dt} \right|$
- $\frac{d(v^2)}{dt}$
- $\frac{d\left(\frac{\vec{v}}{|\vec{v}|}\right)}{dt}$

14. A ball is dropped from a height of 49 m, the wind blows horizontally and imparts a constant acceleration of 4.90 m/s^2 to the ball. Choose the correct statement(s)
- Path of the ball is a straight line.
 - Path of the ball is a curved one.
 - The time taken by the ball to reach the ground is 3.16 s.
 - The angle made by the line joining initial and final positions (on ground after 1st strike) of the ball with horizontal is greater than 45° .
15. An object may have
- varying speed without having varying velocity
 - varying velocity without having varying speed
 - non-zero acceleration without having varying velocity
 - non-zero acceleration without having varying speed
16. From the top of a tower of height 200 m, ball A is projected up with a speed of 10 ms^{-1} and 2 s later, another ball B is projected vertically down with the same speed. Then

24. A body is projected at the angle of 30° and 60° with the same velocity. Their horizontal ranges are R_1 and R_2 and maximum heights are H_1 and H_2 , respectively, then
- a. $\frac{R_1}{R_2} > 1$ b. $\frac{H_1}{H_2} > 1$ c. $\frac{R_1}{R_2} < 1$ d. $\frac{H_1}{H_2} < 1$

For Problems 25–27

A point moves in $x-y$ plane according to the law $x = a \sin \omega t$ and $y = a - a \cos \omega t$, where a is a positive constant and t is the time.

25. The magnitude of velocity of the body at any instant of time t is
a. $a\omega \cos \omega t$ b. $a\omega$ c. $a\omega \sin \omega t$ d. None of these
26. The trajectory represents
a. $x = \sqrt{2ay}$
b. $x = \pm \sqrt{2ay \left(1 - \frac{y}{2a}\right)}$
c. $x \propto y^2$
d. None
27. The magnitude of acceleration of the body at any instant is
a. $a\omega^2$ b. $a\omega$ c. $a\omega^2/2$ d. $a\omega/2$

For Problems 28–29

A helicopter is flying at 200 m and flying at 25 m/s at an angle 37° above the horizontal when a package is dropped from it.

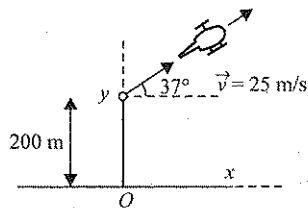


Fig. 6.27

28. Distance of the point from point O where the package lands is
a. 80 m b. 100 m c. 200 m d. 160 m
29. If the helicopter flies at constant velocity, find the x and y coordinates of the location of the helicopter when the package lands.
a. 160 m, 320 m
b. 100 m, 200 m
c. 200 m, 400 m
d. 50 m, 100 m

For Problems 30–31

A particle is moving with a constant acceleration in xy -plane from $x = 4$ m, $y = 3$ m and has velocity $\vec{v} = 2 \text{ m/s } \hat{i} - 9 \text{ m/s } \hat{j}$. The acceleration of the particle is given by vector $\vec{a} = 4 \text{ m/s}^2 \hat{i} + 3 \text{ m/s}^2 \hat{j}$.

30. The velocity vector at $t = 2$ s is
a. $8\hat{i} - 3\hat{j}$
b. $6\hat{i} - 7\hat{j}$
c. $20\hat{i} - 5\hat{j}$
d. $10\hat{i} - 3\hat{j}$
31. The position vector at $t = 4$ s is
a. $44\hat{i} - 9\hat{j}$ b. $4\hat{i} - \hat{j}$
c. $-5\hat{i} + 12\hat{j}$ d. $40\hat{i} - 12\hat{j}$

For Problems 32–34

Two cars approach each other on the highway. Car A moves towards north at 90 m/s. Car B moves towards south at 70 m/s.

32. The velocity of car A as seen from car B , i.e., v_{AB} is
a. 140 m/s b. 160 m/s c. 20 m/s d. 40 m/s
33. The velocity of car B as seen from car A , i.e., v_{BA} is
a. 120 m/s b. 130 m/s c. 170 m/s d. -160 m/s
34. Their velocities relative to car C , which is travelling north at 100 km/h, i.e., v_{AC} and v_{BC} are
a. -10 m/s, -170 m/s
b. -20 m/s, -180 m/s
c. 30 m/s, -140 m/s
d. -40 m/s, 120 m/s

For Problems 35–37

Two particles are thrown simultaneously from points A and B with velocities $u_1 = 2 \text{ ms}^{-1}$ and $u_2 = 14 \text{ ms}^{-1}$, respectively, as shown in Fig. 6.28.

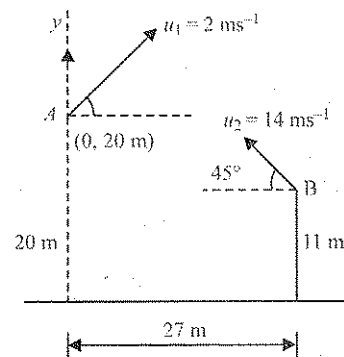


Fig. 6.28

35. The relative velocity of B as seen from A is
a. $-8\sqrt{2}\hat{i} - 6\sqrt{2}\hat{j}$ b. $4\sqrt{2}\hat{i} + 3\sqrt{3}\hat{j}$
c. $3\sqrt{5}\hat{i} + 2\sqrt{3}\hat{j}$ d. $3\sqrt{2}\hat{i} + 4\sqrt{3}\hat{j}$
36. The direction (angle) θ with horizontal at which B will appear to move as seen from A .
a. 37° b. 53° c. 15° d. 90°
37. Minimum separation between A and B .
a. 3 m b. 6 m c. 12 m d. 9 m

For Problems 38–42

Two inclined planes OA and OB having inclination (with horizontal) 30° and 60° respectively, intersect each other at O as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3} \text{ ms}^{-1}$ along a direction perpendicular to plane OA . If the particle strikes plane OB perpendicularly at Q , calculate

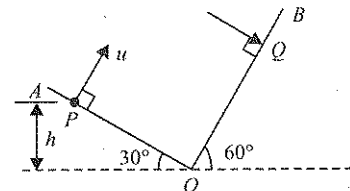


Fig. 6.29

38. the velocity with which particle strikes the plane OB ,
a. 15 m/s b. 30 m/s c. 20 m/s d. 10 m/s

39. time of flight of the particle
a. 8 s b. 6 s c. 4 s d. 2 s
40. the vertical height h of P from O ,
a. 10 m b. 5 m c. 15 m d. 20 m
41. the maximum height attained by the particle (from the line O)
a. 20.5 m b. 5 m c. 16.25 m d. 11.25 m
42. the distance PQ ,
a. 20 m b. 10 m c. 5 m d. 2.5 m

Assertion-Reasoning Type

Solutions on page 6.24

In the following questions, each question contains STATEMENT I (Assertion) and STATEMENT II (Reason). Each question has 4 choices (a), (b), (c), and (d) out of which **only one** is correct.

- (a) Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
(b) Statement I is True, Statement II is True; Statement II is NOT a correct explanation for Statement I.
(c) Statement I is True, Statement II is False.
(d) Statement I is False, Statement II is True.
1. **Statement I:** The dimensional formula for gravitational potential is $[L^2T^{-2}]$.
Statement II: The gravitational potential is the potential energy per unit charge.
2. **Statement I:** The equation representing the distance traversed in n^{th} second, $S_n = u + \frac{1}{2}a(2n - 1)$ is numerically and dimensionally correct.
Statement II: Dimensions of both sides are not matching.
3. **Statement I:** Least count of all screw-based instruments is same.
Statement II: Least count of all screw-based instruments is found using the ratio pitch per division of circular scale.
4. **Statement I:** Backlash error can be minimized by turning the screw in one direction only when the fine adjustment is done.
Statement II: Backlash error is because of the wear and tear or loose fittings in screws.
5. **Statement I:** Screw gauge with a pitch of 0.5 mm is more accurate than 1 mm for same number of circular scale divisions.
Statement II: Higher pitch can make an accurate device.
6. **Statement I:** Pendulum of a clock is made of alloys and not of pure metals.
Statement II: Use of alloys makes the pendulum look good.
7. **Statement I:** The speed of a body may be negative.
Statement II: When a body reverses its direction, its velocity becomes negative of the previous velocity.
8. **Statement I:** Speed and velocity are different physical quantities.
Statement II: Both speed and velocity have same unit (m/s).

9. **Statement I:** Speed is always positive, while velocity may be positive or negative.

Statement II: Speed is the magnitude of velocity and magnitude is always positive.

10. **Statement I:** The $v - t$ graph perpendicular to time axis is not possible in practice.

Statement II: Infinite acceleration cannot be realized in practice.

11. **Statement I:** Plotting the acceleration–time graph from a given position–time graph of a particle moving along a straight line is possible.

Statement II: From the position–time graph, only the sign of acceleration can be determined but no information can be concluded about the magnitude of acceleration.

12. **Statement I:** For uniformly accelerated motion started from rest, the displacement versus time graph is a straight line.

Statement II: For uniformly accelerated motion, the velocity in equal intervals of time changes by the same amount.

13. **Statement I:** An iron ball and a wooden ball are both released from the same height. In the presence of a medium both the balls reach the ground with different velocities and different times.

Statement II: Both the balls reach the ground simultaneously.

14. **Statement I:** If velocity–time graph is a straight line parallel to the time axis, then the acceleration of the body is zero.

Statement II: Acceleration is equal to the rate of change of velocity (constant).

15. **Statement I:** The sum and difference of two non-zero vectors will be equal in magnitude, when the two vectors are perpendicular to each other.

Statement II: If the above two vectors are perpendicular, then their dot product is zero.

16. **Statement I:** The direction of velocity vector is always along the tangent to the path, therefore its magnitude may be given by its slope.

Statement II: The slope of the tangent to the path only measures the direction of velocity at that point.

17. **Statement I:** The relative velocity of A w.r.t. B is greater than the velocity of either, when they are moving in opposite directions.

Statement II: Relative velocity of A w.r.t. $B = \vec{v}_A - \vec{v}_B$.

18. **Statement I:** The relative velocity between any two bodies is equal to the sum of the velocities of the two bodies.

Statement II: Sometimes, the relative velocity between two bodies is equal to the difference in the velocities of the two.

Matching Column Type

Solutions on page 6.24

1. For a particle moving along X -axis, if the acceleration (constant) is acting along $-ve$ X -axis, then match the entries of Column I with the entries of Column II

Column I	Column II
i. Initial velocity > 0	a. Particle may move in +ve X-direction with increasing speed
ii. Initial velocity < 0	b. Particle may move in +ve X-direction with decreasing speed
iii. $x > 0$	c. Particle may move in -ve X-direction with increasing speed
iv. $x < 0$	d. Particle may move in -ve X-direction with decreasing speed

2. The velocity-time graph of a particle moving along X-axis is shown in Fig. 6.30. Match the entries of Column I with the entries of Column II.

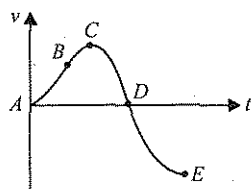


Fig. 6.30

Column I	Column II
i. For AB, the particle is	a. moving in +ve X-direction with an increasing speed
ii. For BC, the particle is	b. moving in +ve X-direction with a decreasing speed
iii. For CD, the particle is	c. moving in -ve X-direction with an increasing speed
iv. For DE, the particle is	d. moving in -ve X-direction with a decreasing speed

3. Fig. 6.31 shows the position-time graph of particles moving along a straight line. Match the entries of Column I with the entries of Column II

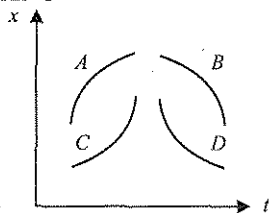


Fig. 6.31

Column I	Column II
i. The particle A is	a. Accelerating
ii. The particle B is	b. Decelerating
iii. The particle C is	c. Speeding up
iv. The particle D is	d. Slowing down

4. The trajectories of the motion of three particles are shown in Fig. 6.32. Match the entries of Column I with the entries of Column II

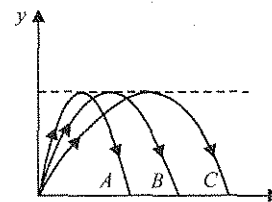


Fig. 6.32

Column I	Column II
i. Time of flight is least for	a. A
ii. Vertical component of the velocity is greatest for	b. B
iii. Horizontal component of the velocity is greatest for	c. C
iv. Launch speed is least for	d. No appropriate match given

5. The path of a projectile is represented by $y = Px - Qx^2$

Column I	Column II
i. Range	a. P/Q
ii. Maximum height	b. P
iii. Time of flight	c. $P^2/4Q$
iv. Tangent of the angle of projection is	d. $\sqrt{\frac{2}{Qg}} P$

6. A body is projected with a velocity of 60 m/s at 30° to horizontal

Column I	Column II
i. Initial velocity vector	a. $60\sqrt{3}\hat{i} + 40\hat{j}$
ii. Velocity after 3 sec	b. $30\sqrt{3}\hat{i} + 10\hat{j}$
iii. Displacement after 2 sec	c. $30\sqrt{3}\hat{i} + 30\hat{j}$
iv. Velocity after 2 sec	d. $30\sqrt{3}\hat{i}$

Archives

Solutions on page 6.25

Fill in the Blanks Type

- A particle moves in a circle of radius R . In half the period of revolution, its displacement is _____ and distance covered is _____. (IIT-JEE, 1983)
- Four persons K, L, M, N are initially at the four corners of a square of side d . Each person now moves with a uniform speed v in such a way that K always moves directly towards L, L directly towards M, M directly towards N , and N directly towards K . The four persons will meet at a time _____. (IIT-JEE, 1984)

3. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 radian/second. The spot of light P moves along the wall at a distance of 3 m. The velocity of the spot P when $\theta = 45^\circ$ is _____ m/s. (IIT-JEE, 1987)

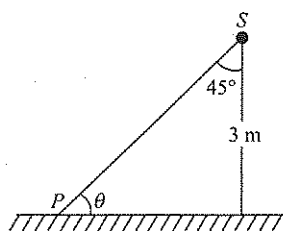


Fig. 6.33

4. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a, b are constants, and x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. The maximum height attained is _____ and the angle of projection from the horizontal is _____. (IIT-JEE, 1997)

True or False

- Two balls of different masses are thrown vertically upward with the same speed. They pass through the point of projection in their downward motion with the same speed (Neglected air resistance). (IIT-JEE, 1983)
- A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of its path. (IIT-JEE, 1984)
- Two identical trains are moving on rails along the equator on the earth in opposite directions with the same speed. They will exert the same pressure on the rails. (IIT-JEE, 1985)
- An electric line of forces in the x - y plane is given by the equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at the point $x = 1, y = 0$ in the x - y plane, will move along the circular line of force. (IIT-JEE, 1988)

Single Correct Answers Type

- A river is flowing from west to east at a speed of 5 m per minute. A man on the south bank of the river, capable of swimming at 10 m per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction
a. due north b. 30° east of north
c. 30° west of north d. 60° east of north (IIT-JEE, 1983)
- A boat which has a speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water in km/h is
a. 1 b. 3 c. 4 d. $\sqrt{41}$ (IIT-JEE, 1988)
- In 1.0 s, a particle goes from point A to point B , moving in a semicircle of radius 1.0 m (Fig. 6.34). The magnitude of the average velocity is (IIT-JEE, 1999)

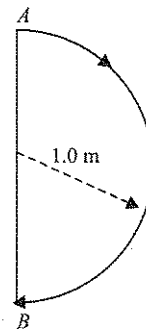
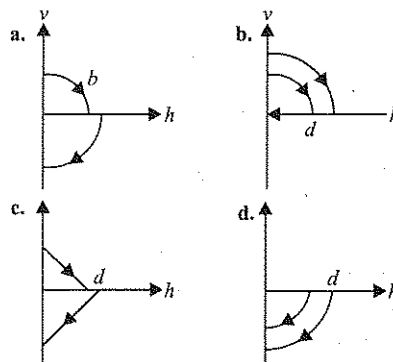


Fig. 6.34

- a. 3.14 m/s b. 2.0 m/s c. 1.0 m/s d. Zero.
4. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as (IIT-JEE, 2000)



5. A particle starts sliding down a frictionless inclined plane. If S_n is the distance travelled by it from time $t = n - 1$ sec to $t = n$ sec, the ratio S_n/S_{n+1} is (IIT-JEE, 2004)
a. $\frac{2n-1}{2n+1}$ b. $\frac{2n+1}{2n}$ c. $\frac{2n}{2n+1}$ d. $\frac{2n+1}{2n-1}$
6. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in Fig. 6.35. The maximum speed of the particle will be (IIT-JEE, 2004)

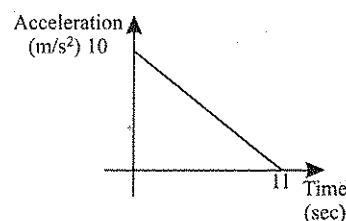


Fig. 6.35

- a. 110 m/s b. 55 m/s c. 550 m/s d. 660 m/s
7. The velocity displacement graph of a particle moving along a straight line is shown in (Fig. 6.36). The most suitable acceleration-displacement graph (Fig. 6.37) will be (IIT-JEE, 2005)

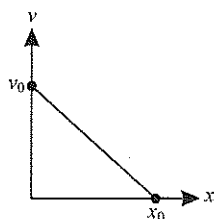


Fig. 6.36

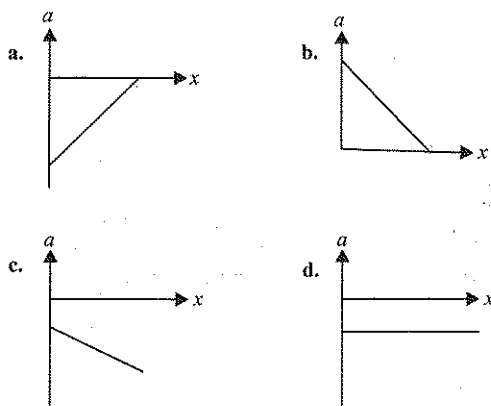


Fig. 6.37

Multiple Correct Answers Type

- A particle is moving eastwards with a velocity of 5 m/s in 10 s, the velocity changes to 5 m/s northwards. The average acceleration in this time is
 - zero
 - $1/\sqrt{2} \text{ m/s}^2$ towards north-west
 - $1/2 \text{ m/s}^2$ towards north-west
 - $1/2 \text{ m/s}^2$ towards north

(IIT-JEE, 1982)
- A particle of mass m moved on the x -axis as follows: it starts from rest $t = 0$ from the point $x = 0$, and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate times ($0 < t < 1$). If α denotes the instantaneous acceleration of the particle, then:
 - α cannot remain positive for all t in the interval $0 \leq t \leq 1$.
 - $|\alpha|$ cannot exceed 2 at any point in its path
 - $|\alpha|$ must be ≥ 4 at some point or points in its path
 - α must change sign during the motion, but no other assertion can be made with the information given

(IIT-JEE, 1993)
- The coordinates of a particle moving in a plane are given by $x(t) = a \cos(pt)$ and $y(t) = b \sin(pt)$ where $a, b (< a)$ and p are positive constants of appropriate dimensions. Then
 - the path of the particle is an ellipse
 - the velocity and acceleration of the particle are normal to each other at $t = \pi/(2p)$
 - the acceleration of the particle is always directed towards a focus
 - the distance travelled by the particle in time interval $t = 0$ to $t = \pi/(2p)$ is a

(IIT-JEE, 1999)

ANSWERS AND SOLUTIONS

Objective Type

- b. We have $v_{av} = \frac{\text{Displacement}}{\text{Time interval}}$
 $= \text{Slope of chord on } x-t \text{ graph.}$

Here, slope of chord between P and Q for all three particles is same, so average velocity of all the three particles would be the same.

- a. $a = v \frac{dv}{ds} = 4 \times \tan 60^\circ = 4\sqrt{3} \text{ m/s}^3.$

- d. We know that $a = \frac{dv}{dt}$
 $\Rightarrow \int dv = \int a dt$

R.H.S. of the above expression represents area under acceleration-time graph.

Area of acceleration-time graph is not getting zero over the interval for which graph is given, hence velocity is not getting zero gain over the interval.

- b. At the instant, when bolt starts falling it acquires the velocity of the lift.

$$v_{BL} = 0 \text{ and } v_{LG} = v$$

$$v_{BG} = 0 + v = v \text{ in upward direction}$$

and $a_{BG} = g$ in downward direction

So, displacement of the bolt w.r.t. ground is given by the equation,

$$y_{BG} = vt - \frac{1}{2}gt^2$$

which is a parabolic equation but this is valid only for the time interval which it takes to the floor of elevator.

After that displacement-time graph of bolt and elevator would be same under the assumption that bolt sticks to the floor of elevator after striking it.

- a. From $a = v \frac{dv}{ds}$, we can find the sign of acceleration at various points. V is positive for all three points 1, 2 and 3. $\frac{dv}{ds}$ is positive for point 1, zero for point 2, negative for point 3.

So, only for point 1, velocity and acceleration have same sign, so the object is speeding up at point 1 only.

- c. As the speed with which the ball has been thrown in the downward direction is greater than the terminal speed.

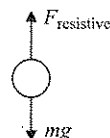


Fig. 6.38

So, $F_{\text{resistive}} > mg$

And hence initially net force acting on the projectile is in upward direction and hence the acceleration also is in the same direction.

But this acceleration is opposite to velocity, so speed decreases and hence $F_{\text{resistive}}$. At a particular instant, the projectile acquires the terminal speed and at this instant $F_{\text{resistive}} = mg$ and hence $a = 0$ and the particle moves with constant speed, i.e., the terminal.

7. c.

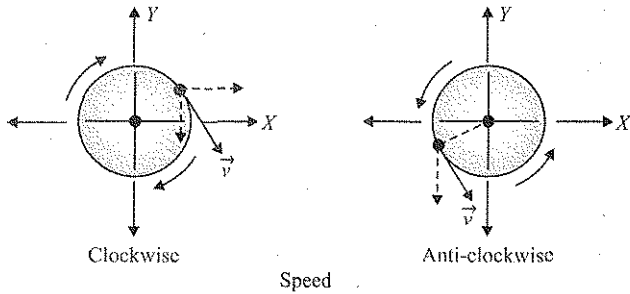


Fig. 6.39

8. b. In this case, velocity and acceleration are acting in opposite directions and velocity is getting zero at $t = 4$ s.

Distance travelled in 10 s

$$= 2 \times |\text{displacement in 4 s}| - \text{displacement in 10 s}$$

$$\text{Required distance} = 2 \times 8 - (-10) = 26 \text{ m.}$$

9. b.

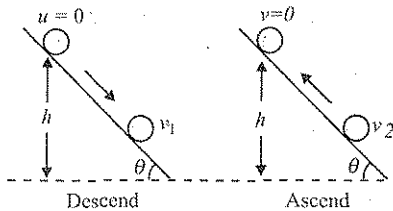


Fig. 6.40

It is clear that $v_1 < v_2$ and from equation of motion, we have

$$\frac{h}{\sin \theta} = \frac{v_1 t_1}{2} = \frac{v_2 t_2}{2}$$

where t_1 and t_2 are the times taken for ascend and descend, respectively.

As $v_1 < v_2$ so $t_1 > t_2$.

10. b. The situation is as shown in Fig. 6.41.

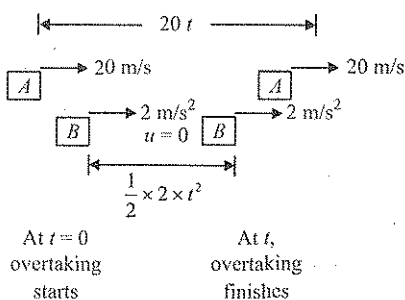


Fig. 6.41

From the diagram, we get

$$20t = 10 + \frac{1}{2} \times 2t^2 \Rightarrow t^2 + 10 - 20t = 0$$

$$t = 0.513 \text{ s, } 19.487 \text{ s}$$

Out of these two, $t_1 = 0.513$ s corresponds to the situation when overtaking has been completed and $t_2 = 19.487$ s corresponds to the same situation as shown in Fig. 6.41, but for $t_1 < t < t_2$ the separation between two cars first increases and then decreases and then B overtakes A.

Total road distance used = $5 + 20t_1 = 15.26$ m.

11. a. At $t = 0$, $v = 0$, $x = 4/3$

$$a = \frac{4 \times 3}{3} - 4 = 0.$$

As both the particle velocity and acceleration are zero at $t = 0$, so it will always remain at rest and hence distance travelled at any time interval would be zero.

12. d. $\vec{u} = 20 \cos 30^\circ \hat{i} + 20 \sin 30^\circ \hat{j}$

Let $\vec{v}$ is perpendicular to $\vec{u}$ at time t

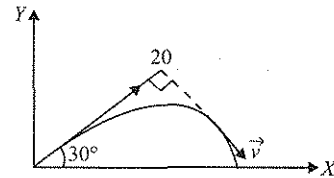


Fig. 6.42

Now, $\vec{v} = 20 \cos 30^\circ \hat{i} + (20 \sin 30^\circ - gt) \hat{j}$

$$\vec{v} \cdot \vec{u} = 0 \Rightarrow t = 4 \text{ s. Here, time of flight } T = 2 \text{ s.}$$

So, it is not possible at any instant.

13. d. Here, the automobile is taking less than 1 min to stop. So first find that time in which the vehicle stops.

$$\text{From } \vec{v} = \vec{u} + \vec{a}t \Rightarrow 0 = 20 - 0.5t \Rightarrow t = 40 \text{ s.}$$

The distance covered by the vehicle in 40 s would be

$$s = 20 \times 40 - \frac{1}{2} \times 0.5 \times (40)^2 = 800 - 400 = 400 \text{ m.}$$

$$\text{Required distance} = 500 - 400 = 100 \text{ m.}$$

14. c. As the parachute inflates, a large impulsive air drag acts in the upward direction, as a result the parachutist suddenly shoots upwards and then under the presence of air drag force and gravity force, he will start falling again but more slowly.

15. d. $\vec{v}_2 = \vec{v}_1 - \vec{v}_0$, $\vec{v}_2 + \vec{v}_0 = \vec{v}_1$

From the law of triangle, we can draw the velocity vector diagram.

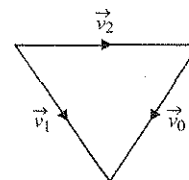


Fig. 6.43

From triangle property, sum of two sides $\geq$ third side
So, $v_1 \leq v_0 + v_2$

16. c.

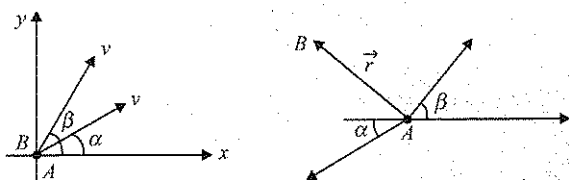


Fig. 6.44

A is fixed and B moves in the direction as shown in Fig. 6.44.

Direction of $\vec{r}$ remains same and magnitude changes with time linearly.

17. b. $x^3 = t^3 + 1 \Rightarrow 3x^2 \frac{dx}{dt} = 3t^2$

$$\frac{dx}{dt} = \frac{t^2}{x^2} \quad (1)$$

$$\frac{d^2x}{dt^2} = \frac{2tx^2 - 2x \frac{dx}{dt} t^2}{x^4}$$

$$a = \frac{1}{x^4} \left[2tx^2 - 2xt^2 \frac{t^2}{x^2} \right]$$

$$= \frac{2tx^2}{x^4} \left[1 - \frac{t^3}{x^3} \right] = \frac{2tx^2}{x^4} \left[1 - \left\{ \frac{x^3 - 1}{x^3} \right\} \right] = \frac{2t}{x^5}$$

18. a. The radius of curvatures will be mutually perpendicular only when the velocity vectors will be mutually perpendicular i.e., after the time $t = \sqrt{2} \frac{u}{g}$.

19. a. $0 = u \cos 30^\circ - g \sin 30^\circ t$

$$t = \frac{u \cos 30^\circ}{g \sin 30^\circ} \quad (1)$$

$$-H \cos 30^\circ = -u \sin 30^\circ t - \frac{1}{2} g \cos 30^\circ t^2 \quad (2)$$

By equation (1) and (2), we get

$$H = \frac{u^2}{g} \left[1 + \frac{\cot^2 \alpha}{2} \right] v = \sqrt{\frac{2gH}{5}}$$

$$\{\alpha = 30^\circ\}$$

20. b. The vertical displacement vs. time graph has the same form

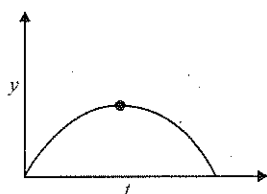


Fig. 6.45

as the trajectory of the particle. At the topmost point the slope is zero, but the curvature is non-zero (Fig. 6.45).

21. a. Time taken by particles to collide

$$t = \sqrt{\frac{2H}{g}} \text{ then } u \sqrt{\frac{2H}{g}} + v \sqrt{\frac{2H}{g}} = d$$

$$\Rightarrow u + v = d \sqrt{\frac{g}{2H}}, \therefore v = d \sqrt{\frac{g}{2H}} - u.$$

22. d. Here $n = 12t = \frac{a}{v - v \cos \frac{2\pi}{n}}$

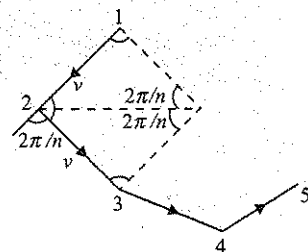


Fig. 6.46

$$t = \frac{a}{v - v \cos \frac{2\pi}{12}} = \frac{2a}{v(2 - \sqrt{3})}$$

23. b. $a = \frac{dv}{dt} = \frac{dv}{dS} \frac{dS}{dt} = 4 + 3v$

$$\int dS = \int \frac{v dv}{4 + 3v}$$

$$S = \frac{v}{3} - \frac{4}{9} \log_e (4 + 3v) + C$$

At $S = 0$, $v = 0$, we get, $C = \frac{4}{9} \log_e 4$

$$S = \frac{v}{3} + \frac{4}{9} \log_e \left(\frac{4}{4 + 3v} \right)$$

Putting $v = -2$ m/s, we get

$$S = \frac{2}{3} + \frac{4}{9} \log_e \frac{4}{10} = 0.26 \text{ m.}$$

24. a. Let d be the river width and u and v the speeds of the water current for P ; time taken

$$t_1 = \frac{2d}{\sqrt{v^2 - u^2}} \quad (1)$$

and for Q ; time taken $t_2 = d \left[\frac{1}{v + u} + \frac{1}{v - u} \right] \quad (2)$

Dividing (1) by (2), we get

$$\frac{t_1}{t_2} = \sqrt{1 - \left(\frac{u}{v} \right)^2} < 1 \therefore t_1 < t_2$$

25. c. $a_x = -g \sin \theta$; $a_y = -g \cos \theta$

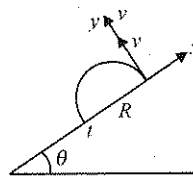


Fig. 6.47

$$u_x = 0; u_y = v$$

Along y-axis, $S_y = u_y t + \frac{1}{2} a_y t^2$

$$0 = v t - \frac{1}{2} g \cos \theta t^2$$

$$t = \frac{2v}{g \cos \theta}$$

Along x-axis, $S_x = u_x t + \frac{1}{2} a_x t^2$

$$R = 0 \times t - \frac{1}{2} g \sin \theta \left(\frac{2v}{g \cos \theta} \right)^2$$

$$R = \frac{-2v \tan \theta \sec \theta}{g}$$

26. a.

$$0.2n = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2 \times 0.2n}{g}}$$

$$0.3n = 4t$$

$$0.3n = 4.5 \sqrt{\frac{2 \times 0.2n}{g}}$$

$$n = 9$$

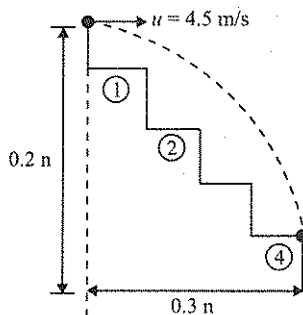


Fig. 6.48

27. a. $\vec{v}_m$ = Velocity of the man

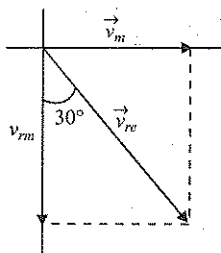


Fig. 6.49

$\vec{v}_{re}$ = Velocity of rain w.r.t. earth

$\vec{v}_{rm}$ = Velocity of rain w.r.t. man

Velocity of man $|\vec{v}_m| = 10 \text{ ms}^{-1}$

Using $\sin 30^\circ = \frac{v_m}{v_{re}}$

$$v_{re} = \frac{v_m}{\sin 30^\circ} = \frac{10}{1/2} = 20 \text{ ms}^{-1},$$

$$\cos 30^\circ = \frac{v_{rm}}{v_{re}}$$

$$v_{rm} = v_{re} \cos 30^\circ = \frac{20 \times \sqrt{3}}{2} = 10\sqrt{3} \text{ ms}^{-1}.$$

28. c.

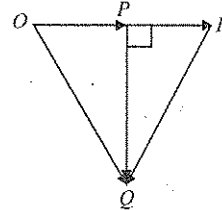


Fig. 6.50

Case I: Let $OP = 3\hat{i}$ be the velocity of man. $\vec{OQ}$ be the velocity of rain.

PQ is the velocity of rain relative to man.

Case II: $\vec{OR} = 6\hat{i}$ is the new velocity of man

$\vec{RQ}$ = new velocity of rain relative to man

Now $OQ^2 + OP^2 + PQ^2$ i.e., $OQ^2 = 3^2 + 3^2$

($\therefore PQ = PR = 6 - 3 = 3$)

i.e., $OQ = 3\sqrt{2} \text{ km h}^{-1}$

and $\tan \theta = \frac{PQ}{OQ} = \frac{3}{2} = 1$, i.e., $\theta = 45^\circ$

29. b. For the maximum range $\theta = 45^\circ$,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2}{g} \sin 90^\circ = \frac{u^2}{g} \text{ or } 500 = \frac{u^2}{g}$$

The distance covered along the inclined plane can be obtained using the equation

$$v^2 - u^2 = 2as$$

$$\text{or } 0 - u^2 = 2(-g \sin 30^\circ)s$$

$$\text{or } s = \frac{u^2}{g} = 500 \text{ m.}$$

30. b. From the graph, velocity-displacement equation can be written as

$$v = v_0 + \alpha x \quad (1)$$

Here v_0 and α are positive constants.

Differentiating (1) with respect to x , we get

$$\frac{dv}{dx} = \alpha = \text{constant}$$

Acceleration of the particle can be written as

$$a = v \frac{dv}{dx} = (v_0 + \alpha x)\alpha$$

$a - x$ equation is a linear equation. Thus, acceleration increases linearly with x .

31. a. Area under $a-t$ graph gives the change in velocity ($dv = adt$)

$$v_f - v_i = \frac{1}{2} \times 2 \times 4 = 4 \text{ ms}^{-1}$$

$$v_f = v_i + 4 = 2 + 4 = 6 \text{ m/s}$$

$$32. \text{ a. } a = \frac{dv}{ds} \frac{ds}{dt} = \frac{dv}{ds} v$$

$$\text{Here } \frac{a}{v} = \frac{dv}{ds} = 1 \text{ s}^{-1}$$

Acceleration versus velocity graph will be a straight line passing through origin with slope 1 s^{-1} .

33. c. Acceleration-velocity equation from the graph is

$$a = kv, \text{ where } k = \text{slope of given line.}$$

This can also be written as

$$v \frac{dv}{ds} = kv \Rightarrow \frac{dv}{ds} = k$$

i.e., slope of velocity-displacement equation is same as slope of acceleration-velocity equation which is a constant.

34. b. Maximum speed is at $t = 8 \text{ s}$

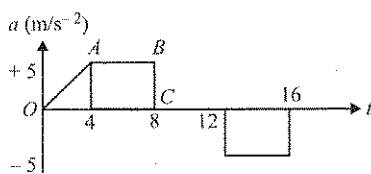


Fig. 6.51

v at $t = 8 \text{ s}$ is given by area under curve $OABC$.

$$\text{Max. speed} = \frac{1}{2}(AB + OC) \times BC$$

$$= \frac{1}{2} \times (12) \times 5 = 30 \text{ ms}^{-1}$$

35. a. Let the velocity of the block is u , then velocity component along the string should be same.

$$u \sin \theta = v \Rightarrow u = v / \sin \theta$$

36. c. $\vec{r} = x\hat{i} + y\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

If $\vec{r}$ and $\vec{v}$ are in opposite direction, definitely the particle will be moving towards origin O , for this

$$\vec{r} \cdot \vec{v} < 0 \Rightarrow xv_x + yv_y < 0$$

37. b. At 12 s , slope of B is same as that of A .

$$38. \text{ d. } x = 7t - 3t^2 \Rightarrow v = \frac{dx}{dt} = 7 - 6t$$

Let us find the time when the velocity becomes zero.

$$\text{Putting } v = 0 \Rightarrow 7 - 6t = 0 \Rightarrow t = 7/6 \text{ s}$$

$$x_{(t=7/6 \text{ s})} = 7 \times \frac{7}{6} - 3 \times \frac{7^2}{6^2} = \frac{49}{12} \text{ m}$$

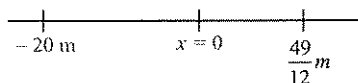


Fig. 6.52

$$\text{at } t = 0 \rightarrow x = 0$$

$$\text{at } t = 45, \rightarrow x = 7 \times 4 - 3(4)^2 = -20 \text{ m}$$

Distance travelled = s

$$= \frac{49}{12} + \frac{49}{12} + 20 = \frac{169}{6 \times 4} = \frac{169}{24} \text{ ms}^{-1}$$

$$\text{Av. speed} = \frac{s}{t} = \frac{169}{6 \times 4} = \frac{169}{24} \text{ ms}^{-1}$$

39. c. In general, $h = \frac{1}{2}gt^2$

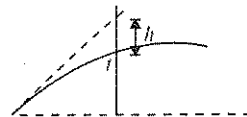


Fig. 6.53

$$x = \frac{1}{2}g(1)^2 = 5 \text{ m}, y = \frac{1}{2}g(2)^2 = 20 \text{ m}, z = \frac{1}{2}g(3)^2 = 45 \text{ m}$$

40. d. $v_1 = v_2 \cos \beta = u \cos \theta$

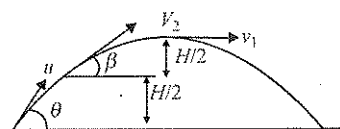


Fig. 6.54

$$0 = (v_2 \sin \beta)^2 - 2g(H/2) \Rightarrow v_1^2 = v_2^2 - gH$$

$$\text{Also } \frac{v_1}{v_2} = \sqrt{\frac{2}{5}}$$

$$\text{From above } v_1 = \sqrt{\frac{2gH}{3}}, v_2 = \sqrt{\frac{5gH}{3}}$$

$$H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \sin^2 \theta = \frac{2gH \cos^2 \theta}{v_1^2}$$

$$\Rightarrow \tan^2 \theta = \frac{2gH}{v_1^2} \Rightarrow \tan^2 \theta = \frac{2gH \times 3}{2gH} \Rightarrow$$

$$\tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

$$41. \text{ b. } \frac{y}{x} = \tan \beta \Rightarrow \frac{u \sin \alpha t - \frac{1}{2}gt^2}{u \cos \alpha t} = \tan \beta$$

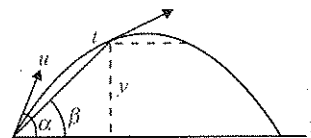


Fig. 6.55

$$\Rightarrow u \sin \alpha - \tan \beta u \cos \alpha = \frac{gt}{2}$$

$$\Rightarrow u = \frac{gt \cos \beta}{2 \sin(\alpha - \beta)}$$

42. c. $u_1 \cos \alpha = u_2 \cos \beta$

$$30\sqrt{3} = \frac{u_2^2 \sin 2\beta}{g} \Rightarrow 30\sqrt{3} = \frac{u_2^2 \sqrt{3}}{2g}$$

$$\Rightarrow u_2^2 = 600 \Rightarrow u_2 = 10\sqrt{6}$$

$$t_2 = \frac{2u_2 \sin 60^\circ}{g} = \frac{(10\sqrt{6})\sqrt{3}/2}{10} \Rightarrow t_2 = \sqrt{18}$$

$$u_1 = \frac{10\sqrt{6} \cos 60^\circ}{\cos 30^\circ} = 10\sqrt{2}$$

$$-h = 10\sqrt{2} \sin 30^\circ \sqrt{18} - \frac{1}{2} 10 (\sqrt{18})^2$$

$$-h = 30 - 90 \Rightarrow h = 60 \text{ m.}$$

43. a. If the particles collide at Q , it means they travel same displacement along the plane in same time. So their velocity components along the plane should be same

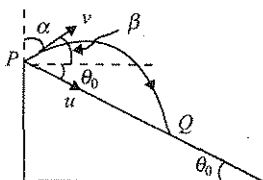


Fig. 6.56

$$v \cos(\beta + \theta_0) = u \text{ and } \beta = 90^\circ - \alpha$$

$$\text{Solve to get: } v \sin(\alpha - \theta_0) = u$$

44. a. Acceleration w.r.t. platform $g + a$

$$\text{So time taken: } T = \frac{2u \sin \theta}{g + a}$$

45. d. If the particles collide in mid-air, they travel same displacement in horizontal direction. So their velocity components along horizontal should be same: $v_1 \cos \theta_1 = v_2 \cos \theta_2$.

Multiple Correct Answers Type

1. a., c., d. Based on thought.
2. a., c., d. Velocity is given by the slope of $x-t$ graph. Here the slope is zero at A and at peak of region CD and the bottommost point of Electric Field.

For AB and EF , the acceleration is positive. For these regions the graph is concave up.

For BC and DE , the acceleration is zero as here the velocity is constant.

For CD , the acceleration is negative, as the graph is concave down for this region.

3. b., c., d. Point of steepest slope corresponds to the maximum speed. Particle will speed up when direction of the acceleration and velocity is the same. For region AB , both the acceleration and velocity are positive while for CD both are negative, so particle is speeding up in these regions.

Average velocity = Slope of chord on $x-t$ graph.

Which is maximum for AB .

4. b., c., d. Based on thought.

5. a., b., c. This is the example of non-uniform acceleration

$$a = \frac{dv}{dt} = -0.5 t$$

$$\int_{16}^v dv = - \int_0^t 0.5 t dt \Rightarrow v = 16 - \frac{0.5 t^2}{2}$$

Direction of velocity changes at the moment when it becomes zero momentarily.

$$0 = 16 - \frac{0.5 t^2}{2} \Rightarrow t = 8 \text{ s}$$

$$\frac{dx}{dt} = v = 16 - \frac{0.5 t^2}{2}$$

Let us consider that at $t = 0$, particle is at $x = 0$

$$\int_0^x dx = \left(16t - \frac{0.5 t^3}{6} \right); \quad x = 16t - \frac{0.5 t^3}{6}$$

Distance travelled = |Displacement| for $t \leq 8$ s. So, the distance travelled in 4 s,

$$x = 16 \times 4 - \frac{0.5 \times 4^3}{6} \simeq 59 \text{ m.}$$

Distance travelled in 10 s

$$= |\text{Displacement in 8 s}| \times 2 - \text{Displacement in 10 s}$$

$$= 85.33 \times 2 - 76.55 = 94 \text{ m}$$

$$v(t = 10 \text{ s}) = -9 \text{ m/s}$$

6. b., c., d. Initially, the FBD of ball would be as shown in Fig. 6.57.

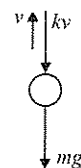


Fig. 6.57

Here, direction of velocity and acceleration is opposite, so particle is slowing down and moving in upward direction, at one instant its velocity becomes zero which would be the highest point of trajectory. Then it starts falling down, for its descent FBD would be as shown in Fig. 6.58.

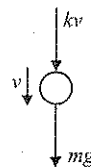


Fig. 6.58

During its descent, the ball will acquire the terminal speed (assuming the ball is not striking ground before acquiring the terminal speed).

As work has been done against the resistive force, so speed of throwing > speed with which the particle lands.

Hence, force of air friction is greatest (when speed is greatest) just after it is thrown.

7. b., c. The particle's velocity is getting zero at $t = 3$ s, where it changes its direction of motion.

For $0 < t < 3$ s, V is negative, a is positive, so particle is slowing down.

For $t > 3$, both V and a are positive, so the particle is speeding up.

8. a., b., d. As the particle is going up, it is slowing down, i.e., speed is decreasing and hence we can say that time taken by the particle to cover equal distances is increasing as the particle is going up.

Hence, $t_1 < t_2 < t_3$.

As $v_{av} = \frac{\text{Distance}}{\text{time}}$, we have $v_{av} \propto \frac{1}{\text{time}}$

So, $v_{av1} > v_{av2} > v_{av3}$

Acceleration throughout the motion remains same from equation,

$$\vec{v} = \vec{u} + \vec{a}t, \quad \Delta v \propto t. \text{ So, } \Delta v_1 < \Delta v_2 < \Delta v_3.$$

9. a., b. $\vec{v}_{AW} = -20 \hat{j}$

$$\vec{v}_{BW} = 32 \hat{i} + 24 \hat{j}$$

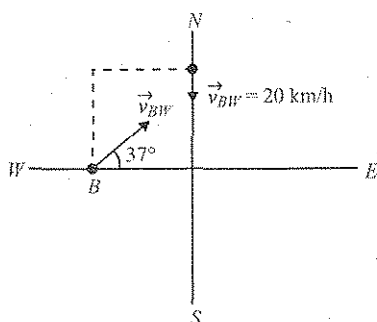


Fig. 6.59

$$\vec{v}_{AB} = \vec{v}_{AW} - \vec{v}_{BW} = -32 \hat{i} - 44 \hat{j}$$

$$\frac{d\vec{r}_{AB}}{dt} = \vec{v}_{AB} = -32 \hat{i} - 44 \hat{j}$$

$$\int_{3\hat{i}+4\hat{j}}^{\vec{r}_{AB}} d(\vec{r}_{AB}) = - \int_0^t 32 dt \hat{i} - \int_0^t 44 dt \hat{j}$$

$$\vec{r}_{AB} = (3 - 32t)\hat{i} + (4 - 44t)\hat{j}$$

Their relative separation is given by $|\vec{r}_{AB}|$

$$x = |\vec{r}_{AB}| = \sqrt{(3 - 32t)^2 + (4 - 44t)^2}$$

For least separation, $\frac{dx}{dt} = 0$.

10. b., c., d. $v_{av} = \frac{\text{Distance travelled}}{\text{Time interval}}$

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}, \quad \vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

11. b., c., d. At O , velocity is non-zero as the slope of $x-t$ graph at $t = 0$ is non-zero, so option (a) is wrong.

For O to A velocity and acceleration are positive so particle is speeding up. Acceleration can be constant or varying so $v-t$ curve can be straight line or curved.

For A to B : Constant velocity, at A and B discontinuity occurs due to large change in velocity, in a very small time.

For B to D : Velocity is negative and acceleration is positive, so particle is slowing down.

For D to E : Constant velocity.

12. a., c. Along X -axis, $v_x = 3$

$$\text{Along } Y\text{-axis, } v_y = 4 + a_y t, \quad |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

13. b. Let $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k}$

As $\vec{a}$ is constant, so its magnitude as well as direction is not changing, but v_x , v_y and v_z all can be varying (any combination of these if a_x , a_y or $a_z = 0$, then corresponding velocity component may be constant).

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\frac{d|\vec{v}|}{dt} = \frac{1}{2\sqrt{v_x^2 + v_y^2 + v_z^2}}$$

$$\left[2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt} + 2v_z \frac{dv_z}{dt} \right] = \frac{\vec{v} \cdot \vec{a}}{v}$$

which is a variable quantity.

$$\left| \frac{d\vec{v}}{dt} \right| = |\vec{a}| \text{ which would be a constant.}$$

$$\frac{dv^2}{dt^2} = \frac{d(v_x^2 + v_y^2 + v_z^2)}{dt^2} = 2 \vec{v} \cdot \vec{a}$$

[a variable quantity].

$$\frac{d(\vec{v}/v)}{dt} = \frac{d}{dt} \left[\frac{v_x \hat{i} + v_y \hat{j} + v_z \hat{k}}{\sqrt{v_x^2 + v_y^2 + v_z^2}} \right]$$

[a variable quantity].

14. a., c., d. $u_x = u_y = 0$, $a_x = 4.9 \text{ m/s}^2$, $a_y = 9.8 \text{ m/s}^2$

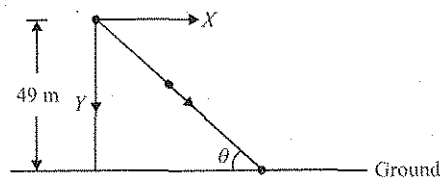


Fig. 6.60

$$\text{So, } x = \frac{1}{2} a_x t^2, \text{ and } y = \frac{1}{2} a_y t^2$$

$$\frac{x}{y} = \frac{a_x}{a_y} = \frac{1}{2}$$

So, path of the ball is a straight line.

Let t be the time taken by the ball to reach ground, then

$$49 = \frac{1}{2} a_y t^2.$$

$$t^2 = 10 \Rightarrow t = 3.16 \text{ s.}$$

$$\tan \theta = \frac{a_y}{a_x} = 2 \Rightarrow \theta = \tan^{-1}(2).$$

15. **b., d.** A body moving on a circular path with uniform speed must have varying velocity as well as acceleration.
16. **a., c., d.** Total displacement travelled by ball A in 2 s,

$$S = ut - \frac{1}{2}gt^2 = 10 \times 2 - \frac{1}{2} \times 10 \times (2)^2 = 0$$

Hence both A and B will reach the ground simultaneously and strike with the same velocity.

17. **b., c.**

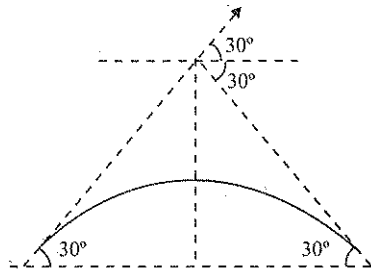


Fig. 6.61

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 10 \times \frac{1}{2}}{10} = 1 \text{ s.}$$

18. **b., c.** $\frac{dx}{dt} = t^2$ (i), $y = \frac{1}{2}t^3 \Rightarrow \frac{dy}{dt} = \frac{t^2}{2}$ (ii)

$$t = 1, v_x = 1, v_y = \frac{1}{2} \Rightarrow \vec{v} = \hat{i} + \frac{1}{2}\hat{j}.$$

$$\frac{d^2x}{dt^2} = 2t \quad \text{(iii)} \quad \frac{d^2y}{dt^2} = t \quad \text{(iv)}$$

$$\text{At } t = 1 \text{ s, } a_x = 2 \text{ and } a_y = 1 \Rightarrow \vec{a} = 2\hat{i} + \hat{j}.$$

Comprehensive Type

For Problems 1-3

1. **b., 2. d., 3. c.**

Sol.

1. **b.** For 0-5 s, initially, for both the particles, velocity and acceleration both are positive, so both the particles are speeding up. But approximately from 2.3 s onwards, direction of acceleration reverses for both the particles, so the particles slow down.
2. **d.** Make a chord to both the particles on $x-t$ graph, so that chord from 0 to $t(s)$ becomes a tangent at t s.
3. **c.** At $t = 0$, slope $|_A$ is steeper than slope $|_B$ so,

$$v_A > v_B$$

There is no time for which tangent drawn to both curves have same slope.

For 5-15 s, $v_{av, A} = 0$ but for B it is non-zero.

All the things can be directly concluded from the graph.

For Problems 4-8

4. **a., 5. c., 6. b., 7. c., 8. b.**

Sol.

4. **a.** $h = \frac{1}{2}gt^2$

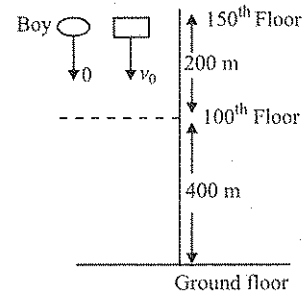


Fig. 6.62

$$200 = \frac{1}{2}g \times (6.39)^2$$

$$\Rightarrow g = 9.798 = 9.80 \text{ m/s}^2.$$

5. **c.** Let time taken by rocketeer in catching the boy at the top of 100th floor be t , then time from which boy is in free fall motion is $(t + 5)$ s.

$$200 = \frac{1}{2}g(t + 5)^2 = v_0t + \frac{gt^2}{2}$$

where v_0 is the initial downward speed of the rocketeer.

$$\Rightarrow t = 1.33 \text{ s}$$

which gives $v_0 = 143.7 \text{ m/s}$.

6. **b.** Initially, the rocketeer is under free fall and acceleration is along vertical downward, direction, i.e., along positive Y -axis, so $y-t$ graph is concave up but when he uses his jet pack the acceleration starts acting in upward direction and hence $y-t$ graph would be concave down.
7. **c.** From solution 2, velocity of rocketeer at the time of catching the boy is

$$v = v_0 + gt = 157 \text{ m/s}$$

For safe landing, final velocity = 0

$$\Rightarrow 0 = v^2 - 2 \times a \times s [s = 400 \text{ m}] \Rightarrow a \approx 3g$$

So, the acceleration produced by jet pack alone would be $(3g + g) \text{ m/s}^2$ in upward direction, as due to gravity, acceleration g is acting in the downward direction.

8. **b.** Initially, the rocketeer's velocity increases with time linearly having acceleration g and then decreases due to the use of jet pack with an acceleration (acting upwards) of $\leq 5g$.

So, the most suitable option is (b).

For Problems 9-13

9. **a., 10. c., 11. d., 12. b., 13. c.**

Sol.

9. **a.** Initial velocity of ball w.r.t. ground.

$$\vec{v}_{BG} = \vec{v}_{BE} + \vec{v}_{EG} = 15 + 10 = 25 \text{ ms}^{-1} \uparrow$$

$$\vec{v}_{BE} = 15 \text{ ms}^{-1} \uparrow, \vec{a}_{BG} = 10 \text{ ms}^{-2} \downarrow$$

$$\vec{a}_{BE} = \vec{a}_{BG} - \vec{a}_{EG} = 10 - (-5) = 15 \text{ ms}^{-2} \downarrow$$

$$\vec{s}_{BE} = 2m \downarrow. \text{ Using } s = ut + \frac{1}{2}at^2 \text{ w.r.t. elevator frame 2}$$

$$= 15t - \frac{1}{2} \times 15t^2 \Rightarrow t = 2.13 \text{ s.}$$

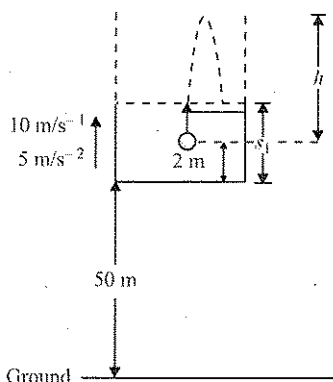


Fig. 6.63

10. c. The height of the ball from ground at the time of projection is 52 m. Maximum height from point of projection is

$$h = \frac{|\vec{v}_{BG}|^2}{2a_{BG}} = \frac{(25)^2}{2 \times 10} = 31.25 \text{ m}$$

Required height = 52 + 31.25 = 83.25 m.

11. d. Displacement of floor of elevator in 2.13 s is

$$s_1 = 10t + \frac{1}{2} \times 5 \times t^2$$

$$s_1 = 32.64 \text{ m}$$

So, displacement of ball w.r.t. ground (Fig. 6.67)

$$\vec{s}_{BG} = \vec{s}_{BE} + \vec{s}_{EG} = -2 + 32.64 = 30.64 \text{ m}$$

12. b. From the diagram above (Fig. 6.63), it is very clear that distance travelled by the ball is

$$H = [h - (s_1 - 2)] = 2h - s_1 + 2 = 31.86 \text{ m.}$$

13. c. Displacement of the ball w.r.t. elevator is given by

$$s = 15t - \frac{1}{2} \times 15t^2$$

Here s is nothing but separation between the floor of elevator and ball.

$$\text{For } s \text{ to be maximum, } \frac{ds}{dt} = 0$$

which gives $t = 1 \text{ s}$

So, the maximum value of s is

$$\left(15 \times 1 - \frac{1}{2} \times 15 \times (2)^2 \right) = 7.5 \text{ m.}$$

For Problems 14–15

14. a., 15. c.,

Sol.

$$14. \text{ a. Av. velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{r\sqrt{2}}{t}$$

where $t = \frac{2\pi r/4}{v}$

$$15. \text{ c. Average acceleration} = \frac{\text{Change in vel}}{t} = \left| \frac{\vec{v}_2 - \vec{v}_1}{t} \right|$$

where $\vec{v}_1 = 2.5\hat{i}$, $\vec{v}_2 = -2.5\hat{j}$.

For Problems 16–17

16. c., 17. b.

Sol.

$$16. \text{ c. } y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}. \text{ Put } x = 30 \text{ m, } y = 15 \text{ m}$$

We get $u = 10\sqrt{6} \text{ m/s.}$

$$17. \text{ b. } v_x = u_x = u \cos \theta, v_y^2 = u_y^2 + 2a_y s_y, s_y = 15 \text{ m and}$$

$$a_y = -g. \text{ Apply } v = \sqrt{v_x^2 + v_y^2}$$

For Problems 18–21

18. a., 19. b., 20. b., 21. d.

Sol.

$$18. \text{ a. } \sin \beta = \frac{u}{v} = \frac{2}{4} = \frac{1}{2} \text{ or } \beta = 30^\circ$$

$$\text{Required angle} = 90^\circ + 30^\circ = 120^\circ.$$

$$19. \text{ b. } t = \frac{d}{\sqrt{v^2 - u^2}}$$

20. b. For the shortest time, one should head the boat perpendicular to flow

$$21. \text{ d. } t = \frac{2}{v - u} + \frac{2}{v + u} = \frac{4}{3} \text{ h.}$$

For Problems 22–24

22. b., 23. d., 24. d.

Sol.

$$22. \text{ b. } y = ax - bx^2 \text{ or } \frac{dy}{dx} = a - 2bx \Rightarrow \frac{dy}{dt} = a - 2bx \frac{dx}{dt} = (a - 2bx) \frac{dx}{dt}$$

Initially $x = 0$, $v_y = av_x$

$$\frac{d^2y}{dx^2} = (a - 2bx) \frac{d^2x}{dx^2} + \frac{dx}{dx} \left[-2b \frac{dx}{dx} \right]$$

$$= (a - 2bx) \frac{d^2x}{dx^2} - 2b \left(\frac{dx}{dx} \right)^2$$

$$\text{at } t = 0, \frac{d^2y}{dx^2} = -\alpha, x = 0, \frac{d^2x}{dx^2} = 0$$

$$\Rightarrow -\alpha = -2b \left(\frac{dx}{dx} \right)^2 = -2b(v_x)^2 \text{ or } v_x = \sqrt{\frac{\alpha}{2b}}$$

$$\Rightarrow v = \sqrt{v_x^2 + v_y^2} = v_x \sqrt{1 + a^2} = \sqrt{\frac{\alpha(1 + a^2)}{2b}}$$

$$23. \text{ d. } \vec{v} = a\hat{i} + bx\hat{j}, v_x = a \text{ or } dx = a dt \Rightarrow x = at$$

$$v_y = bx = bat \text{ or } y = \frac{bat^2}{2}$$

$$\Rightarrow y = \frac{ba}{2} \left(\frac{x}{a} \right)^2 \Rightarrow y \propto x^2 \rightarrow \text{parabolic.}$$

$$24. \text{ d. Ranges will be same. } \frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2}.$$

For Problems 25–27

25. b., 26. b., 27. c.

Sol.

$$25. \text{ b. } x = a \sin \omega t, y = a - a \cos \omega t$$

$$v_x = \frac{dx}{dt} = a\omega \cos \omega t, v_y = \frac{dy}{dt} = a\omega \sin \omega t$$

$$\text{Now } v = \sqrt{v_x^2 + v_y^2} = a\omega.$$

26. b. $\sin \omega t = \frac{x}{a}, \cos \omega t = 1 - \frac{y}{a}$

Now $\sin^2 \omega t + \cos^2 \omega t = 1$

$$\Rightarrow x = \pm \sqrt{2ay \left(1 - \frac{y}{2a}\right)}$$

27. c. $a_x = \frac{dv_x}{dt} = -a\omega^2 \sin \omega t, a_y = \frac{dv_y}{dt} = a\omega^2 \cos \omega t$

$$a = \sqrt{a_x^2 + a_y^2} = a\omega^2$$

For Problems 28-29

28. d., 29. a.

Sol.

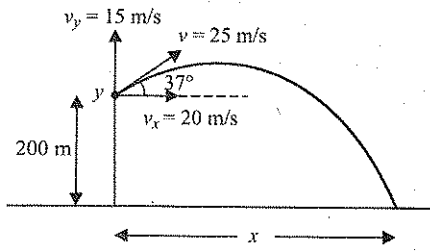


Fig. 6.64

28. d. $x - X_0 = v_x t$

$$y - Y_0 = v_y t - \frac{1}{2} g t^2$$

$$-200 = 15t - 5 + 2$$

$$t = 8 \text{ s}$$

$$x = 20 \times 8 = 160 \text{ m.}$$

29. a $x = 20 \times 8 = 160 \text{ m}$

$$y = 200 + 15 \times 8 = 320 \text{ m.}$$

For Problems 30-31

30. d., 31. a.

Sol.

30. d. $v = u + at$

$$= (2\hat{i} - 9\hat{j}) + (4\hat{i} + 3\hat{j}) \times 2 = 10\hat{i} - 3\hat{j}.$$

31. a. $S = ut + \frac{1}{2}at^2$

$$= (2\hat{i} - 9\hat{j})4 + \frac{1}{2}(4\hat{i} + 3\hat{j}) \times 16$$

$$= 8\hat{i} - 36\hat{j} + 32\hat{i} + 24\hat{j} = 40\hat{i} - 12\hat{j}$$

$$S_i = 44\hat{i} - 9\hat{j}.$$

For Problems 32-34

32. b., 33. d., 34. a.

Sol.

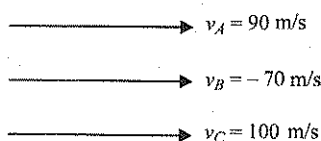


Fig. 6.65

$$v_C = 100 \text{ km/hr}$$

32. b. $v_{AB} = v_A - v_B = 90 + 70 = 160 \text{ m/s}$

33. d. $v_{BA} = v_B - v_A = -160 \text{ m/s}$

34. a. $\vec{v}_{AC} = \vec{v}_A - \vec{v}_C = 90 - 100 = -10 \text{ m/s}$

$$\vec{v}_{BC} = \vec{v}_B - \vec{v}_C = -70 - 100 = -170 \text{ m/s}$$

For Problems 35-37

35. a., 36. a., 37. d.

Sol.

35. a. $V_{B,A} = V_B - V_A$

$$= -14 \cos 45^\circ \hat{i} + 14 \sin 45^\circ \hat{j} - (2 \cos 45^\circ \hat{i}) - 2 \sin 45^\circ \hat{j}$$

$$= -\frac{16}{\sqrt{2}} \hat{i} + \frac{12}{\sqrt{2}} \hat{j} = -8\sqrt{2} \hat{i} - 6\sqrt{2} \hat{j}.$$

36. a. $\tan \phi = \frac{V_2 \sin \theta_2 - V_1 \sin \theta_1}{V_2 \cos \theta_2 + V_1 \cos \theta_1} = \frac{\frac{14}{\sqrt{2}} - \frac{2}{\sqrt{2}}}{\frac{14}{\sqrt{2}} + \frac{2}{\sqrt{2}}} = \frac{3}{4}$

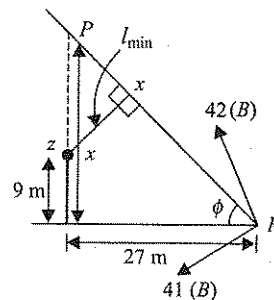


Fig. 6.66

$$\tan \phi = \frac{3}{4} \Rightarrow \phi = 37^\circ.$$

37. d. $\angle QPR = 57^\circ, \frac{3}{4} = \frac{x}{27} \Rightarrow x = \frac{81}{4}$

$$ZP = \frac{81}{4} - 9 = \frac{45}{4} \text{ m.}$$

$$\text{For } \triangle PZX, \sin 53^\circ = \frac{l_{\min}}{ZP}, l_{\min} = \frac{4}{5} \times \frac{45}{4} = 9 \text{ m}$$

For Problems 38-42

38. d., 39. d., 40. b., 41. c., 42. a.

Sol. Two given planes are mutually perpendicular and the particle is projected perpendicularly from plane OA. It means $\vec{u}$ is parallel to plane OB.

At the instant of collision of the particle with OB, its velocity is perpendicular to OB or velocity component parallel to OB is zero.

First considering motion of particle parallel to plane OB, $u = 10\sqrt{3} \text{ ms}^{-1}$, acceleration $= -g \sin 60^\circ$

$$= -5\sqrt{3} \text{ ms}^{-2}$$

$$v = 0, t = ? \text{ s} = ?$$

$$\text{Using, } v = u + at, t = 2 \text{ s}$$

$$s = ut + \frac{1}{2}at^2$$

$$\text{or } OQ = 10\sqrt{3} \text{ m}$$

Now considering motion of the particle normal to plane OB .

Initial velocity = 0, acceleration = $g \cos 60^\circ = 5 \text{ ms}^{-2}$

$t = 2 \text{ s}$, $v = ?$, $s = PO = ?$

Using, $v = u + at$, $v = 10 \text{ ms}^{-1}$

$s = ut + \frac{1}{2}at^2$ or $PO = 10 \text{ m}$

$h = PO \sin 30^\circ = 10 \times \sin 30^\circ = 5 \text{ m}$

Inclination of point P with the vertical is 30° , therefore its vertical component is,

$u \cos 30^\circ = 15 \text{ ms}^{-1}$ (upward)

Considering vertically upward motion of the particle from P , initial velocity = 15 ms^{-1} ,

acceleration = $-g = -10 \text{ ms}^{-2}$, $v = 0$, $s = H = ?$

Using $v^2 = u^2 + 2as$, $H = 11.25 \text{ m}$

Maximum height reached by particle above

$O = h + H = 16.25 \text{ m}$
Distance $PQ = \sqrt{(PO)^2 + (OQ)^2}$

$$= \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ m}$$

Assertion-Reasoning Type

- c.** The gravitational potential is the potential energy per unit mass.
- d.** $S_n = u + \frac{1}{2}a(2n-1)$ is numerically correct but dimensionally incorrect, because dimensions of both sides are not matching.
- d.** Least count depends upon the scale.
- a.** Backlash error is caused due to wear and tear or loose fittings in screws and can be minimized by turning the screw in one direction only.
- c.** Least count = pitch/no. of circular scale divisions.
Less is the pitch, less is least count so more is the accuracy.
- c.** Alloys have least variation in length with temperature.
- d.** Speed of a body is always +ive, its velocity may be +ive or -ive.
- b.** Speed and velocity are different physical quantities. Speed is a scalar quantity and velocity is a vector quantity.
- a.** Speed of a body is always +ive, its velocity may be +ive or -ive.
- a.** If $v-t$ graph is perpendicular to time axis, its slope will be infinite which will indicate infinite acceleration which is not possible in practice.
- c.** Slope of position-time graph will give the velocity and from here we can find both direction and magnitude of acceleration.
- d.** For uniformly accelerated motion started from rest, the displacement versus time graph is parabolic.
And for uniformly accelerated motion, the velocity in equal intervals of time changes by the same amount.

- c.** Due to air resistance, the accelerations of both the balls will be different. Hence, they will reach at different times and with different velocities.
- a.** Slope of $v-t$ graph is zero, hence acceleration will be zero.
- b.** $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$
 $\Rightarrow A^2 + B^2 + 2\vec{A} \cdot \vec{B} = A^2 + B^2 - 2\vec{A} \cdot \vec{B}$
 $\Rightarrow 4\vec{A} \cdot \vec{B} = 0$
 $\Rightarrow \vec{A} \perp \vec{B}$
- d.** The direction of velocity vector is always along the tangent to the path but its slope will not give the magnitude of velocity. Magnitude of velocity is given by the slope of position-time graph.
- b.** When objects move in opposite directions, their velocities are added while calculating relative velocity w.r.t. each other.
- d.** Depends upon the angle between the velocities of two bodies.

Matching Column Type

- i. $\rightarrow$ b., c., ii. $\rightarrow$ c., iii. $\rightarrow$ b., c., iv. $\rightarrow$ b., c.**

If initial velocity and acceleration are in opposite directions, velocity reaches zero and then increases in opposite direction.

In B , initial velocity and acceleration are in same direction so velocity increases continuously and particle move along the direction of acceleration.

In C , $x > 0$ but velocity can be along either positive or negative X -axis.

- i. $\rightarrow$ a., ii. $\rightarrow$ a., iii. $\rightarrow$ b., iv. $\rightarrow$ c.** Slope of velocity-time graph gives acceleration. If direction of acceleration and velocity is the same, then particle is speeding up, otherwise slowing down.
Particle moves in the direction of velocity.
- i. $\rightarrow$ b., d., ii. $\rightarrow$ a., c., iii. $\rightarrow$ a., c., iv. $\rightarrow$ b., d.**
Particle is accelerating when both velocity and acceleration are having the same direction (sign), and decelerating when velocity and acceleration have opposite directions (sign).

- i. $\rightarrow$ d., ii. $\rightarrow$ d., iii. $\rightarrow$ c., iv. $\rightarrow$ a.**

Here, maximum height for all the particles is same.

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u_y^2}{2g}$$

So, all three particles have same u_y .

$$T = \frac{2u \sin \theta}{g} = \frac{2u_y}{g}$$

So, all three particles have the same time period.

Range (R) is maximum for C

R = horizontal component of velocity $\times T$

So, horizontal component of velocity is greater for C .

$$u = \sqrt{u_x^2 + u_y^2}$$

u_x is least for A and u_y is same for all, so u is least for A .

5. i.
- $\rightarrow$
- a., ii.
- $\rightarrow$
- c., iii.
- $\rightarrow$
- d., iv.
- $\rightarrow$
- b.

$$y = Px - Qx^2$$

Equation of trajectory of the projectile motion is given

$$\text{by } y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

On comparing, $\tan \theta = P$ So, iv. $\rightarrow$ b.

$$\frac{g}{2u^2 \cos^2 \theta} = Q; \quad u \cos \theta = \sqrt{\frac{g}{2Q}}$$

$$\text{Range } R = \frac{2u^2}{g} \sin \theta \cos \theta = \frac{2}{g} \frac{g}{2Q} P = \frac{P}{Q}$$

So i. $\rightarrow$ a.

$$\text{Maximum height } H = \frac{u^2}{2g} \sin^2 \theta$$

$$= \frac{1}{2g} \frac{g}{2Q \cos^2 \theta} \sin^2 \theta = \frac{1}{4Q} P^2 = \frac{P^2}{4Q^2}$$

So, ii. $\rightarrow$ c.

$$\begin{aligned} \text{Time of flight } T &= \frac{2u \sin \theta}{g} = \frac{2}{g} \sqrt{\frac{g}{2Q}} P \\ &= \sqrt{\frac{g}{Qg}} P \end{aligned}$$

So, iii. $\rightarrow$ d.

6. i.
- $\rightarrow$
- c., ii.
- $\rightarrow$
- d., iii.
- $\rightarrow$
- a., iv.
- $\rightarrow$
- b.

$$\text{Velocity } u = u \cos \theta \hat{i} + u \sin \theta \hat{j}$$

$$= 60 \times \frac{\sqrt{3}}{2} \hat{i} + 60 \times \frac{1}{2} \hat{j} = 30\sqrt{3} \hat{i} + 30 \hat{j}$$

Displacement after 2 s, $x = \cos \theta \hat{i}$

$$y = (u \sin \theta t - \frac{1}{2} g t^2) \hat{j}, \quad x = 60 \times \frac{\sqrt{3}}{2} \times 2 \hat{i} = 60\sqrt{3} \hat{i}$$

$$y = \left(60 \times \frac{1}{2} \times 2 - \frac{1}{2} \times 10 \times 4 \right) \hat{j} = 40 \hat{j}$$

Displacement $+ y = 60\sqrt{3} \hat{i} + 40 \hat{j}$ Velocity after 2 s, $v_x = u_x = u \cos \theta \hat{i}$

$$= 60 \times \frac{\sqrt{3}}{2} \hat{i} = 30\sqrt{2} \hat{i}$$

$$v_y = (u \sin \theta - gt) \hat{j} = (60 \times \frac{1}{2} - 10 \times 2) = 10 \hat{j}$$

$$v = v_x \hat{i} + v_y \hat{j} = 30\sqrt{3} \hat{i} + 10 \hat{j}$$

Archives

Fill in the Blanks Type

1. Displacement =
- $2r$

Distance = πr

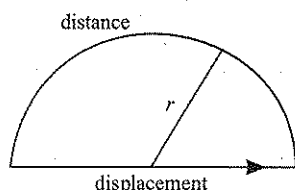


Fig. 6.67

2. The velocity of
- K
- throughout the motion towards the centre of the square is
- $v \cos 45^\circ$
- and the displacement covered by this velocity will be
- KO
- (Fig. 6.68).

$$t = \frac{KO}{v \cos 45^\circ} = \frac{(\sqrt{2}d/2)}{v/\sqrt{2}} = \frac{d}{v}$$

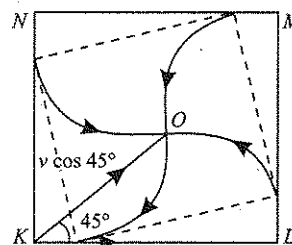


Fig. 6.68

Alternatively:

$$\vec{v}_{KN} = \vec{v}_K - \vec{v}_N = \vec{v}_K + (-\vec{v}_N) \text{ along the line}$$

$$KN = 0 + [-(-v)] = v$$

Time taken for K and N to meet will be $= d/v$.

- 3.
- $v = r\omega = 3 \tan 45^\circ \times 0.1 = 0.3 \text{ m/s}$

4. Given that
- $y = ax - bx^2$

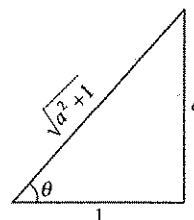


Fig. 6.69

Comparing it with the equation of a projectile, we get

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} \Rightarrow \tan \theta = a \quad \text{(i)}$$

$$\text{and } \frac{g}{2u^2 \cos^2 \theta} = b \quad \text{(ii)}$$

$$\text{From (ii), } \frac{g}{2b \cos^2 \theta} = \frac{g(a^2 + 1)}{2b} \quad \text{(iii)}$$

$$\left[\because \cos \theta = \frac{1}{\sqrt{a^2 + 1}} \right]$$

$$\text{Maximum height attained } H = \frac{u^2 \sin^2 \theta}{2g} \quad \text{(iv)}$$

$$\text{From (ii), } \frac{g}{2b} = u^2 \cos^2 \theta \Rightarrow \frac{g}{2b} = u^2 - u^2 \sin^2 \theta$$

$$\Rightarrow u^2 \sin^2 \theta = u^2 - \frac{g}{2b}$$

$$\Rightarrow u^2 \sin^2 \theta = \frac{g(a^2 + 1)}{2b} - \frac{g}{2b} = \frac{ga^2}{2b} \quad \text{(v)}$$

$$\text{From (iv) and (v), we get } H = \frac{ga^2}{2b \times 2g} = \frac{a^2}{4b}$$

Alternatively:

$$\frac{dy}{dx} = a - 2bx = 0 \text{ (For maximum height)}$$

$$\Rightarrow x = \frac{a}{2b}$$

Substituting the value of x in $y = ax - bx^2$ to find maximum height

$$H = a\left(\frac{a}{2b}\right) - b\left(\frac{a^2}{4b^2}\right) = \frac{a^2}{2b} - \frac{a^2}{4b} = \frac{a^2}{4b}$$

True or False

1. **True.** When the two balls are thrown vertically upward with the same speed then

$$\begin{aligned} &+ \uparrow \uparrow u_1 \uparrow u_2 \\ &- \downarrow u_1 = u \end{aligned}$$

$$s_1 = 0; u_2 = 0$$

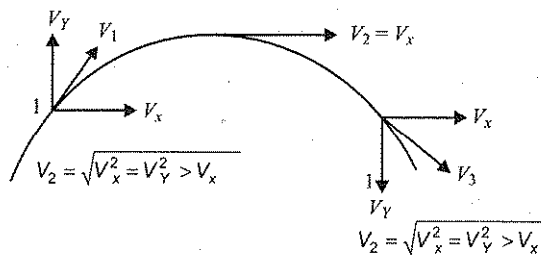
$$a_1 = g; s_2 = 0 \text{ (as ball reaches back to the same point)}$$

$$v_1 = ?; a_2 = -g$$

$$v^2 - u^2 = 2as; v_2 = ?$$

$v_1 = \sqrt{-2gs + u^2} = u$. Thus, we find that g is independent of mass.

2. **True.**

**Fig. 6.70**

As shown in the Fig. 6.74, the velocity at 1 and 3, i.e., at any arbitrary points before and after the topmost point is greater than v_x .

Alternatively:

$$T.E. = P.E. + K.E.$$

$$T.E. = \text{Constant}$$

At P , $K.E.$ is minimum and $P.E.$ is maximum

Since $K.E.$ is minimum velocity is also minimum at P , the topmost point.

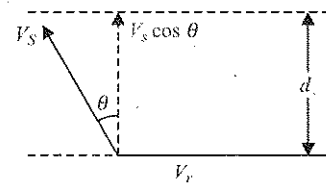
3. **False.** The pressure exerted will be different because one train is moving in the direction of earth's rotation and the other in the opposite direction.
4. **False.** For a particle to move in a circular motion, we need a centripetal force which is not available.
The statement is false.

Single Correct Answers Type

1. a. Time taken to cross the river $t = \frac{d}{v_s \cos \theta}$

For time to be minimum
 $\cos \theta = \max$

$$\Rightarrow \theta = 0^\circ$$

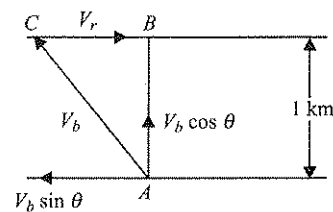
**Fig. 6.71**

$\Rightarrow$ The swimmer should swim due north.

2. b. Vertical displacement = 1 km

$$t = \frac{15}{60} = \frac{1}{4} \text{ hr}$$

$$v_b \cos \theta = \frac{1}{1/4} = 4 \text{ km/h}$$

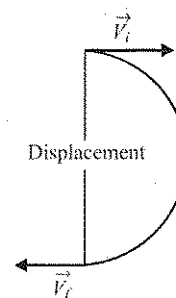
**Fig. 6.72**

$$\cos \theta = \frac{4}{v_b} = \frac{4}{5} \Rightarrow \sin \theta = \frac{3}{5}$$

By velocity triangle ABC , $\sin \theta = \frac{V_r}{V_b}$

$$\frac{V_r}{V_b} = \frac{3}{5} \Rightarrow \frac{V_r}{5} = \frac{3}{5} \Rightarrow V_r = 3 \text{ km/h.}$$

3. b. $|\text{Average velocity}| = \frac{|\text{Displacement}|}{\text{Time}}$

**Fig. 6.73**

$$= \frac{2r}{t} = 2 \times \frac{1}{1} = 2 \text{ m/s.}$$

4. a. Before hitting the ground, the velocity v is given by $v^2 = 2gd$ (quadratic equation and hence parabolic path)
Downwards direction means negative velocity. After collision, the direction becomes positive and velocity decreases.

$$\text{Further, } v'^2 = 2g \times \left(\frac{d}{2}\right) = gd; \left(\frac{v}{v'}\right) = \sqrt{2}$$

$$\text{or } v = v'\sqrt{2}.$$

As the direction is reversed and speed is decreased and hence the graph (a) represents these conditions correctly.

$$5. \text{ a. } S_n = \frac{a}{2}(2n-1); S_{n+1} = \frac{a}{2}(2n+1)$$

$$\frac{S_n}{S_{n+1}} = \frac{2n-1}{2n+1}$$

6. b. Till 11 s, acceleration is positive, so velocity will go on increasing up to 11 s and maximum velocity will happen at 11 s.

The area under acceleration-time graph gives change in velocity. Since particle starts with $u = 0$, therefore change in velocity

$$= v_f - v_i = v_{\max} - 0 = \text{area under } a-t \text{ graph}$$

$$\text{or } v_{\max} = \frac{1}{2} \times 10 \times 11 = 55 \text{ m/s}$$

$$7. \text{ a. } v = -\frac{v_0}{x_0}x + v_0, a = v \frac{dx}{dv} = \left(-\frac{v_0}{x_0}x + v_0\right) \left(-\frac{v_0}{x_0}\right)$$

$$\Rightarrow a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Multiple Correct Answers Type

$$1. \text{ b. Average acceleration } \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{\vec{v}_f + (-\vec{v}_i)}{t} = \frac{\vec{v}}{t}$$

To find the resultant of $\vec{v}_f$ and $-\vec{v}_i$, we draw the following (Fig. 6.74)

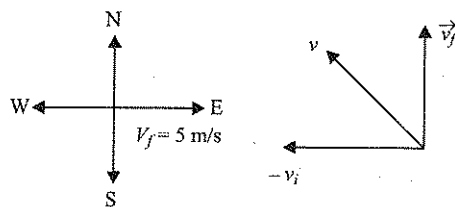


Fig. 6.74

$$|\vec{v}| = \sqrt{v_f^2 + v_i^2} = \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ m/s}$$

$$\text{Since, } |\vec{v}_f| = |-\vec{v}_i|$$

$\vec{v}$ is directed in between $\vec{v}_f$ and $-\vec{v}_i$.

Therefore, $\vec{v}$ is directed towards N-W

$$\vec{a} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$$

2. a., d. α cannot remain positive for all t in the interval $0 \leq t \leq 1$. This is because since the body starts from rest, it will first accelerate, finally it stops therefore α will become negative. Therefore α will change its direction. Hence, (a) and (d) are the correct options.

3. a., b., c.

$$x = a \cos pt \Rightarrow \cos(pt) = \frac{x}{a} \quad (i)$$

$$y = b \sin pt \Rightarrow \sin(pt) = \frac{y}{b} \quad (ii)$$

Squaring and adding (i) and (ii), we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Path of the particle is an ellipse.

Hence option (a) is correct.

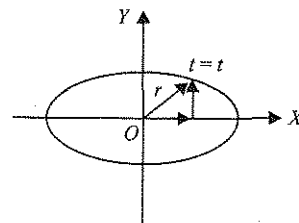


Fig. 6.75

From the given equations, we find

$$\frac{dx}{dt} = v_x = -ap \sin pt$$

$$\frac{d^2x}{dt^2} = a_x = -ap^2 \cos pt$$

$$\frac{dy}{dt} = v_y = bp \cos pt$$

$$\text{and } \frac{d^2y}{dt^2} = a_y = -bp^2 \sin pt$$

$$\text{At time } t = \frac{\pi}{2p} \text{ or } pt = \frac{\pi}{2}$$

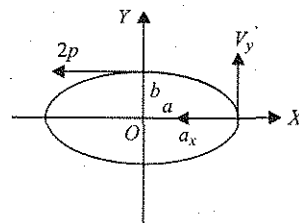


Fig. 6.76

a_x and v_y become zero (because $\cos \pi/2 = 0$), only v_x and a_y are left (Fig. 6.76).

Or we can say that velocity is along negative x -axis and acceleration along y -axis.

Hence, at $t = \frac{\pi}{2p}$, velocity and acceleration of the particle are normal to each other. So option (b) is also correct. At $t = t$, position of the particle $\vec{r}(t) = x\hat{i} + y\hat{j} = a \cos pt\hat{i} + b \sin pt\hat{j}$ and acceleration of the particle is

$$\vec{a}(t) = a_x\hat{i} + a_y\hat{j} = -p^2 [a \cos pt\hat{i} + b \sin pt\hat{j}]$$

$$= -p^2 [x\hat{i} + y\hat{j}] = -p^2 \vec{r}(t)$$

Therefore, acceleration of the particle is always directed towards origin.

Hence, option (c) is also correct.

At $t = 0$, particle is at $(a, 0)$ and at $t = \frac{\pi}{2p}$, particle is at $(0, b)$. Therefore, the distance covered is one-fourth of the elliptical path and not a .

Hence, option (d) is wrong.

Newton's Laws of Motion

- Introduction
- The Concept of Force
- Classification of Forces
- Newton's Laws of Motion
- Impulse
- Free Body Diagrams
- Non-inertial Frame of Reference and Pseudo (fictitious) Force
- Equilibrium of a Particle
- Constraint Relation
- Spring Force and Combinations of Springs
- Analysis of Friction Force
- Dynamics of Circular Motion

INTRODUCTION

In kinematics we dealt with motion of particles based on the definitions of position, velocity, and acceleration without analysing its cause of motion. We would like to be able to answer general questions related to that cause of motion such as “What mechanism causes change in motion?” and “why do some objects accelerate at higher rates than others?” In this section we shall discuss the causes of the change in motion of particles using the concepts of force and mass. We will discuss the three fundamental laws of motion which are based on experimental observations and were formulated by **Sir Isaac Newton**.

THE CONCEPT OF FORCE

How a body moves? It is determined by the interaction of the body with its environment. These interactions are called forces. The concept of force gives us a quantitative description of the interaction between two bodies or between a body and its environment. As a result of everyday experiences, everybody has a basic understanding of concept of force. When you push or pull an object you exert a force on it. You exert a force when you throw or kick a ball. In these examples, the word ‘force’ is associated with the result of muscular activity and with some change in the state of motion of an object. Force does not always cause an object to move, however.

For example, as you sit reading a book, the gravitational force acts on your body and yet you remain stationary. You can push on a heavy block of stone and yet fail to move it (Fig. 7.1).

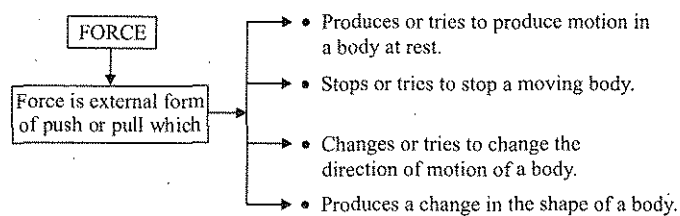


Fig. 7.1

CLASSIFICATION OF FORCES

Based on the nature of the interaction between two bodies, forces may be broadly classified as under:

- 1. Contact Forces:** Tension, normal reaction, friction, etc. Forces that act between bodies in contact.
- 2. Field Forces (Non-Contact Forces):** Weight, electrostatic forces, etc. Forces that act between bodies separated by a distance without any actual contact.

Since we're going to encounter these forces in our analysis, we will briefly discuss each force and how it acts between two bodies, its nature, etc., and how we are going to take it into account. We are going to discuss some special forces.

If you pull on a spring, as in Fig. 7.2(a), the spring stretches. If the spring is calibrated, the distance it stretches can be used to measure the strength of the force. If a child pulls on a wagon, as

in Fig. 7.2(b), the wagon moves. When a football is kicked, as in Fig. 7.2(c), it is both deformed and set in motion. These examples show the results of a class of forces called contact forces. That is, these forces represent the result of physical contact between the two objects.

There exist other forces that do not involve physical contact between the two objects. These forces, known as field forces, can act through empty space. The gravitational force between two objects that causes the free-fall acceleration described in chapters 2 and 3 is an example of this type of force and is illustrated in Fig. 7.2(d). This gravitational force keeps objects bound to the Earth and gives rise to what we commonly call the weight of an object. The planets of our solar system are bound to the Sun under the action of gravitational forces. Another common example of a field force is the electric force that one electric charge exerts on another electric charge, as in Fig. 7.2(e). These charges might be an electron and proton forming a hydrogen atom. A third example of a field force is the force that a bar magnet exerts on a piece of iron, as shown in Fig. 7.2(f).

Some examples of forces applied to various objects. In each case, a force is exerted on the particle or object within the boxed area. The environment external to the boxed area provides this force.

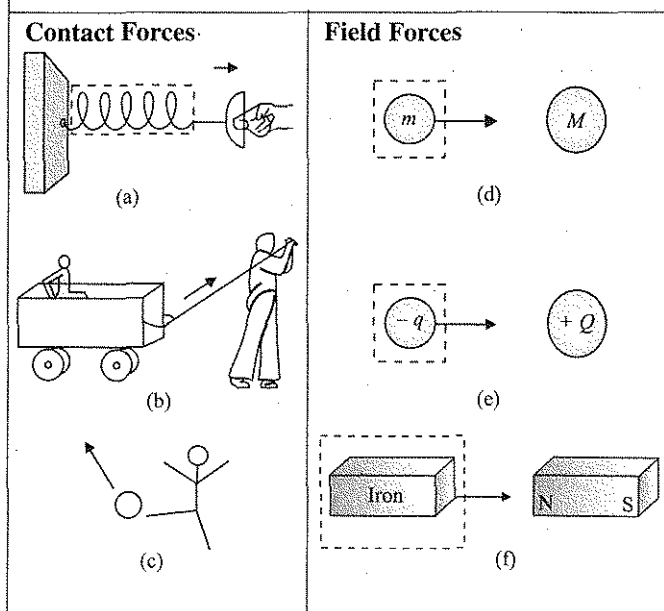


Fig. 7.2

NEWTON'S LAWS OF MOTION

The entire classical mechanics is based upon Newton's laws of motion. They, in fact, are simply known as **Laws of motion**. These laws provide the basis for understanding the effect that forces have on an object.

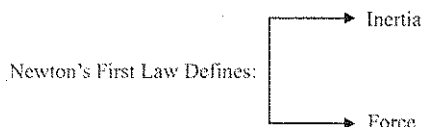
Newton's First Law of Motion

According to this law: A body continues to be in its state of rest or uniform motion along a straight line, unless it is acted upon by some external force to change the state.

Every body continues to be in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed thereon. This law, also called the law of inertia, is really a statement about reference frames, in the sense that it defines the kind of reference frame in which the laws of Newtonian Mechanics hold. If you find that the net force on a body is zero, it is possible for you to find a reference frame in which the body has no acceleration.

$$\vec{a} = 0 \Rightarrow \sum \vec{F} = 0$$

$$\Rightarrow \sum \vec{F}_x = 0, \sum \vec{F}_y = 0, \sum F_z = 0.$$



Force

According to Newton's first law of motion, a body continues to be in a state of rest or of uniform motion along a straight line, unless it is acted upon by an external force to change the state. This means force applied on a body alone can change its state of rest or state of uniform motion along a straight line. Hence we define force as an external effort in the form of push or pull which moves or tries to move a body at rest, stops or tries to stop a body in motion, changes or tries to change the direction of motion of a body. Hence Newton's first law of motion defines force.

Inertia

We observe in routine that an object lying anywhere keeps on lying there only unless some one moves it. For example, a chair, a table, a bed, etc. cannot change their positions on their own, i.e., a body at rest cannot start moving on its own. The reverse is also true, though it is slightly difficult to perceive. If there were no forces of friction and air resistance, etc., a body moving uniformly along a straight line shall never stop, on its own.

According to Newton's first law of motion, a body continues to be in state of rest or of uniform motion along a straight line, unless it is acted upon by an external force to change its state. This means a body, on its own, cannot change its state of rest or state of uniform motion along a straight line. This inability of a body to change by itself its state of rest or state of uniform motion along a straight line is called inertia of the body. Hence Newton's first law defines inertia and is rightly called the law of inertia.

Quantitatively, inertia of a body is measured by the mass of the body. Heavier the body, greater is the force required to change its state and hence greater is its inertia. The reverse is also true (Fig. 7.3).

This inherent property of all bodies by virtue of which they cannot change their state of rest or of uniform motion along a straight line on their own is called Inertia. It depends on the mass of the body.

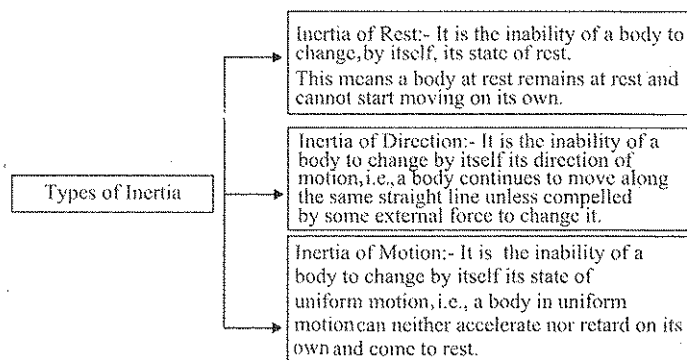


Fig. 7.3

Newton's Second Law of Motion

According to this law: The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the force applied.

When an unbalanced force is applied on a body, the momentum of the body changes; the rate of change of momentum with respect to time is defined as the net external force acting on

the body. i.e. $\vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt}$; where linear momentum $\vec{p} = m\vec{v}$, m = mass of the body, $\vec{v}$ = instantaneous velocity

$$\Rightarrow \vec{F}_{\text{ext}} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

$$\text{If } m = \text{constant, i.e., } \frac{dm}{dt} = 0$$

$$\Rightarrow \vec{F}_{\text{ext}} = m\vec{a}$$

If mass of a body is constant, the acceleration of the body is inversely proportional to its mass and directly proportional to the resultant force acting on it, i.e., $\sum \vec{F} = m\vec{a}$. This vector equation is equivalent to three algebraic equations,

$$\sum F_x = ma_x \quad (i)$$

$$\sum F_y = ma_y \quad (ii)$$

$$\sum F_z = ma_z \quad (iii)$$

Points to Remember

1. If $\vec{F} = 0$, second law gives $\vec{a} = 0$, therefore it is consistent with the first law.
2. The second law of motion is a vector law. It means whatever be the direction of the instantaneous velocity of a particle, if any net external force is acting on it, this force will change only that component of the velocity which is in the direction of the force.
3. Strictly speaking, this law applies to particles, i.e., point masses only. However, with the introduction of the concept of center of mass, this law can now be applied in case of extended bodies or a system of point masses

also. You will study this in the chapter on systems of particles and rotational motion.

- This is a local law. This means that it applies to a particle at a particular instant without taking into consideration any part history of the particle or its motion.
- As $\vec{F} = \frac{d\vec{p}}{dt}$, where $\vec{p}$ denotes momentum, slope of a momentum versus time graph gives force. In Fig. 7.4, $\tan \alpha$ gives the force at $t = t_1$ and $\tan \beta$ gives the force at $t = t_2$.

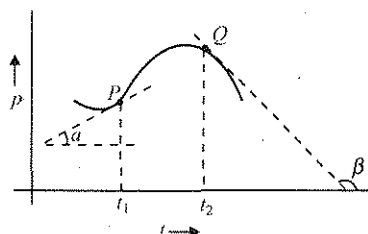


Fig. 7.4

The force $\vec{F}$ in the law stands for the net external force. Any internal forces in the system are not included in $\vec{F}$.

Newton's Third Law of Motion

According to this law: To every action, there is always an equal and opposite reaction, i.e., the forces of action and reaction are always equal and opposite.

To every action there is an equal and opposite reaction. Whenever a body exerts a force on another body, the second body also exerts a force on the first that is equal in magnitude, is in the opposite direction and has the same line of action, acting simultaneously.

Two point masses act on each other with forces which are always equal in magnitude and oppositely directed along the straight line connecting these points.

Any of the two forces making action–reaction pair can be called action, and the other reaction.

If two bodies are in contact with each other, the action and reaction forces are the contact forces. But Newton's third law also applies to long range forces that do not require physical contact, such as the force of gravitational attraction or electromagnetic interaction between two charged bodies.

To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always directed to contrary parts.

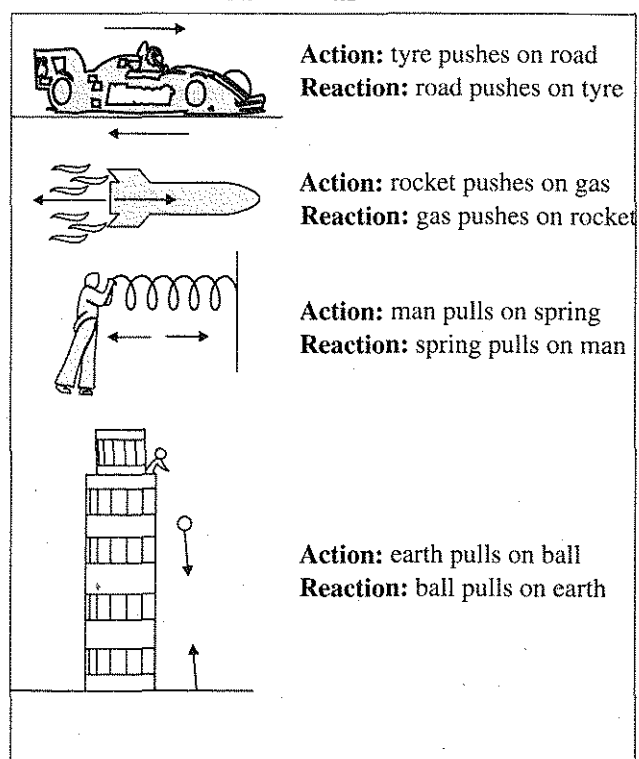
or

To every action there is always an equal and opposite reaction.

It appears to be the easiest law to remember, but what students do not appreciate is that although action and its reaction are always equal and opposite, yet they never cancel each other out in case of any of the particles or bodies of the system. Why? Because they always act on different bodies.

Forces always occur in pairs. If body A exerts a force F on body B, then B will exert equal and opposite force on body A.

We can write it as $\vec{F}_{BA} = -\vec{F}_{AB}$.



To every action there is always an opposed equal reaction.

It doesn't matter which force we call action and which we call reaction. The important thing is that they are co-pairs of a single interaction, and that neither of forces exists without the other.

When you walk, you interact with the floor. You push against the floor, and the floor pushes against you. The pair of forces occurs at the same time (they are simultaneous). Likewise, the tyres of a car push against the road while the road pushes back on the tyres—the tyres and road simultaneously push against each other. In swimming, you interact with the water, pushing the water backward, while the water simultaneously pushes you forward—you and the water push against each other. The reaction forces are what account for our motion in these examples. These forces depend on friction; a person or car on ice, for example, may be unable to exert the action force to produce the needed reaction force. Neither force exists without the other.

IMPULSE

According to Newton's second law, the momentum of a particle changes if the net force acts on the particle. Knowing that the change in momentum caused by a force is useful in solving some types of problems. To build a better understanding of this important concept, let us assume that a net force $\sum \vec{F}$ acts on a particle and that this force may vary with time. According to Newton's second law,

$$\sum \vec{F} = d\vec{p}/dt \text{ or } d\vec{p} = \sum \vec{F} dt \quad (i)$$

We can integrate this expression to find the change in the momentum of a particle when the force acts over some time interval. (If the momentum of the particle when the force acts over

some time interval.) If the momentum of the particle changes from $\vec{p}$, at time t_i , to $\vec{p}_f$ at time t_f , integrating equation (i) gives

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \sum \vec{F} dt \quad (\text{ii})$$

To evaluate the integral, we need to know how the net force varies with time. The quantity on the right side of this equation is a vector called the impulse of the net force $\sum \vec{F}$ acting on a particle over the time interval $\Delta t = t_f - t_i$.

$$\vec{I} = \int_{t_i}^{t_f} \sum \vec{F} dt \quad (\text{iii})$$

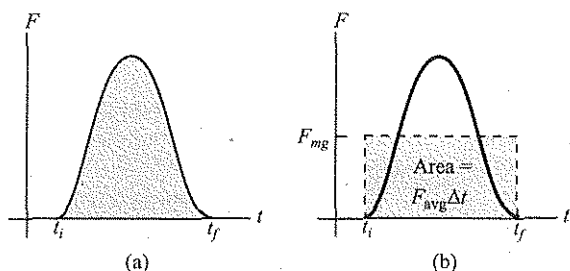
From its definition, we see that impulse $\vec{I}$ is a vector quantity having a magnitude equal to the area under the force–time curve as described in Fig. 7.5. It is assumed the force varies in time in the general manner shown in the figure and is nonzero in the time interval $\Delta t = t_f - t_i$. The direction of the impulse vector is the same as the direction of the change in momentum. Impulse has the dimensions of momentum, that is, ML/T . Impulse is not a property of a particle; rather, it is a measure of the degree to which an external force changes the particle's momentum.

Equation (ii) is an important statement known as the impulse-momentum theorem.

The change in the momentum of a particle is equal to the impulse of the net force acting on the particle.

$$\Delta \vec{p} = \vec{I} \quad (\text{iv})$$

A large force acting for a short time to produce a finite change in momentum is called an impulsive force. In the history of science, impulsive forces were put in a conceptually different category from ordinary forces. Now, there is no such distinction. Impulsive force is like any other force with the only difference that it is large and acts for a short time. Even if the net force is small and acts for long time, we can still calculate the impulse imparted by it and equate it with the total change in the momentum of the body on which it has acted.



(a) A net force acting on a particle may vary with time. The impulse is the area under the curve of the magnitude of the net force versus time.

(b) The average force (horizontal dashed line) gives the same impulse to the particle in the time interval Δt as the time-varying force described in part (a). The area of the rectangle is the same as the area under the curve.

Fig. 7.5

If we plot a graph between average force and time (Fig. 7.6), the area under the curve will give impulse imparted during the time interval under consideration.

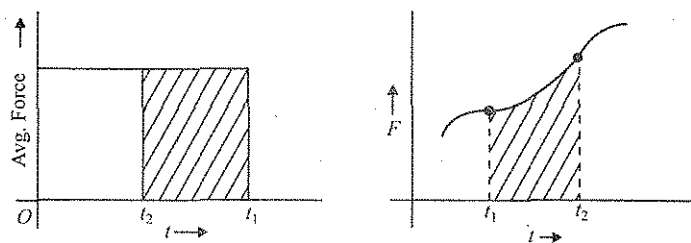


Fig. 7.6

Note: Area under force F versus time t graph gives total change in momentum, i.e., impulse. Hatched area in figure.

$$J = \int_{t_i}^{t_f} F dt = \int_{t_i}^{t_f} dp = \text{Total change in } p = \text{Impulse.}$$

Illustration 7.1 In a particular crash test, a car of mass 1500 kg collides with a wall as shown in Fig. 7.7. The initial and final velocities of the car are $\vec{v}_i = -15.0 \hat{i}$ m/s and $\vec{v}_f = 5.00 \hat{i}$ m/s, respectively. If the collision lasts 0.150 s, find the impulse caused by the collision and the average force exerted on the car.

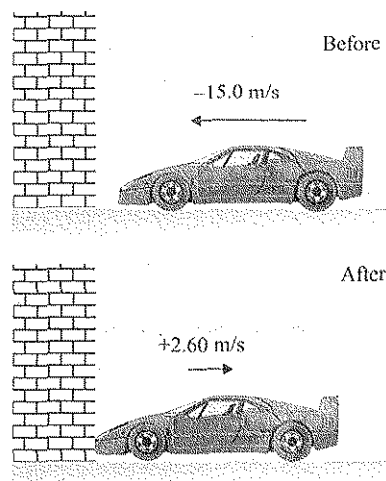


Fig. 7.7

Sol. The collision time is short, so we can imagine the car being brought to rest very rapidly and then moving back in the opposite direction with a reduced speed.

Let us assume that the force exerted by the wall on the car is large compared with other forces on the car (such as friction and air resistance). Furthermore, the gravitational force and the normal force exerted by the road on the car are perpendicular to the motion and therefore do not affect the horizontal momentum. Therefore, we categorise the problem as one in which we can apply the impulse approximation in the horizontal direction.

Now to evaluate the initial and final momenta of the car;

$$\vec{p}_i = m \vec{v}_i = (1500 \text{ kg})(-15.0 \hat{i} \text{ m/s}) = -2.25 \times 10^4 \hat{i} \text{ kg m/s}$$

$$\vec{p}_f = m \vec{v}_f = (1500 \text{ kg})(5.00 \hat{i} \text{ m/s}) = 0.75 \times 10^4 \hat{i} \text{ kg m/s}$$

Impulse on the car:

$$\begin{aligned}\vec{I} &= \Delta \vec{p} = \vec{p}_f - \vec{p}_i \\ &= 0.75 \times 10^4 \hat{i} \text{ kg m/s} - (-2.25 \times 10^4 \hat{i} \text{ kg m/s}) \\ &= 3.00 \times 10^4 \hat{i} \text{ kg m/s}\end{aligned}$$

The average force exerted by the wall on the car:

$$\vec{F}_{\text{avg}} = \frac{\Delta \vec{p}}{\Delta t} = \frac{3.00 \times 10^4 \hat{i} \text{ kg m/s}}{0.150 \text{ s}} = 2.00 \times 10^5 \hat{i} \text{ N}$$

Some Examples of Impulse: Force Exerted by Liquid Jet on Wall

Liquid Jet Striking a Vertical Wall Normally

Consider a liquid jet of area A that strikes the vertical wall with a velocity v . The liquid after striking the wall moves parallel to the wall.

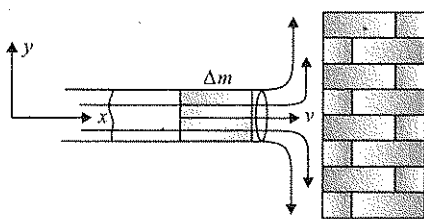


Fig. 7.8

Consider small element of liquid of mass Δm . The change in its momentum after strike is,

$$[\Delta P_x]_{\text{jet}} = 0 - \Delta m v = -\Delta m v$$

But, $[\Delta \vec{P}_x]_{\text{jet}} + [\Delta \vec{P}_x]_{\text{wall}} = 0$

So the transfer of momentum to the wall $[\Delta P_x]_{\text{wall}} = \Delta m v$

If Δt is the duration of strike, then the force exerted by jet on the wall

$$F = \frac{[\Delta P_x]_{\text{wall}}}{\Delta t} = \frac{\Delta m v}{\Delta t} = \left(\frac{\Delta m}{\Delta t} \right) v;$$

$$\text{but } \Delta m = \rho \Delta V = \left(\rho \frac{\Delta V}{\Delta t} \right) v = \rho \left(\frac{\Delta V}{\Delta t} \right) v;$$

ρ is density of the liquid.

$$\text{If the rate of flow of liquid is } \frac{\Delta V}{\Delta t} = av, \quad F = \rho a v^2.$$

Liquid Jet Striking the Wall at the Same Angle

Now consider the jet strikes the wall at an angle θ with the normal and rebounds with the same angle as in Fig. 7.9.

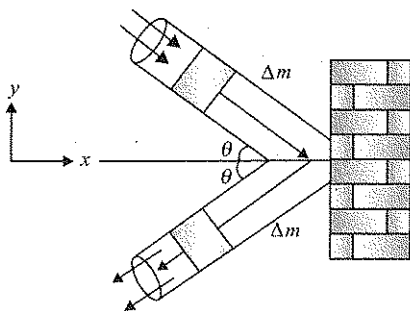


Fig. 7.9

The transfer of momentum to the wall

$$[\Delta P_x]_{\text{wall}} = 2 \Delta m v \cos \theta \quad \text{and} \quad [\Delta P_y]_{\text{wall}} = 0$$

If Δt is the duration of strike, then the force exerted by the jet on the wall is,

$$F = \frac{[\Delta P_x]_{\text{wall}}}{\Delta t} = \frac{2 \Delta m v \cos \theta}{\Delta t} = \left(\frac{\Delta m}{\Delta t} \right) 2v \cos \theta$$

$$= \rho \left(\frac{\Delta V}{\Delta t} \right) 2v \cos \theta$$

$$F = 2 \rho v^2 \cos \theta$$

Illustration 7.2 Wind with a velocity of 100 km h^{-1} blows normally against one wall of house with an area of 108 m^2 . Calculate the force exerted on the wall if the air moves parallel to the wall after striking it and has a density of 1.2 kg m^{-3} .

Sol. Let us first deduce a general formula.

Change in velocity of air along the normal to the wall $= v - 0 = v$. (Velocity along the normal after the air strikes the wall is zero because it moves parallel to it.)

Amount of air striking the wall per second $= A \times v$, where A = area of wall

Mass of air striking the wall per second $= A \times v \times \rho$ where ρ = density of air.

$$\therefore \text{change in momentum per second} = (A v \rho) v = A v^2 \rho$$

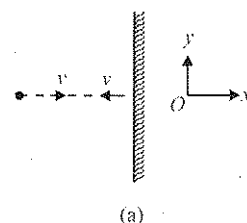
By laws of motion the wall exerts this force and the air exerts the same force on the wall.

$$\therefore \text{force exerted by the air on the wall} = A v^2 \rho$$

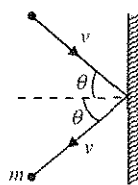
$$= 108 \times \left(\frac{100 \times 1000}{3600} \right)^2 \times 1.2 = 10^5 \text{ N}$$

Concept Application Exercise 7.1

- Explain why?
 - A horse cannot pull a cart and run in empty space.
 - Passengers are thrown forward from their seats when a speeding bus stops suddenly,
 - A cricketer moves his hands backwards when holding a catch.
- As shown in Fig. 7.10, two identical balls strike a rigid wall with equal speeds but at different angles of incidence. They are reflected back without any loss in the speed.



(a)



(b)

Fig. 7.10

- Determine the direction of force exerted by each ball on the wall.
 - Determine the ratio of impulse imparted by the two balls on the wall in both cases.
3. Fig. 7.11 shows the position–time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between the two consecutive impulses received by the particle? What is the magnitude of each impulse?

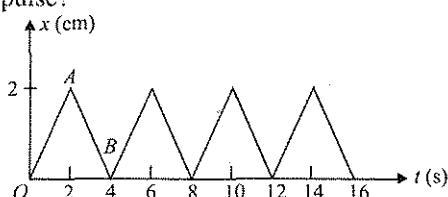


Fig. 7.11

4. A rubber ball of mass 50 g falls from a height of 5 m and rebounds to a height of 1.25 m. Find the impulse and the average force between the ball and the ground if the time for which they are in contact is 0.1 s.

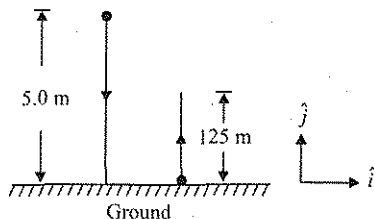


Fig. 7.12

5. Water falls without splashing at a rate of 0.250 L/s from a height of 2.60 m into a 0.750 kg bucket on a scale. If the bucket is originally empty, what does the scale read 3.00 s after water starts to accumulate in it?
6. A liquid of density ρ is flowing with a speed V through a pipe of cross-sectional area A . The pipe is bent in the shape of a right angle as shown in Fig. 7.13. What force should be exerted on the pipe at the corner to keep it fixed?



Fig. 7.13

7. A 3.00 kg steel ball strikes the wall with a speed of 10.0 m/s at an angle of 60.0° with the surface. It bounces off with the same speed and angle (Fig. 7.14). If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball?

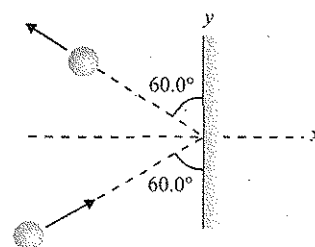


Fig. 7.14

8. The magnitude of the net force exerted in the x -direction on a 2.50 kg particle varies with time as shown in Fig. 7.15. Find
- the impulse of the force,
 - the final velocity the particle attains if it is originally at rest,
 - its final velocity if its original velocity is -2.0 m/s, and
 - the average force exerted on the particle for the time interval between 0 and 5.00 s.

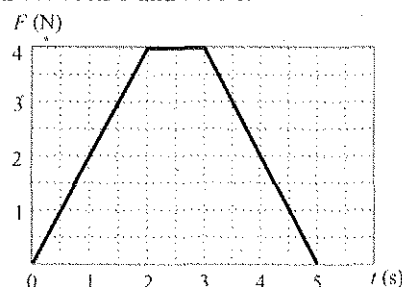


Fig. 7.15

FREE BODY DIAGRAMS

In free body diagrams (FBD) the object of interest is isolated from its surroundings and the interactions between the object and the surroundings are represented in terms of forces.

After knowing the nature of different forces, let us draw a “free body diagram”. The phrase itself reveals that we must mentally free (isolate) the bodies (or particles) in the system and then consider all the force acting on all the particles of the system.

The Common Forces Encountered in Mechanics and Representation of these Forces Through Free Body Diagrams

- | | |
|-------------------------|---------------------|
| 1. Weight | 2. Normal Force |
| 3. Tension | 4. Frictional Force |
| 5. Elastic Spring Force | |

Weight

It is the gravitational force with which the earth pulls an object. The weight of an object can be written as mg , where m is the mass of the object and g is the acceleration due to gravity, which is always directed towards the centre of the earth. For a small stretch of the earth's surface which can be considered to be flat, the acceleration due to gravity g can be taken to be uniform,

pointing vertically downwards and in this situation weight of the object is mg directed vertically downwards (Fig. 7.16).

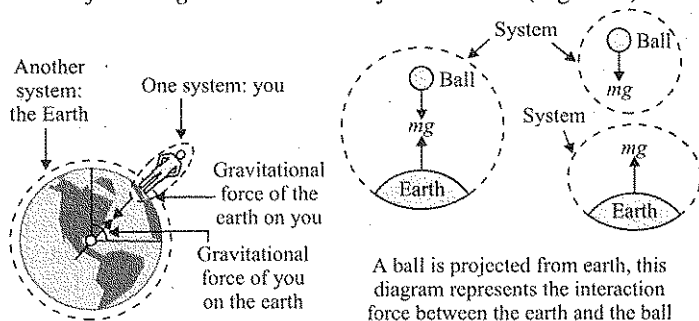


Fig. 7.16

Normal Force

Whenever two surfaces are in contact, they press (or push) each other by a force called contact force. The component of the contact force perpendicular to the surface is normal reaction and along the surface in contact is frictional force. The normal reaction to the contact surface can be computed by using $\sum \vec{F} = m\vec{a}$. Figure 7.17 shows action–reaction pairs of normal forces for two physical situations.

Normal forces on a block are drawn normal to the contact surface directed towards the block; and $N \geq 0$.

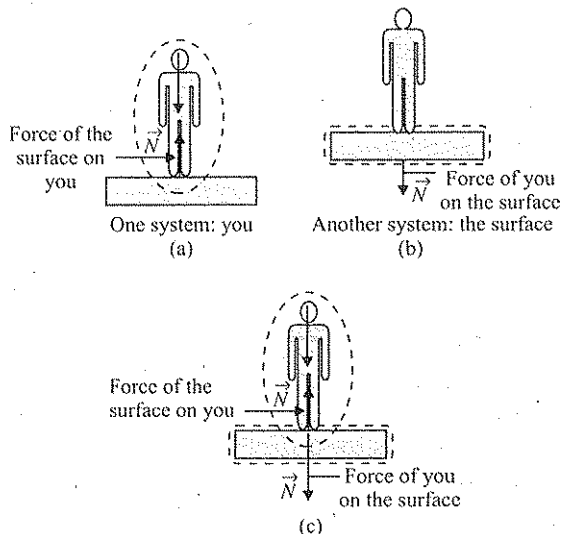


Fig. 7.17

Representing Normal Reaction in Different Situations

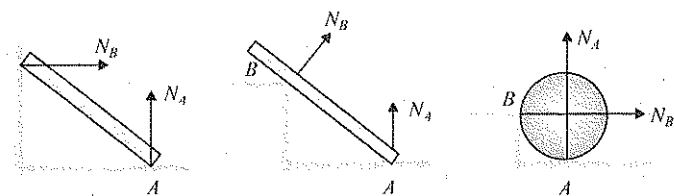


Fig. 7.18

- If direction of contact force cannot be determined, it should be shown as two components (as shown in Fig. 7.19).

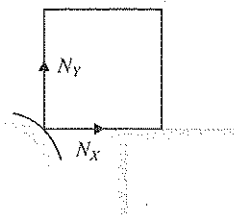


Fig. 7.19

Points to Remember

1. Normal force will be perpendicular to the surface of contact.
2. If normal force perpendicular to the surface of contact cannot be drawn, it will act perpendicular to the surface of the body.
3. If neither can be done, normal force has to be drawn as two components — one in the X -direction and one in the Y -direction. Remember that there is no relation between the forces acting along the X - and Y -directions. They are independent of each other.
4. The number of normal forces acting on a body depends on the number of points or surfaces of contact.

Representing Normal Reactions and Weight in Free Body Diagrams

Let us consider two blocks of mass M and m that are placed as shown in Fig. 7.20 and a force of magnitude F is acting on M . Our aim is to represent the forces in free body diagrams in different situations.

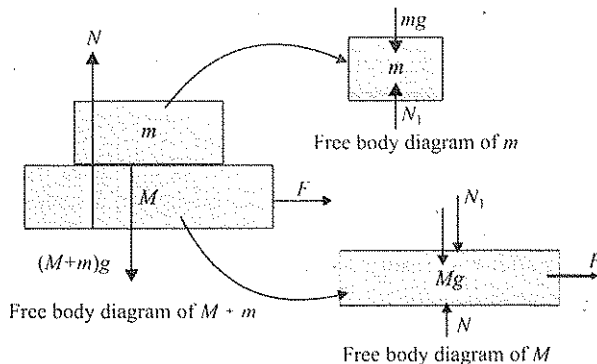


Fig. 7.20

Tension

It is quite practical that we can pull the objects by a string, but we cannot push the object by a string. This gives us an idea that a string can pull but cannot push.

Consider a light rope AB subjected to two equal and opposite forces at ends A and B . Let us observe the state of affairs at the cross-section (or the point) C . For this, consider the equilibrium of the parts AC and CB of the rope. Part CB of the rope applies a force on AC at point C , $F_{AC,CB}$. In turn AC also applies a force on CB at point C , $F_{CB,AC}$.

We introduce a physical quantity, **tension**, which is a property of rope or a point of the rope, and which equals the magnitude

of the force with which the rope pulls the body connected to it. Tension at point C of the rope in the above example:

$$T_C = |\vec{F}_{AC,CB}| = |\vec{F}_{CB,AC}|$$

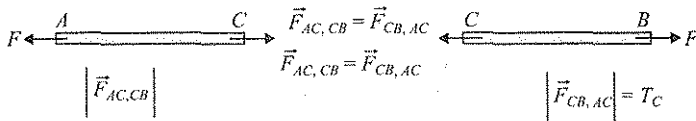


Fig. 7.21

Whenever a body is connected to a light rope, it is pulled with a force which equals the tension in the rope at the point where the body is attached to the rope.

Consider another example. Let a particle of mass m be hung in the vertical position by a light rope whose one end is attached to a rigid support. The forces acting on the particle, shown in the free body diagram in Fig. 7.22, are: mg , the weight of the particle, and a force by the rope, T .

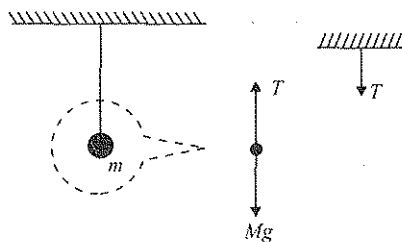


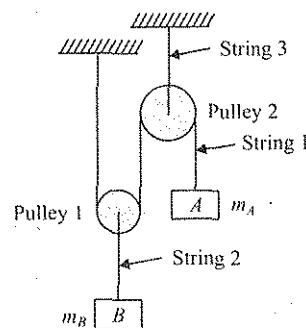
Fig. 7.22

The rope pulls the particle with a force which is equal to the tension in the rope at the point where the particle is attached to it.

Also, the rope pulls the support by a force represented by T . Tension in the rope at the point where it is attached to the support is the magnitude of the force.

Illustration 7.3 Two blocks of mass m_A and m_B are arranged in the diagram as shown in Fig. 7.23. Draw the free body diagram of

1. Block m_A
2. Block m_B
3. Pulley 1
4. Pulley 2



Sol.

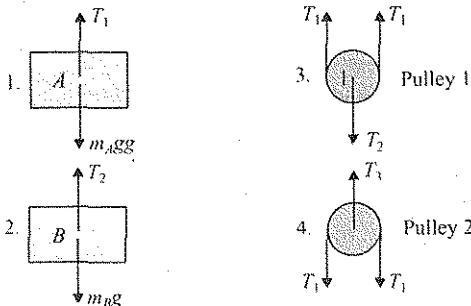
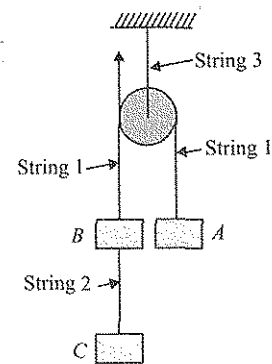


Fig. 7.23

Illustration 7.4 Two blocks of mass m_A and m_B are arranged in the diagram as shown in Fig. 7.24. Draw the free body diagram of

1. Block m_A
2. Block m_B
3. Block m_C
4. Pulley



Sol.

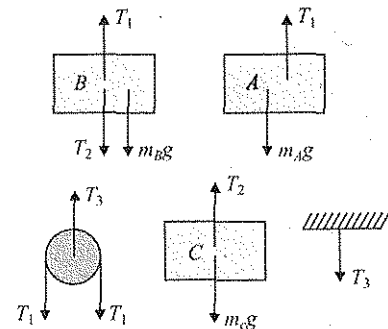


Fig. 7.24

Points to Remember

1. Tension force always pulls a body.
2. Tension can never push a body or rope.
3. Tension across massless pulley or frictionless pulley remains constant.
4. Ropes become slack when tension force becomes zero.

Friction

When a surface of a solid body slides (or has a tendency to slide if it does not actually slide) on a surface of another solid body, its (surface's) motion (or tendency of motion) is opposed. The force which opposes the relative motion (or tendency of the relative motion) between the contact surfaces is called frictional force.

The origin of frictional force is a complicated matter. Friction is a consequence of molecular interaction, originating in the realm of molecules and atoms because of electrical reasons. On an atomic level, both surfaces of contact are irregular. There are many points of contact where the atoms seem to cling together, and when the surface is pulled along, the atoms snap apart and vibration ensues.

If we push a block on a table, the table starts pushing the block back along the surface in contact (tangentially) (Fig. 7.25). Hence we can call this contact force **tangential contact**.

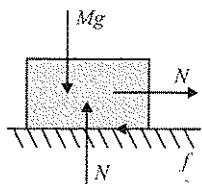


Fig. 7.25

Elastic Spring Forces

Tension in an extended (or compressed) spring is proportional to the magnitude of extension (or compression): $T \propto x$, in magnitude, but opposite in direction. So, $T = -kx$, where k is a positive constant, also known as the spring constant of the spring (Fig. 7.26).

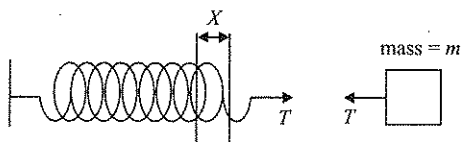
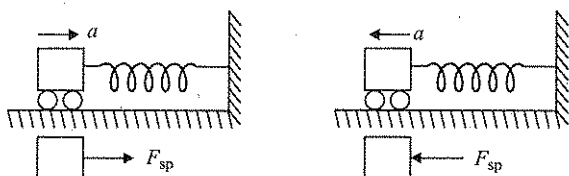


Fig. 7.26

Let us connect a body (a block, say) to one end of a spring and pull its other end. We observe that the block receives a pull. If we push the spring, the block will receive the push. In other cases, the spring gets deformed. That means, a deformed spring can pull or push an object.

The force exerted by a deformed spring on the connected bodies is known as "spring force" (Fig. 7.27).



Elongated spring pulls the block A compressed spring pushes the object

Fig. 7.27

While drawing force diagram, first of all we will mark the points of the connected bodies at which the spring is connected. If the spring is really elongated (or assumed to be elongated), it pulls the points (P and Q) of contact (like a string). If we assume that the spring is compressed, it pushes both the bodies (points P and Q) in contact, such as a rod (Fig. 7.28).

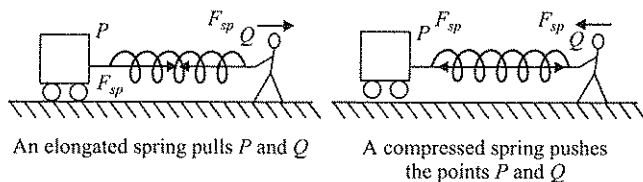
An elongated spring pulls P and Q A compressed spring pushes the points P and Q

Fig. 7.28

NON-INERTIAL FRAME OF REFERENCE AND PSEUDO (FICTITIOUS) FORCE

Motion of a particle (P) is studied from two frames of references S and S' . S is an inertial frame of reference and S' is a non-inertial

frame of reference. At any time, position vectors of the particle with respect to those two frames are $\vec{r}$ and $\vec{r}'$, respectively. At the same moment, position vector of the origin of S' is $\vec{R}$ with respect to S as shown in Fig. 7.29.

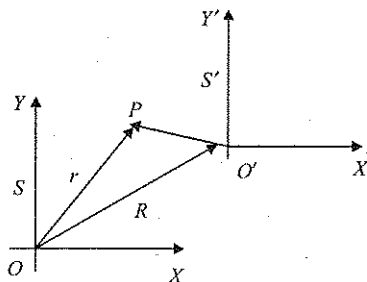


Fig. 7.29

From the vector triangle $OO'P$, we get $\vec{r}' = \vec{r} - \vec{R}$

Differentiating this equation twice with respect to time we get,

$$\frac{d^2 \vec{r}'}{dt^2} = \frac{d^2 (\vec{r})}{dt^2} - \frac{d^2 (\vec{R})}{dt^2} \Rightarrow \vec{a}' = \vec{a} - \vec{A}$$

Here, $\vec{a}'$ = Acceleration of the particle P relative to S'

$\vec{a}$ = Acceleration of the particle relative to S

$\vec{A}$ = Acceleration of S' relative to S .

Multiplying the above equation by m (mass of the particle) we get, $m \vec{a}' = m \vec{a} - m \vec{A}$

$$\Rightarrow \vec{F}' = \vec{F}_{(\text{real})} - m \vec{A} \Rightarrow \vec{F}' = \vec{F}_{(\text{real})} + (-m \vec{A})$$

Suppose a box is placed on a railway platform. The only forces acting on it are its weight W acting downward and the normal reaction R acting on it upward. W and R are thus equal and opposite. Thus net external force acting on the box is zero. The box is at rest.

Let us consider the forces acting on this box and its state with respect to three observers, namely

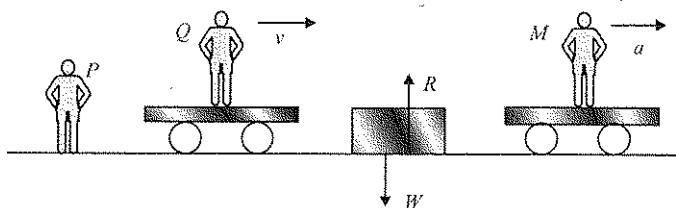


Fig. 7.30

- A person P standing on the platform.
- A person Q who is sitting in a train which is moving with uniform velocity in a straight line parallel to the platform.
- A person M sitting in a train which is moving parallel to the platform but with some acceleration.
- This box is at rest w.r.t. observer P .
- This box is in uniform motion in a straight line w.r.t. observer Q [moving with $-v$ velocity].

- This box is having $(-)$ acceleration w.r.t. the observer M . For all the three observers (or frames of reference), the force acting on the box is the same: W and R . As W and R are equal and opposite, the net external force acting on the box is zero.

Let us reconsider the example of the box lying at rest on a railway platform. We saw that it has $(-\vec{a})$ acceleration w.r.t. a train which is moving with an acceleration $(\vec{a})$ even though no net external material force is acting on it. Neither first nor second law remains valid on the box with respect to the accelerated train.

But still, if we want to apply Newton's laws of motion, in a non-inertial reference frame, we are allowed to do so, provided we include an additional force, the pseudo force, in the free body diagram.

If we apply an imaginary force ma in the direction opposite to the direction of motion of observer M , the observer M can write equation of motion of box.

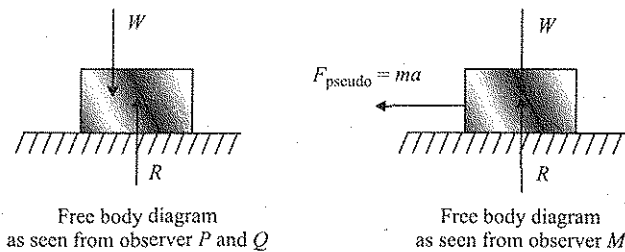


Fig. 7.31

Thus, Newton's law can also be applied with respect to a non-inertial reference frame, provided we include an "extra" force of $(-ma)$ on the system. This force has no existence, in reality, but has been included only to suit the calculations involved, by Newton's second law, while working out a problem w.r.t. a non-inertial reference frame. This imaginary force is known as "pseudo force" or "fictitious force" or "inertial force". [The term "pseudo" means something which is not real.]

Illustration 7.5 A man of mass M stands on a weighing machine in an elevator accelerating upwards with an acceleration a_0 . Draw the free body diagram of the man as observed by the observer A (stationary on the ground) and observer B (stationary on the elevator). Also, calculate the reading of the weighing machine.

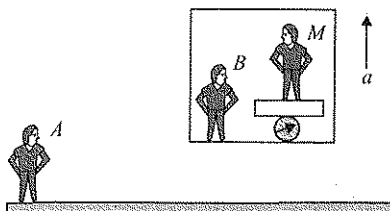


Fig. 7.32

Sol. Using Newton's second law,

$$\begin{aligned} N - Mg &= Ma \\ N &= M(g + a) \end{aligned}$$

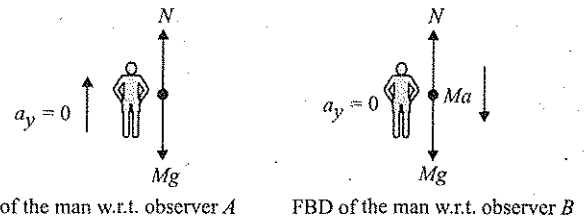


Fig. 7.33

EQUILIBRIUM OF A PARTICLE

Equilibrium of a particle in mechanics refers to the situation when the net external force acting on the particle is zero.

The above condition is correct and complete as far as equilibrium of a particle (i.e., a point mass) is concerned. In case of rigid bodies (i.e., extended bodies), there are two conditions to be satisfied for such bodies to have equilibrium. First, net external force acting on the body should be zero. Second, net external torque acting on the body should be zero.

Concurrent Forces

If two or more forces act on the same particle, we call them concurrent forces.

Lamy's Theorem

If three concurrent forces P , Q and R acting on a particle keep the particle in equilibrium, then Lamy's (Lami's) theorem states:

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

Here α = angle opposite to $\vec{P}$

β = angle opposite to $\vec{Q}$

γ = angle opposite to $\vec{R}$

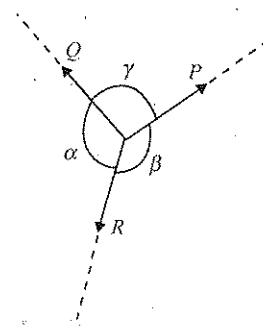


Fig. 7.34

Note: If the concurrent forces are coplanar but more than three, then it is generally convenient to resolve all of them along two mutually perpendicular directions and then the resultant of each set of these resolved components will be zero.

$$\sum F_x = 0; \quad \sum F_y = 0$$

If the force are not coplanar, then they can be resolved along any three mutually perpendicular directions and then the following conditions shall apply

$$\sum F_y = 0; \quad \sum F_z = 0$$

Problem Solving Strategy: Applying Newton's Laws

The following procedure is recommended when dealing with problems involving Newton's law.

- 1. Conceptualize:** Draw a simple, neat diagram of the system to help establish the mental representation. Establish convenient coordinate axes for each object in the system.
- 2. Categorize:** If an acceleration component for an object is zero, it is modelled as a particle in equilibrium in this direction and $\sum F = 0$. If not, the object is modelled as a particle under a net force in this direction and $\sum F = ma$.
- 3. Analyses:** Isolate the object whose motion is being analysed. Draw a free body diagram for this object. For systems containing more than one object, draw separate free body diagrams for each object. Do not include in the free body diagram forces exerted by the object on its surroundings.

Find the components of the forces along the coordinate axes. Apply Newton's second law, $\sum \vec{F} = m\vec{a}$, in component form. Check your dimensions to make sure all terms have units of force.

- 4. Solve the component equations for the unknowns.** Remember that to obtain a complete solution, you must have as many independent equation as you have unknowns.
- 5. Finalize:** Make sure your results are consistent with the free body diagram. Also check the predictions of your solutions for extreme values of the variables. By doing so, you can often detect errors in your results.

Illustration 7.6 A block of mass 30 kg is suspended by three strings as shown in Fig. 7.35. Find the tension in each string.

Sol. Method I: Considering equilibrium of each part of system (Fig. 7.36):

The whole system is in equilibrium, therefore for part $\sum \vec{F} = 0$

From free body diagram of block C, $T_C = 300$ N

Now consider the equilibrium of point O;

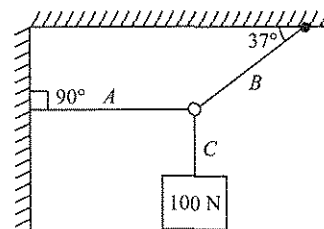


Fig. 7.35

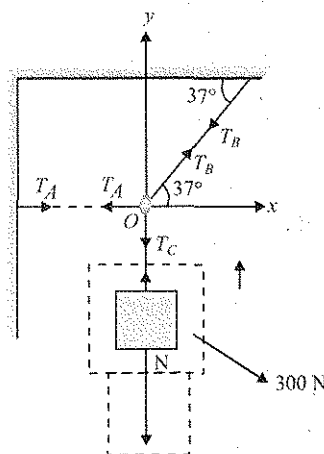


Fig. 7.36

$$\sum F_x = 0; \text{ or } T_B \cos 37^\circ - T_A = 0$$

$$T_A = T_B \cos 37^\circ = T_B \frac{4}{5} \quad (i)$$

$$\text{and } \sum F_y = 0; \text{ or } T_B \sin 37^\circ - T_C = 0 \quad (ii)$$

$$\therefore T_B = \frac{T_C}{\sin 37^\circ} = \frac{300}{3/5} = 500 \text{ N}$$

$$\text{From equation (i), we get } T_A = \frac{4}{5} T_B = \frac{4}{5} \times 500 = 400 \text{ N}$$

Method II: Using Lami's theorem

By Lami's theorem, we have

$$\frac{T_A}{\sin(90 + 37^\circ)} = \frac{T_B}{\sin 90^\circ} = \frac{T_C}{\sin(180 - 37^\circ)}$$

But $T_C = 300$ N

$$\text{and } T_B = \frac{T_C}{\sin 37^\circ} = \frac{300}{3/5} = 500 \text{ N}$$

$$T_A = T_C \left(\frac{\sin(90 - 37^\circ)}{\sin(180 - 37^\circ)} \right) = 300 \left(\frac{\cos 37^\circ}{\sin 37^\circ} \right) = 400 \text{ N}$$

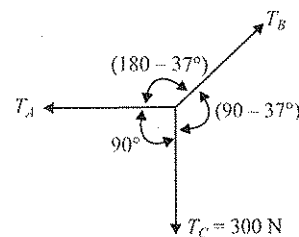


Fig. 7.37

Illustration 7.7 Two particles of masses 10 kg and 35 kg are connected with four strings at points B and D as shown in Fig. 7.38.

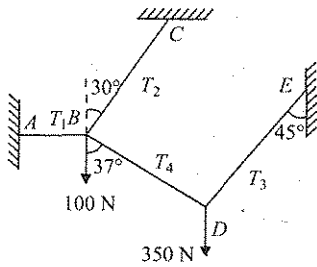


Fig. 7.38

Determine the tensions in various segments of the string.

Sol. Free body diagram of whole system is shown in Fig. 7.39.

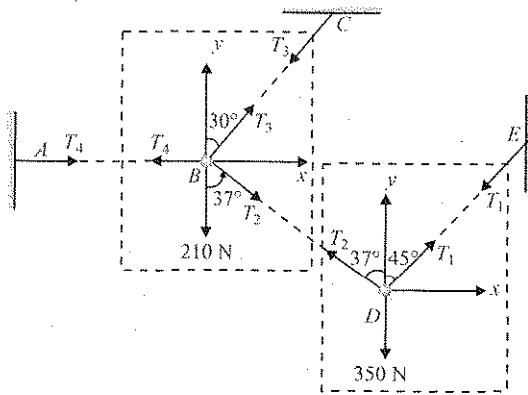


Fig. 7.39

$$\sum F_x = 0, \text{ or } T_1 \sin 45^\circ - T_2 \sin 37^\circ = 0 \quad (i)$$

$$\text{and } \sum F_y = 0, \text{ or } T_1 \cos 45^\circ + T_2 \cos 37^\circ = 350 \quad (ii)$$

$$\text{From equation (i), we have } T_2 = \frac{T_1 \sin 45^\circ}{\sin 37^\circ}$$

Now from equation (ii),

$$T_1 \cos 45^\circ + \frac{T_1 \sin 45^\circ}{\sin 37^\circ} \times \cos 37^\circ = 350$$

$$\text{or } \frac{T_1}{\sqrt{2}} + \frac{T_1}{\sqrt{2}} \times \frac{4}{3} = 350 \text{ or } \frac{T_1}{\sqrt{2}} \left[1 + \frac{4}{3} \right] = 350$$

$$\Rightarrow T_1 = 150\sqrt{2} \text{ N and } T_2 = \frac{T_1 \sin 45^\circ}{\sin 37^\circ} = \frac{150\sqrt{2} \times \left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{3}{5}\right)} = 250 \text{ N}$$

Analysing the equilibrium of point B:

$$\sum F_x = 0; \text{ or } T_2 \sin 37^\circ + T_3 \sin 30^\circ - T_4 = 0 \quad (i)$$

$$\text{and } \sum F_y = 0; \text{ or } T_3 \cos 30^\circ - T_2 \cos 37^\circ - 100 = 0 \quad (ii)$$

$$\text{From equation (ii), } T_3 \cos 30^\circ - 250 \times \frac{4}{5} - 100 = 0$$

$$\Rightarrow T_3 = 200\sqrt{3} \text{ N}$$

$$\text{From equation (i), } T_4 = 150 + 100\sqrt{3} = 50(3 + 2\sqrt{3}) \text{ N.}$$

Illustration 7.8 Three blocks A, B, and C of masses m_1 , m_2 , and m_3 are resting one on top of the other as shown in Fig. 7.40. A horizontal force F is applied on block B. Assuming all the surfaces are frictionless.

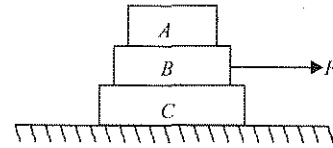


Fig. 7.40

Calculate:

1. Acceleration of block A, block B, and block C.
2. Normal reactions between A and B, B and C, and between C and the ground.

Sol:

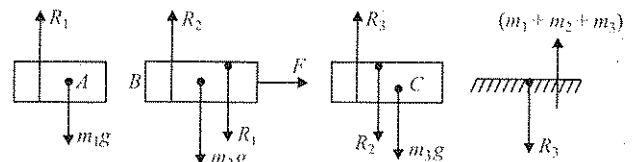


Fig. 7.41

This system cannot be in equilibrium because in the horizontal direction, the system has a net external force F . As far as the vertical direction is concerned, all the forces are internal action and reaction forces, hence equal and opposite, hence will cancel each other out.

Body A: No external force is acting on it, hence it will remain stationary in equilibrium, hence acceleration of block A, $a_A = 0$, and $R_1 = m_1g$. (i)

Body B: There is no external force acting on it vertically, hence it will not have any acceleration in the vertical direction.

$$R_2 = R_1 + m_2g \quad (ii)$$

However, there is one external force F which is acting on it. So it will have some acceleration a in the horizontal direction of $\vec{F}$ such that

$$F = m_2a_B \quad (iii)$$

which gives acceleration of block B, $a_B = \frac{F}{m_2}$

Body C and earth: Same comments as in case of body A above

$$\therefore R_3 = R_2 + m_3g \quad (iv)$$

$$\text{and } R_3 = (m_1 + m_2 + m_3)g \quad (v)$$

No external force acts on block C in horizontal direction hence a_C is equal to 0.

Illustration 7.9 Two blocks of masses m_1 and m_2 are placed side by side on a smooth horizontal surface as shown in the Fig. 7.42. A horizontal force F is applied on block m_1 .

1. Find the acceleration of each block.

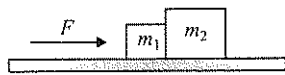


Fig. 7.42

2. Find the normal reaction between the two blocks.

Sol. Method 1: Since the two blocks always remain in contact with each other so they must move with the same acceleration.

Using Newton's second law,

$$\text{For block } m_1 \quad F - N = m_1 a \quad (i)$$

$$\text{For block } m_2 \quad N = m_2 a \quad (ii)$$

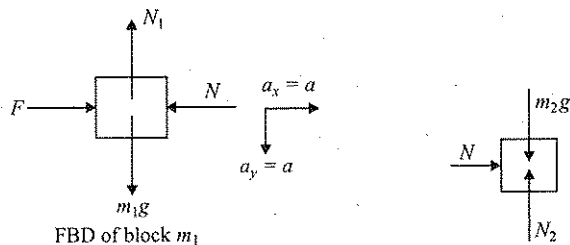


Fig. 7.43

On adding the two equations, we get

$$F = (m_1 + m_2) a \Rightarrow a = \frac{F}{m_1 + m_2}$$

Substituting the value of a in equation (ii), we get

$$N = m_2 a = \frac{F m_2}{m_1 + m_2}$$

Method 2: The situation may be considered as follows: Instead of drawing the free body diagrams of each block we can draw the free body diagram of both blocks together as shown in Fig. 7.44.

The net force acting on the system is F and the total mass of the system is $(m_1 + m_2)$, thus $a = \frac{F}{m_1 + m_2}$.

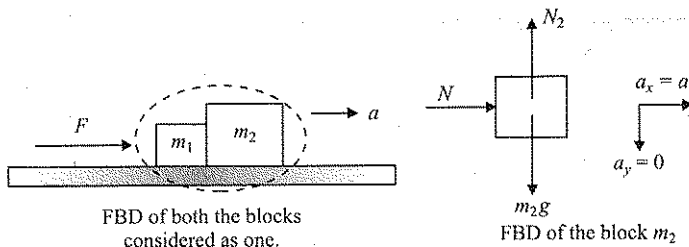


Fig. 7.44

To find out the normal reaction N between the two blocks we can imagine that, the block m_2 is moving with an acceleration a , therefore, the net force acting on it must be $(m_2 a)$ which is nothing but the normal reaction applied by the block m_1 .

$$\text{Thus, } N = m_2 a = \frac{m_2 F}{m_1 + m_2}$$

Illustration 7.10 Three boys, each of mass 45 kg, pull simultaneously a block on a smooth surface. Mass of block is 20.0 kg.

1. Find the acceleration of the block.
2. Find the instantaneous acceleration of the boy exerting the force F_1 .

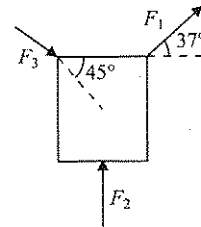


Fig. 7.45

Sol. First we will resolve all the forces acting on the block into x and y components.

$$\sum F_x = F_1 \cos 37^\circ + F_3 \cos 45^\circ + F_2 \cos 90^\circ$$

$$\sum F_y = F_1 \sin 37^\circ + F_2 \cos 0^\circ - F_3 \sin 45^\circ$$

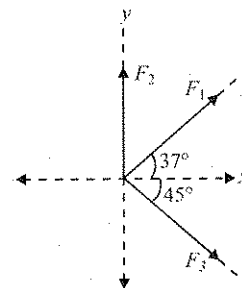


Fig. 7.46

$$\text{Now, } a_x = \frac{\sum F_x}{m} \quad \text{and} \quad a_y = \frac{\sum F_y}{m}$$

$$a_x = \frac{(90.0) \cos 37^\circ + 128\sqrt{2} \cos 45.0^\circ}{20.0} = 10 \text{ m/s}^2;$$

$$a_y = \frac{(90.0) (\sin 37^\circ) + 114 - (128\sqrt{2} \sin 45.0^\circ)}{20.0} = 2 \text{ m/s}^2$$

According to Newton's third law, the force exerted on the block by the boy must be equal to the force exerted on the boy by the block.

$$\text{Therefore, } a_1 = \frac{F'_1}{m} = \frac{90.0}{45} = 2 \text{ m/s}^2$$

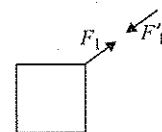


Fig. 7.47

Illustration 7.11 A block of mass M is being pulled with the help of a string of mass m and length L . The horizontal force applied on the string is F .

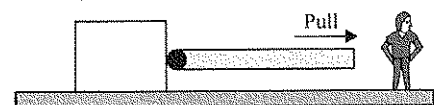


Fig. 7.48

Determine:

- a. force exerted by the string on the block and acceleration of system

- b. tension at the mid-point of the string.
 c. tension at a distance x from the end at which force is applied.
 d. assume that the block is kept on a frictionless horizontal surface and the mass is uniformly distributed in the string.
- Sol.**

- a. The force applied by the string to the block is T . For part (a) consider one system is block and other string. Let the acceleration of the system (block + string) be a

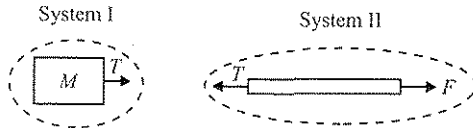


Fig. 7.49

Now we apply Newton's law to each of them.

$$\text{For system II : } F - T = ma \quad (i)$$

$$\text{For system I : } T = Ma \quad (ii)$$

After solving equations (i) and (ii), $a = \frac{F}{M + m}$;

$$T = \frac{MF}{M + m}$$

- b. Now we have to redefine our system. Choose system 1 as block and half string and system 2 as other half string. On applying Newton's second law to system 1 and system 2, we have

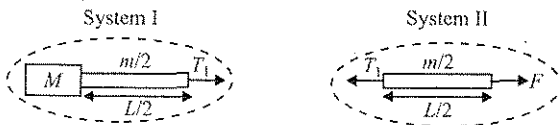


Fig. 7.50

$$\text{System II : } F - T_1 = \frac{M}{2} \times a \quad (iii)$$

Mass per unit length of string = m/M .

Hence, mass of $L/2$ length of string = $\frac{m}{L} \times \frac{L}{2} = \frac{m}{2}$

$$\text{System I : } T_1 = \left(M + \frac{m}{2}\right) a \quad (iv)$$

After solving equations (iii) and (iv) we get,

$$a = \frac{F}{M + m}; T_1 = \frac{(M + m/2)F}{(M + m)}$$

- c. Now we can redefine our system in the block and string of length $(L - x)$ (system I) and string of length x (system II)

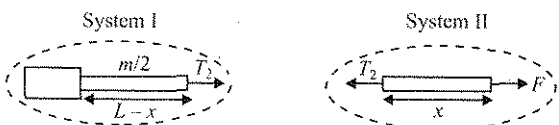


Fig. 7.51

$$\text{System II : } F - T_2 = \left(\frac{m}{L} \times x\right) a \quad (v)$$

$$\text{System I : } T_2 = \left\{\frac{m}{L}(L - x) + M\right\} a \quad (vi)$$

Solving equations (v) and (vi) we get

$$a = \frac{F}{M + m} \text{ and } T_2 = \frac{\{m(L - x)/L + M\}F}{M + m}$$

Hence, we can say the string having mass tension v charges along the length.

Illustration 7.12 In the diagram shown, the blocks A and B are connected together by a string and placed on a smooth inclined plane. B is connected to C (which is suspended vertically) by another string which passes over a smooth pulley fixed to the plane. The mass of A is $m_A = 1$ kg; mass of B, $m_B = 2$ kg.

- a. If the system is at rest. Find the value of mass of C.
 b. If mass of C is twice the mass calculated in (a) then find the acceleration of the system.

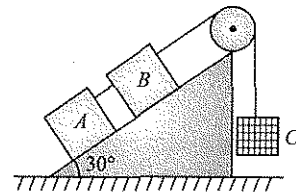


Fig. 7.52

Sol.

- a. From the free body diagram of A:

T_1 : tension in string between A and B

N_A : normal reaction between A and incline

As A is at rest, net force parallel and perpendicular to the incline should be balanced.

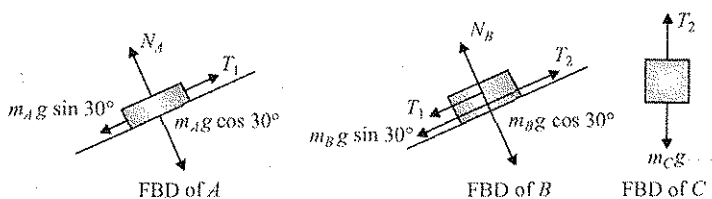


Fig. 7.53

$$N_A = m_A g \cos \theta = 10 \cos 30^\circ = 5\sqrt{3} \text{ N}$$

$$T_1 = m_A g \sin \theta = 10 \sin 30^\circ = 5 \text{ N}$$

From the free body diagram of B:

As B is connected to both strings, two tensions T_1 and T_2 will act on it.

T_2 = tension (force) of string between B and C acting upwards

T_1 = tension of string between A and B acting downwards

$W_B \sin \theta$, $W_B \cos \theta$ are components of weight W_B

Balancing forces:

$$N_B = m_B g \cos 30^\circ = 20 \cos 30^\circ = 10\sqrt{3} \text{ N}$$

$$T_2 = T_1 + m_B g \sin 30^\circ = 5 + 20 \sin 30^\circ = 5 + 20 \times \frac{1}{2} = 15 \text{ N}$$

Force diagram of C: T_2 is the pulling force of string on block C
 $m_C g = T_2$. Hence $m_C = 1.5$ kg

Note: You should remember that components of the weight of a body on the inclined plane are $W \cos \theta$.

- b. Let the acceleration of the system be a . We can assume an arbitrary direction of motion, let the blocks be moving up the incline.

From FBD of A: $T_1 - 10 \sin 30^\circ = 1a$ (i)

From FBD of B: $T_2 - T_1 - 20 \sin 30^\circ = 2a$ (ii)

From FBD of C: $30 - T_2 = 3a$ (iii)

Solving equations (i), (ii), and (iii), $a = 2.5 \text{ m/s}^2$

Illustration 7.13 Blocks A and B rest on a horizontal surface in contact with each other. Pushing forces F_1 and F_2 are applied as shown. Weight of A is 30 N and of B is 20 N. Find the force F_1 and the normal reactions between all the contact surfaces.

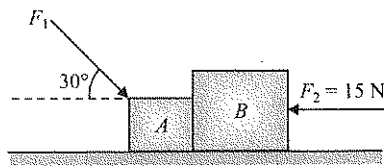


Fig. 7.54

Sol. Force diagram of B:

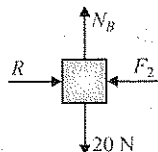


Fig. 7.55

N_R = normal reaction between B and the ground

R = normal reaction between B and A (on B by A)

$R = F_2 = 15 \text{ N}$, $N_B = 20 \text{ N}$

Force diagram of A: R = normal force on A by B

N_A = normal reaction between A and the ground

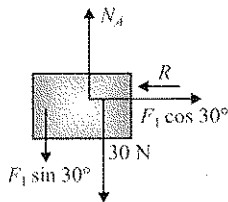


Fig. 7.56

Balancing forces: $F_1 \cos 30^\circ = R$

$F_1 \sin 30^\circ + 30 = N_A$

$F_1 = 2R/\sqrt{3} = 30/\sqrt{3} = 10\sqrt{3} \text{ N}$;

$N_A = 10\sqrt{3} \frac{1}{2} + 30 = (30 + 5\sqrt{3}) \text{ N}$

Illustration 7.14 A body hangs from a spring balance supported from the ceiling of an elevator.

- a. If the elevator has an upward acceleration of 2.5 m/s^2 and the balance reads 50 N, what is the true weight of the body?

- b. Under what circumstances will the balance read 30 N?

- c. What will be the reading in the balance if the cable of the elevator breaks?

Sol.

- a. Reading in the spring balance is equal the tension in the spring = 50 N.

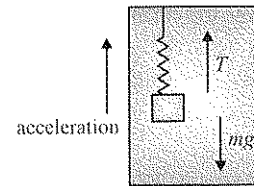


Fig. 7.57

As the elevator is accelerating in upward direction with 2.45 m/s^2 the acceleration of the block = $a = 2.5 \text{ m/s}^2 = g/4$.

$T - mg = ma$

$50 - mg = m(g/4)$

$mg = 40 \text{ N}$ (i)

When the elevator has an upward acceleration, reading is greater than the actual weight.

- b. Reading of balance = $T = 30 \text{ N}$.

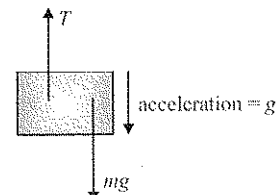


Fig. 7.58

As $T < mg$ (Actual weight), the block and elevator must have a downward acceleration a .

$mg - T = ma$

$a = 2.5 \text{ m/s}^2$ is in downward direction. It is possible in two ways:

1. the elevator is going up and slowing down, or
2. the elevator is going down and its speed is increasing.

The reading of balance is less than actual weight if elevator has a downward acceleration.

- c. If the cable breaks, the acceleration of the block and the elevator = g (downwards)

Net force = mass $\times$ acceleration

$mg - T = mg$, $T = 0$

the reading of the balance = 0 N

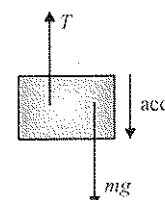


Fig. 7.59

Illustration 7.15 Two masses m_1 and m_2 are attached to a flexible inextensible massless rope which passes over a frictionless and massless pulley. Find the accelerations of the masses and tension in the rope.

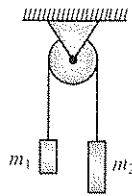


Fig. 7.60

Sol. Fix an inertial reference frame to the ground to observe the motion of the masses.

Let the tension in the rope be T . (Infact, tension is the property of a point of the rope). Tension may be same at all the points of the rope or may vary from point to point. In this case with ideal pulley (massless and frictionless) and ideal rope (inextensible, massless and flexible), the tension will remain constant throughout the rope).

Let the acceleration of m_2 be vertically downwards and acceleration of m_1 will be vertically upwards.

This is because the rope is inextensible; during motion, the length of the rope must not change, and the rope must not slacken either.

From the above statement you must not conclude that the acceleration of the masses connected by a rope are always equal. The relationship between the accelerations of the masses depends on the configuration of the pulley-rope system, which can be obtained from the fact that the length of an ideal rope must not change and the rope must not slacken.

Using equation $\sum \vec{F} = m\vec{a}$ for the force diagrams of m_1 and m_2 ,

$$T - m_1g = m_1a \quad (i)$$

$$m_2g - T = m_2a \quad (ii)$$

Adding equations (i) and (ii),

$$m_2g - m_1g = m_1a + m_2a$$

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1} \right) g \quad (iii)$$

Substituting this value of a in equation (i),

$$T = m_1g + m_1 \left(\frac{m_2 - m_1}{m_2 + m_1} \right) g = \frac{2m_1m_2}{(m_1 + m_2)} g \quad (iv)$$

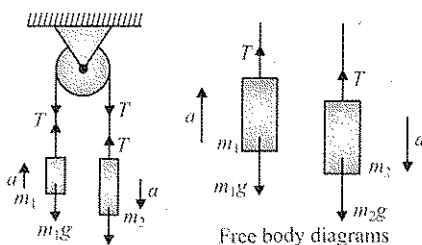


Fig. 7.61

Illustration 7.16 A solid sphere of mass 2 kg is resting inside a cube as shown in Fig. 7.62. The cube is moving with a velocity, $\vec{v} = (5t\hat{i} + 2t\hat{j})$ m/s. Here t is the time in seconds. All the surfaces are smooth. The sphere is at rest with respect to the cube. What is the total force exerted by the sphere on the cube?

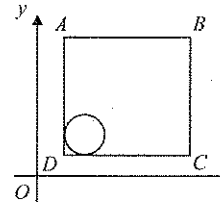


Fig. 7.62

Sol.

$$\vec{v} = 5t\hat{i} + 2t\hat{j}$$

$$\vec{a} = 5\hat{i} + 2\hat{j}$$

$$N_x = 2 \times 5 = 10 \text{ N}$$

$$N_y = 2 \times 10 + 2 \times 2 = 24 \text{ N}$$

$$\text{Total force} = \sqrt{N_x^2 + N_y^2} = \sqrt{(10)^2 + (24)^2} = 26 \text{ N}$$

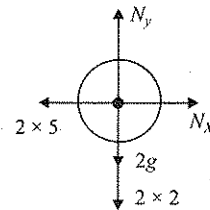


Fig. 7.63

Concept Application Exercise 7.2

1. As per the diagram given in Fig. 7.64(a),

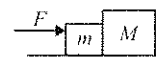


Fig. 7.64(a)

Free body diagram of M is (True or false). Assume all surfaces are frictionless.

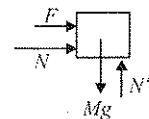


Fig. 7.64(b)

2. Mass m is placed on a body of mass M . There is no friction. Force F is applied on M and it moves with acceleration a . Find the force (along X -axis) on the top body.

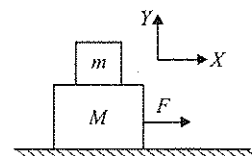


Fig. 7.65

Here a is acceleration of lower body.

3. In the problem shown below, the mass of man is M . The weight of the man as registered by weighing machine will be _____, assuming that the weighing machine, man, and wedge all are stationary.

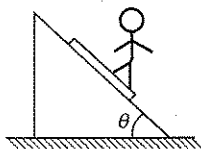


Fig. 7.66

4. A small object is suspended at rest from two strings as shown in Fig. 7.67. The magnitude of the force exerted by each string on the object is $10\sqrt{2}$ N. Find the magnitude of the mass of the object.

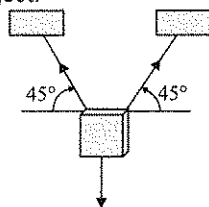


Fig. 7.67

5. Figure 7.68 shows a platform on which a man of mass M is standing and holding a string passing over a system of ideal pulleys. Another mass m is hanging as shown in figure. Find the force man has to exert to maintain the equilibrium of system. Also find the force exerted by the platform on the man.

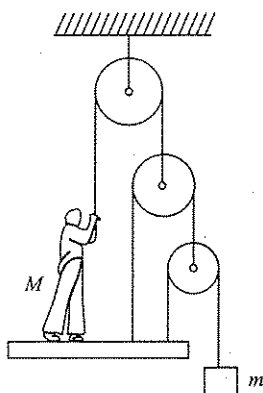


Fig. 7.68

6. The object in Fig. 7.69 weighs 40 kg and hangs at rest. Find the tensions in the three cords that hold it.

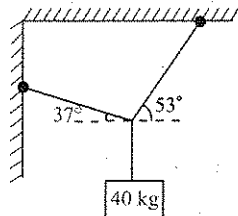


Fig. 7.69

7. A block of mass $m = 10$ kg is suspended with the help of three strings as shown in the Fig. 7.70. Find the tensions T_1 , T_2 , and T_3 .

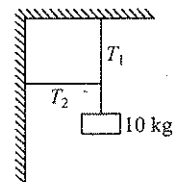


Fig. 7.70

8. Determine the tension T_4

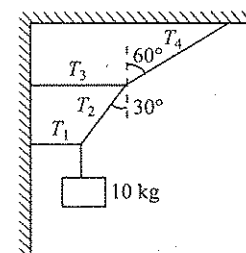


Fig. 7.71

9. Two trolleys A and B are moving with accelerations a and $2a$, respectively, in the same direction as shown in Fig. 7.72. To an observer in trolley A, the magnitude of pseudo force acting on a block of mass m on the trolley B is _____

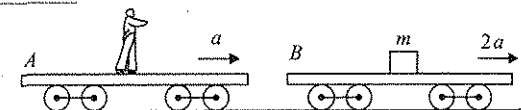


Fig. 7.72

10. a. A 10 kg block is supported by a cord that runs to a spring scale, which is supported by another cord from the ceiling as shown in Fig. 7.73. What is the reading on the scale?

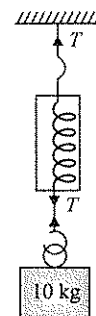


Fig. 7.73

- b. In Fig. 7.74 the block is supported by a cord that runs around a pulley and to a scale. The opposite end of the scale is attached by a cord to a wall. What is the reading of the scale?

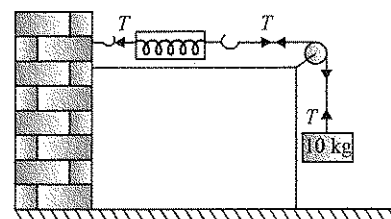


Fig. 7.74

- c. In Fig. 7.75 the wall has been replaced with a second 10 kg block on the left. What is the reading on the scale now?

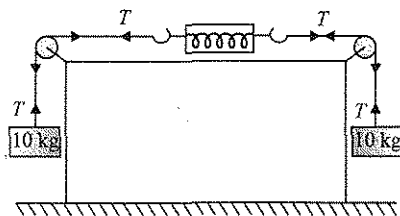


Fig. 7.75

11. What is the reading of the spring balance in the following device?

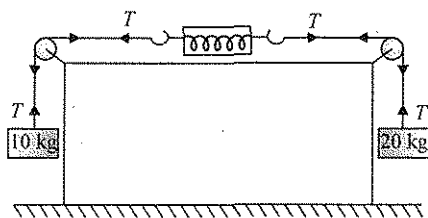


Fig. 7.76

12. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig. 7.77. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding.

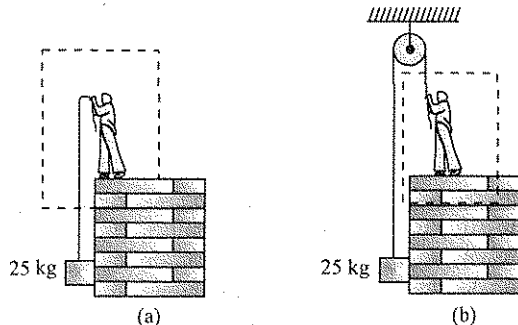


Fig. 7.77

13. Two blocks of masses 1 kg and 2 kg are placed in contact on a smooth horizontal surface as shown in Fig. 7.78. A horizontal force of 3 N is applied

a. on 1 kg block

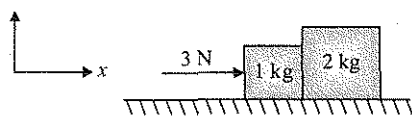


Fig. 7.78

- b. on 2 kg block. Find force of interaction between the blocks (see Fig. 7.79).

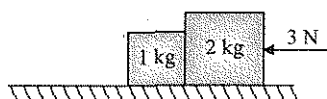


Fig. 7.79

14. Consider the system shown in Fig. 7.80. The system is released from rest, find the tension in the cord connected between 1 kg and 2 kg blocks.

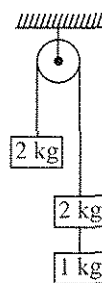


Fig. 7.80

15. The system shown in Fig. 7.81 is released from rest. Calculate the tension in the strings and force exerted by the strings on the pulleys. Assuming pulleys and strings are massless.

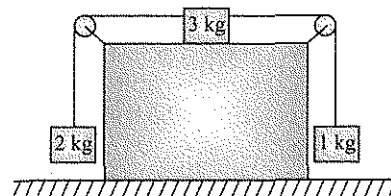


Fig. 7.81

16. Find acceleration of blocks and tension in the cord in the device shown in the Fig. 7.82.

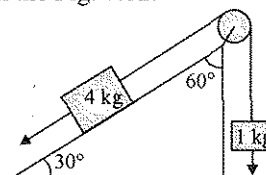


Fig. 7.82

17. Two monkeys of masses 10 kg and 8 kg are moving along a vertical rope as shown in Fig. 7.83. The former climbing up with an acceleration of 2 m/s^2 , while the latter coming down with a uniform velocity of 2 m/s . Find the tension in the rope at the fixed support.

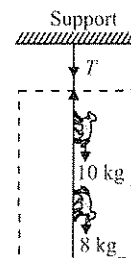


Fig. 7.83

18. A student standing on the large platform of a spring scale notes his weight. He then takes a step on this platform and notices that the scale reads less than his weight at the beginning of the step and more than his weight at the end of the step. Explain.

19. A homogeneous rod of length L is acted upon by two forces F_1 and F_2 applied to its ends and directed opposite to each other. With what force F will the rod be stretched at the cross-section at a distance l from the end where F_1 is applied?
20. In the arrangement shown in Fig. 7.84, a wedge of mass $M = 4$ kg moves towards left with an acceleration of $a = 2$ m/s². All surfaces are smooth. Find the acceleration of mass $m = 1$ kg relative to the wedge.

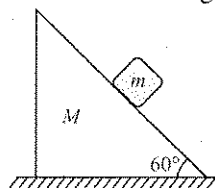


Fig. 7.84

21. A 20-kg monkey has a firm hold on a light rope that passes over a frictionless pulley and is attached to a 20-kg bunch of bananas (as shown in Fig. 7.85). The monkey looks up, sees the bananas, and starts to climb the rope to get them.
- As the monkey climbs, do the bananas move up, down or remain at rest?
 - As the monkey climbs, does the distance between the monkey and the bananas decrease, increase, or remain constant?

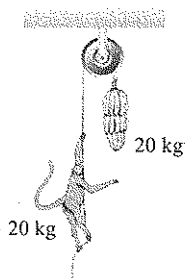


Fig. 7.85

- The monkey releases her hold on the rope. What happens to the distance between the monkey and the bananas while she is falling?
 - Before reaching the ground, the monkey grabs the rope to stop her fall. What do the bananas do?
22. A lift is going up. The total mass of the lift and the passengers is 1,500 kg. The variation in the speed of the lift is given by the graph (Fig. 7.86).

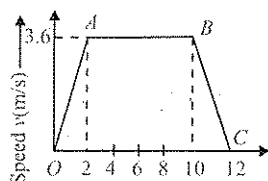


Fig. 7.86

- What will be the tension in the rope pulling the lift at time t equal to (i) 1 s (ii) 6 s (iii) 11 s?
- What will be the average velocity and the average acceleration during the course of the entire motion?

23. A body hangs from a spring balance supported from the roof of an elevator.
- If the elevator has an upward acceleration of 2.45 m/s² and the balance reads 50 N, what is the true weight of the body?
 - Under what circumstances will the balance read 30 N?
 - What will the balance read if the elevator cable breaks?
24. An object of mass 5.00 kg attached to a spring scale, rests on a frictionless, horizontal surface as shown in Fig. 7.87. The spring scale, attached to the front end of a boxcar, has a constant reading of 18.0 N when the car is in motion.
- The spring scale reads zero when the car is at rest. Determine the acceleration of the car.
 - What constant reading will the spring scale show if the car moves with constant velocity?
 - Describe the forces on the object as observed by someone in the car and by someone at rest outside the car.

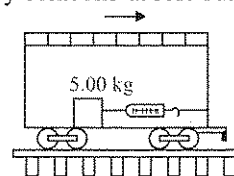


Fig. 7.87

CONSTRAINT RELATION

The equations showing the relation of the motions of a system of bodies, in which once motion is constrained by the others motion, are called the constrained relations.

First we start our analysis with simple cases of pulleys. Consider the situation shown in Fig. 7.88. Two bodies are connected with a string which passes over a pulley at the corner of a table. Here if string is inextensible, we can directly state that the displacement of A in downward direction is equal to the displacement of B in horizontal direction on table, and if displacements of A and B are equal in equal time, their speeds and acceleration magnitude must also be equal.

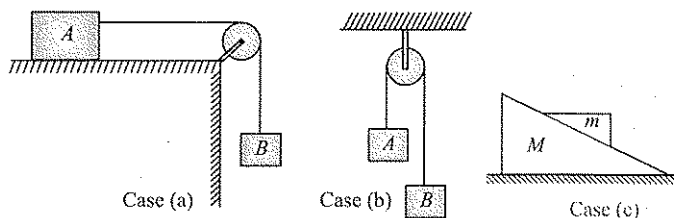


Fig. 7.88

In this case (c) if the wedge and block are free to move it is obvious that the acceleration of the block and the wedge are related.

Applying Newton's law alone is not sufficient in most cases. If you apply Newton's laws based on the methods shown above, you will get a few equations. You may however find that the

number of unknowns are much larger than the number of equations. That is, you will have a situation where, say the number of variables are three but the number of equations are only two. It is, therefore obvious that you cannot solve the equations to get the value of the variables.

Look at the diagram in Fig. 7.88(a)

In this case let us say that you have to find the acceleration of the masses.

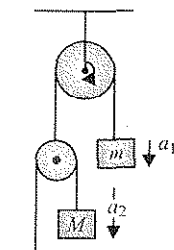
The number of unknowns will be

1. tension T ,
2. acceleration a_1 of the mass, and
3. acceleration of the other body a_2 .

There are three unknowns. However we will get only two equations – one for one mass and another for the other mass.

Clearly you can see that Newton's laws are not sufficient to solve the problem. In such cases we need additional equations. These are provided by what are called as constraint equations.

Constraints mean that two bodies (in this case the bodies which are attached to the pulley) are not free to move the way they want. The accelerations between them are dependent on each other. We need to find out the relationship to be able to solve these equations; therefore, constraints provide additional equations.



The need for constraints

Fig. 7.89

Constraints are the geometrical restrictions imposed on the motion of a body, which also governs the trajectory of the body. For example, a block placed on the table cannot move normal to the surface, it is bound to move parallel to the surface.

We have to use the method of constraint equations to relate the accelerations between the bodies.

In many cases you can write down the relation of acceleration by just looking at the situation. In other cases, for complex relationships, we can think of four types of constraints.

1. General constraints
2. Wedge constraint
3. Pulley constraint
4. Combination of the wedge and pulleys

General Constraints

Illustration 7.17 Fig. 7.90 shows a rod of length l resting on a wall and floor. Its lower end A is pulled towards left with a constant velocity u , as a result of this end B starts moving down along the wall. Find the velocity with which

the other end B moves downward when the rod makes an angle θ with the horizontal.

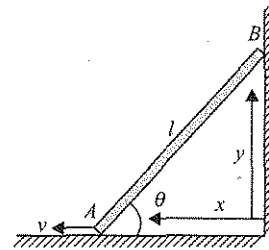


Fig. 7.90

Sol. Let us first find the relation between the two displacements then differentiate with respect to time. Here if the distance from the corner to the point A is x and that up to B is y . Now the left velocity of point A can be given as $v_A = \frac{dx}{dt}$ and that of B can

be given as $v_B = -\frac{dy}{dt}$ (–sign indicates, y is decreasing).

If we relate x and y : $x^2 + y^2 = l^2$

Differentiating with respect to t : $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$$\Rightarrow xv_A = yv_B \Rightarrow xu = yv_B \Rightarrow v_B = u \frac{x}{y} = u \cot \theta$$

Alternatively: In cases where distance between two points is always fixed, we can say the relative velocity of one point of an object with respect to any other point of the same object in the direction of the line joining them will always remain zero, as their separation always remains constant.

Here in the above example, the distance between points A and B of the rod always remains constant, thus the two points must have same velocity components in the direction of the line joining, i.e., along the length of the rod.

If point B is moving down with velocity v_B , its component along the length of the rod is $v_B \sin \theta$. Similarly, the velocity component of point A along the length of rod is $v \cos \theta$. Thus we have

$$v_B \sin \theta = v \cos \theta \text{ or } v_B = v \cot \theta$$

Illustration 7.18 In Fig. 7.91, a ball of mass m_1 and a block of mass m_2 are joined together with an inextensible string. The ball can slide on a smooth horizontal surface. If v_1 and v_2 are the respective speeds of the ball and the block, determine the constraint relation between the two.

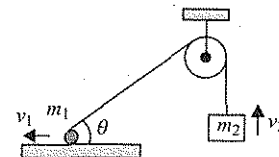


Fig. 7.91

Sol. Method 1: Distances are assumed from the center of the pulley as shown in Fig. 7.92.

Constraint: Length of the string remains constant.

$$\sqrt{x_1^2 + h_1^2} = x_2 \text{ constant}$$

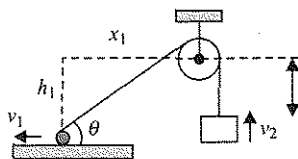


Fig. 7.92

Differentiating both the sides w.r.t. the time, we get

$$\frac{2x_1}{2\sqrt{x_1^2 + h_1^2}} \frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

Since the ball moves so as to increase x_1 with time and block moves so as to decrease x_2 with time, therefore,

$$\frac{dx_1}{dt} = +v_1 \quad \text{and} \quad \frac{dx_2}{dt} = -v_2 \quad \text{also,} \quad \frac{x_1}{\sqrt{x_1^2 + h_1^2}} = \cos \theta$$

or $v_2 = v_1 \cos \theta$

Method 2:

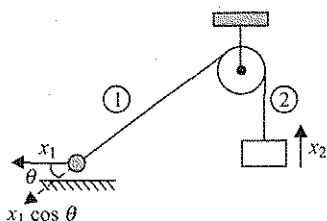


Fig. 7.93

Change in the length of segment (1)

$$= x_1 \cos \theta - 0 = x_1 \cos \theta$$

Change in the length of segment (2) = $-x_2$

Total change in the string length should be zero.

$$x_1 \cos \theta - x_2 = 0 \Rightarrow x_2 = x_1 \cos \theta$$

Hence $v_2 = v_1 \cos \theta$

Method 3: The problem can be solved very easily if we look at the problem from a different viewpoint and identify a different constraint; i.e., velocity of any two points along the string is same. Obviously, from Fig. 7.94, $v_1 \cos \theta = v_2$.

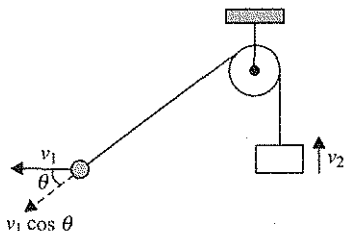


Fig. 7.94

Writing Down Constraints-Pulley

Pulley constraints are applicable when the bodies concerned are connected through pulleys and the rope connecting them is inextensible.

Case-I

Mass A is connected with a string which passes through a fixed pulley. The other end of string is connected with a movable pulley N. Block B is connected with another string which passes through the pulley N as shown in Fig. 7.95.

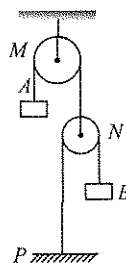


Fig. 7.95

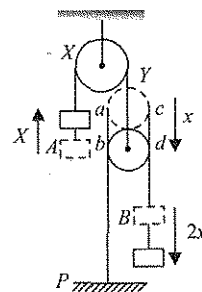


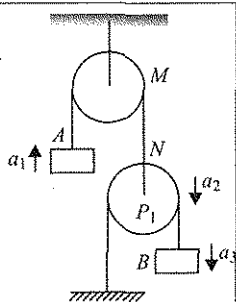
Fig. 7.96

Consider the situation shown in Fig. 7.96. If we consider that mass A is going up by distance x , pulley N which is attached to the same string will go down by the same distance x . Due to this the string which is connected to mass B will now have free lengths ab and cd ($ab = cd = x$) which will go on the side of mass B due to its weight as the other end is fixed at point P. Thus mass B will go down by $2x$ hence its speed and acceleration will be twice that of block B.

Hence,

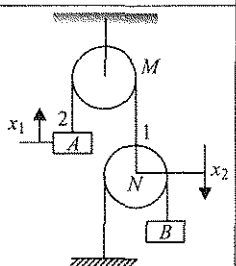
$$v_B = 2v_A \quad \text{and} \quad a_B = 2a_A$$

Above example can be understood by another approach in which we will consider the total length of the string will always be constant.



To relate the acceleration of the bodies, assume that the various bodies move by a distance $x_1, x_2, \dots$, and so on. Calculate the number of segments in the rope.

The segments in the first rope are marked 1 and 2. The distance moved by the various elements are also marked (as shown in the figure). Note that the pulley, which is connected to the ceiling, cannot move.



Relate the distance moved. (total change in length of the string must be zero). To do this, calculate the change in length of each segment of the string. then add these changes to get the total change in length of the rope.

Change in length of segment 1 = $-x_1$

Change in length of segment 2 = $+x_2$.

Therefore total change in length of string $-x_1 + x_2 = 0 \Rightarrow x_1 = x_2$

Once we have the relation between the distances, the relation between accelerations is simple. For the first string it is $a_1 = a_2$.

For second string

Let us apply steps 3, 4, and 5 for the second inextensible string.

The distances moved by the pulley N and block B are x_2 and x_3 as shown in figure. The ground is at rest. Therefore the end of the string that is connected to the ground will not move.

Change in length of segment 3 = $-x_2$ (Why negative? Because as the pulley moves down, the rope comes closer to the ground and the length of the segment decreases).

Change in length of segment 4 = $+x_3 - x_2$.

Rope with either parts moving by a distance x_2 and x_1 .

To see why this is so, let us consider a string with either points moving by a distance x_2 and x_3 . This is shown in the figure.

Because the other end moves by x_3 , the length of the string increases by x_3 .

When the other end moves by x_2 , the length reduces by x_2 .

The change in length is therefore $x_3 - x_2$.

The total change in length of the rope is $-x_2 + x_3 - x_2 = 0$
 $\Rightarrow x_3 - 2x_2 = 0 \Rightarrow x_3 = 2x_2$

Once we get the relation between the distances moved, the acceleration relation will be the same. The acceleration relation is also, $a_3 = 2a_2$

Case-II

In the given situation M is a fixed pulley and N is a movable pulley. The blocks A and B are tied to strings and arranged as shown.

If mass A goes up by the distance x , we can observe that the string lengths ab and cd are slack, due to the weight of block B , this length ($ab + cd = 2x$) will go on this side and block B will descend by a distance $2x$. As in equal time duration B has travelled a distance twice that of A .

So, $v_A = 2v_B$ and $a_A = 2a_B$

Case-III

Here three blocks A , B , and C are connected with strings and pulleys as shown in figure.

Here we develop constraint relation between the motion of masses A , B and C . Let us assume that masses A and C would go up by distance x_A and x_C , respectively, these lengths of the string will slack as length $ab - cd$ below the pulley Z . Thus this will go down by a distance x_B as shown in Fig.(b). Thus we have

$$ab + cd = x_A + x_C \text{ or } 2x_B = x_A + x_C$$

differentiating w.r.t. time. we get $2v_B = v_A + v_C$ (i)

differentiating again w.r.t. time $2a_B = a_A + a_C$ (ii)

Equations (i) and (ii) are the constraint relations for motion of masses A , B , and C .

Your Shortcut

In the cases where pulley moves along with the blocks connected on both the sides, we can say that the displacement of the pulley is the average of the displacement on both sides of the pulley.

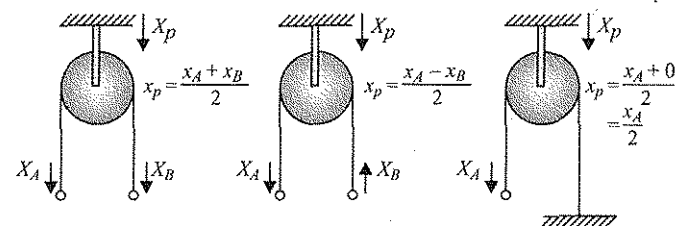


Fig. 7.97

If one end of the string is connected with the fixed end, the displacement of that end can be considered as zero.

Analysis of Case-I using shortcut method

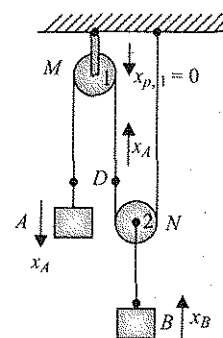


Fig. 7.98

- As pulley 1 is fixed, hence the displacement should be 0. If the displacement of block A is x_A (down) then the displacement of other end should be x_A (up) (see Fig. 7.98).

$$X_{P,1} = 0 = \frac{x_A + x_D}{2} \Rightarrow x_D = -x_A$$

- Displacement of block B = Displacement of pulley 2
- $x_{p2} = \frac{x_A + 0}{2} \Rightarrow x_A = 2x_{p2} \Rightarrow x_A = 2x_B$

Analysis of Case-II using Shortcut Method

- As pulley 1 is fixed $|x_{p,2}| = |x_A|$
If block A moves up by x_A , pulley 2 should move x_A in downward direction as shown in Fig. 7.99.
- For pulley 2

$$x_{p,2} = x_A = \frac{x_B + 0}{2}$$

$$x_B = 2x_A$$

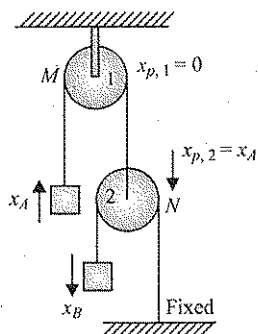


Fig. 7.99

Analysis of Case-III using Shortcut Method

- As pulley M is fixed $x_M = 0$ (see Fig. 7.100)

$$= \frac{x_A + x_W}{2} \Rightarrow x_W = -x_A$$

- Pulley N is also fixed

$$X_N = 0 = \frac{x_C + x_Z}{2} \Rightarrow x_Z = -x_C$$

$$x_B = \frac{x_W + x_Z}{2}$$

$$2x_B = x_W + x_Z = x_A + x_C$$

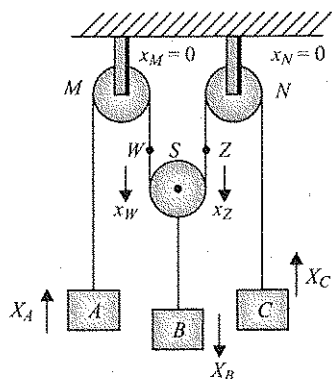


Fig. 7.100

Illustration 7.19 In the arrangement of three blocks as shown in the Fig. 7.101, the string is inextensible. If the directions of accelerations are as shown in the given figure, then determine the constraint relation.

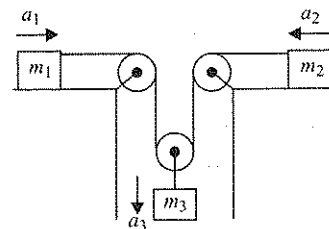


Fig. 7.101

Sol. Method 1: Let us assume the respective distance of each block as shown in Fig. 7.101. Since the length of the string is constant, therefore, $x_1 + x_2 + 2x_3 = \text{constant}$

On differentiating twice w.r.t. time, we get

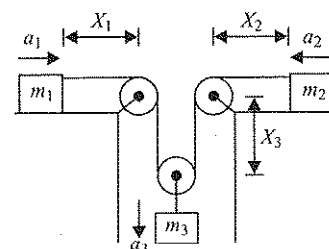


Fig. 7.102

$$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} + 2\frac{d^2x_3}{dt^2} = 0$$

Since x_1 and x_2 are assumed to be decreasing with time, therefore,

$$\frac{d^2x_1}{dt^2} = -a_1 \text{ and } \frac{d^2x_2}{dt^2} = -a_2$$

and x_3 is assumed to be increasing with time, therefore,

$$\frac{d^2x_3}{dt^2} = +a_3$$

Thus $-a_1 - a_2 = 2a_3$ or $a_1 + a_2 = 2a_3$.

Method 2: As total change in the length of string should be zero. The summation of the change in the lengths of segment should be zero.

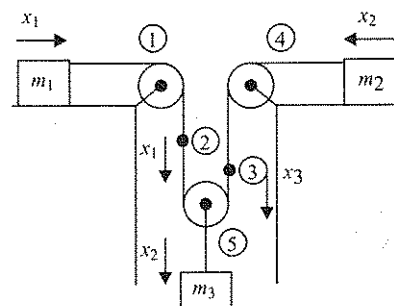


Fig. 7.103

$$\begin{aligned} -x_1 + x_2 + x_3 - x_2 &= 0 \\ -x_1 + 2x_3 - x_2 &= 0 \\ 2x_3 &= x_1 + x_2 \end{aligned}$$

Hence, $2a_3 = a_1 + a_2$

Shortcut method: $x_3 = \frac{x_1 + x_2}{2}$

$\Rightarrow 2x_3 = x_1 + x_2$

Illustration 7.20 In the pulley-rope-mass arrangement shown in the Fig. 7.104 $m_1 = 2$ kg, $m_2 = 2$ kg, and $m_3 = 4$ kg. Find the tension in the ropes and the accelerations of the masses when the masses are set free to move. Assume that the pulley and the ropes are ideal. Take $g = 10$ m/s².

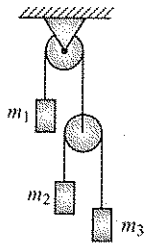


Fig. 7.104

Sol. Step 1: Constraint relations

Now, $l_1 = \text{constant}$ gives,

$$l_1 = x_1 + x_p + \pi R$$

$$\frac{d^2 l_1}{dt^2} = \frac{d^2 x_1}{dt^2} + \frac{d^2 x_p}{dt^2} + \frac{d^2 \pi R}{dt^2}$$

$$0 = a_1 + a_p \Rightarrow |a_1| = |a_p| \quad (1)$$

and $l_2 = \text{constant}$ gives,

$$l_2 = (x_2 - x_p) + (x_3 - x_p) + \pi R$$

$$\Rightarrow l_2 = x_2 + x_3 - 2x_p + \pi R$$

$$\frac{d^2 l_2}{dt^2} = \frac{d^2 x_2}{dt^2} + \frac{d^2 x_3}{dt^2} - \frac{2d^2 x_p}{dt^2} + \frac{d^2 \pi R}{dt^2}$$

$$\Rightarrow 0 = a_2 + a_3 - 2a_p \Rightarrow 2a_1 = a_2 + a_3 \quad (2)$$

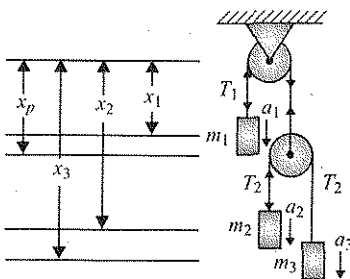


Fig. 7.105

Step 2: Free body diagrams (Fig. 7.106)

Step 3 : Equations of motion

$$20 - T_1 = 2a_1 \quad (3)$$

$$20 - T_2 = 2a_2 \quad (4)$$

$$40 - T_2 = 4a_3 \quad (5)$$

$$2T_2 - T_1 = 0 \quad (6)$$

Solving these equations for the required quantities,

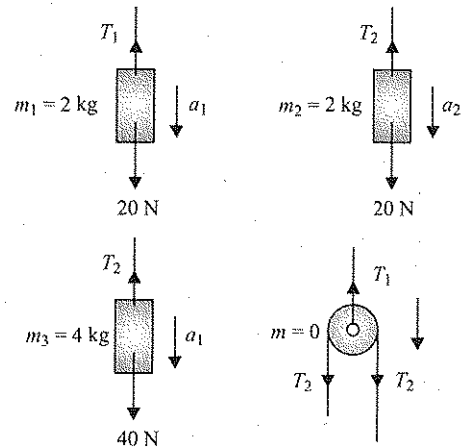


Fig. 7.106

$$a_1 = \frac{-50}{11} \text{ m/s}^2, a_2 = \frac{30}{11} \text{ m/s}^2, a_3 = \frac{70}{11} \text{ m/s}^2, a_p = \frac{50}{11} \text{ m/s}^2$$

$$T_1 = \frac{320}{11} \text{ N}, T_2 = \frac{160}{11} \text{ N}$$

Note: You can also establish equations (1) and (2) somewhat indirectly in this case.

1. For the rope length l_1 not to change and for this rope not to slacken acceleration of m_1 w.r.t the fixed pulley = - acceleration of the movable pulley w.r.t the fixed pulley

$$\text{or } a_1 = -a_p \Rightarrow |a_1| = |a_p|$$

2. For the rope length l_2 not to change and for this rope not to slacken-acceleration of m_2 w.r.t movable pulley = - acceleration of m_3 w.r.t the movable pulley

$$\text{or } (a_2 - a_p) = -(a_3 - a_p)$$

$$a_2 + a_3 - 2a_p = 0$$

$$\Rightarrow 2a_1 = a_2 + a_3$$

3. You can even use any of the methods given in previous discussions.

Illustration 7.21 In the arrangement shown in the Fig. 7.107 find the tensions in the rope and the acceleration of the masses m_1 and m_2 and pulleys P_1 and P_2 when the system is set free to move. Assume the pulleys to be massless and strings to be are light and inextensible.

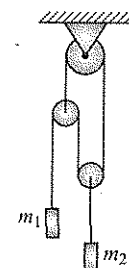


Fig. 7.107

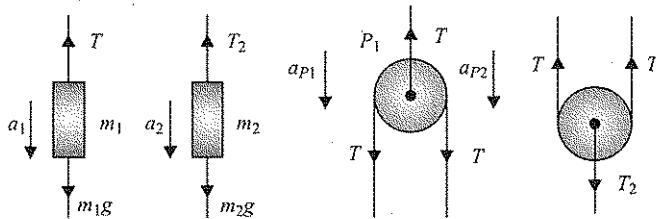
Sol. Step 1: Free body diagrams

Fig. 7.108

Step 2: Applying the equations of motion for pulleys and blocks

For m_1 : $m_1g - T = m_1a_1$ (1)

For m_2 : $m_2g - T_2 = m_2a_2$ (2)

For pulley p_1 : $2T - T = 0 \times a_{p1}$ (3)

For pulley p_2 : $T_2 - 2T = 0 \times a_{p2}$ (4)

From equation (3), $T = 0$

From equation (4) $T_2 = 0$ and from equations (1) and (2), $a_1 = g$, $a_2 = g$.

Calculating a_{p1} and a_{p2} :

Step 3: Constraint relations: Consider the reference line and the position vectors of the pulleys and masses as shown in the Fig. 7.109. Write the length of the rope in terms of the position vectors and differentiate it to obtain the relations between the accelerations of the masses and the pulleys.

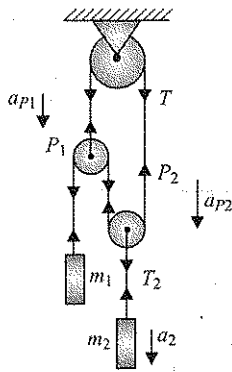


Fig. 7.109

1. For the length of the string connecting P_2 and m_2 not to be change and for this rope not to slacken.

$$a_{p2} = a_2 = g.$$

2. Length of the string

Connecting P_1 to m_1 not to change and for this rope not to slacken:

$$l = (x_1 - X_{p1}) + X_{p1} + (X_{p2} - X_{p1}) + X_{p2}$$

$$\Rightarrow l = x_1 - X_{p1} + 2X_{p2}$$

Differentiating this equation w.r.t, twice,

$$\frac{d^2l}{dt^2} = \frac{d^2x_1}{dt^2} - \frac{d^2x_{p1}}{dt^2} + 2\frac{d^2x_{p2}}{dt^2} \Rightarrow 0 = a_1 = a_{p1} - 2a_{p2}$$

$$\Rightarrow 0 = g - a_{p1} + 2a_{p2} \Rightarrow a_{p1} = 3g$$

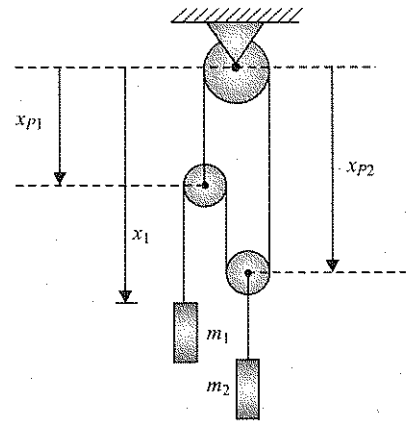


Fig. 7.110

Wedge Constraint

We can observe that the wedge M can only move in horizontal direction towards left, and the block m can slide on an inclined surface of M which is always in contact with the wedge.

- Let us define our x and y axis parallel to the inclined and perpendicular to inclined, respectively, as in Fig. 7.111.

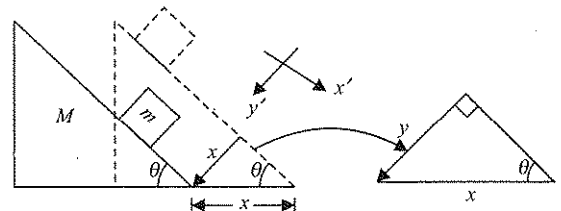


Fig. 7.111

- We can observe that the displacement of m and M in x' -direction will be same as the block never loses contact with the wedge.
- If the wedge moves in horizontal direction by a distance of x during this time the block will move x in x -direction
- We can relate these displacement x and X as

$$\frac{x}{X} = \sin \theta \Rightarrow x = X \sin \theta \quad (i)$$

Hence velocity relation can be written as

$$V_x = V \sin \theta \quad (ii)$$

and acceleration relation can be written as

$$a_x = A \sin \theta \quad (iii)$$

Here v_x and a_x are the velocity and acceleration of the block in the direction perpendicular to the inclined surface.

Illustration 7.22 A block of mass m is placed on the inclined surface of a wedge as shown in Fig. 7.112. Calculate the acceleration of the wedge when the block is released. Assume that all the surfaces are frictionless.

Sol.

Step 1: Constraint relation (Fig. 7.113)

From the previous discussion we can write

$$a_x = A \sin \theta \quad (i)$$

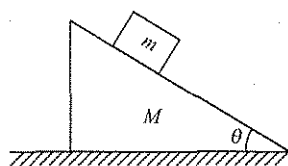


Fig. 7.112

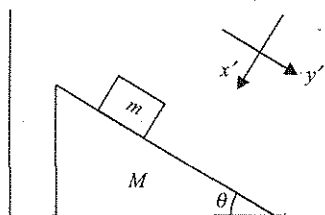


Fig. 7.113

Step 2: Free body diagrams shown in Fig. 7.114.

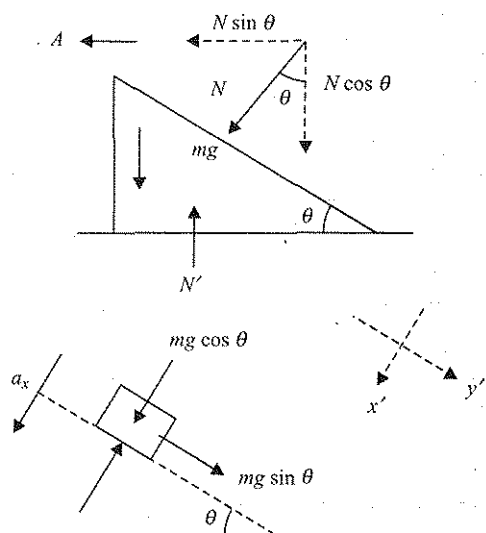


Fig. 7.114

Step 3: Equation of motions is

$$\text{For } M: N \sin \theta = MA$$

$$\text{For } m \text{ in } x' \text{ direction}$$

$$mg \cos \theta - N = ma_x = m(A \sin \theta)$$

$$\text{From equations (ii) and (iii)}$$

$$A = \frac{mg \sin \theta \cos \theta}{(M + m \sin^2 \theta)}$$

Pulley and Wedge Constraint

Consider a block of mass m is placed on the inclined surface of the wedge. The block is connected with a string and arranged as shown in Fig. 7.115. Now the system is released from the rest.

We can divide the string in two segments (1) and (2). Let at any interval of time the wedge move a distance X towards right and the block move a distance x parallel to inclined direction.

From the diagrams it is clear that the length of the segment (1) will decrease by X while the length of the segment (2) will

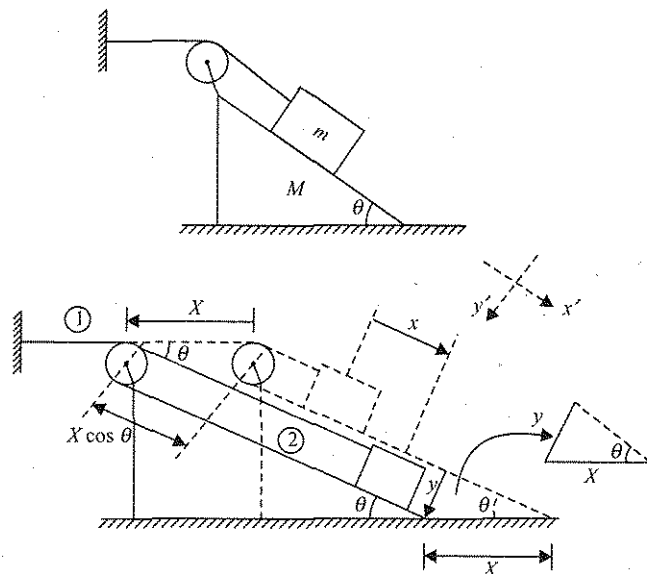


Fig. 7.115

increase by $(x + X \cos \theta)$. As overall length of the string is constant, then we can write

$$-X + (x + X \cos \theta) = 0$$

$$x = X(1 - \cos \theta), \text{ and}$$

$$a_x = A(1 - \cos \theta)$$

$$\text{From wedge constant it is clear that } \frac{y}{x} = \sin \theta$$

$$\Rightarrow y = X \sin \theta \text{ and } a_y = A \sin \theta$$

Illustration 7.23 Find the accelerations of rod A and wedge B in the arrangement shown in the Fig. 7.116 if the mass of the rod is m and that of the wedge is M . The friction between all the contact surfaces is negligible.

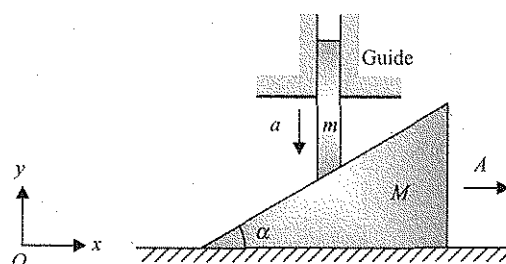


Fig. 7.116

Sol. The rod (mass m) is constrained to move in the vertical direction (with the help of the guides), and the wedge will move along the surface in the horizontal direction.

Let the acceleration of m w.r.t. ground be a , vertically downwards and acceleration of M w.r.t. ground be A , horizontally towards right.

Constraint relation: The motion of the system is constrained by the fact that the "bottom face of the rod must always be in contact with the inclined plane". If in time t , X be the displacement of the wedge and x be the displacement of the rod, then the constraint demands that (see Fig. 7.117).

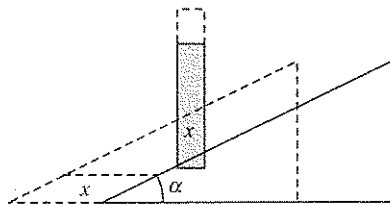


Fig. 7.117

$$\frac{x}{X} = \tan \alpha$$

$$\Rightarrow x = X \tan \alpha \quad (1)$$

Differentiating equation (1) w.r.t. t twice,

$$\left(\frac{d^2 x}{dt^2} \right) = \left(\frac{d^2 X}{dt^2} \right) \tan \alpha \quad [\tan \alpha = \text{constant}]$$

$$\Rightarrow \alpha \cos \alpha = A \sin \alpha$$

The fact that the rod (or a particle on the wedge) and the wedge must not lose contact is usually called “wedge constraints”. For this, the component of the acceleration of the rod perpendicular to the wedge plane should be equal to the component of acceleration of the wedge perpendicular to the wedge plane.

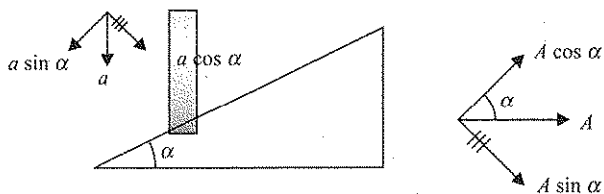


Fig. 7.118

$$a \cos \alpha = A \sin \alpha$$

$$a = A \tan \alpha$$

Solving the above equation for a and A .

$$a = \frac{mg \tan \alpha}{m \tan \alpha + M \cot \alpha}$$

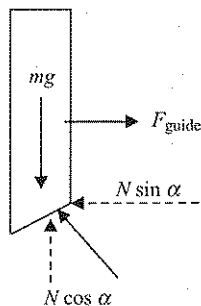


Fig. 7.119

The forces acting on the rod are:

1. the weight mg , vertically downwards,
2. the normal force N , normal to the bottom surface of the rod, and
3. the force (F_{guide}) exerted by the guide to nullify the horizontal component of N as for the rod $a_{\text{horizontal}} = 0$.

Motion in vertical direction

$$mg - N \cos \alpha = ma \quad (2)$$

and the forces acting on the wedge are (Fig. 7.120):

1. the weight, Mg ,
2. N , reaction of N acting on the rod, and
3. N_1 , normal force by the surface.

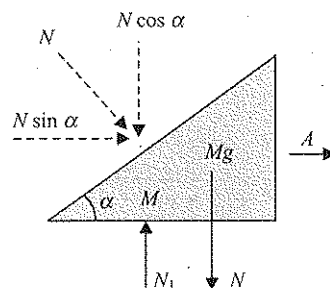


Fig. 7.120

The force equations are:

$$N_1 - Mg - N \cos \alpha = 0, \text{ and} \quad (3)$$

$$N \sin \alpha = MA \quad (4)$$

Illustration 7.24 A block of mass m is placed on an inclined plane (Fig. 7.121). With what acceleration A towards right should the system move on a horizontal surface so that m does not slide on the surface of the inclined plane? Assume that all the surfaces are smooth.

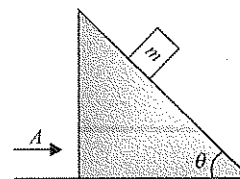


Fig. 7.121

Sol. If the motion of m is analysed from ground, its acceleration is A and the forces acting on it are: its weight mg and normal reaction R .

If the motion of m is analysed from the view point of an observer standing on the inclined plane (i.e., relative to the plane), and its acceleration is 0 m/s^2 and the forces acting on it are: its weight, the normal reaction, and a pseudo force of magnitude mA towards left. dividing, we get, $A = g \tan \theta$.

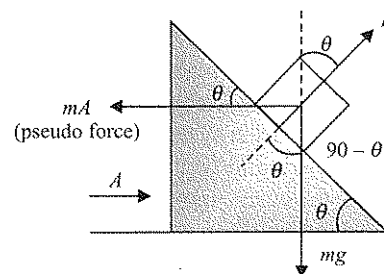


Fig. 7.122

Analysis of forces on m relative to the inclined plane:

As m is at rest, forces must balance each other along both directions

$$R \cos \theta = mg \quad (i)$$

$$R \sin \theta = mA \quad (ii)$$

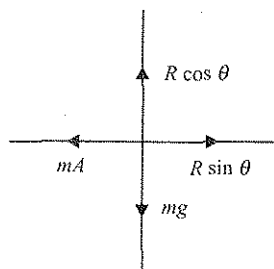


Fig. 7.123

Illustration 7.25 Find the acceleration of the blocks in Fig. 7.124. The pulley and the strings are massless.

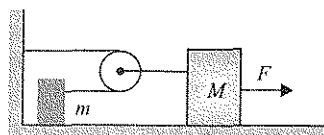


Fig. 7.124

Sol. Let a = acceleration of block M and pulley.
 a = acceleration of m .

Let x, X be the co-ordinates of m and pulley as shown.

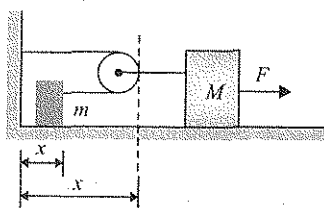


Fig. 7.125

Length of string passing over the pulley is:

$$L = X + X - x$$

$$L = 2X - x$$

Differentiating twice with respect to time,

$$0 = 2A - a \Rightarrow a = 2A \quad (1)$$

Note: We can generalise this result. If one end of a string, passing over a moving pulley, is fixed then the acceleration of other end is twice the acceleration of pulley.

From the force diagrams of m and M , we have

$$T = ma, \text{ and} \quad (2)$$

$$F - 2T = MA \quad (3)$$

Comparing equations 1, 2, and 3, we get

$$A = \frac{F}{M + 4m} \text{ and}$$

$$a = \frac{2F}{(M + 4m)}$$

Illustration 7.26 Mass of wedge as shown in Fig. 7.126 is $M = 42 \text{ kg}$ and that of the block is $m = 5 \text{ kg}$. Neglecting friction at all the places and mass of the pulley, calculate the acceleration of the wedge. Thread is inextensible.

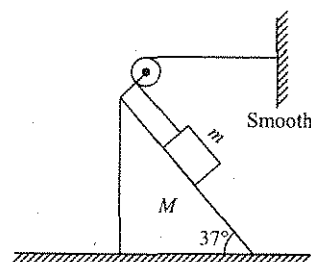


Fig. 7.126

Sol. Let the acceleration of the wedge be a rightward then acceleration of block relative to wedge will also be a but down the incline. Hence, net acceleration of the block will be equal to the vector sum of these two as shown in Fig. 7.127.

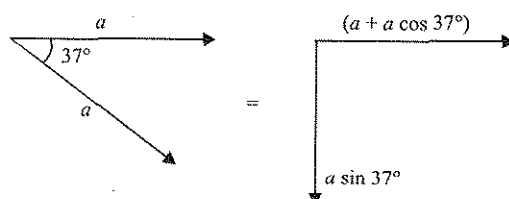


Fig. 7.127

Now considering FBDs (see Fig. 7.128).

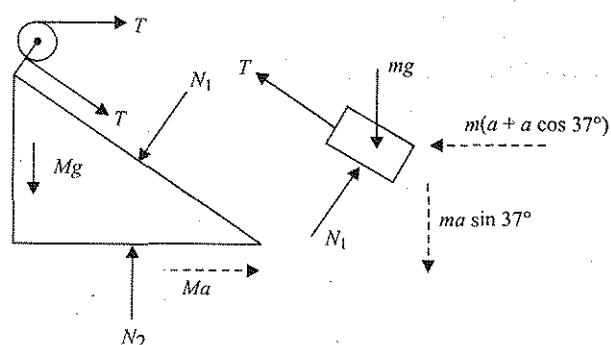


Fig. 7.128

For the wedge,

$$N_2 = N_1 \cos 37^\circ + T \sin 37^\circ \quad (i)$$

$$T - T \cos 37^\circ - N_1 \sin 37^\circ = Ma \quad (ii)$$

For the block,

$$N_1 \sin 37^\circ - T \cos 37^\circ = m(a + a \cos 37^\circ) \quad (iii)$$

$$mg - N_1 \cos 37^\circ - T \sin 37^\circ = ma \sin 37^\circ \quad (iv)$$

Solving the above equations, $N_1 = 41.5 \text{ N}$, $T = 25.5 \text{ N}$, and $a = 0.5 \text{ ms}^{-2}$.

Concept Application Exercise 7.3

1. Figure 7.129 shows a system of four pulleys with two masses A and B . Find, at an instant:

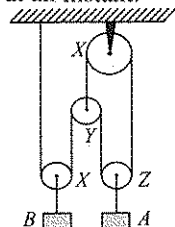


Fig. 7.129

- Speed of the block A when the block B is going up at 1 m/s and the pulley Y is going up at 2 m/s .
 - Acceleration of block A if block B is going up at 3 m/s^2 and the pulley Y is going down at 1 m/s^2 .
2. Figure 7.130 shows a pulley over which a string passes and is connected to two masses A and B . Pulley moves up with a velocity v_P and mass B is also going up at a velocity v_B . Find the velocity of mass A if:

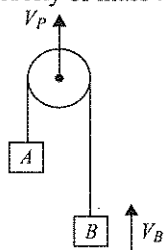


Fig. 7.130

- $v_P = 5 \text{ m/s}$ and $v_B = 10 \text{ m/s}$
 - $v_P = 5 \text{ m/s}$ and $v_B = -20 \text{ m/s}$
3. Find the relation in the acceleration of the three masses shown in Fig. 7.131(a) and Fig. 7.131(b).

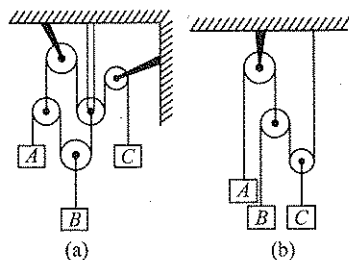


Fig. 7.131

4. Figure 7.132 shows a small mass m hanging over a pulley. The other end of the thread is being pulled in horizontally with a uniform speed u . Find the speed with which the mass ascends at the instant the string makes an angle θ with the horizontal.

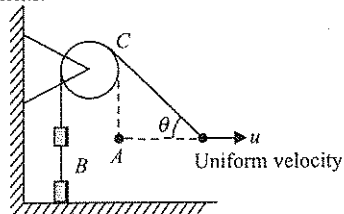


Fig. 7.132

5. A ring A which can slide on a smooth wire is connected to one end of a string as shown in Fig. 7.133. Other end of the string is connected to a hanging mass B . Find the speed with the ring when the string makes an angle θ with the wire and mass B is going down with a velocity v .

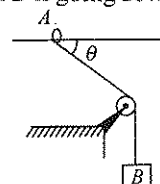


Fig. 7.133

6. Figure 7.134 shows a block A constrained to slide along the incline plane of the wedge B . Block A is attached with a string which passes through three ideal pulleys and is connected to the wedge B . If the wedge is pulled towards right with an acceleration ' a '.

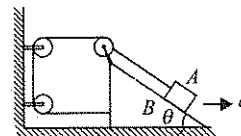


Fig. 7.134

- find the acceleration of the block with respect to wedge.
 - find the acceleration of the block with respect to ground.
7. Find the acceleration of the block B as shown in Fig. 7.135 (a) and (b) relative to the block A and relative to the ground if the block A is moving towards left with an acceleration a .

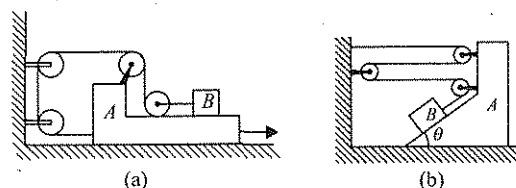


Fig. 7.135

8. If all the surfaces are smooth, to keep the block stationary with respect to the wedge, the wedge should be given a horizontal acceleration towards _____. The magnitude of the acceleration is given by the horizontal force to be applied on the wedge would be _____.

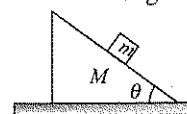


Fig. 7.136

9. If the string is inextensible, determine the velocity u of each block in terms of v and θ .

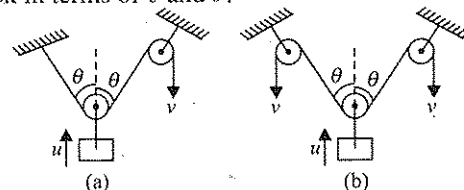


Fig. 7.137

- Fig. 7.137(a) (A) $u =$ _____
- Fig. 7.137(b) $u =$ _____

10. Calculate the accelerations of the block A and B in cases of Figs 7.138 (i), (ii), and (iii).

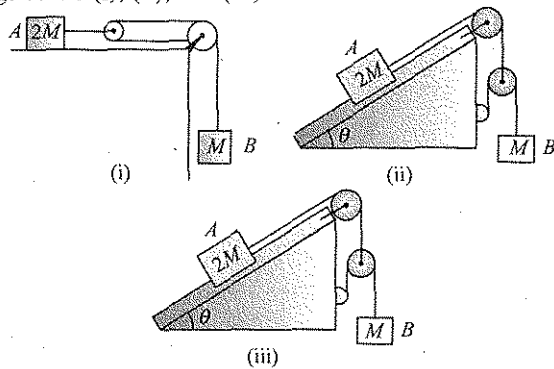


Fig. 7.138

11. The rod of mass m is constrained to move along the guide and always in contact with the wedge of mass M as shown in Fig. 7.139. Assume that there is no friction anywhere. Calculate the acceleration of the wedge and rod.

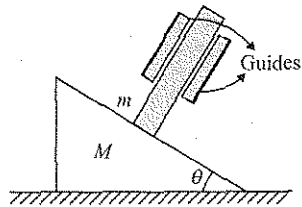


Fig. 7.139

12. A man (mass = m) is standing on a platform (mass = M). By pulling the string he causes the motion of platform. What is the ratio of m/M such that the man and the platform move together?

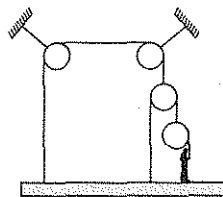


Fig. 7.140

13. In Fig. 7.141 no relative motion takes place between the wedge and the block placed on it. The rod slides downwards over the wedge and pushes the wedge to move in the horizontal direction. The mass of the wedge is the same as that of the block and is equal to M . If $\tan \theta = \frac{1}{\sqrt{3}}$, find the mass of the rod. (Neglect the rotation of the rod).

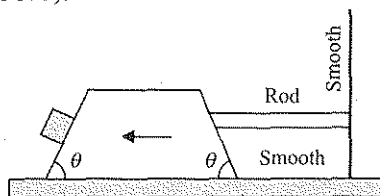


Fig. 7.141

14. The velocities of A and B are shown in the Fig. 7.142. Find the speed (in m/s) of block C . (Assume that the pulleys and the string are ideal.)

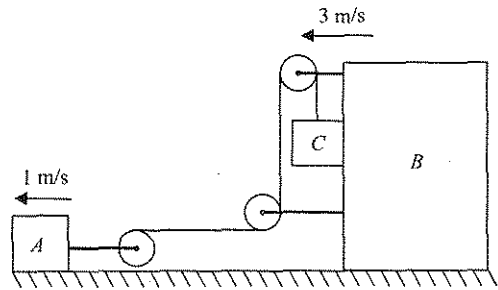


Fig. 7.142

15. A 20 kg block B is suspended from an ideal string attached to a 40 kg block A . The ratio of the acceleration of the block B in cases (I) and (II) shown in Figs. 7.143 and 7.144 immediately after the system is released from rest is $\frac{3n}{2\sqrt{2}}$. Find value of n . (Neglect friction.)

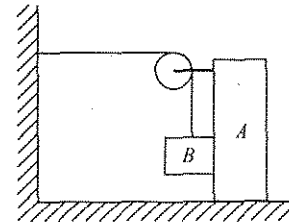


Fig. 7.143

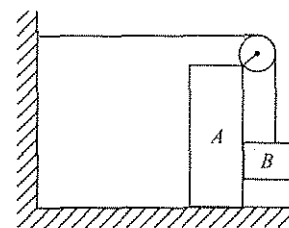


Fig. 7.144

SPRING FORCE AND COMBINATIONS OF SPRINGS

Springs can be of many types such as helical [Fig. 7.145(a)] or spiral [Fig. 7.145(b)]. These springs are either (b) and are stretchable or compressible.

Regarding springs it is worth nothing that

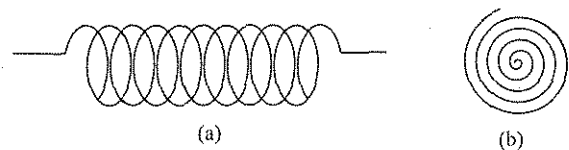


Fig. 7.145

Springs are assumed to be the same everywhere.

Springs can be stretched or compressed and the stretch or compression is always taken to be positive. [A string cannot be compressed.]

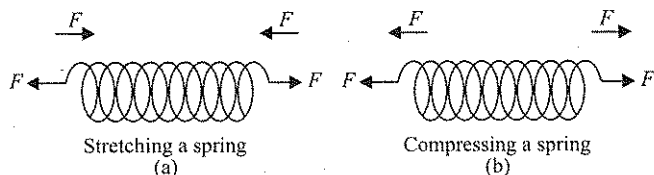


Fig. 7.146

To produce extension or compression in a spring two equal and opposite forces are applied and equilibrium restoring force is developed due to the elasticity of spring which is equal to either force.

i.e., $F = F'$ and always opposite to applied force.

For small stretch or compression, springs obey Hook's law, i.e., for a spring

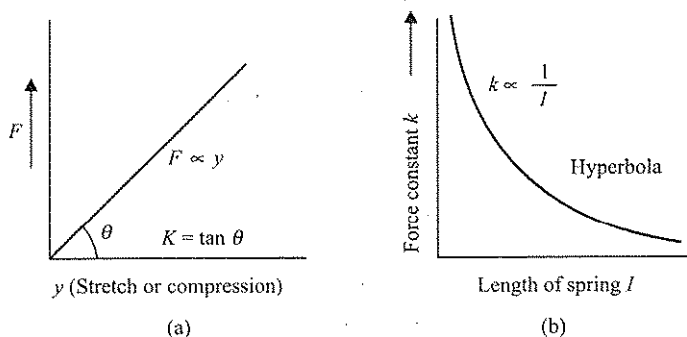


Fig. 7.147

Force $\propto$ stretch (or compression), i.e., $F = ky$ (i)

This means that the restoring force is linear. This force in a spring is not constant and depends on stretch (or compression) y . Greater the stretch (or compression) greater will be the force and vice-versa.

k is called the **force constant** of the spring and is equal to the slope of the force versus the stretch curve. It has dimensions $[F/L]$ which is $[MT^{-2}]$ and units N/m. Greater the force, constant of a spring lesser will be the stretch (or compression) for a given force and more stiffer is said to be the spring. The force constant k of a spring depends on wire (its length, radius r , and material) used to make the spring (radius of spring R and length l of spring). It is well established that for a given spring

$$k \propto (1/l) \quad (\text{ii})$$

This means that smaller the length of the spring, greater will be the force constant and vice-versa.

Force Constant of Composite Springs

If a number of springs are connected to a body and we want to produce it to a single spring, following three cases of common interest are possible

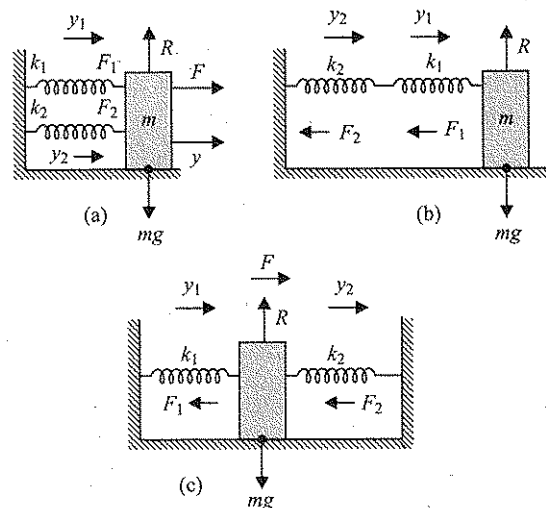


Fig. 7.148

Spring in Parallel

This situation is shown in Fig. 7.148(a). If the force F pulls the mass m by y , the stretch in each spring will be y ,

$$y_1 = y_2 = y \quad (\text{i})$$

Now as for a spring $F = ky$ and as k s are not equal so $F_1 \neq F_2$ but for equilibrium

$$F = F_1 + F_2, \text{ i.e., } ky = k_1 y_1 + k_2 y_2 \quad [\text{as } F = ky]$$

which in the light of equation (i) reduces to

$$k = k_1 + k_2 + \dots$$

This is like capacitors in parallel or resistance in series.

Spring in Series

This situation is shown in Fig. 7.148(b), as springs are massless, so force in these must be the same, i.e.,

$$F = F_1 = F_2 \quad (\text{i})$$

Now as $F = ky$ and k s are not equal so stretches will not be equal, i.e., $y \neq y_2$

$$\text{But, } y = y_1 + y_2 \quad \text{or, } \frac{F}{k} = \frac{F_1}{k_1} + \frac{F_2}{k_2}$$

[as for $F = ky$, $y = (F/k)$]

which in the light of equation (i) reduces to

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$$

Spring in Series with a Mass between them

As shown in Fig. 7.148 (c), if force F stretches a spring by y , the other will be compressed by the same amount, so

$$y_1 = y_2 = y \quad (\text{i})$$

Now as $F = ky$ and k s are not equal so, $F_1 \neq F_2$

$$\text{But, } F = F_1 + F_2, \text{ i.e., } ky = k_1 y_1 + k_2 y_2 \quad [\text{as } F = ky]$$

which in the light of equation (i) reduces to

$$k = k_1 = k_2$$

Illustration 7.27 Two blocks of masses m_1 and m_2 are in equilibrium. The block m_2 hangs from a fixed smooth pulley by an inextensible string that is fitted with a light spring of stiffness k as shown in the Fig. 7.149 neglecting the friction and mass of the string. Find the acceleration of the bodies just after the string S is cut.

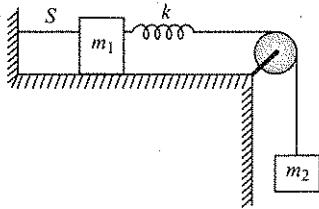


Fig. 7.149

Sol. FBD: Let the spring forces be $F = kx$ just after cutting the spring. Hence, at that instant the forces acting on m_1 are $T = kx \rightarrow m_1 g \downarrow$ and $N \uparrow$; on m_2 the forces are $m_2 g \downarrow$ and $T \uparrow$.

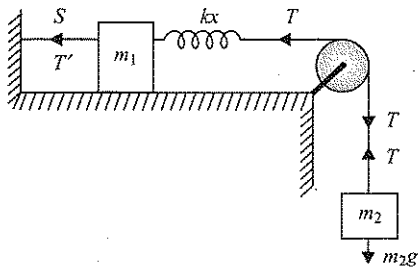


Fig. 7.150

Force equation : Initially, all the particles are stationary; hence $a_1 = a_2 = 0$. Applying Newton's second law

$$\text{For } m_1: \sum F = T' - kx = 0 \quad (i)$$

$$\text{For } m_2: \sum F = T - m_2 g = 0 \quad (ii)$$

$$\text{For spring } \sum F = T - kx = 0 \quad (iii)$$

Solving equations (i), (ii), and (iii), we have

$T' = T = kx = m_2 g$ as shown in the Fig. 7.150 just after the string S is cut, tension T' vanishes immediately, but the spring cannot regain its shape and size instantaneously. Therefore, the spring force remains as it is then, just after cutting the string, the net force acting on m_1 is equal to kx whereas the net force acting on m_2 is equal to $T - m_2 g$. Hence, the acceleration of m_1 and m_2 just after cutting the string is given by

$$\sum F = m_1 a_1 = kx = m_2 g, \quad \sum F = m_2 a_2 = T - m_2 g = 0$$

$$\text{This gives, } a_1 = \left(\frac{m_2}{m_1} \right) g \text{ and } a_2 = 0.$$

ANALYSIS OF FRICTION FORCE

In the previous section, we had introduced Newton's laws of motion and applied them to situations in which we ignored friction. In this section, we shall expand our investigation to objects moving in the presence of friction, which will allow us to model situations more realistically.

Forces of Friction When an object moves either on a surface or through a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance as force of friction.

Imagine you are trying to slide a heavy box on horizontal concrete surface by applying a force $\vec{F}$. You try to drag the box across the surface of your concrete floor. The concrete surface is real, not an idealized, frictionless surface in a simplification model. If we apply an external horizontal force $\vec{F}$ to the box acting to the right, the box remains stationary if $\vec{F}$ is small. The force that counteracts $\vec{F}$ and keeps the box from moving acts to the left and is called the force of static friction $\vec{f}_s$. As long as the box is not moving, it is modeled as a particle in equilibrium and $f_s = F$. Therefore, if $\vec{F}$ decrease, $\vec{f}_s$ also decreases. Experiments show that the friction force arises from the nature of the two surfaces; because of their roughness, contact is made only at a few points, as shown in the magnified surface view in Fig. 7.151(a).

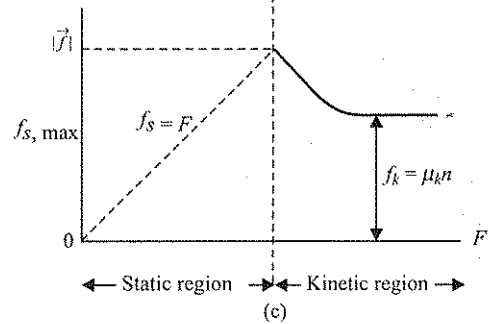
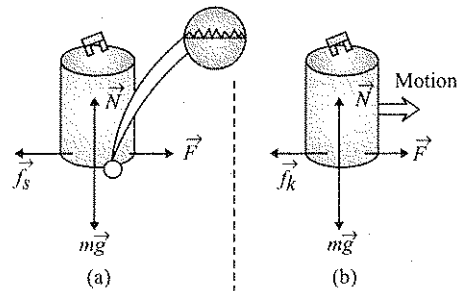


Fig. 7.151

If we increase the magnitude of $\vec{F}$, as in Fig. 7.151(b) the box eventually slips. When the box is on the verge of slipping, f_s is a maximum as shown in fig. If F excess $f_{s, \max}$ the box moves and accelerates to the right. While the box is in motion, the friction force is less than $f_{s, \max}$. The friction force for an object in motion is called the force of kinetic friction. The net force $F - f_k$ in the x direction produces an acceleration to the right, according to Newton's second law. If we reduce the magnitude of $\vec{F}$ so that $F = f_k$, the acceleration is zero and the box moves to the right with a constant speed. If the applied force is removed, the friction force acting to the left provides an acceleration of the box in the $-x$ direction and eventually brings it to rest.

Experimentally, one finds that, to a good approximation, both $f_{s, \max}$ and f_k for an object on a surface are proportional to the normal force exerted by the surface on the object; thus, we adopt a simplification model in which this approximation is assumed

to the exact. The assumption in this simplification model can be summarized as follows:

- The magnitude of the force of static friction between any two surfaces in contact can have the values.

$$f_s \leq \mu_s N \quad (i)$$

where the dimensionless constant μ_s is called the coefficient of static friction and N is the magnitude of the normal force. The equality in equation (i) holds when the surfaces are on the verge of slipping, that is, when $f = f_{s, \max} \equiv \mu_s N$. This situation is called impending motion. The inequality holds when the component of the applied force parallel to the surfaces is less than this value.

- The magnitude of the force of kinetic friction acting between two surfaces is

$$f_k = \mu_k n$$

where μ_k is the coefficient of kinetic friction. In our simplification model, this coefficient is independent of the relative speed of the surfaces.

- The values of μ_k and μ_s depend on the nature of the surfaces, but μ_k is generally less than μ_s .
- The direction of the friction force on an object is opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface with which it is in contact.

Laws of Limiting Friction

The force of friction always acts along a direction so as to oppose the motion of the body relative to the other.

The force of friction depends upon the nature and the roughness of the two surfaces in contact. The rougher the surface the more will be the frictional forces.

Frictional forces are independent of the area of contact between the two bodies, as long as the normal force remains the same.

Illustration 7.28 The following illustration shows a simple method of measuring coefficient of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Fig. 7.152. The incline angle θ is increased until the block starts to move.

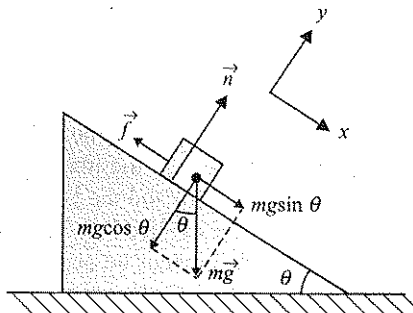


Fig. 7.152

- How is the coefficient of static friction related to the critical angle θ_c at which the block begins to move?

2. How could we find the coefficient of kinetic friction? Sol.

- The forces on the block, as shown in Fig. 7.152, are the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of static friction $\vec{f}_s$. As long as the block is not moving, these forces are balanced and the block is in equilibrium. We choose a coordinate system with the positive x -axis parallel to the incline and downhill and the positive y -axis upward perpendicular to the incline. Applying Newton's second law in component form to the block gives

$$a. \sum F_x = mg \sin \theta - f_s = 0 \quad (1)$$

$$b. \sum F_y = n - mg \cos \theta = 0 \quad (2)$$

These equations are valid for any angle of inclination θ .

At the critical angle θ_c at which the block is on the verge of slipping, the friction force has its maximum magnitude $\mu_s n$, so we rewrite (a) and (b) for this condition as

$$c. mg \sin \theta_c = \mu_s n \quad (3)$$

$$d. mg \cos \theta_c = n \quad (4)$$

Dividing equations (c) and (d), we have $\tan \theta_c = \mu_s$

Therefore, the coefficient of static friction is equal to the tangent of the angle of the incline at which the block begins to slide.

- Once the block begins to move, the magnitude of the friction force is the kinetic value $\mu_k n$, which is smaller than that of the force of static friction. As a result, if the angle is maintained at the critical angle, the block accelerates down the incline. To restore the equilibrium situation in equation (1), with f_s replaced by f_k , the angle must be reduced to a value θ'_c such that the block slides down the incline at constant speed. In this situation, equations (3) and (4), with θ_c replaced by θ'_c and μ_s by μ_k , give us $\tan \theta'_c = \mu_k$.

Angle of Friction

Normal reaction N and friction force f are two components of the total reaction force of the surface on the block. Angle between total reaction R and normal reaction N is called angle of friction.

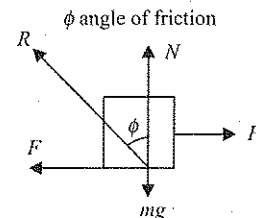


Fig. 7.153

When block is in impending state, $\tan \phi_s = \frac{\mu_s N}{N} = \mu_s$ (ϕ_s is maximum angle of friction).

When block is static, $\phi \leq \phi_s$.

When block is sliding, $\tan \phi_k = \frac{\mu_k N}{N} = \mu_k$. Since $\mu_s > \mu_k$, it follows that $\phi_s > \phi_k$.

Figure 7.153 shows a block kept on an incline whose angle of inclination can be varied. At a certain value of θ sliding motion of the block is impending. The equilibrium equations are

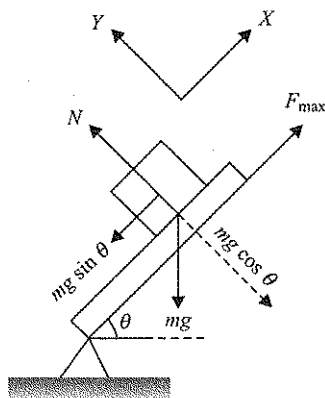


Fig. 7.154

$$\sum F_x = -mg \sin \theta + \mu_s N = 0$$

$$\sum F_y = -mg \cos \theta + N = 0$$

On eliminating N between the two equations, we get,

$$\tan \theta_{\max} = \mu; \quad \theta_{\max} = \tan^{-1} \mu$$

Friction force opposes relative motion between two surfaces. In order to decide the direction of static friction, try to imagine the likely direction in which the body will tend to move; friction force is opposite to it. In Fig. 7.154, force P pulls block B towards left and A is pulled towards right. Friction force on B is towards right and A is towards left. Important point to notice is that for two contact surfaces friction force is in opposite direction. It is internal force for two contact surfaces, so it must be an action-reaction pair.

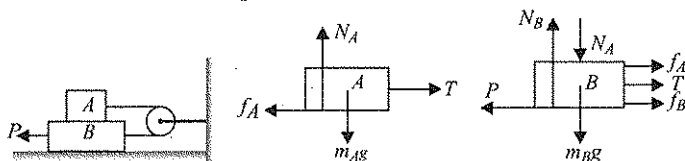


Fig. 7.155

Points to Remember

1. μ has no unit.
2. If frictional force be limiting frictional force then $\mu = \mu_s$ (static frictional coefficient).
3. If frictional force be kinetic then $\mu = \mu_k$ (kinetic frictional coefficient).
4. Static friction $\leq$ limiting friction, i.e., the frictional force acting on a body at rest can be anything from zero to limiting friction.
5. It is incorrect to write $\vec{f} = \mu \vec{N}$. Since $\vec{f}$ is not parallel to $\vec{N}$.
6. If the body is at rest with respect to the surface then $f < \mu_s N$.
7. If the body is just in motion with respect to the surface $f = \mu_s N$.
8. If the body is in motion with respect to the surface $f = \mu_k N$.

Illustration 7.29 A block of weight 100 N lying on a horizontal surface just begins to move when a horizontal force of 25 N acts on it. Determine the coefficient of static friction.

Sol. As the 25 N force brings the block to the point of sliding, the frictional force $= \mu_s N$ from the force diagram (see Fig. 7.156).

$$N = 100 \text{ N}$$

$$\mu_s N = 25 \quad \mu_s = 0.25$$

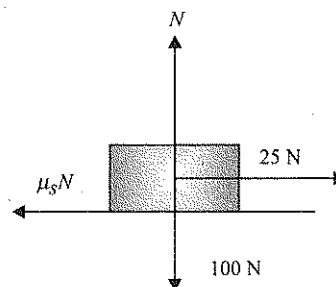


Fig. 7.156

Illustration 7.30 A block lying on an inclined plane has a weight of 50 N. It just begins to slide down when the inclination of plane with the horizontal is 30° . Find μ_s .

Sol. The block reaches the point of sliding when the plane makes an angle of 30° with the horizontal (see Fig. 7.157).

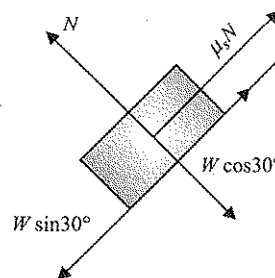


Fig. 7.157

Hence in this situation, frictional force $= \mu_s N$

Balancing the forces:

$$N = W \cos 30^\circ$$

$$\mu_s N = W \sin 30^\circ$$

$$\mu_s W \cos 30^\circ = W \sin 30^\circ$$

$$\mu_s = 1/\sqrt{3}$$

Note: Minimum angle for which a block starts sliding down an inclined plane is known as angle of repose.

Illustration 7.31 A block of weight 100 N lying on a horizontal surface is pushed by a force F acting at an angle 30° with horizontal. For what value of F will the block begin to move if $\mu_s = 0.25$?

Sol. Consider the force diagram (Fig. 7.158) of the block at the moment when it has just started to move.

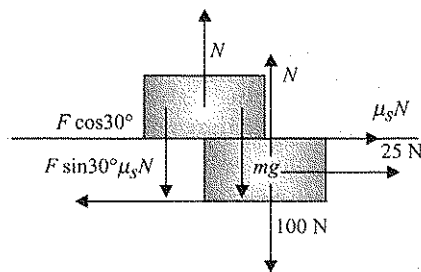


Fig. 7.158

Balancing the forces:

$$N = mg F \sin 30^\circ$$

$$F \cos 30^\circ = \mu_s N$$

$$F \cos 30^\circ = \mu_s (mg + F \sin 30^\circ)$$

$$F = \frac{\mu_s mg}{\cos 30^\circ - \mu_s \sin 30^\circ} = \frac{0.25(100)}{\sqrt{3} - 0.25}$$

$$F = 33.74 \text{ N}$$

Illustration 7.32 A 5 kg block slides down a plane inclined at 30° to the horizontal. Find

1. the acceleration of the block if the plane is frictionless.
2. the acceleration if the coefficient of kinetic friction is $\frac{1}{2\sqrt{3}}$.

Sol.

$$1. N = mg \cos 30^\circ$$

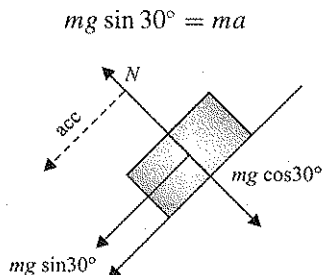


Fig. 7.159

$a = g \sin 30^\circ$, down the plane if plane is smooth.

$$a = g/2 = 5 \text{ m/s}^2$$

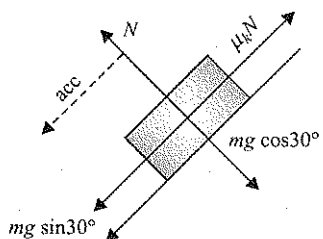


Fig. 7.160

$$1. N = mg \cos 30^\circ$$

$$mg \sin 30^\circ - \mu_k N = ma$$

$$a = g \sin 30^\circ - \mu_k g \cos 30^\circ$$

$$a = 5/2 \text{ m/s}^2$$

Illustration 7.33 A 5 kg block is projected upwards with an initial speed of 10 m/s from the bottom of a plane inclined at 30° to the horizontal. The coefficient of kinetic friction between the block and the plane is 0.2.

1. How far does the block move up the plane?
2. How long does it move up the plane?
3. After what time from its projection does the block again come back to the bottom? With what speed does it arrive?

Sol.

- While the block is moving up, the frictional force acts down the incline.
- As the block is slowing down, the velocity and the acceleration must be in opposite directions.
- Velocity in this case is upwards, so the acceleration is in downward direction and hence negative. The magnitude of acceleration = $\frac{mg \sin 30^\circ + \mu mg \cos 30^\circ}{m}$
 $= g(\sin 30^\circ + \mu \cos 30^\circ) = a = -g(\sin 30^\circ + \mu \cos 30^\circ) = -6.6 \text{ m/s}^2$

For the motion of block from the bottom to up the plane:

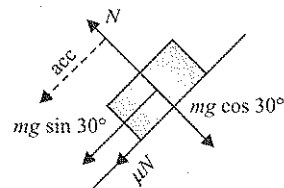


Fig. 7.161

$u = +10 \text{ m/s}$, using $v^2 = u^2 + 2as$, we get

$$0^2 = 10^2 + 2(-6.6)(s)$$

$$\Rightarrow s = 7.58 \text{ m}$$

$$v = u + at$$

$$0 = 10 - 6.6 \times t \Rightarrow t = 1.5 \text{ s}$$

Hence the block moves up the plane for 1.5 s covering 7.58 m.

For the motion of block down the plane:

$$\text{the magnitude of acceleration} = \frac{mg \sin 30^\circ + \mu mg \cos 30^\circ}{m}$$

$$= g(\sin 30^\circ - \mu \cos 30^\circ) \Rightarrow a = 3.2 \text{ m/s}^2$$

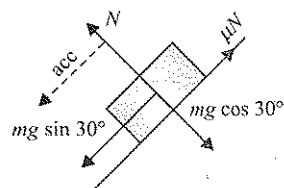


Fig. 7.162

As acceleration is in downward direction, $a = -3.2 \text{ m/s}^2$

$s = -7.58 \text{ m}$ (down the plane) and $u = 0 \text{ m/s}$

$$s = ut + \frac{1}{2}at^2$$

$$-7.58 = (0) + \frac{1}{2}(-3.2)t^2 \Rightarrow t = 2.18 \text{ s}$$

So the total time taken to come back:

$$t_{\text{up}} + t_{\text{down}} = 1.5 + 2.18 = 3.68 \text{ s}$$

$$v = u + at$$

$v = -6.8 \text{ m/s}$. So the block arrives at the bottom with a speed of 6.8 m/s .

Illustration 7.34 Find the least pulling force which is acting at an angle of 45° with the horizontal, will slide a body weighing 5 kg along a rough horizontal surface. The coefficient of friction $\mu_s = \mu_k = 1/3$. If a force of double this is applied along the same direction, find the resulting acceleration of the block.

Sol. When the block is about to start sliding, frictional force is at its limiting value $= \mu_s R$

Balancing forces in vertical direction:

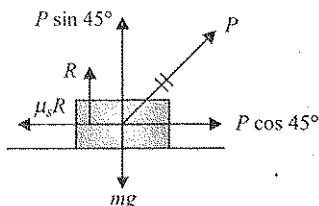


Fig. 7.163

$$R + P \sin 45^\circ = mg$$

$$R = mg - P \sin 45^\circ$$

Balancing forces in horizontal direction:

$$P \cos 45^\circ = \mu_s R$$

$$P \cos 45^\circ = \mu_s (mg - P \sin 45^\circ)$$

$$\Rightarrow P = \frac{\mu_s mg}{\cos 45^\circ + \mu_s \sin 45^\circ}$$

If applied force is $2P$:

then $R = mg - 2P \sin 45^\circ$ and $2P \cos 45^\circ - \mu_k R = ma$

$$2P \cos 45^\circ - \mu_k (mg - 2P \sin 45^\circ) = ma$$

$$\Rightarrow a = \frac{2P}{m\sqrt{2}}(1 + \mu)g = 5.72 \text{ m/s}^2$$

Illustration 7.35 A block of mass m lying on a horizontal surface (coefficient of static friction $= \mu_s$) is to be brought into motion by a pulling force F . At what angle θ with the horizontal should the force F be applied so that its magnitude is minimum? Also find this minimum magnitude.

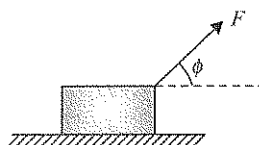


Fig. 7.164

Sol. Let us first calculate the force F required to bring m into motion in terms of angle θ

Equation using laws of motion are as follows:

$$N = mg - F \sin \theta$$

$$F \cos \theta = \mu_s N$$

$$F \cos \theta = \mu (mg - F \sin \theta)$$

$$F = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta}$$

We have to find the angle θ for which this force F is minimum. Substituting $\mu_s = \tan \mu$ (for simplification), we get

$$F = \frac{mg \tan \lambda}{\cos \theta + \tan \lambda \sin \theta} = \frac{mg \sin \lambda}{\cos(\theta - \lambda)}$$

F is minimum if $\cos(\theta - \lambda)$ is maximum. Hence F is minimum for $\theta = \tan^{-1} \mu_s$ and $F_{\min} = mg \sin \lambda$.

To bring m into motion with least effort, force should be applied at an angle $\tan^{-1} \mu_s$ and should have a magnitude equal to:

$$F_{\min} = mg \sin \lambda = \frac{\mu_s mg}{\sqrt{1 + \mu_s^2}}$$

Illustration 7.36 A block of mass $m = 10 \text{ kg}$ is to be pulled on a horizontal rough surface with the minimum force.

1. The block should be pulled at an angle θ which is equal to _____
2. The magnitude of the force F is equal to _____

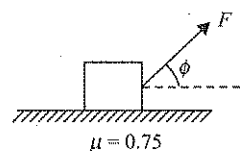


Fig. 7.165

Sol.

1. The block should be pulled at the angle of friction, i.e., $\theta = \tan^{-1}(0.75) = 37^\circ$
2. The magnitude of external force is

$$f_{\min} = mg \sin \phi = (10)(10) \sin 37^\circ = (100) \left(\frac{3}{5} \right) = 60 \text{ N}$$

Illustration 7.37 Determine the magnitude of frictional force and acceleration of the block in each of the following cases:

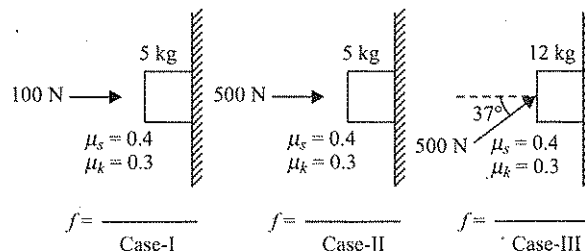


Fig. 7.166

Sol. Case-I: Limiting value of friction force

$$f_{\max} = \mu N = (0.4)(100) = 40 \text{ N}$$

Force responsible for sliding of two blocks (parallel to wall surface) = $mg = 5 \times 10 = 50 \text{ N}$

The force parallel to the surface of the wall will act as driving force. ($F_{\text{driving}} = mg = 50 \text{ N}$). Here the driving force is the weight which is greater than the maximum resisting force for a $f_{\text{max}} = F_{\text{resisting}} = f_{\mu m} = \mu_s N = 0.4 \times 100 = 40 \text{ N}$.

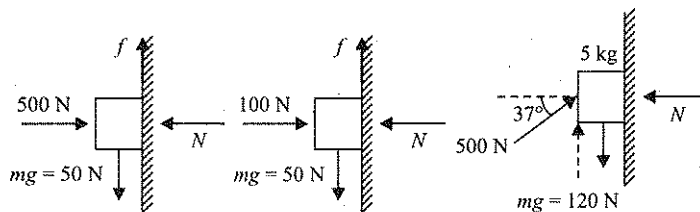


Fig. 7.167

$$F_{\parallel} = 500 \cos 37^\circ = 400 \text{ N} \quad F_{\parallel} = 500 \times \frac{3}{5} = 300 \text{ N}$$

Here $F_{\text{driving}} > F_{\text{resisting}}$

Hence the block will slide and friction will be $f_{\text{kinetic}} = f = \mu_k N = 0.3 \times 100 = 30 \text{ N}$ (upward)

Hence the equation of the motion of the block

$$Mg - f = ma \Rightarrow 50 - 30 = 5 \times a$$

That gives acceleration of the block

$$a = \frac{50 - 30}{5} = 4 \text{ m/s}^2 \text{ (downward)}$$

Case II: In this case the resisting force is

$$F_{\text{resisting}} = f_{\text{max}} = f_{\mu m} = \mu_s N = 0.4 \times 500 = 200 \text{ N}$$

Driving force (force parallel to the wall surface)

$$F_{\text{driving}} = mg = 50 \text{ N}$$

Here, $F_{\text{driving}} < F_{\text{resisting}}$

Therefore, the friction will be of static nature. As static friction force is self adjusting, it takes the value between 0 and $\mu_s N$ ($0 \leq f_s \leq \mu_s N$).

Hence friction force in this case

$$f = mg = 50 \text{ N (upward)}$$

Hence the acceleration of the block will be zero.

Case III: In this case the resisting force is

$$F_{\text{resisting}} = f_{\text{max}} = f_{\mu m} = \mu_s N = 0.4 \times 400 = 160 \text{ N}$$

Driving force parallel to the wall; component of 500 N is $F_{\parallel} = 300 \text{ N}$ (upward) and the weight of the block, $mg = 120 \text{ N}$ (downward)

Net driving force $F_{\text{driving}} = 300 - 120 = 180 \text{ N}$ (upward)

Here $F_{\text{driving}} > F_{\text{resisting}}$

Hence friction is of kinetic nature.

$$F = f_{\text{kin}} = \mu_k N = 0.3 \times 400 = 120 \text{ N (downward)}$$

The friction will act downward as driving force on the block acts upward. Hence the acceleration of the block is

$$a = \frac{180 - 120}{12} = 5 \text{ m/s}^2 \text{ (upward)}$$

Illustration 7.38 The Fig. 7.168 shows two blocks m and M which are pushed together on a smooth horizontal sur-

face. If the coefficient of friction between the blocks is μ , then determine the minimum value of the horizontal force F required to hold the blocks together.

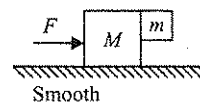


Fig. 7.168

Sol. Acceleration of system $a = \frac{F}{M + m}$

Free body diagrams as shown in Fig. 7.169

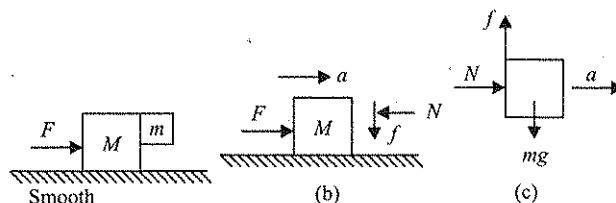


Fig. 7.169

From the free body diagram of m , $N = m \left(\frac{F}{M + m} \right)$ (i)

m is not sliding, therefore, $f = mg$ (ii)

For no sliding case $f \leq \mu N$

$$mg \leq \mu \frac{mF}{(M + m)}$$

$$\Rightarrow F \geq \frac{(M + m)g}{\mu}$$

Using the method of pseudo force: Observing m in frame of M

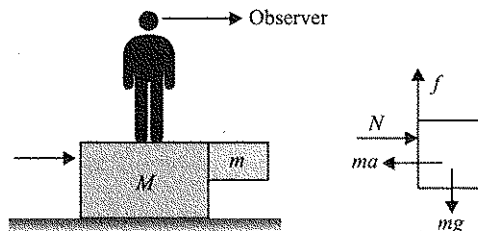


Fig. 7.170

In the frame of M the block m is not sliding.

$$N = ma \quad (i)$$

$$f = mg \quad (ii)$$

If f is of static nature $f \leq \mu N$

$$mg \leq \mu ma$$

Since $f = \mu N = \mu ma$, therefore,

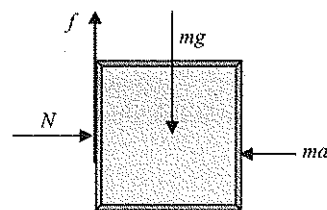


Fig. 7.171

$$\mu ma = mg \quad a = \frac{g}{\mu}$$

For block M : $F - N = Ma$ or $F = (M + m)a$

$$\text{Then } F = (M + m) \frac{g}{\mu}$$

Illustration 7.39 A block of mass m_1 is placed on another block of mass m_2 as shown in Fig. 7.172. The blocks have initial velocities v_0 and v_0 , respectively. If $v_0 > v_0$, find

1. the acceleration of the blocks.

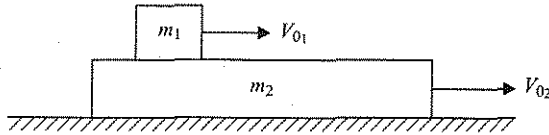


Fig. 7.172

2. the variation of velocities of the blocks versus time.

Sol.

1. Since, the body 1 moves with a velocity v_{021} ($v_{01} - v_{02}$) towards right relative to the body 2, the kinetic friction on the body 1 is directed towards left the same amount of friction acts on the body 2, towards right. Apart from this, we have shown the weights m_1g and m_2g and the normal contact forces N and N' .

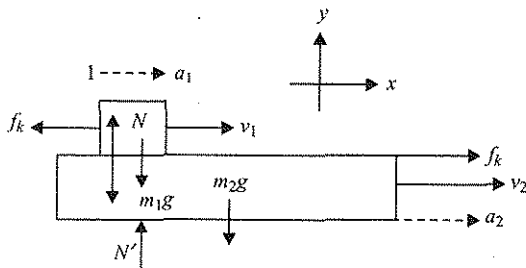


Fig. 7.173

Equation of motion:

$$\text{For } m_1: \sum F_x = -f_k = m_1 a_1 \quad (i)$$

$$\text{For } m_2: \sum F_y = f_k = m_2 a_2 \quad (ii)$$

Law of kinetic friction $f_k = \mu_k N$, where $N = m_1 g$

$$\text{This gives } f_k = \mu_k m_1 g \quad (iii)$$

Substituting f_k from eqs. (iii), (i), and (ii) we have

$$a_1 = -\mu_k g \text{ and } a_2 = \mu_k \frac{m_1}{m_2} g$$

2. **Kinematics:** Applying kinematics equation for m_1 , we have

$$v = u + at$$

Then by substituting, $v = v_1$, $u = v_{01}$ and $a = -\mu_k g$, we have

$$v_1 = v_{01} - \mu_k g t$$

Similarly, applying $v = u + at$ for m_2 by

substituting $v = v_2$, $u = v_{02}$ and $a = \mu_k \frac{m_1}{m_2} g$,

$$\text{we have, } v_2 = v_{02} + \mu_k \frac{m_1}{m_2} g t.$$

Illustration 7.40 Two blocks M and m are arranged shown in Fig. 7.174. If $M = 50$ kg, then determine the min-

imum and maximum values of mass of block m to keep the heavy block M stationary.

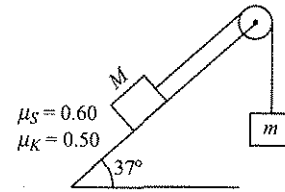


Fig. 7.174

Sol. Since angle of inclination of the plane is more than the angle of repose, i.e., $30^\circ > \tan^{-1} 0.5$ or $\tan 30^\circ > 0.5$, therefore, the block M has a tendency to slide down. In order to keep it stationary the necessary force is applied by the tension in the string.

If the block M also has a tendency to slide down, the friction will act in up the plane.

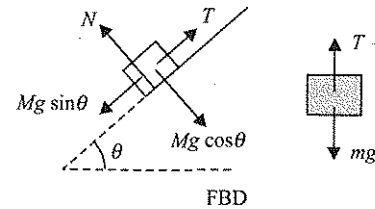


Fig. 7.175

For equilibrium of M : $Mg \sin \theta = T + f$

$$f = Mg \sin \theta - T \quad (i)$$

$$N = Mg \cos \theta \quad (ii)$$

$$\text{For equilibrium of } m: T = mg \quad (iii)$$

$$\text{From equations (i) and (ii), } f = Mg \sin \theta - mg \quad (iv)$$

For friction to be static, $f < f_{\max}$

$$Mg \sin \theta - mg < \mu_s (Mg \cos \theta)$$

$$\Rightarrow M (\sin \theta - \mu_s \cos \theta) < m$$

$$\text{Hence } m > M (\sin \theta - \mu_s \cos \theta) \quad (v)$$

$$\text{Or } m > 50 \left(\frac{3}{5} - \frac{3}{5} \times \frac{4}{5} \right) \Rightarrow m > 6$$

Hence minimum value of mass, $m_{\min} = 6$ kg.

If the block M has a tendency to slide down, the friction will act in downward direction. By this relation, we will get the maximum value of m . This value of m can be obtained by simply substituting $(-\mu_s)$ in place of μ_s in equation (v), finally we will get the relation

$$m < M (\sin \theta - (-\mu_s) \cos \theta)$$

$$\Rightarrow m < M (\sin \theta + \mu_s \cos \theta)$$

$$m < 50 \left(\frac{3}{5} + \frac{3}{5} \times \frac{4}{5} \right) \Rightarrow m < 54 \text{ kg}$$

Hence maximum value of mass, $m_{\max} = 54$ kg.

Illustration 7.41 A bar of mass m is placed on a triangular block of mass M as shown in Fig. 7.176. The friction coefficient between the two surfaces is μ and the ground is

smooth. Find the minimum and maximum horizontal force F required to be applied on block so that the bar will not slip on the inclined surface of block.

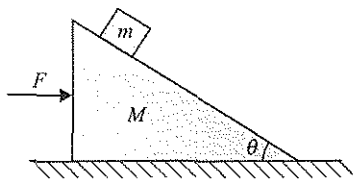


Fig. 7.176

Sol. If both the masses are moving together then the acceleration of the system will be $\frac{F}{M+m}$. If we observe the mass m relative to M , it experiences a pseudo force ma towards left. Along the incline it experiences two forces, $mg \sin \theta$ downward and $ma \cos \theta$ upward. If $mg \sin \theta$ is more than $ma \cos \theta$, it has a tendency of slipping downward so friction on it will act in the upward direction. Here if block m is in equilibrium on the inclined surface, we must have

$$mg \sin \theta - ma \cos \theta \leq \mu (mg \cos \theta + ma \sin \theta)$$

$$\text{or } a \geq \frac{\sin \theta - \mu \cos \theta}{\cos \theta + \mu \sin \theta} g$$

$$\text{or } F \geq \frac{\sin \theta - \mu \cos \theta}{\cos \theta + \mu \sin \theta} (M+m) g \quad (i)$$

If force is more than the value obtained in equation (i), $ma \cos \theta$ will increase on m and the static friction on it will decrease. At $a = g \tan \theta$ [when $F = (M+m)g \tan \theta$], we know that the force $mg \sin \theta$ will be balanced by $ma \cos \theta$ at this acceleration no friction will act on it. If applied force increases beyond this value, $ma \cos \theta$ will exceed $mg \sin \theta$ and friction starts acting in upward direction.

Illustration 7.42 An object of mass 10 kg is kept at rest on an inclined plane making an angle of 37° with the horizontal by applying a force F along the plane upwards as shown in Fig. 7.177. The coefficient of static friction between the object and the plane is 0.2. Find the magnitude of force F . [Take $g = 10 \text{ m/s}^2$.]

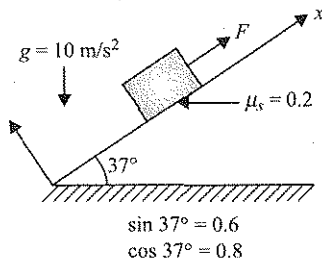


Fig. 7.177

Sol. For the object to remain at rest on the incline the resultant force on the object must be equal to zero $\sum \vec{F} = 0$ is, and equivalent to $\sum F_x = 0$, and $\sum F_y = 0$. Choose the coordinate system with x -axis along the plane upwards and y -axis perpendicular to it, as shown in the Fig. 7.178.

The forces acting on the object are:

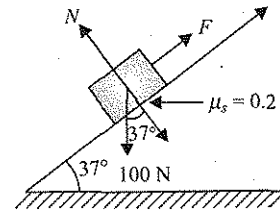


Fig. 7.178

1. the weight, 100 N, acting vertically downwards,
2. the normal force, acting perpendicular to the surface as shown,
3. the applied force F , and
4. a frictional force, static in nature because the object remains at rest.

The law for the static friction is $f_s < \mu_s N$. The static friction can take all the values in the range $0 - \mu_s N$, either along the plane upwards or along the plane downwards, depending upon which way the object tends to move under the combined action of the applied force F and the component of weight down the incline $mg \sin \theta$, ($mg \sin \theta = 10 \times 10 \times 0.6 = 60 \text{ N}$). If $mg \sin \theta > F$, the object tends to move downwards, the frictional force on it must be upwards along the incline; and when $mg \sin \theta < F$, the object will tend to move upwards, the frictional force on it will be downwards along the incline; and $f_s < \mu_s N$. You can conclude from this that many distinct possibilities develop.

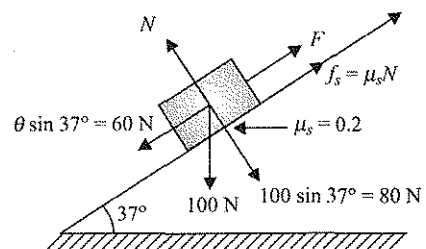


Fig. 7.179

When $mg \sin \theta > F$ and the object is about to move down the incline, the static friction on it will be at its peak value (limiting friction). Under this condition for the equilibrium of the object,

$$N - 80 = 0, \text{ and} \quad (1)$$

$$F - 60 + 16 = 0 \quad (2)$$

Solving for F ,

$$F = 44 \text{ N}$$

When $mg \sin \theta > F$ and $(mg \sin \theta - F) < \mu_s N$ (Fig. 7.180), the frictional force will be equal to $(mg \sin \theta - F)$, and $0 < f_s < 16 \text{ N}$.

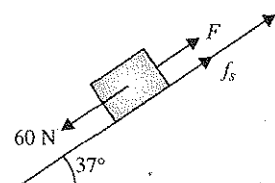


Fig. 7.180

or $F = mg \sin \theta - f_s (0 < f_s < 16 \text{ N})$
 $F = 60 - f_s, 44 \text{ N} < F < 60 \text{ N} (0 < f_s < 16 \text{ N})$

When $mg \sin \theta = F, f_s = 0, F = 60 \text{ N}$.

When $mg \sin \theta < F$ and $(F - mg \sin \theta) < \mu_s N$, the frictional force will be $f_s = F - mg \sin \theta$ (Fig. 7.181).

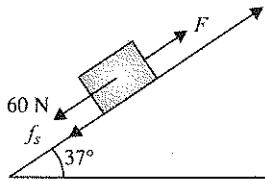


Fig. 7.181

or $F = f_s + mg \sin \theta$

$F = f_s + 60 (0 < f_s < 60 \text{ N})$

$60 \text{ N} < F < 76 \text{ N} (0 < f_s < 16 \text{ N})$

and when $mg \sin \theta < F$, and the object is about to move up the incline, the frictional force on it will be at its peak value ($= \mu_s N$) along the plane downwards, and for the equilibrium,

$N - 80 = 0$, and (3)

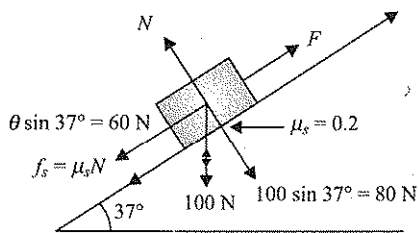


Fig. 7.182

$F - 60 - 16 = 0$ (4)

$F = 76 \text{ N}$

Therefore, $44 \text{ N} < F < 76 \text{ N}$

Now see how the frictional force on the object varies between 0 and 16 N along the plane upwards or downwards depending on the requirement, $(-16 \text{ N} \leq f \leq +16 \text{ N})$. And that is why the popular phrase "static friction is self adjusting in nature" has been coined.

Illustration 7.43 The masses of the blocks A, B, and C shown in the Fig. 7.183 weigh 4 kg, 1.5 kg, and 1.5 kg [Fig. 7.183(a)] respectively. Block A moves with an acceleration of 2.5 m/s^2 .

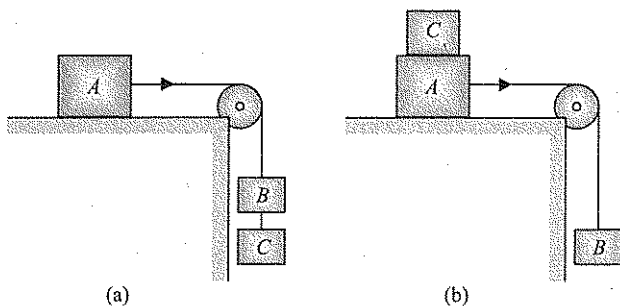


Fig. 7.183

1. Block C is removed from its position and placed on block A, shown in Fig. 7.183(b). What is now the acceleration of block C?
2. The positions of the blocks A and B is subsequently interchanged. Find the new acceleration of C. The coefficient of friction is the same for all the contact surfaces (Fig. 7.184).

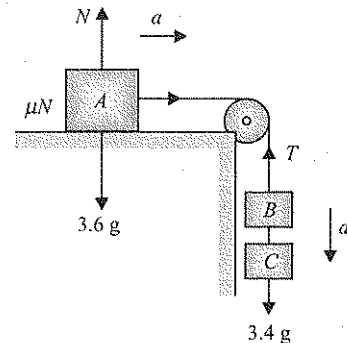


Fig. 7.184

Sol. It is given that A moves with an acceleration of 2.45 m/s^2 . Acceleration of B and C each will be 2.45 m/s^2 downwards. The force equation for A, B, and C put together are

$N - 4 \times 10 = 0$ (1)

$T - \mu N = 4 \times 2.5$ (2)

$5 \times 10 - T = 3 \times 2.5$ (3)

Solving these equations for $\mu = 0.25$

1. Fig. 7.185 shows the system after block c is removed from its position and placed on A. Assume the accelerations of A, B, and C and tension in the string. Calculate the numerical values of the accelerations and the tension. A close scrutiny will show that the calculated values contradict the very basics on which the accelerations were assumed.

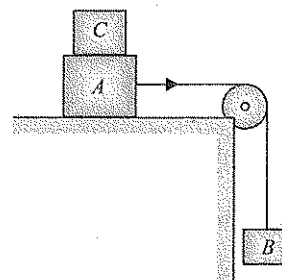


Fig. 7.185

The value of T which can overcome the frictional force on A and make it move is

$T > 0.25 \times (3.6 + 1.2) \times 9.8 \text{ N}$

and, as B will never move upward in the given condition.

$T < 1.2 \times 9.8 \text{ N}$

Therefore, A and B will not move, so will C, $a_C = 0$

2. For the motion of blocks in this arrangement, there are two distinct possibilities.
 - a. C slips on B.

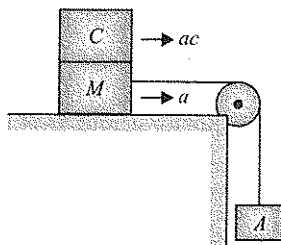


Fig. 7.186

b. C does not slip on B .

Let C slip on B $3.6 \times 9.8 - T = 3.6 a$

$$T - \mu \times 1.2 \times 9.8 - \mu \times 2.4 \times 9.8 = 1.2 a \quad (4)$$

$$\text{and } 0.25 \times 1.2 \times 9.8 = 1.2 a_c \quad (5)$$

From equations (4) and (5)

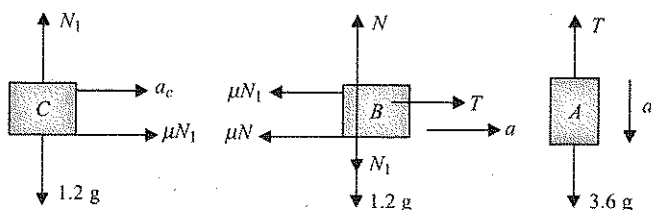


Fig. 7.187

$$a = \times 9.8 = 5.5125 \text{ m/s}^2$$

and from equation (6)

$$a_c = \frac{0.25 \times 1.2 \times 9.8}{1.2} = 2.45 \text{ m/s}^2$$

Now $5.5125 \text{ m/s}^2 > 2.45 \text{ m/s}^2$

$\Rightarrow a > a_c$ will slip on A

Therefore, $a_c = 2.45 \text{ m/s}^2$.

Illustration 7.44 Two blocks of mass M and m are arranged as shown in Fig. 7.188. There is no friction between ground and M .

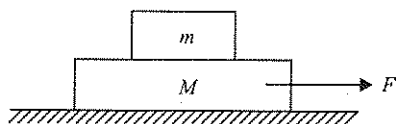


Fig. 7.188

Coefficient of friction between M and m is μ_s and μ_k .

1. Calculate the maximum possible value of F so that both the bodies move together.

2. Maximum possible friction force between the surfaces

Sol. Free body diagrams of m and M (Fig. 7.189)

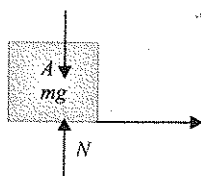


Fig. 7.189

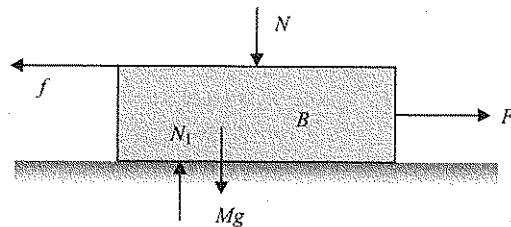


Fig. 7.190

- Static friction force is a self adjusting force $0 \leq f \leq \mu_s N$, $f_{\max} = \mu_s N$ (Fig. 7.190).

After friction force reaches limiting value, the motion starts and the friction force becomes $f_k = \mu_k N$

- Assume system moves together, i.e., no sliding between M and m (Fig. 7.191).

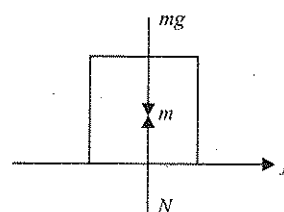
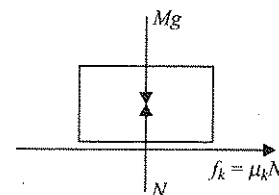


Fig. 7.191

$$\text{Acceleration of system in this case } a = \frac{F}{(M + m)} \quad (i)$$

Now FBD of m (see Fig. 7.192).

Fig. 7.192 FBD of m

Equation of motion of m .

$$f = ma = m \frac{F}{(M + m)} \quad (ii)$$

If there is no sliding between M and m $f \leq f_s$

$$\frac{mF}{(M + m)} \leq \mu_s (mg) \Rightarrow F \leq \mu_s (M + m) g$$

If $F > \mu_s (M + m) g$

- there will be relative sliding between M and m .

Calculate the acceleration of M and m in this case.

When relative sliding between M and m starts

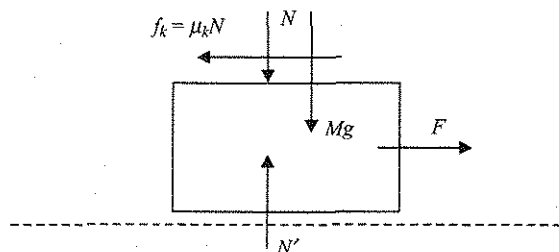
Free body diagram of M (Fig. 7.193).

Equation of motion of m .

$$\mu_k (mg) = ma_1 \Rightarrow a_1 = \mu_k g$$

Equation of motion of M .

$$F - \mu_k mg = Ma_2 \Rightarrow a_2 = \frac{F - \mu_k mg}{M}$$


 Fig. 7.193 FBD of M

Case II

There is no friction between the ground and M . The coefficient of static and kinetic friction are μ_s and μ_k , respectively.

- What is the maximum possible value of F so that system moves together?
- If there is relative sliding between M and m then calculate acceleration of M and m ?

Let the systems moves together

$$a = \frac{F}{(M + m)}$$

FBD of m and M

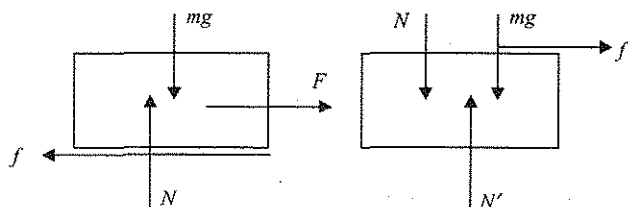


Fig. 7.194

From FBD of M , $f = ma$

$$f = M \left(\frac{F}{M + m} \right) \quad (i)$$

As there is no sliding between M and m , $\therefore f \leq f_s$

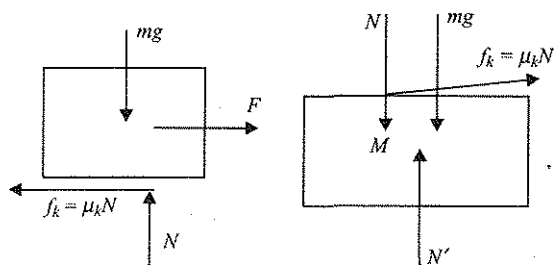


Fig. 7.195

$$M \left(\frac{F}{M + m} \right) \leq \mu_s mg \Rightarrow F \leq \mu_s \frac{m}{M} (M + m) g$$

If the relative sliding between the block starts.

$$\Rightarrow a_1 = \frac{F - \mu_k mg}{m}$$

$$\Rightarrow a_2 = \frac{\mu_k mg}{M}$$

Concept Application Exercise 7.4

- A block weighing 20 N rests on a horizontal surface. The coefficient of static friction between block and surface is 0.4 and the coefficient of kinetic friction is 0.20.
 - How much is the friction force exerted on the block?
 - How much will the friction force be if a horizontal force of 5 N is exerted on the block?
 - What is the minimum force that will start the block in motion?
 - What is the minimum force that will keep block in motion once it has been started?
 - If the horizontal force is 10 N, what is the friction force?
- A block of mass m rests on a rough floor. The coefficient of friction between the block and the floor is μ .
 - Two boys apply force P at an angle θ to the horizontal. One of them pushes the block; the other one pulls. Which one would require less efforts to cause impending motion of the block?
 - What is the minimum force required to move the block by pulling it?
 - Show that if the block is pushed at a certain angle θ_0 it cannot be moved whatever the value of P be.
- What is the value of friction f for the following value of applied force F .



Fig. 7.196

- a. 1 N b. 2 N c. 3 N d. 4 N e. 20 N

Assume the coefficient of friction to be $\mu_s = 0.3$; $\mu_k = 0.25$. Mass of the body is $m = 1$ kg. (Assume $g = 10$ m/s²)

- A block of mass 5 kg rests on a rough horizontal surface. It is found that a force of 10 N is required to make the block just move. However, once the motion begins, a force of only 8 N is enough to maintain the motion. Find the coefficients of kinetic and static friction between the block and the horizontal surface.



Fig. 7.197

- A body of mass m is kept on a rough horizontal surface of friction coefficient μ . A force is applied horizontally, but the body is not moving. Find the net force F exerted by the surface on the body.
- Determine the magnitude of frictional force in each of the following cases:

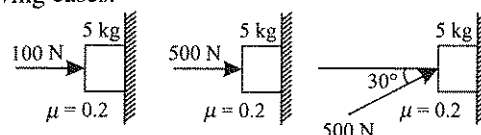


Fig. 7.198

$$f = \quad f = \quad f =$$

4. When a real force pushes a particle radially outward, we cannot call it centrifugal force of the first kind.
5. The centrifugal force is directed “radially” outward from the axis of rotation of the reference frame (or observer) along the line drawn from the particle perpendicular to the axis of rotation.
6. The centrifugal force is a pseudo-force which is equal to $-m \vec{a}_{CP}$ and centripetal force is a real force. The centrifugal force is adopted to solve the problems in the rotating frame.

Illustration 7.45 A small ball of mass m is suspended from a string of length L . The ball revolves with a constant speed v in a horizontal circle of radius r as shown in Fig. 7.211. (Because the string sweeps out the surface of a cone, the system is known as a conical pendulum.) Find an expression for v .

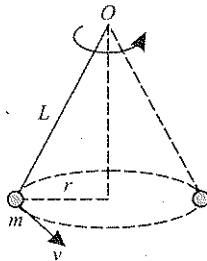


Fig. 7.211

Sol. Imagine the motion of the ball in Fig. 7.212 and convince yourself that the string sweeps out a cone and that the ball moves in a circle.

The ball in Fig. 7.212 does not accelerate vertically. Therefore, we model it as a particle in equilibrium in the vertical direction. It experiences a centripetal acceleration in the horizontal direction, so it is modeled as a particle in uniform circular motion in this direction.

Let θ represents the angle between the string and the vertical. In the FBD shown in Fig. 7.212, the force $\vec{T}$ exerted by the string is resolved into a vertical component $T \cos \theta$ and a horizontal component $T \sin \theta$ acting towards the center of the circular path.

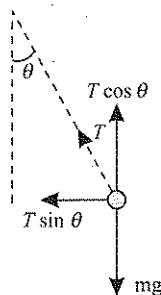


Fig. 7.212

Apply the particle in equilibrium model in the vertical direction:

$$\sum F_y = T \cos \theta - mg = 0$$

$$T \cos \theta = mg \quad (i)$$

$$\sum F_x = T \sin \theta = ma_c = \frac{mv^2}{r} \quad (ii)$$

Divide equation (ii) by equation (i) and use

$$\sin \theta / \cos \theta = \tan \theta, \tan \theta = \frac{v^2}{rg}$$

$$\text{Solve for } v: v = \sqrt{rg \tan \theta}$$

Incorporate, $r = L \sin \theta$, from the geometry in the Fig. 7.213.

$$v = \sqrt{Lg \sin \theta \tan \theta}$$

Illustration 7.46 A particle of mass m is moving with a constant speed v in a circular path in a smooth horizontal plane (plane of the paper) by a spring force as shown in Fig. 7.213. If the natural length of the spring is l_0 and stiffness of the spring is k , find the elongation of the spring.

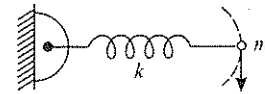


Fig. 7.213

Sol. If the particle executes uniform circular motion then its speed must be uniform because there is no tangential force to speed it up. Here, the spring force kx acting on the particle is the centripetal force caused by the elongation x of the spring.

$$\text{Equation of motion: } \sum F_r = F_{SP} = ma_r \quad (i)$$

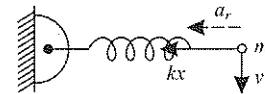


Fig. 7.214

$$\text{But centripetal acceleration } a_r = \frac{v^2}{R} \quad (ii)$$

(where R = radius of circular path)

$$\text{Radius of rotation: } R = l_0 + x \quad (iii)$$

$$|F_{SP}| = kx$$

Using the above equations, we have the quadratic equations,

$$kx^2 + kl_0x - mv^2 = 0$$

$$\text{This gives } x = \frac{\sqrt{x^2 l_0^2 + 4mv^2k} - kl_0}{2k}$$

Illustration 7.47 A coin is pushed down tangentially from an angular position θ on a cylindrical surface, with a velocity v as shown in the Fig. 7.215. If the coefficient of friction between the coin and the surface is μ , find the tangential acceleration of the coin.

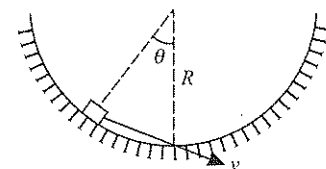


Fig. 7.215

Sol. As the coin slides down, the friction is kinetic and acts up along the plane.

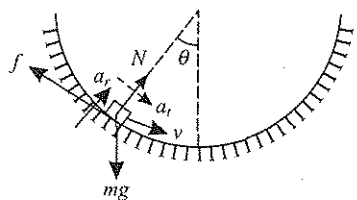


Fig. 7.216

Equation of motion:

$$\sum F_r = N - mg \cos \theta = ma_r \quad (i)$$

$$\sum F_t = mg \sin \theta - f_k = ma_t \quad (ii)$$

$$\text{But } f_k = \mu N \quad (iii)$$

$$\text{and centripetal acceleration is } a_r = \frac{mv^2}{R} \quad (iv)$$

Substituting f_k from equation (iii), a_r from equation (iv), in equation (ii),

$$\text{We get } a_r = g \sin \theta - \frac{\mu N}{m} \quad (v)$$

Now, substituting N from equation (i) in equation (v),

$$\text{We get } a_t = g (\sin \theta - \mu \cos \theta) - \frac{\mu mv^2}{R}$$

Illustration 7.48 A small block is connected to one end of two identical massless strings of length $16\frac{2}{3}$ cm each with their other ends fixed to a vertical rod. If the ratio of tensions T_1/T_2 be 4 : 1, then what will be the angular velocity ω of the block.

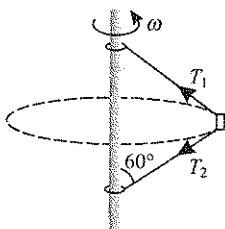


Fig. 7.217

Sol. For horizontal equilibrium of the block,

$$T_1 \sin 60^\circ + T_2 \sin 60^\circ = m\omega^2 r = m\omega^2$$

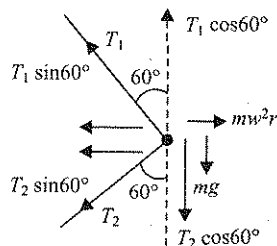


Fig. 7.218

$$T_1 + T_2 = m\omega^2 l \quad (i)$$

For vertical equilibrium of the block,

$$T_1 \cos 60^\circ = T_2 \cos 60^\circ + mg$$

$$T_1 - T_2 = \frac{mg}{\cos 60^\circ} = 2mg \quad (ii)$$

Dividing equation (i) by (ii)

$$\frac{T_1 + T_2}{T_1 - T_2} = \frac{\omega^2 l}{2g} \Rightarrow \frac{\frac{T_A}{T_2} + 1}{\frac{T_1}{T_2} - 1} = \frac{\omega^2 l}{2g} \Rightarrow \frac{4 + 1}{4 - 1} = \frac{\omega^2 l}{2g}$$

$$\omega^2 = \frac{10g}{3l} = \frac{10 \times 9.8 \times 3}{3 \times 50 \times 10^{-2}} = 196 \Rightarrow \omega = 14 \text{ rads}^{-1}$$

Illustration 7.49 Two wires AC and BC are tied at C of a small sphere of mass 5 kg, which revolves at a constant speed v in the horizontal speed v in the horizontal circle of radius 1.6 m. Find the minimum value of v .

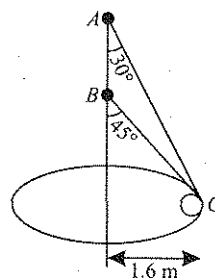


Fig. 7.219

Sol. From force diagram shown in Fig. 7.220,

$$T_1 \cos 30^\circ + T_2 \cos 45^\circ = mg \quad (i)$$

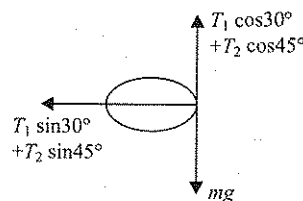


Fig. 7.220

$$T_1 \sin 30^\circ + T_2 \sin 45^\circ = \frac{mv^2}{r} \quad (ii)$$

$$\text{After solving equations (i) and (ii), } T_1 = \frac{mg - \frac{mv^2}{r}}{(\sqrt{3} - 1)}$$

$$\text{But } T_1 > 0, \frac{mg - \frac{mv^2}{r}}{\sqrt{3} - 1} > 0 \Rightarrow mg > \frac{mv^2}{r}, v < \sqrt{rg}$$

$$v_{\max} = \sqrt{rg} = \sqrt{1.6 \times 9.8} = 3.96 \text{ m/s}$$

Bending of a Cyclist

When a cyclist takes a turn, he also requires some centripetal force. If he keeps himself vertical while turning, his weight is balanced by the normal reaction of the ground. In that event, he has to depend on the force of friction between the tyres and the

road for obtaining the necessary centripetal force. As the force of friction is small and uncertain, dependence on it is not safe.

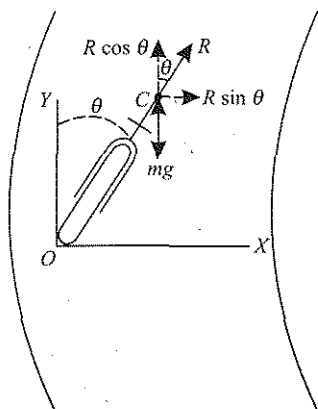


Fig. 7.221

To avoid dependence on the force of friction for obtaining centripetal force, the cyclist has to bend a little inward from his vertical position, while turning. By doing so, a component of normal reaction in the horizontal direction provides the necessary centripetal force. To calculate the angle of bending with vertical, suppose,

m = mass of the cyclist, v = velocity of the cyclist while turning, r = radius of the circular path, θ = angle of bending with vertical

In Fig. 7.222, we have shown weight of the cyclist (mg) acting vertically downwards at the center of gravity C . R is force of reaction of the ground on the cyclist. It acts at an angle θ with the vertical.

R can be resolved into two rectangular components:

$R \cos \theta$, along the vertical upward direction, $R \sin \theta$, along the horizontal, towards the center of the circular track.

In equilibrium, $R \cos \theta$ balances the weight of the cyclist, i.e.,

$$R \cos \theta = mg \quad (i)$$

and $R \sin \theta$ provides the necessary centripetal force (mv^2/r)

$$R \sin \theta = \frac{mv^2}{r} \quad (ii)$$

Dividing equation (i) by equation (ii), we get

$$\frac{R \sin \theta}{R \cos \theta} = \frac{\frac{mv^2}{r}}{mg}; \tan \theta = \frac{v^2}{rg} \quad (iii)$$

Banking of Roads

Perhaps you have noticed that when a road is straight, it is horizontal too. However, when a sharp turn comes, the surface of the road does not remain horizontal. Figure 7.222 depicts it. This is called banking of the roads.

Purpose of Banking

Banking is done:

1. to reduce frictional wear and tear of tyres.
2. to avoid skidding, and
3. to avoid overturning of vehicles.

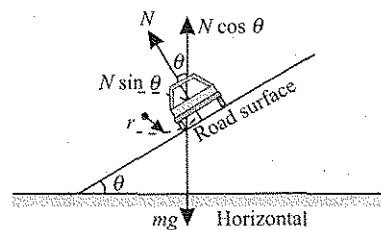


Fig. 7.222

Banking to Avoid Frictional Wear and Tear

What we really wish is that even if there is no friction between the tyres and the road, we should be able to take a round-turn. Let us attend to Fig. 7.223. Vertical $N \cos \theta$ component of the normal reaction N will be equal to mg and the horizontal $N \sin \theta$ component will provide for the necessary centripetal force. [Please note that as we are assuming μ to be zero here, the total reaction of the road will be the normal reaction.] Frictional forces will not act in such a case.

$$N \cos \theta = mg \quad (i)$$

$$N \sin \theta = \frac{mv^2}{r} \quad (ii)$$

Dividing equation (ii) by (i), we get $\tan \theta = \frac{v^2}{rg}$; where θ is the angle of banking.

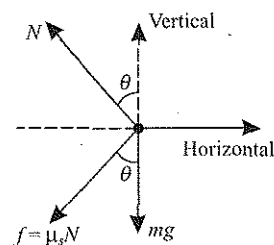


Fig. 7.223

- This particular formula is very important because it comes back again and again. It had given the angle of tilt of the cyclist and the minimum value of μ required also while negotiating a turn.
- The value of v here will be the maximum value of the velocity of the vehicle permissible on this banked road, i.e., if we assume $\mu = 0$.

Motion Along a Circular Track

When a vehicle negotiates a curved (circular) track, centripetal force must be active acting towards the centre of the circular path.

Consider the situation shown in Fig. 7.224, where a vehicle negotiates a horizontal circular curve of radius of curvature r with a uniform speed v . The forces acting on the vehicle are (i) its weight mg , (ii) normal reaction N offered by a road on to the vehicle, and (iii) friction f between the road and the tyres of the vehicle.

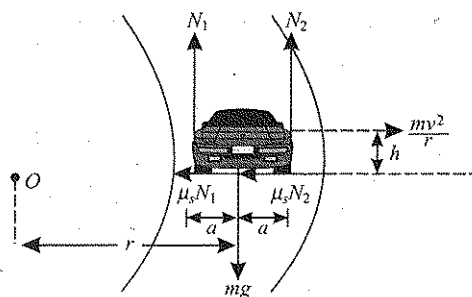


Fig. 7.224

For vertical equilibrium of the vehicle

$$N = mg \quad (i)$$

For horizontal equilibrium of the vehicle (as shown in Fig. 7.225),

$$f = \frac{mv^2}{r} \quad (ii)$$

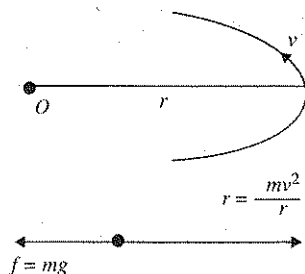


Fig. 7.225

Evidently, the speed of a vehicle is variable and so also the frictional force which is a "self adjusting force". For speeds within a maximum value, the frictional force automatically sets its value, so as to just prevent the vehicle from skidding radially outwards. However for speeds too large, the centrifugal force, exceeds to such an extent, that even the limiting friction (maximum static friction) is also unable to prevent the vehicle from skidding.

Thus, for horizontal equilibrium (with respect to a rotating frame), the speed v should be such that

$$\frac{mv^2}{r} \leq f_{\text{limiting}} \Rightarrow \frac{mv^2}{r} \leq \mu_s mg \quad [\text{where } \mu_s \text{ is the static coefficient of friction between tyres and ground}].$$

$$\Rightarrow v^2 \leq \mu_s gr \Rightarrow v \leq \sqrt{\mu_s gr} \quad (iii)$$

Thus, the maximum speed limit for a vehicle with coefficient of friction μ_s , for successful negotiation, of a curve of radius of curvature r is given by, $v_{\text{max}} = \sqrt{\mu_s gr}$.

Overturning

With the increase in the speed of the car N_2 increases while N_1 decreases. For a particular value of speed $v = v_{\text{max}}$, N_1 become zero, and the car is about to overturn. Thus to prevent overturning, $N_1 \geq 0$.

$$\text{or } \frac{mg}{2} \left(1 - \frac{v^2 h}{rga} \right) \geq 0 \quad \text{or } v \leq \sqrt{\frac{rga}{h}}$$

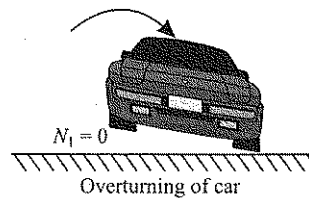
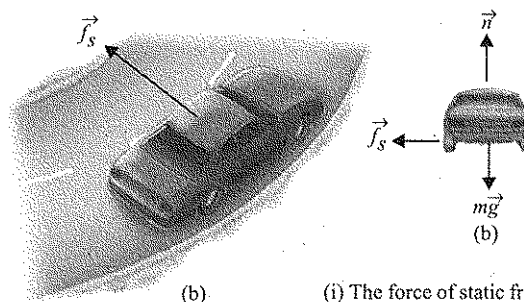


Fig. 7.226

Illustration 7.50 A 1,500-kg car moving on a flat, horizontal road negotiates a curve as shown in Fig. 7.227. If the radius of the curve is 20.0 m and the coefficient of static friction between the tires and dry pavement is 0.50, find the maximum speed the car can have and still make the turn successfully.



- (i) The force of static friction directed toward the center of the curve keeps the car moving in a circular path.
(ii) Free body diagram for the car.

Fig. 7.227

Sol. Imagine that the curved roadway is part of a large circle so that the car is moving in a circular path.

Based on the conceptualize step of the problem, we model the car as a particle in uniform circular motion in the horizontal direction. The car is not accelerating vertically, so it is modeled as a particle in equilibrium in the vertical direction.

The force that enables the car to remain in its circular path is the force of static friction. (It is static because no slipping occurs at the point of contact between road and tires. If this force of static friction were zero, for example, if the car were on an icy road, the car would continue in a straight line and slide off the road.) The maximum speed v_{max} the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,\text{max}} = \mu_s N$.

$$f_{s,\text{max}} = \mu_s N = m \frac{v_{\text{max}}^2}{r} \quad (i)$$

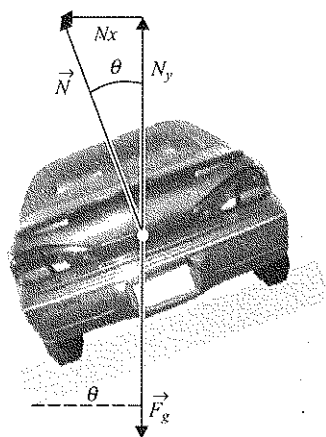
Apply the particle in equilibrium model to the car in the vertical direction:

$$\sum F_y = 0 \rightarrow N - mg = 0 \rightarrow N = mg$$

Solve equation (i) for the maximum speed and substitute for N :

$$\begin{aligned} v_{\text{max}} &= \sqrt{\frac{\mu_s N r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r} \\ &= \sqrt{(0.50)(10 \text{ m/s}^2)(20 \text{ m})} = 10.0 \text{ m/s} \end{aligned} \quad (ii)$$

Illustration 7.51 A civil engineer wishes to redesign a curved roadway in such a way that the car will not have to rely on friction to round the curve without skidding. In other words, a car moving at a designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually banked, which means that the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 10.0 m/s and the radius of the curve is 20.0 m. At what angle should the curve be banked?



A car rounding a curve on a road banked at an angle θ to the horizontal. In the absence of friction the force that causes the centripetal acceleration and keeps the car moving in its circular path is the horizontal component of the normal force.

Fig. 7.228

Sol. The difference between this example and the previous illustration is that the car is no longer moving on a flat roadway. Figure 7.228 shows the banked roadway, with the center of the circular path of the car far to the left of the Fig. 7.228. Notice that the horizontal component of the normal force participates in causing the car's centripetal acceleration.

As in previous illustration, the car is modeled as a particle in equilibrium in the vertical direction and a particle in uniform circular motion in the horizontal direction.

On a level (unbanked) road, the force that causes the centripetal acceleration is the force of static friction between the car and the road as we saw in the preceding example. If the road is banked at an angle θ as in Fig. 7.229, however, the normal force $\vec{n}$ has a horizontal component towards the center of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $N_x = N \sin \theta$ curves the centripetal acceleration.

Write Newton's second law for the car in the radial direction, which is the x direction

$$\sum F_r = N \sin \theta = \frac{mv^2}{r} \quad (i)$$

Apply the particle in equilibrium model to the car in the vertical direction

$$\begin{aligned} \sum F_y &= N \cos \theta - mg = 0 \\ N \cos \theta &= mg \end{aligned} \quad (ii)$$

Dividing equation (i) by equation (ii)

$$\tan \theta = \frac{v^2}{rg} \quad (iii)$$

Solve for the angle θ

$$\theta = \tan^{-1} \left(\frac{(10.0 \text{ m/s}^2)}{(20.0 \text{ m})(10.0 \text{ m/s}^2)} \right) = \tan^{-1} \left(\frac{1}{2} \right)$$

Concept Application Exercise 7.5

- Three masses are attached to strings rotating in the horizontal plane. The strings pass over two nails as shown in the Fig. 7.229. Will this system be in equilibrium?

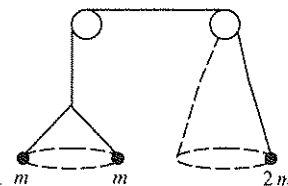
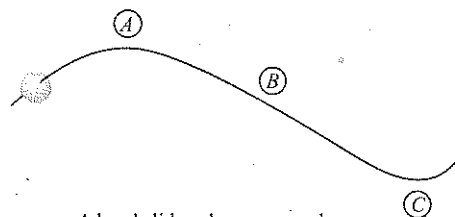


Fig. 7.229

- Can force determine the direction of motion and direction of acceleration?
- You are riding on a Ferris wheel that is rotating with a constant speed. The car in which you are riding always maintains its correct upward orientation; it does not invert.
 - What is the direction of the normal force on you from the seat when you are at the top of the wheel?
 - upward
 - downward
 - impossible to determine
 - From the same choices, what is the direction of the net force on you when you are at the top of the wheel?
- A bead slides freely along a curved wire lying on a horizontal surface at a constant speed as shown in Fig. 7.230.
 - Draw the vectors representing the force exerted by the wire on the bead at points A, B and C. Suppose the bead in Fig. 7.230 speeds up with a constant tangential acceleration as it moves toward the right. Draw the vectors representing the force on the bead at point A, B and C.



A bead slides along a curved curve

Fig. 7.230

- A smooth block loosely fits in a circular tube placed on a horizontal surface. The block moves in a uniform circular motion along the tube (Fig. 7.231). Which wall (inner or outer) will exert a nonzero normal contact force on the block?

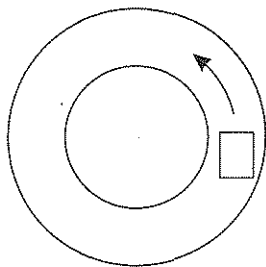


Fig. 7.231

6. An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away. The coefficient of the static friction between the person and the wall is μ_s , and the radius of the cylinder is R (see Fig. 7.232).

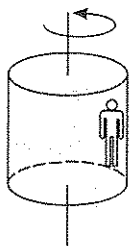


Fig. 7.232

- Show that the maximum period of revolution necessary to keep the person from falling is $T = (4\pi^2 R \mu_s / g)^{1/2}$.
 - Obtain a numerical value for T , taking $R = 4.00$ m and $\mu_s = 0.400$. How many revolutions per minute does the cylinder make?
 - If the rate of revolution of the cylinder is made to be somewhat larger, what happens to the magnitude of each one of the forces acting on the person? What happens to the motion of the person?
 - If instead the cylinder's rate of revolution is made to be somewhat smaller, what happens to the magnitude of each one of the forces acting on the person. What happens to the motion of the person?
7. Tarzan ($m = 85.0$ kg) tries to cross a river by swinging on a vine. The vine is 10.0 m long, and his speed at the bottom of the swing (as he just clears the water) will be 8.00 m/s. Tarzan doesn't know that the vine has a breaking strength of 1000 N. Does he make it across the river safely?
8. A roller-coaster car has a mass of 500 kg when fully loaded with passengers.
- If the vehicle has a speed of 20.0 m/s at point A, what is the force exerted by the track on the car at this point?
 - What is the maximum speed the vehicle can have at point B and still remain on the track?
9. An air puck of mass m_1 is tied to a string and allowed to revolve in a circle of radius R on a frictionless horizontal table (see Fig. 7.233). The other end of the string passes through a small hole in the center of the table, and a load of mass m_2 is tied to the string. The suspended load remains in the equilibrium while the puck on the tabletop revolves. What is

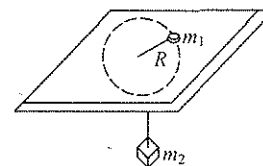


Fig. 7.233

- the tension in the string;
 - the radial force acting on the puck, and
 - the speed of the puck?
- Qualitatively describe what will happen in the motion of the puck if the value of m_2 is somewhat increased by placing an additional load on it.
 - Qualitatively describe what will happen in the motion of the puck if the value of m_2 is instead decreased by removing a part of the hanging load.
10. A sleeve A can slide freely along a smooth rod bent in the shape of a half circle of radius R . The system is set in rotation with a constant angular velocity ω about a vertical axis OO' . Find the angle θ corresponding to the steady position of the sleeve (see Fig. 7.234 for reference).

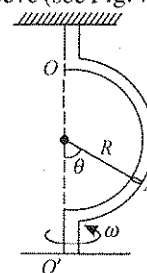


Fig. 7.234

- A ball suspended by a thread swing in a vertical plane so that its acceleration values in the extreme and the lowest position are equal. Find the thread deflection angle in the extreme position.
- A simple pendulum is oscillating with an angular displacement of 90° . For what angle with the vertical the acceleration of bob directed horizontally?
- A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1,500 at full speed. Consider a particle of mass 1 g sticking at the outer end of a blade.

How much force does it experience when the fan runs at full speed?

Who exerts this force on the particle?

How much force does the particle exert on the blade along its surface?
- A block of mass m is kept on a horizontal ruler. The friction coefficient between the ruler and the block is μ . The ruler is fixed at one end and the block is at a distance L from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end.
 - What can the maximum angular speed be for which the block does not slip?
 - If the angular speed of the ruler is uniformly increased from zero at an angular acceleration α , at what angular speed will the block slip?

15. An old record player of 15.0 cm radius turns at 33.0 rev/min while mounted on a 30° incline as shown in the Fig. 7.235.

a. If a mass m can be placed anywhere on the rotating record, which is the most critical place on the disc where slipping might occur?

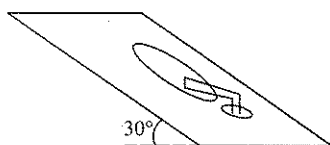


Fig. 7.235

- b. Calculate the least possible coefficient of friction that must exist if no slipping occurs.
16. A 60 kg woman is on a large vertical swing of radius 20 m. The swing rotates with a constant speed.
- a. At what speed would she feel weightless at the top?
- b. At this speed, what is her apparent weight at the bottom?
17. A rod OA rotates about a horizontal axis through O with a constant anticlockwise velocity $\omega = 3$ rad/sec. As it passes the position $\theta = 0$ a small block of mass m is placed on it at a radial distance $r = 450$ mm (see Fig. 7.236). If the block is observed to slip at $\theta = 50^\circ$, find the coefficient of static friction between the block and the rod. (Given that $\sin 50^\circ = 0.766$, $\cos 50^\circ = 0.64$).

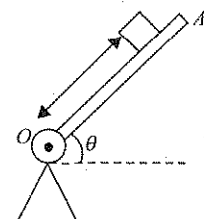


Fig. 7.236

18. A particle rests on the top of a smooth hemisphere of radius r . It is imparted a horizontal velocity of $\sqrt{\eta gr}$ (see Fig. 7.237). Find the angle made by the radius vector joining the particle with the vertical, at the instant, the particle loses contact with the sphere.

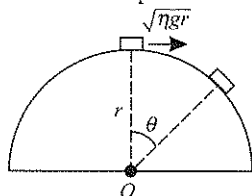


Fig. 7.237

Solved Examples

Example 7.1 Two blocks of masses 5 kg and 2 kg (see Fig. 7.238) are initially at rest on the floor. They are connected by a light string, passing over a light frictionless pulley. An

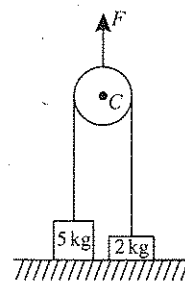


Fig. 7.238

upward force F is applied on the pulley and maintained at a constant. Calculate the acceleration a_1 and a_2 of the 5 kg and 2 kg masses, respectively, when F is

1. 30 N, 2. 60 N, 3. 110 N ($g = 10 \text{ ms}^{-2}$).

Sol. Apart from the constraint that the string is unstretchable, the additional constraint is that neither of the masses can go downward. So the block will be lifted only when the tension of the string exceeds the gravitational pull on them.

1. Considering the FBD of the pulley (Fig. 7.239)
- $$30 - 2T = 0 \text{ (pulley is massless)}$$
- or $T = 15 \text{ N}$

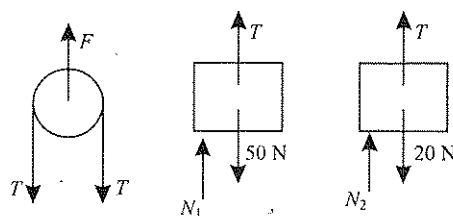


Fig. 7.239

So tension is less than gravitational pull on both the blocks.

So no acceleration is produced in them.

$$\therefore a_1 = a_2 = 0$$

2. Now $60 - 2T = 0$, or $T = 30 \text{ N}$

So the 5 kg weight will not be lifted but the 2 kg weight will be lifted.

$$30 - 20 = 2 \times a_2 \text{ or } a_2 = 5 \text{ ms}^{-2}$$

$$\text{Thus } a_1 = 0 \text{ and } a_2 = 5 \text{ ms}^{-2}$$

3. Now $140 - 2T = 0$, or $T = 70 \text{ N}$

So both the weights are lifted.

$$70 - 50 = 5a_1, \text{ or } a_1 = 4 \text{ ms}^{-2} \text{ and } 70 - 20 = 2a_2,$$

$$\text{or } a_2 = 25 \text{ ms}^{-2}$$

Example 7.2 In the arrangement shown in Fig. 7.240 $m_1 = 1 \text{ kg}$, $m_2 = 2 \text{ kg}$. Pulleys are massless and strings are light. For what value of M the mass m_1 moves with a constant velocity.

Sol. Mass m_1 moves with a constant velocity if tension in the lower string is

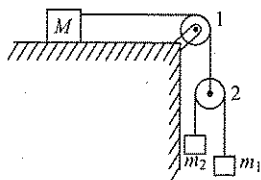


Fig. 7.240

$$T_1 = m_1 g = (1)(10) = 10 \text{ N}$$

Tension in the upper string is

$$T_2 = 2T_1 = 20 \text{ N}$$

Acceleration of block M is, therefore,

$$a = \frac{T_2}{M} = \frac{20}{M}$$

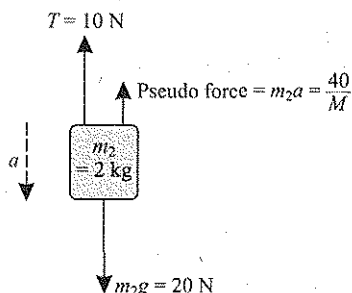


Fig. 7.241

This is also the acceleration of pulley 2.

Absolute acceleration of mass m_1 is zero. Thus, acceleration of m_1 relative to pulley 2 is a (upwards) or acceleration of m_2 with respect to pulley 2 is a (downwards). Draw free body diagram of m_2 with respect to pulley 2 (Fig. 7.241).

Equation of motion gives

$$20 - \frac{40}{M} - 10 = 2a = \frac{40}{M}$$

Solving this, we get, $M = 8 \text{ kg}$.

Example 7.3 A block of mass 1 kg is placed over a plank of mass 2 kg. The length of the plank is 2 m. Coefficient of friction between the block and the plank is 0.5 and the ground over which the plank is placed is smooth. A constant force $F = 30 \text{ N}$ is applied on the plank in horizontal direction. Find the time after which the block will separate from the plank.

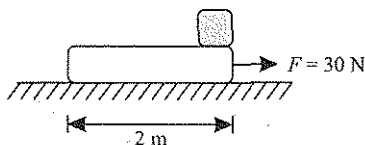


Fig. 7.242

Sol. Maximum frictional force between the block and the plank is

$$f_{\max} = \mu mg = (0.5)(1)(10) = 5 \text{ N}$$

The FBD of the block and the plank are shown in Fig. 7.243.

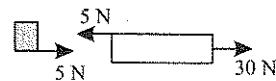


Fig. 7.243

$$\text{Acceleration of block, } a_1 = \frac{5}{1} = 5 \text{ m/s}^2$$

$$\text{Acceleration of plank, } a_2 = \frac{30 - 5}{2} = \frac{25}{2} \text{ m/s}^2$$

Relative acceleration of plank, $a = a_2 - a_1$ or,

$$a = \frac{25}{2} - 5 = 7.5 \text{ m/s}^2$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 2}{7.5}} = 0.73 \text{ s}$$

Example 7.4 Consider a system of a small body of mass m kept on a large body of mass M placed over an inclined plane of angle of inclination θ to the horizontal. Find the acceleration of m when the system is set in motion. Assume inclined plane to be fixed. All the contact surfaces are smooth.

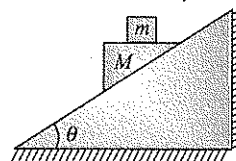


Fig. 7.244

Sol. (i) Analysis in an inertial reference frame attached to the ground:

Let a be the acceleration of m w.r.t. M directed horizontally towards right.

Let A be the acceleration of M w.r.t. the ground directed along the incline m downward direction.

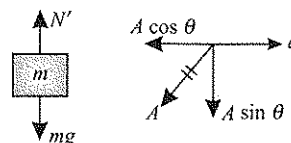


Fig. 7.245

Acceleration of m w.r.t ground will be the vector sum of the acceleration of m w.r.t M and acceleration of M w.r.t ground.

Force equations for m are,

$$0 = m(a - A \cos \theta) \quad (1)$$

$$mg - N = m(A \sin \theta) \quad (2)$$

Force equations for M are,

$$N' \sin \theta + Mg \sin \theta = MA \quad (3)$$

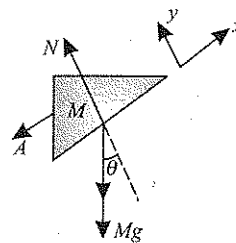


Fig. 7.246

$$N' \cos \theta + Mg \cos \theta - N = 0 \quad (4)$$

From equation (2)

$$N' = mg - mA \sin \theta$$

Substituting N' in equation (3),

$$(mg - mA \sin \theta) \sin \theta + Mg \sin \theta = MA$$

$$\Rightarrow A = \frac{(M + m)g \sin \theta}{M + m \sin^2 \theta}$$

Acceleration of m w.r.t ground is $A \sin \theta$ (since, $A \cos \theta - a = 0$) thus the acceleration of m w.r.t ground is

$$= \frac{(M + m)g \sin \theta}{M + m \sin^2 \theta}$$

(ii) Analysis of motion of m in the non inertial reference frame attached to M :

Let the acceleration of M w.r.t. ground be A along the inclined plane downwards. Consider that ground is an inertial reference frame, the reference frame attached to M will be non-inertial. For applying Newton's second law of motion to any object w.r.t. M , you have to apply an inertial force (also called pseudo force) on the object which equals mass of the object times acceleration of M , directed opposite to A .

Force equations for m :

Let the acceleration of m w.r.t. M be a , in the horizontal direction towards right.

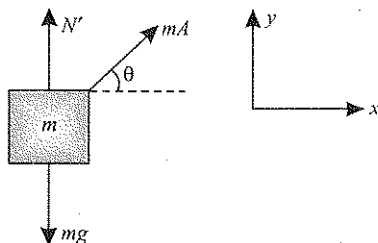


Fig. 7.247

The forces acting on m are:

- weight of m , mg , acting vertically downwards,
- normal force on m by M , N' , vertically upwards, and
- (mA) , the inertial force acting along the incline upwards, as the acceleration of M is A along the incline downwards.

$$\text{From } \sum \vec{F} = m\vec{a}$$

$$N' + (mA) \sin \theta - mg = 0 \quad (1)$$

$$[\sum F_y = ma_y]$$

$$(mA) \cos \theta = ma$$

$$[\sum F_x = ma_x]$$

Force equations for M :

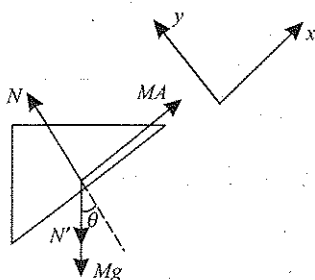


Fig. 7.248

The forces acting on M are

- the weight Mg ,
- N' , normal force exerted by m ,
- N , normal force exerted by the incline, and
- (MA) , the inertial force along the plane upwards.

acceleration of M w.r.t. M (w.r.t. itself) = 0,

$$Mg \sin \theta + N' \sin \theta - MA = 0 \quad (3)$$

(along the incline)

$$Mg \cos \theta + N' \cos \theta - N = 0 \quad (4)$$

(perpendicular to the incline)

From equations (1), (3), and (4),

$$A = \frac{(M + m)g \sin \theta}{M + m \sin^2 \theta}$$

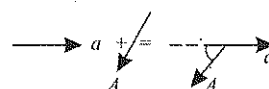


Fig. 7.249

Acceleration of m relative to ground,

$$\vec{a}_{mG} = \vec{a}_{mM} + \vec{a}_{MG}$$

But $a = A \cos \theta$, (from equation 2), therefore

$$\vec{a}_{mG} = A \sin \theta, \text{ vertically downwards} = \frac{(M + m)g \sin^2 \theta}{M + m \sin^2 \theta}$$

Example 7.5

Pulleys shown in the system of the Fig. 7.250 are massless and frictionless. Threads are inextensible. Mass of blocks A, B, and C are $m_1 = 2$ kg, $m_2 = 4$ kg and $m_3 = 2.75$ kg, respectively. Calculate acceleration of each block.

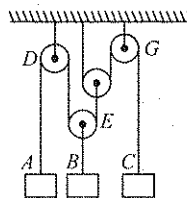


Fig. 7.250

Sol. Let acceleration of blocks A and B be a and b vertically upwards, respectively. Then according to geometry of the given figure downward acceleration of block C will be equal to $2a + 4b$. Now considering the FBDs (see Fig. 7.251).

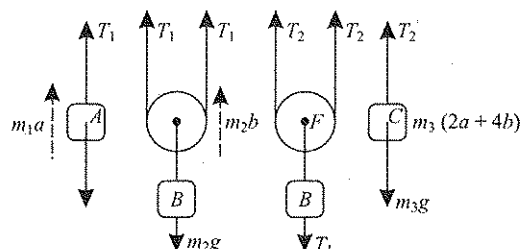


Fig. 7.251

$$\text{For block A, } T_1 - m_1 g = m_1 a \quad (i)$$

$$\text{For block B, } 2T_1 - m_2 g = m_2 b \quad (ii)$$

$$\text{For pulley F, } T_1 = 2T_2 \quad (iii)$$

For block C, $m_3g - T_2 = m_3(2a + 4b)$ (iv)

Solving above equations, $T_1 = 22 \text{ N}$, $T_2 = 11 \text{ N}$,

$a = 1 \text{ ms}^{-2}$, $b = 1 \text{ ms}^{-2}$

Hence, acceleration of block A, $a = 1 \text{ ms}^{-2}$ ($\uparrow$)

acceleration of block B, $b = 1 \text{ ms}^{-2}$ ($\uparrow$)

acceleration of block C, $= (2a + 4b) = 6 \text{ ms}^{-2}$ ($\uparrow$)

Example 7.6 A motorcycle has to move with a constant speed on an overbridge which is in the form of a circular arc of radius R and has a total length L . Suppose the motorcycle starts from the highest point.

1. What can its maximum velocity be for which the contact with the road is not broken at the highest point?
2. If the motorcycle goes at speed $1/2$ times the maximum found in part (a), where will it lose the contact with the road?
3. What maximum uniform speed can it maintain on the bridge if it does not lose contact anywhere on the bridge?

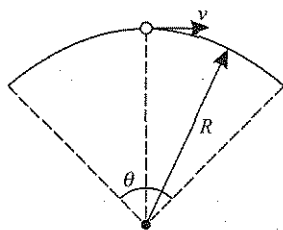


Fig. 7.252

Sol.

$$1. \theta = \frac{\text{Arc length}}{\text{radius}} = \frac{L}{R}$$

Free body diagram (Fig. 7.253) at top position.

$$\text{Equation of motion, } mg - N = \frac{mv^2}{R}$$

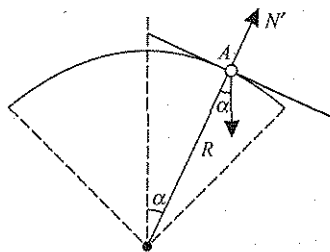


Fig. 7.253

$$\Rightarrow N = mg - \frac{mv^2}{R}$$

If contact at highest point does not loose $N > 0$,

$$mg > \frac{mv^2}{R}$$

$$v^2 < gR \Rightarrow v < \sqrt{gR}$$

$$2. \text{ If velocity of motor cyclist } v' = \frac{1}{\sqrt{2}}v = \frac{\sqrt{gR}}{\sqrt{2}}$$

At position A,

$$mg \cos \alpha - N' = \frac{mv'^2}{R}$$

$$\Rightarrow N' = mg \cos \alpha - \frac{m}{R} \left(\frac{\sqrt{gR}}{\sqrt{2}} \right)^2$$

$$\Rightarrow N' = mg \cos \alpha - \frac{mgR}{2R} = mg \cos \alpha - \frac{mg}{2}$$

If he does not loose contact with road,

$$N' > 0$$

$$mg \cos \alpha - \frac{mg}{2} > 0 \Rightarrow \cos \alpha > \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$$

Hence arc length, $l' = \frac{R\pi}{3}$ from top.

3. The equation of motion of motor cyclist in function of a measured from vertical

$$mg \cos \alpha - N'' = \frac{mv''^2}{R}$$

The normal reaction will be minimum at the end of the arc. Hence critical position is at the end of the arc. where,

$$\alpha = \frac{\theta}{2} = \frac{L}{2R}$$

$$mg \cos \left(\frac{L}{2R} \right) - N'' = \frac{mv''^2}{R}$$

$$\Rightarrow N'' = mg \cos \left(\frac{L}{2R} \right) - \frac{mv''^2}{R}$$

If he does not loose contact even at the end of the arc, $N'' > 0$

$$mg \cos \left(\frac{L}{2R} \right) - \frac{mv''^2}{R} > 0$$

$$\Rightarrow g \cos \left(\frac{L}{2R} \right) > \frac{v''^2}{R} \Rightarrow v'' < \sqrt{gR \cos \left(\frac{L}{2R} \right)}$$

Example 7.7 A track consists of two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in Fig. 7.254. Each part subtends a right angle at its centre. A cycle weighing 100 kg together with the rider travels at a constant speed of 18 kg/h on the track.

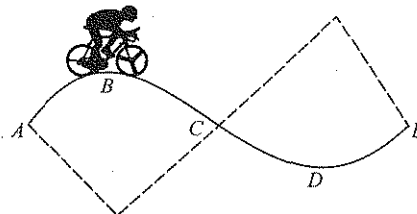


Fig. 7.254

1. Find the normal contact force by the road on the cycle when it is at B and at D.
2. Find the force of friction exerted by the track on the tyres when the cycle is at B, C, and D.
3. Find the normal force between the road and the cycle just before and just after the cycle crosses C.
4. What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed?

Sol. $v = 18 \text{ km/h} = \frac{1800}{60 \times 60} = 5 \text{ m/s}$

1. a. At point B, various forces acting on the cyclist are:

i. the weight mg , vertically downward, and

ii. normal force N_B by the road upward.

Since, under the action of these two forces, the cyclist moves in a circular path of radius r , hence from the dynamics of circular motion,

$$\frac{mv^2}{r} = mg - N_B \Rightarrow N_B = m \left(g - \frac{v^2}{r} \right)$$

$$= 100 \left(10 - \frac{5^2}{100} \right) = 975 \text{ N}$$

- b. At point D, various forces acting on the cyclist are:

i. the weight mg , vertically downward, and

ii. normal force N_D by the road upward.

$$N_D - mg = \frac{mv^2}{r}$$

$$\Rightarrow N_D = m \left(g + \frac{v^2}{r} \right) = 100 \left(10 + \frac{5^2}{100} \right) = 975 \text{ N}$$

2. At the points B and D, the tracks are almost horizontal, therefore, there is no components of g along the track at those points that will change the speed of the cyclist. Since the cyclist moves with a constant speed, the frictional force at these two points is necessarily zero.

At point C, however, the component of mg along the track that tends to accelerate the motion of the cyclist $= mg \sin \alpha$.

But since his speed remains constant, a force, equal and opposite to $mg \sin \alpha$, must be acting on it and this force is the force of friction μN_c , where μ is the friction coefficient between the road and the tyre and N_c is the normal reaction at the point C (see Fig. 7.255).

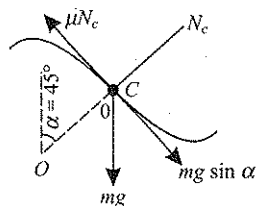


Fig. 7.255

Hence frictional force at C is, $\mu N_c = mg \sin \alpha$
 $= 100 \times 10 \times \sin 45^\circ = 707.12 \text{ N}$

3. Just before the cyclist crosses the point C, various forces acting on him are:

i. Weight mg , vertically downward, and

ii. Normal force N_b of the track along.

Hence, from the dynamics of circular motion, we have

$$mg \cos \alpha - N_b = \frac{mv^2}{r}$$

$$N_b = m \left(g \cos \alpha - \frac{v^2}{r} \right) = 682.11 \text{ N}$$

Just after the cyclist crosses the point C, various forces acting on him are:

i. weight mg , vertically downward, and

ii. normal force N_d of the track along.

Hence, from the dynamics of circular motion,

$$N_d - mg \cos \alpha = \frac{mv^2}{r}$$

$$\Rightarrow N_d = m \left(g \cos \alpha + \frac{v^2}{r} \right) = 732.11 \text{ N}$$

$$= 100 \left(10 \cos 45^\circ + \frac{5^2}{100} \right)$$

4. The tendency of the cyclist to skid is maximum just before the point C, the point where the radius of curvature changes its direction. This is so because the curve is steepest at this point.

Force along the track at this point $= mg \sin \alpha$.

frictional force $= \mu N_b$

where N_b normal force of the track just before the point C.

The cyclist can move with constant speed only if there two forces are equal and opposite,

$$\mu N_b = mg \sin \alpha \Rightarrow \mu \times 632.11$$

$$= 100 \times 10 \times \sin 45^\circ \Rightarrow \mu = 1.037$$

Example 7.8

A plank of mass m rests symmetrically on two wedges B and C of mass M . What is the acceleration of the plank? Neglect the friction between all the contact surfaces.

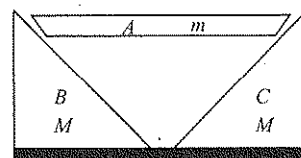


Fig. 7.256

Sol. When the plank is released it falls through a distance y and both the wedges move through a distance x . From Fig. 7.257.

$$y = x \tan \theta \quad (i)$$

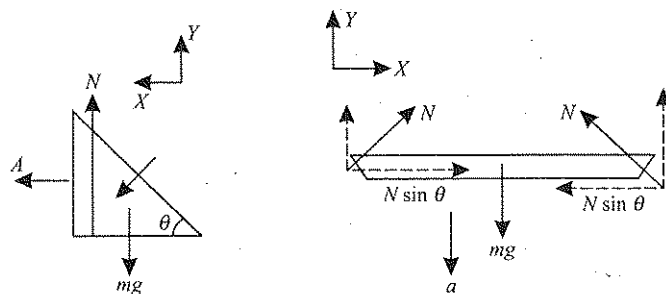


Fig. 7.257

On differentiating this expression twice, we obtain

$$a = A \tan \theta$$

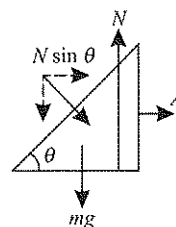


Fig. 7.258

Equations of wedge:

$$\sum F_x = N \sin \theta = MA$$

$$\sum F_y = N' - N \cos \theta - Mg = 0$$

Equations of block:

$$\sum F_x = N \sin \theta - N \sin \theta = 0$$

$$\sum F_y = mg - 2N \cos \theta = ma$$

From equation (ii), $N = \frac{MA}{\sin \theta}$

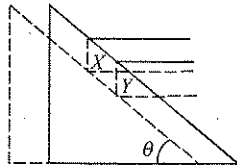


Fig. 7.259

On substituting expression for N and a in equation (v), we obtain

$$mg - \frac{2MA \cos \theta}{\sin \theta} = \frac{mA \sin \theta}{\cos \theta}$$

$$\Rightarrow A = \frac{mg \sin \theta \cos \theta}{m \sin^2 \theta + M \cos^2 \theta}$$

Example 7.9 In Fig. 7.260, mass m is being pulled on the incline of a wedge of mass M . All the surfaces are smooth. Find the acceleration of the wedge.

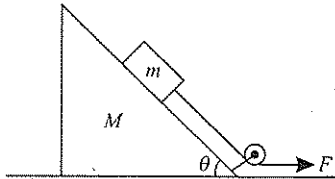


Fig. 7.260

Sol. Figure 7.261(a) shows force diagram of the wedge and the block. Let acceleration of block relative to wedge be $\vec{a}_{mM} = \vec{a}$ and acceleration of wedge on ground is $\vec{a}_M = \vec{a}$.

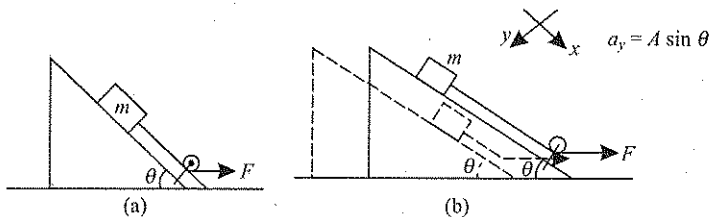


Fig. 7.261

Free body diagrams as in Figs. 7.262 and 7.263

Equation of motion of M

$$N \sin \theta = F - F \sin \theta = MA$$

Equation of motion of m

In y -direction

$$mg \cos \theta - N = ma_y = mA \sin \theta \quad (ii)$$

Substituting the value of N from equation (ii) in equation (i) we can get the value of A ;

$$\vec{a}_m = \vec{a}_{mM} + \vec{a}_M$$

(ii)

(iii)

(iv)

(v)

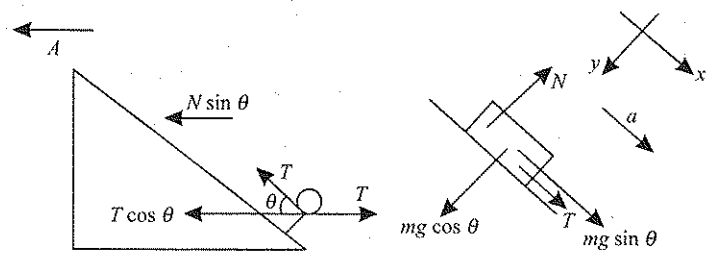


Fig. 7.262

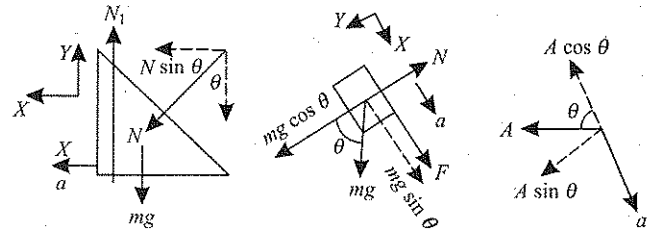


Fig. 7.263

$$(a_m)_x = a - A \cos \theta$$

$$(a_m)_y = A \sin \theta$$

Equations of wedge:

$$\sum F_x = N \sin \theta = MA \quad (iii)$$

$$\sum F_y = N' - N \cos \theta - Mg = 0 \quad (iv)$$

Equations of block:

$$\sum F_x = F + mg \sin \theta = m(a - A \cos \theta) \quad (v)$$

$$\sum F_y = mg \cos \theta - N = mA \sin \theta \quad (vi)$$

From equation (iii), $N = \frac{MA}{\sin \theta}$

On substituting expression for N in equation (vi), we obtain

$$mg \cos \theta - \frac{MA}{\sin \theta} = mA \sin \theta,$$

$$mg \cos \theta = mA \sin \theta + \frac{MA}{\sin \theta}$$

$$\Rightarrow A = \frac{mg \sin \theta \cos \theta}{m \sin^2 \theta + M}$$

Example 7.10

In the arrangement shown in the Fig. 7.264, mass of the rod M exceeds the mass m of the ball. The ball has an opening permitting it to slide along the thread with some friction. The mass of the pulley and the friction in its axle are negligible. At the initial moment, the ball was located opposite the lower end of the rod. When set free, both bodies began moving with constant accelerations. Find the friction force between the ball and the thread if t seconds after the beginning of motion, the ball got opposite the upper end of the rod. The rod length equals l .

Sol. The friction force between the ball and the thread is f_r .

Acceleration of both the bodies is downward.

$$mg - f_r = ma_1 \quad (i)$$

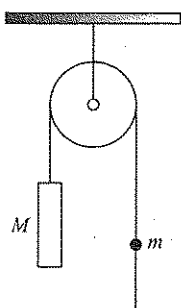


Fig. 7.264

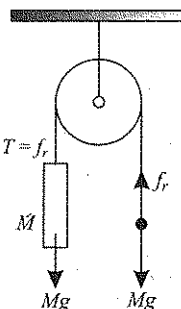


Fig. 7.265

but $Mg - T = Ma_2$ (ii)
 $f_r = T$

$$a_2 - a_1 = \left(\frac{Mg - f_r}{M} \right) - \left(\frac{mg - f_r}{m} \right)$$

According to the problem,

$$l = \frac{1}{2}(a_2 - a_1)t^2 \text{ or } (a_2 - a_1) = \frac{2l}{t^2}$$

$$\frac{m(Mg - f_r) - M(mg - f_r)}{Mm} = \frac{2l}{t^2}$$

$$(M - m)f_r = \frac{2lmM}{t^2} \Rightarrow f_r = \frac{2lmM}{(M - m)t^2}$$

Example 7.11 In the Fig. 7.266 shown force $F = \alpha t$ applied on the block m_2 . Here α is a constant and t is the time. Find the acceleration of the block.

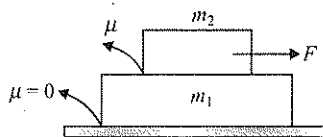


Fig. 7.266

Sol. If $F \leq \mu \frac{m_2}{m_1}(m_1 + m_2)g$ then both blocks will move together.

Here, $t \leq \mu \frac{m_2}{\alpha m_1}(m_1 + m_2)g$

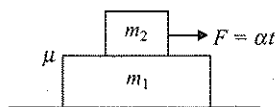


Fig. 7.267

During this time acceleration of both blocks

$$a_1 = a_2 = \frac{F}{(m_1 + m_2)} + \frac{\alpha t}{(m_1 + m_2)}$$

If $t \geq \frac{\mu m_2}{\alpha m_1}(m_1 + m_2)g$

Friction will be of kinetic nature.

Free body diagrams (Fig. 7.268):

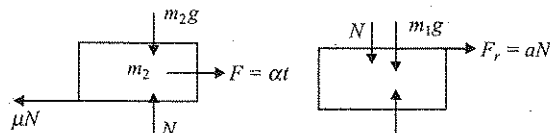


Fig. 7.268

Acceleration of m_1 :

$$a_1 = \frac{\mu N}{m_1} = \frac{\mu m_2 g}{m_1}$$

Acceleration of m_2 : $a_2 = \frac{\alpha t - \mu m_2 g}{m_2}$

Draw the acceleration of m , v_s time.

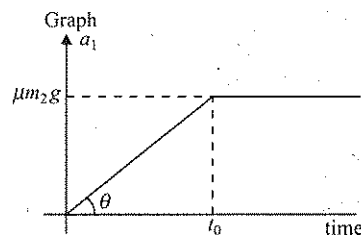


Fig. 7.269

$$t_0 = \frac{\mu m_2}{\alpha m_1}(m_1 + m_2)g$$

$$a_1 = \frac{\alpha t_0}{(m_1 + m_2)} = \frac{\mu m_2 g}{m_1}$$

Acceleration v_s time graph for m_2 (Fig. 7.271)

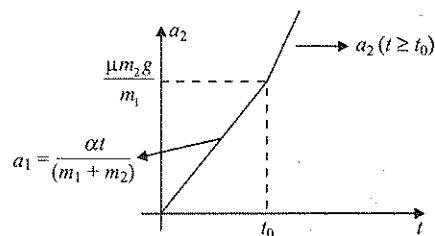


Fig. 7.270

After time t_0 ; $a_2 = \frac{\alpha t - \mu m_2 g}{m_2}$

$$a_2 = \frac{\alpha t}{m_2} - \frac{\mu m_2 g}{m_2}; \quad a_2 = \frac{\alpha t}{m_2} - \mu g$$

Example 7.12 Find the acceleration a_1 , a_2 , and a_3 of the three blocks shown in Fig. 7.271, if a horizontal force of 10 N of is applied on

(1) 2 kg block (2) 3 kg block (3) 7 kg block (take $g = 10 \text{ m/s}^2$)

Sol.

1. When force of 10 N is applied on 2 kg block.

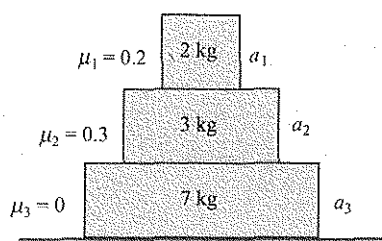


Fig. 7.271

The limiting frictional force between 2 kg and 3 kg blocks

$$f_1 = 0.2 \times 2g = 0.2 \times 2 \times 10 = 4 \text{ N}$$

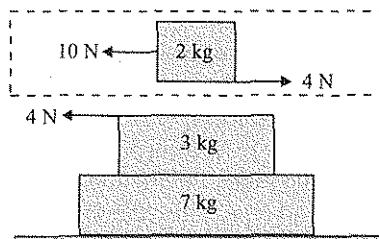


Fig. 7.272

The limiting frictional force between 3 kg and 7 kg blocks that can be

$$f_2 = 0.3 \times 5g = 0.3 \times 5 \times 10 = 15 \text{ N}$$

As applied force 10 N is greater than f_1 but less than f_2 , so 2 kg block will slide over 3 kg

$$\text{Thus we have: } a_1 = \frac{10 - 4}{2} = 3 \text{ m/s}^2;$$

$$a_2 = a_3 = \frac{4}{3 + 7} = 0.4 \text{ m/s}^2$$

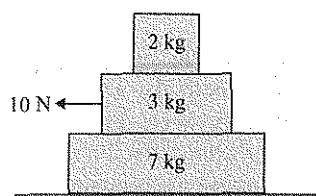


Fig. 7.273

2. When force of 10 N is applied on 3 kg block.

As the applied force is less than the friction between 3 kg and 7 kg blocks (that can be 15 N) so that block will move together as one unit with an acceleration

$$a = \frac{10}{2 + 3 + 7} = \frac{5}{6} \text{ m/s}^2$$

Now find pseudo force on 2 kg block because of acceleration of 3 kg block

$$F_{\text{pseudo}} = 2 \times \frac{5}{6} = \frac{5}{3} \text{ N}$$

As the F_{pseudo} is smaller than the friction between 2 kg and 3 kg (that can be 4 N). So 2 kg will move together with other blocks.

3. When force of 10 N is applied on 7 kg block.

Suppose 3 kg and 2 kg blocks move together with a 7 kg block.

$$\text{The acceleration of the whole system, } a = \frac{10}{2 + 3 + 7} = \frac{5}{6} \text{ m/s}^2$$

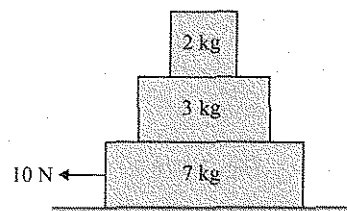


Fig. 7.274

The pseudo force on 2 kg block = $\frac{2 \times 5}{6} = \frac{5}{3} \text{ N}$ which is less than the frictional force between 2 kg and 3 kg block, so they move together.

Now check whether 2 kg and 3 kg blocks move together over 7 kg block. The pseudo force on (2 + 3) kg is $= 5 \times \frac{5}{6} = \frac{25}{6} \text{ N}$ which is also less than frictional force between 3 kg and 7 kg block, so all the blocks move together with a common acceleration of $\frac{5}{6} \text{ m/s}^2$.

$$\text{Therefore } a = a_1 = a_2 = a_3 = \frac{10}{2 + 3 + 7} = \frac{5}{6} \text{ m/s}^2.$$

Example 7.13 A smooth semicircular wire-track of radius R is fixed in a vertical plane (see Fig. 7.275). One end of a massless spring of natural length $\left(\frac{3}{4}\right)R$ is attached to the lowest point O of the wire track. A small ring of mass m , which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant $k = \frac{mg}{R}$. Consider the instant when the ring is released, and

1. The FBD of the ring is drawn.

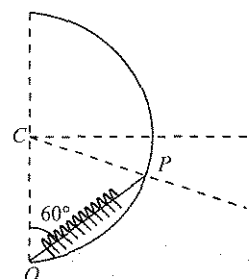


Fig. 7.275

2. Determine the tangential acceleration of the ring and the normal reaction. (IIT-JEE, 1996)

Sol.

1. The FBD of the ring is shown in the Fig. 7.276. The forces acting on the ring are:

- a. the weight, mg acting vertically downwards, and

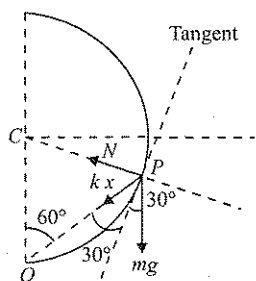


Fig. 7.276

- b. the normal force N by the wire track. Normal force on the ring could be either radially outwards or radially inwards depending on whether the ring presses against the inner surface or outer surface of the track. To ascertain whether normal force is inward or outward assume that, to begin with, it is inwards, then from $\sum \vec{F} = m\vec{a}$ find the value of normal force, if it is +ve it is inward and if it is -ve, it is outwards.
- c. Force of the spring kx . In the given physical situation, the spring is extended, it will pull the ring. So the spring force kx is along the spring towards O .

2. Length of the spring in the position shown = R .
($CP = CO = R$; $\angle COP = \angle OPC = 60^\circ$; $\triangle COP$ is equilateral)

$$\text{Change in length of the spring} = R - \left(\frac{3}{4}\right)R = \left(\frac{R}{4}\right)$$

$$kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$$

$$\begin{aligned} \text{Now from } F_r &= ma_r, \left(\frac{mg}{4}\right)\cos 30^\circ + mg \cos 30^\circ \\ &= ma_r a_r = \frac{5\sqrt{3}}{8}g \end{aligned}$$

Example 7.14 Blocks A , B , and C are placed as shown in Fig. 7.277 and connected by ropes of negligible mass. Both A and B weigh 25.0 N each, and the coefficient of kinetic friction between each block and the surface is 0.35. Block C descends with a constant velocity.

1. Draw two separate FBDs showing the forces acting on A and B .

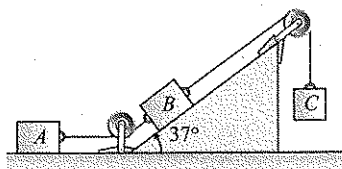


Fig. 7.277

2. Find the tension in the rope connecting blocks A and B .
3. What is the weight of block C ?
4. If the rope connecting A and B were cut, what would be the acceleration of C ? (IIT-JEE 1981)

Sol.

1.

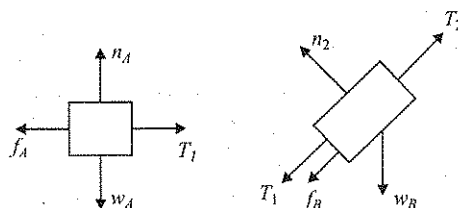


Fig. 7.278

2. The blocks move with a constant speed, so there is no net force on block A ; the tension in the rope connecting A and B must be equal to the frictional force on block A ,

$$\mu_k = (0.35)(25.0 \text{ N}) = 9 \text{ N}$$

3. The weight of block C will be the tension in the rope connecting B and C . This is found by considering the forces on block B . The components of force along the ramp are the tension in the first rope [9 N, from part (i)], the component of the weight along the ramp, the friction on block B , and the tension in the second rope. Thus, the weight of block C is

$$\begin{aligned} w_C &= 9 \text{ N} + w_B(\sin 36.9^\circ + \mu_k \cos 36.9^\circ) \\ &= 9 \text{ N} + (25.0 \text{ N})[\sin 36.9^\circ + (0.35) \cos 36.9^\circ] = 31.0 \text{ N} \end{aligned}$$

or 31 N to two figures. The intermediate calculation of the first tension may be avoided to obtain the answer in terms of the common weight w of blocks A and B ,

$$w_C = w[\mu_k + (\sin \theta + \mu_k \cos \theta)],$$

giving the same result.

4. Applying Newton's second law to the remaining masses (B and C) gives

$$a = \frac{g(w_C - \mu_k w_B \cos \theta - w_B \sin \theta)}{(w_B + w_C)} = 1.54 \text{ m/s}^2$$

Example 7.15 In the Fig. 7.279 m_1 , m_2 , and M are 20 kg, 5 kg, and 50 kg, respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the spring are massless. The string is perfectly horizontally between P_1 and P_2 and also between P_2 and m_2 . The string is perfectly vertical between P_1 and P_2 . An external horizontal force F is applied to mass M . Take $g = 10 \text{ m/s}^2$ between P_1 and P_2 . An external horizontal force F is applied to mass M . Take $g = 10 \text{ m/s}^2$

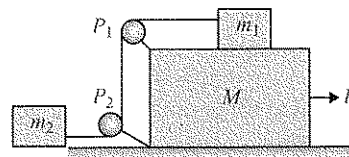


Fig. 7.279

1. Draw a free-body diagram of mass m , clearly showing all the forces.
2. Let the magnitude of the force of the friction between m_1 and M be f_1 and that between m_2 and ground be f_2 . For a particular F it is found that $f_1 = 2f_2$. Find f_1 and f_2 .

Write down equations of motion of all the masses. Find F , tension in the string and the accelerating of the masses.

(IIT-JEE, 2000)

Sol.

1. Free body diagram is shown in the Fig. 7.280.

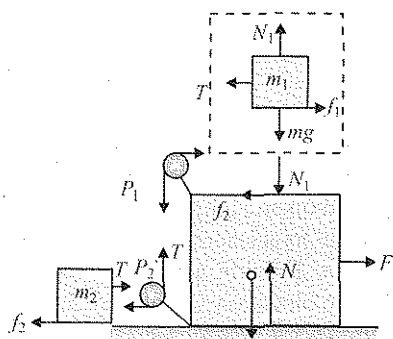


Fig. 7.280

2. Supposing all the blocks are in motion

- a. $f_{1\max} = \mu_1 N_1 = \mu_1 m_1 g = 0.3 \times 20 \times 10 = 60 \text{ N}$
 and $f_{2\max} = \mu_2 N_2 = \mu_2 m_2 g = 0.3 \times 5 \times 10 = 15 \text{ N}$
 Given that $f_1 = 2f_2 \therefore f_1 = 2 \times 15 = 30 \text{ N}$
 The block m_1 cannot move.

b. Let all the blocks are at rest

$F - f_1 = 0$ and $T - f_1 = 0$ and $T - f_2 = 0$
 which gives $f_1 = f_2$ which does not satisfy the given condition.

c. Since m_1 cannot move over the block M , therefore all the blocks move together and $f_1 = 30 \text{ N}$ and $f_2 = 15 \text{ N}$.

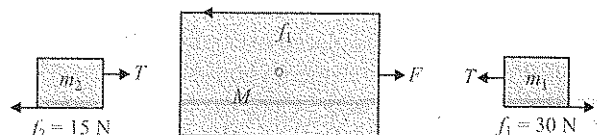


Fig. 7.281

$$30 - T = 20a \quad (i)$$

$$F - 30 = 50a, \text{ and} \quad (ii)$$

$$\text{and} \quad T - 15 = 5a \quad (iii)$$

After solving these equations, we get $a = \frac{3}{5} \text{ m/s}^2$,

$$T = 18 \text{ N}, F = 60 \text{ N}.$$

Example 7.16 Two block A and B of equal masses are placed on rough inclined plane as shown in Fig. 7.282. When and where will the two blocks come on the same line on the inclined plane if they are released simultaneously? Initially the block A is $\sqrt{2} \text{ m}$ behind the block B . Coefficient of kinetic friction for the blocks A and B are 0.2 and 0.3 , respectively ($g = 10 \text{ m/s}^2$). (IIT-JEE, 2004)

$$\text{Sol. } a = \frac{mg \sin \theta - \mu_k mg \cos \theta}{m}$$

$$a_A = g \sin \theta - \mu_{k,A} g \cos \theta, \text{ and} \quad (i)$$

$$a_B = g \sin \theta - \mu_{k,B} g \cos \theta \quad (ii)$$

Putting values we get

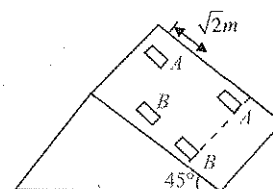


Fig. 7.282

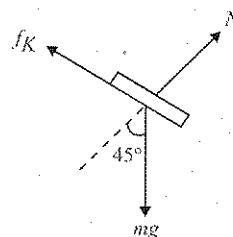


Fig. 7.283

$$a_A = \frac{0.89}{\sqrt{2}} \text{ and } a_B = \frac{0.79}{\sqrt{2}}$$

a_{AB} is relative acceleration of A w.r.t. $B = a_A - a_B$

$$L = \sqrt{2} \text{ m} \Rightarrow L = \frac{1}{2} \sqrt{2} m a_{A/B} t^2$$

[where L is the relative distance between A and B]

$$\text{or } t^2 = \frac{2L}{a_{A/B}} = \frac{2L}{a_A - a_B}$$

Putting values we get, $t^2 = 4$ or $t = 2 \text{ s}$.

Distance moved by B during that time is given by

$$S = \frac{1}{2} a_B t^2 = \frac{1}{2} \frac{0.79}{\sqrt{2}} \times 4 = \frac{2 \times 0.7}{\sqrt{2}} \times 10 = 7\sqrt{2} \text{ m}$$

Similarly for $A = 8\sqrt{2} \text{ m}$.

Example 7.17 A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The coefficient of friction between the block and all surfaces of groove in contact is $\mu = 2/5$. The disc has an acceleration of 25 m/s^2 (see Fig. 7.284). Find the acceleration of the block with respect to disc. (IIT-JEE, 2006)

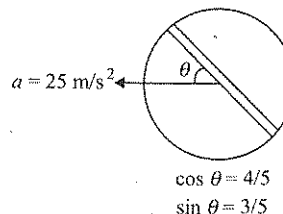


Fig. 7.284

Sol. Normal reaction in vertical direction $N_1 = mg$

Normal reaction from side to the groove $N_2 = ma \sin 37^\circ$

Therefore, acceleration of block with respect to disc

$$a_r = \frac{ma \cos 37^\circ - \mu N_1 - \mu N_2}{m}$$

Substituting the values we get,

$$a_r = -10 \text{ m/s}^2$$

EXERCISES

Subjective Type

Solutions on page 7.118

1. Block B , with mass m_B , rests on block A , with mass m_A , which in turn is on a horizontal tabletop (as shown in Fig. 7.285). The coefficient of kinetic friction between block A and the tabletop is μ_k , and the coefficient of static friction between block A and block B is μ_s . A light string attached to block A passes over a frictionless, massless pulley and block C is suspended from the other end of the string. What is the largest mass m_C that block C can have so that blocks A and B still slide together when the system is released from the rest?

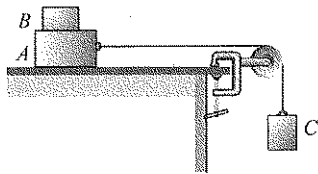


Fig. 7.285

2. A block of mass $m = 4$ kg is placed over a rough inclined plane as shown in Fig. 7.286. The coefficient of friction between the block and the plane is $\mu = 0.5$. A force $F = 10$ N is applied on the block at an angle of 30° . Find the contact force between the block and the plane.

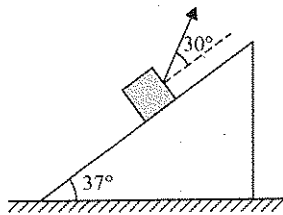


Fig. 7.286

3. A block of mass $m = 2$ kg is resting on a rough inclined plane of inclination 30° as shown in Fig. 7.287. The coefficient of friction between the block and the plane is $\mu = 0.5$. What minimum force F should be applied perpendicularly to the plane on the block, so that block does not slip on the plane?

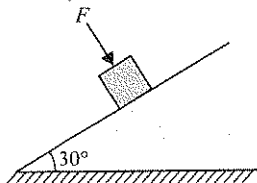


Fig. 7.287

4. Block A as shown in Fig. 7.288 weighs 1.40 N and block B weighs 4.20 N. The coefficient of kinetic friction between all the surfaces is 0.30. Find the magnitude of the horizontal force necessary to drag block B to the left at a constant speed if A and B are connected by a light, flexible cord passing around a fixed, frictionless pulley.

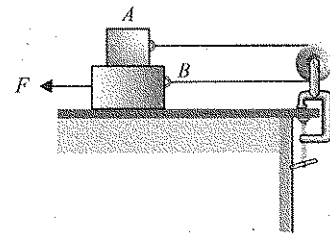


Fig. 7.288

5. A block is placed on an inclined plane moving towards right horizontally with an acceleration $a_0 = g$. The length of the plane $AC = 1$ m. Friction is absent everywhere. Find the time taken by the block to reach from C to A .

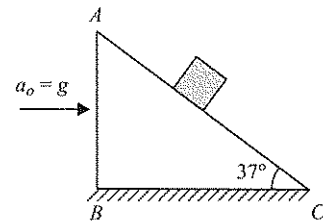


Fig. 7.289

6. You are designing an elevator for a hospital. The force exerted on a passenger by the floor of the elevator is not to exceed 1.60 times the passenger's weight. The elevator accelerates upward with a constant acceleration for a distance of 3.0 m and then starts to slow down. What is the maximum speed of the elevator?
7. A block A of mass m is placed over a plank B of mass $2m$. Plank B is placed over a smooth horizontal surface. The coefficient of friction between A and B is $1/2$. Block A is given a velocity v_0 towards right. Find acceleration of B relative to A .

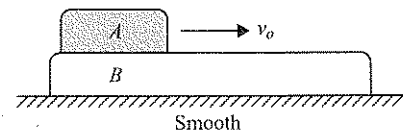


Fig. 7.290

8. Block A as shown in Fig. 7.291 has a mass of 4.00 kg and block B has mass 12.0 kg. The coefficient of kinetic friction between block B and the horizontal surface is 0.25.

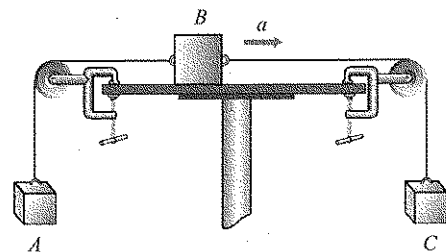


Fig. 7.291

- a. What is the mass of block C if block B is moving to the right and speeding up with an acceleration 2.00 m/s^2 ?
- b. What is the tension in each cord when block B has this acceleration?

9. Two blocks, with masses m_1 and m_2 , are stacked as shown in Fig. 7.292 and placed on a frictionless horizontal surface. There is a friction between the two blocks. An external force of magnitude F is applied to the top block at an angle below the horizontal.

- a. If the two blocks move together, find their acceleration.
- b. Calculate the maximum value of force so that the blocks will move together.

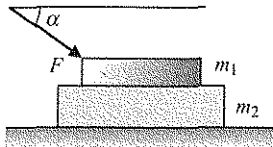


Fig. 7.292

10. A hot-air balloon consists of a basket, one passenger, and some cargo. Let the total mass be M . Even though there is an upward lift force on the balloon, the balloon is initially accelerating downward at a rate of $g/3$.

- a. Draw an FBD for the descending balloon.
- b. Find the upward lift force in terms of the initial total weight Mg .
- c. The passenger notices that he is heading straight for a waterfall and decides he needs to go up. What fraction of the total weight must he drop overboard so that the balloon accelerates upward at a rate of $g/2$? Assume that the upward lift force remains the same.

11. A student tries to raise a chain consisting of three identical links. Each link has a mass of 300 g. The three-piece chain is connected to a string and then suspended vertically with the student holding the upper end of the string and pulling upward. Because of the student's pull, an upward force of 12 N is applied to the chain by the string.

- a. Draw a free body diagram for each of the links in the chain and also for the entire chain considered as a single body.
- b. Use the results of part (a) and Newton's laws to find (i) the acceleration of the chain; and (ii) the force exerted by the top link on the middle link.

12. Two men of masses M and $M + m$ start simultaneously from the ground and climb with uniform accelerations up from the free ends of a massless inextensible rope which passes over a smooth pulley at a height h from the ground.

- a. Which man reaches the pulley first?
- b. If the man who reaches first takes time t to reach the pulley, then find the distance of the second man from the pulley at this instant.

13. Block A has a mass of 30 kg and block B a mass of 15 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.15$ and $\mu_k = 0.10$ (see Fig. 7.293). Knowing that $\theta = 30^\circ$ and that the magnitude of the force F applied to block A is 250 N, determine (1) acceleration of block A , and (2) the tension in the rope.

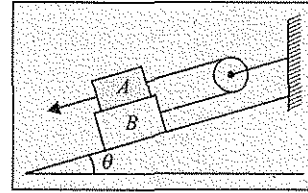


Fig. 7.293

14. With what force F a man pulls a rope in order to support the platform on which he stands, if the mass of man is 60 kg and that at platform is 20 kg. With what forces N does the man press the platform. What is the maximum weight of the platform that the man can support?

15. A monkey A (mass = 6 kg) is climbing up a rope tied to a rigid support. The monkey B (mass = 2 kg) is holding on the tail of monkey A (see Fig. 7.294). If the tail can tolerate a maximum tension of 30 N what force should monkey A apply on the rope in order to carry monkey B with it? ($g = 10 \text{ m/s}^2$)

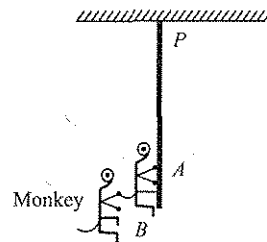


Fig. 7.294

16. Figure 7.295 represent a painter in a crate which hangs alongside a building. When the painter of mass 10 kg pulls the rope, the force exerted by him on the floor of the crate is 450 N. If the crate weighs 25 kg, find the acceleration and tension in the rope ($g = 10 \text{ m/s}^2$).

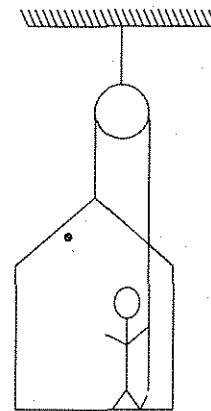


Fig. 7.295

17. A smooth ring A of mass m can slide on a fixed horizontal rod XY . A string tied to the ring passes over a fixed pulley B and carries a block C of mass $M (= 2m)$ as shown in Fig. 7.296. At an instant the string between the ring and pulley makes an angle θ with the rod. Now

- a. show that if the ring slides with speed v , the block descends with speed $v \cos \theta$.

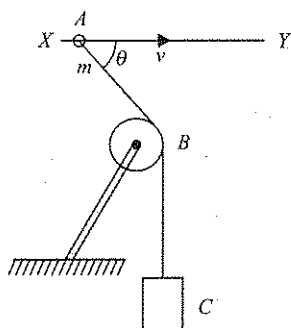


Fig. 7.296

- b. with what acceleration will the ring start moving if the system is released from rest with $\theta = 30^\circ$.
18. A rod AB of length 2 m is hinged at point A and B is attached to a platform on which a block of mass m is kept. Rod rotates about point A maintaining angle $\theta = 30^\circ$ with the vertical in such a way that platform remains horizontal and revolves on the horizontal circular path (see Fig. 7.297). If the coefficient of static friction between the block and platform is $\mu = 0.1$ then find the maximum angular velocity in rad/s of rod so that block does not slip on the platform. [$g = 10 \text{ m/s}^2$]

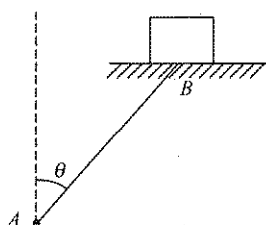


Fig. 7.297

19. A block weighing 20 kg is placed on a smooth surface. A weight of 2 kg is mounted on the block. The coefficient of friction between the block and the weight is 0.25. Calculate the acceleration of the block and the weight and also the frictional force between the block and the weight when a horizontal force of 2 N is applied to the weight as shown in the Fig. 7.298. What will these quantities be if the horizontal force is 20 N? ($g = 10 \text{ ms}^{-2}$).

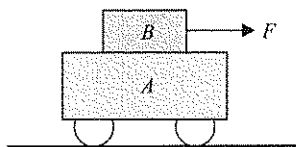


Fig. 7.298

20. In Fig. 7.299, find the acceleration of m assuming that there is friction between m and M , and all other surfaces are smooth and pulleys light and $\mu =$ coefficient of friction between m and M .
21. A block A of mass M rests on a smooth horizontal surface over which it can move without friction. A cube B of mass m lies on the block at one edge. The coefficient of friction between the block and the cube is k (see Fig. 7.300). At what force F applied to the block in the horizontal direction will

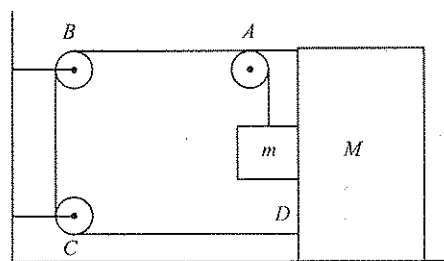


Fig. 7.299

the cube begin to slide towards the other end of the block? In what time will the cube fall from the block if the length of the latter is l ?

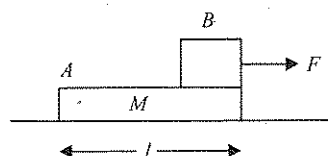


Fig. 7.300

22. Two bars 1 and 2 are placed on an inclined plane forming an angle α with the horizontal (see Fig. 7.301). The masses of the bars are equal to m_1 and m_2 , and the coefficient of friction between the plane and the bars are equal to k_1 and k_2 respectively, with $k_1 > k_2$. Find:
- the force of interaction of the bars in the process of motion,
 - the minimum value of α at which the bars start sliding down.

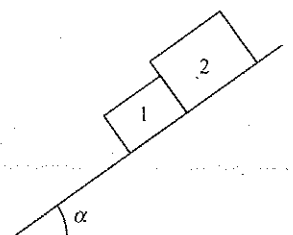


Fig. 7.301

23. A block A , of weight W , slides down an inclined plane S of slope 37° at a constant velocity while the plank B , also of weight W ; rests on top of A (Fig. 7.302). The plank B is attached by a cord to the top of the plane. If the coefficient of kinetic friction is the same between the surfaces A and B and between the surfaces A and B and S and A , determine its value.

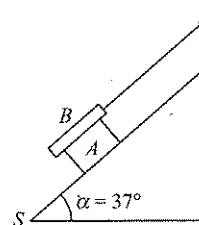


Fig. 7.302

24. A particle A of mass $2m$ is held on a smooth horizontal table and is attached to one end of an inelastic string which runs

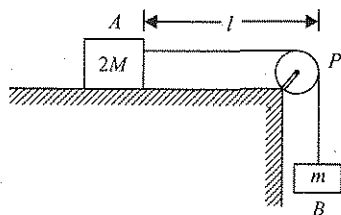


Fig. 7.303

over a smooth light pulley at the edge of the table. At the other end of the string there hangs another particle B of mass m , the distance from A to the pulley is l (Fig. 7.303). The particle A is then projected towards the pulley with velocity u . Find

- the time before the string becomes taut; and show that after the string becomes taut, the initial velocity of A and B is $4u/3$.
 - the common velocity when A reaches the pulley (assume that B has not yet reached the ground).
25. The masses of the blocks A and B are m and M . Between A and B there is a constant frictional force F , but B can slide frictionlessly on the horizontal surface (Fig. 7.304). A is set in motion with velocity v_0 while B is at rest. What is the distance moved by A relative to B before they move with the same velocity?

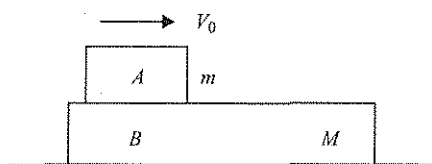


Fig. 7.304

26. A smooth pulley A of mass M_0 is lying on a frictionless table. A massless rope passes round the pulley and has masses M_1 and M_2 tied to its ends, the two portions of the string being perpendicular to the edge of the table so that the masses hang vertically (see Fig. 7.305). Find the acceleration of the pulley.

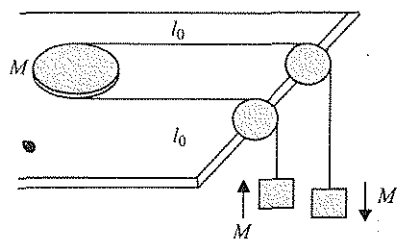


Fig. 7.305

27. Two blocks of masses m and M are connected by a chord passing around a frictionless pulley which is attached to a rotating frame, which rotates about a vertical axis with an angular velocity ω (see Fig. 7.306). If the coefficient of friction between the two masses and the surface be μ_1 and μ_2 , respectively, determine the value of ω , at which the block starts sliding radially ($M > m$).
28. A block with mass m_1 is placed on an inclined plane with a slope angle α and is connected to a second hanging block with mass m_2 by a cord passing over a small, frictionless

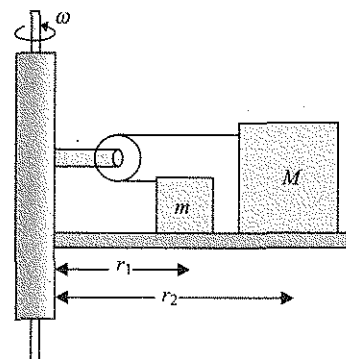


Fig. 7.306

pulley as shown in Fig. 7.307. The coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k .

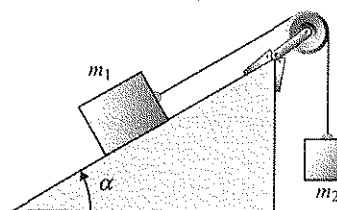


Fig. 7.307

- Find the mass m_2 for which block m_1 moves up the plane at constant speed once it is set in motion.
- Find the mass m_2 for which block m_1 moves down the plane at constant speed once it is set in motion.
- For what range of values of m_2 will the blocks remain at rest if they are released from rest?

Objective Type

Solutions on page 7.125

- When a body is stationary
 - there is no force acting on it
 - the forces acting on its are not in contact with it
 - the combination of forces acting on it balance each other
 - the body is in vacuum
- A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet releasing water at a rate of 1 kg/s and a speed of 5 m/s. The initial acceleration of the block will be

a. 2.5 m/s^2	b. 5 m/s^2
c. 10 m/s^2	d. 20 m/s^2
- Two persons are holding a rope of negligible weight tightly at its ends so that it is horizontal. A 15 kg weight is attached to the rope at the mid point which how no longer remains horizontal. The minimum tension required to completely straighten the rope is

a. 15 kg	b. $15/2 \text{ kg}$
c. 5 kg	d. Infinitely large

4. Three equal weights A , B , and C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the Fig. 7.308. The tension in the string connecting weights B and C is

- a. zero
c. 3.3 N
- b. 13 N
d. 19.6 N

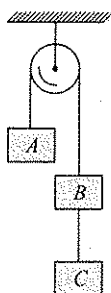


Fig. 7.308

5. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m . Force P is applied at one end of rope. The force which the rope exerts on the block is

- a. $\frac{P}{(M - m)}$ b. $\frac{P}{M(m + M)}$
c. $\frac{PM}{(m + M)}$ d. $\frac{PM}{(M - m)}$

6. The elevator shown in Fig. 7.309 is descending with an acceleration of 2 ms^{-2} . The mass of the block $A = 0.5 \text{ kg}$. The force exerted by the block A on the block B is (take $g = 10 \text{ m/s}^2$)

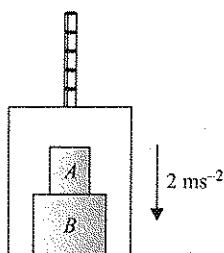


Fig. 7.309

- a. 2 N b. 4 N c. 6 N d. 8 N

7. A mass M is suspended by a rope from a rigid support at A as shown in Fig. 7.310. Another rope is tied at the end B and it is pulled horizontally with a force F . If the rope AB make an angle θ with the vertical, then the tension in the string AB is

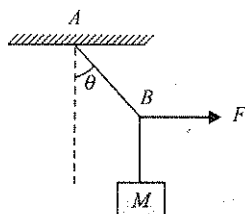


Fig. 7.310

- a. $F \sin \theta$
c. $F \cos \theta$

8. Two bodies of mass 4 kg and 6 kg are attached to the ends of a string passing over a pulley (see Fig. 7.311). The 4 kg mass is attached to the table top by another string. The tension in this string T_1 is equal to (take $g = 10 \text{ m/s}^2$)

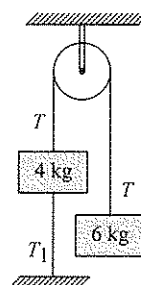


Fig. 7.311

- a. 20 N b. 25 N c. 10.6 N d. 10 N

9. In the Fig. 7.312, the pulley P_1 is fixed and the pulley P_2 is movable. If $W_1 = W_2 = 100 \text{ N}$, what is the angle $\angle AP_2P_1$? The pulleys are frictionless

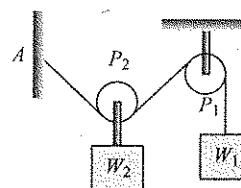


Fig. 7.312

- a. 30° b. 60°
c. 90° d. 120°

10. A man sits on a chair supported by a rope passing over a frictionless fixed pulley. The man who weighs 1,000 N exerts a force of 450 N on the chair downwards while pulling the rope on the other side. If the chair weighs 250 N, then the acceleration of the chair is

- a. 0.45 m/s^2 b. 0
c. 2 m/s^2 d. $9/25 \text{ m/s}^2$

11. In the Fig. 7.313, the ball A is released from rest, when the spring is at its natural (unstretched) length. For the block B of mass M to leave contact with ground at some stage, the minimum mass of A must be

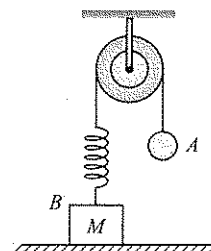


Fig. 7.313

- a. $2M$ b. M
c. $M/2$ d. $M/4$
12. Two skaters weighing in the ratio 4 : 5 and 9 m apart are skating on a smooth frictionless surface. They pull on a rope stretched between them. The ratio of the distance covered by them when they meet each other will be
- a. 5 : 4 b. 4 : 5
c. 25 : 16 d. 16 : 25
13. Three forces are acting on a particle of mass m initially in equilibrium. If the first 2 forces (R_1 and R_2) are perpendicular to each other and suddenly the third force (R_3) is removed, then the acceleration of the particle is
- a. $\frac{R_3}{m}$ b. $\frac{R_1 + R_2}{m}$
c. $\frac{R_1 - R_2}{m}$ d. $\frac{R_1}{m}$
14. n balls each of mass m impinge elastically each second on a surface with velocity u . The average force experienced by the surface will be
- a. $mn u$ b. $2mn u$
c. $4mn u$ d. $mn u/2$
15. A ball of mass m moving with a velocity u rebounds from a wall. The collision is assumed to be elastic and the force of interaction between the ball and wall varies as shown in the Fig. 7.314. Then the value of F_0 is

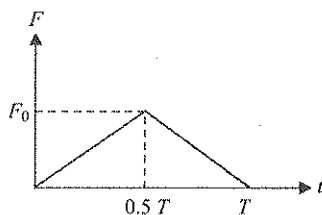


Fig. 7.314

- a. mu/T b. $2mu/T$
c. $4mu/T$ d. $mu/2T$
16. A unidirectional force F varying with time t as shown in the Fig. 7.315 acts on a body initially at rest for a short duration $2T$. Then the velocity acquired by the body is

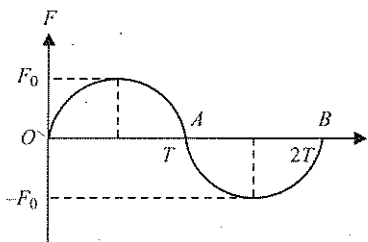


Fig. 7.315

- a. $\frac{\pi F_0 T}{4m}$ b. $\frac{\pi F_0 T}{2m}$
c. $\frac{F_0 T}{4m}$ d. zero

17. A body of mass 2 kg has an initial velocity of 3 m/s along OE and it is subjected to a force of 4 N in a direction perpendicular to OE (see Fig. 7.316). The distance of body from O after 4 s will be

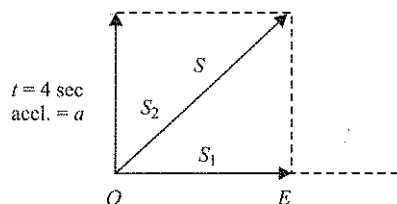


Fig. 7.316

- a. 12 m b. 20 m
c. 8 m d. 48 m
18. In order to raise a mass of 100 kg a man of mass 60 kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with acceleration $5g/4$ relative to the rope (see Fig. 7.317). The tension in the rope is (take $g = 10 \text{ m/s}^2$).

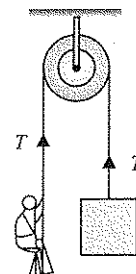


Fig. 7.317

- a. 1432 N b. 928 N
c. 1219 N d. 642 N
19. A plumb bob is hung from the ceiling of a train compartment. The train moves on an inclined track of inclination 30° with horizontal. Acceleration of train up the plane is $a = g/2$. The angle which the string supporting the bob makes with normal to the ceiling in equilibrium is
- a. 30° b. $\tan^{-1}(2/\sqrt{3})$
c. $\tan^{-1}(\sqrt{3}/2)$ d. $\tan^{-1}(2)$
20. Two particles A and B, each of mass m , are kept stationary by applying a horizontal force $F = mg$ on particle B as shown in Fig. 7.318. Then

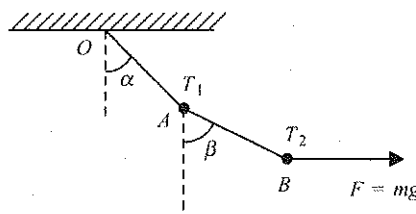


Fig. 7.318

a. $2 \tan \beta = \tan \alpha$

b. $2T_1 = 5T_2$

c. $T_1 = T_2$

d. None of these

21. A lift is going up, the total mass of the lift and the passengers is 1500 kg. The variation in the speed of lift is shown in Fig. 7.319. Then the tension in the rope at $t = 1$ s will be

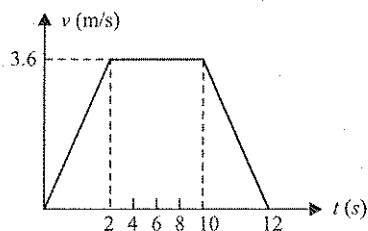


Fig. 7.319

- a. 17400 N
b. 14700 N
c. 12000 N
d. None of the above
22. In the above problem the tension in the rope will be least at
- a. $t = 1$ s
b. $t = 4$ s
c. $t = 9$ s
d. $t = 11$ s
23. A block is placed on a rough horizontal plane attached with an elastic spring as shown in Fig. 7.320

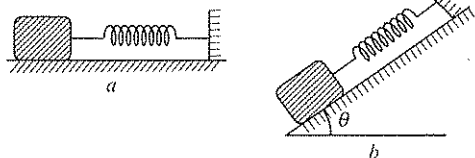
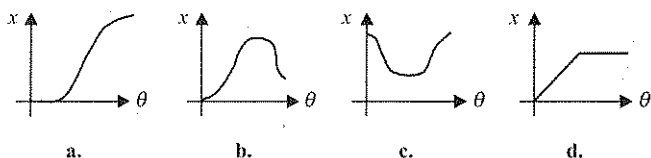


Fig. 7.320

Initially spring is unscratched. If the plane is gradually lifted from $\theta = 0^\circ$ to $\theta = 90^\circ$, then the graph showing extension in the spring (x) versus angle (θ) is



- a. $\frac{\alpha}{\alpha + g} M$
b. $\frac{2\alpha}{\alpha + g} M$
c. $\frac{\alpha + g}{\alpha} M$
d. $\frac{\alpha + g}{2\alpha} M$
24. A balloon of mass M is descending at a constant acceleration α . When a mass m is released from the balloon it starts rising with the same acceleration a . Assuming that its volume does not change, what is the value of m ?

- a. $(M + m)g \tan \beta$
b. $g \tan \beta$
c. $mg \cos \beta$
d. $(M + m)g \operatorname{cosec} \beta$

25. The system shown in Fig. 7.321 is released from rest. The spring gets elongated

a. if $M > m$

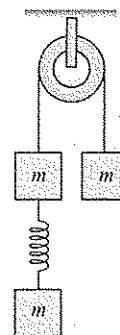


Fig. 7.321

- b. if $M > 2m$
c. if $M > m/2$
d. For any value of M (Neglect friction and masses of pulley, string and spring)
26. A trolley T of mass 5 kg on a horizontal smooth surface is pulled by a load of 2 kg through a uniform rope ABC of length $2m$ and mass 1 kg (see Fig. 7.322). As the load falls from $BC = 0$ to $BC = 2m$, its acceleration (in m/s^2) changes from

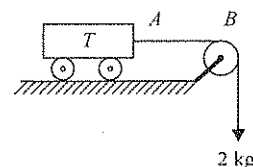


Fig. 7.322

- a. $\frac{20}{6}$ to $\frac{30}{6}$
b. $\frac{20}{8}$ to $\frac{30}{8}$
c. $\frac{20}{5}$ to $\frac{30}{6}$
d. none of these
27. Two wooden blocks are moving on a smooth horizontal surface such that the mass m remains stationary with respect to block of mass M as shown in the Fig. 7.323. The magnitude of force P is

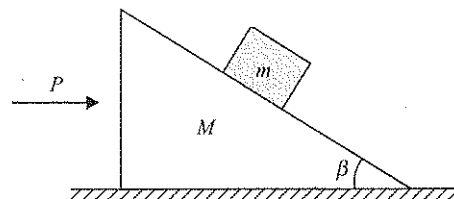


Fig. 7.323

- a. $(M + m)g \tan \beta$
b. $g \tan \beta$
c. $mg \cos \beta$
d. $(M + m)g \operatorname{cosec} \beta$
28. A bead of mass m is attached to one end of a spring of natural length R and spring constant $K = \frac{(\sqrt{3} + 1)mg}{R}$. The other end of the spring is fixed at a point A on a smooth vertical ring of radius R as shown in the Fig. 7.324. The normal reaction at B just after it is released to move is

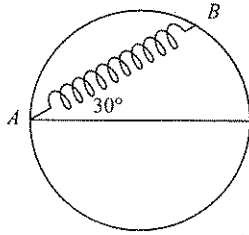


Fig. 7.324

- a. $mg/2$
 b. $\sqrt{3}mg$
 c. $3\sqrt{3}mg$
 d. $\frac{3\sqrt{3}mg}{2}$

29. An inclined plane makes an angle 30° with the horizontal. A groove (OA) of length 5 m cut, in the plane makes an angle 30° with OX . A short smooth cylinder is free to slide down the influence of gravity. The time taken by the cylinder to reach from A to O is ($g = 10 \text{ m/s}^2$)

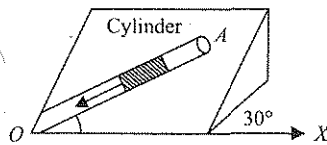


Fig. 7.325

- a. 4 s b. 2 s c. 2 s d. 1 s

30. A man is raising himself and the crate on which he stands with an acceleration of 5 m/s^2 by a massless rope-and-pulley arrangement. Mass of the man is 100 kg and that of the crate is 50 kg. If $g = 10 \text{ m/s}^2$, then the tension in the rope is

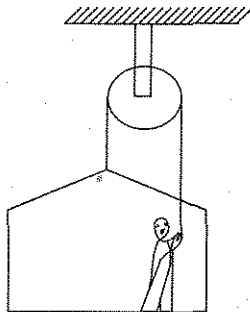


Fig. 7.326

- a. 2250 N b. 1125 N
 c. 750 N d. 375 N

31. In question 30, contact force between man and the crate is
 a. 2250 N b. 1125 N
 c. 750 N d. 375 N

32. Two objects A and B each of mass m are connected by a light inextensible string. They are restricted to move on a frictionless ring of radius R in a vertical plane (as shown in Fig. 7.327). The objects are released from rest at the position shown. Then, the tension in the cord just after release is

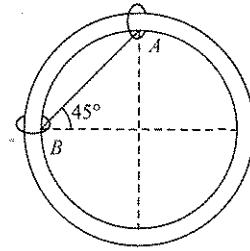


Fig. 7.327

- a. 0 b. mg
 c. $\sqrt{2}mg$ d. $mg/\sqrt{2}$

33. Blocks A and C start from rest and move to the right with acceleration $a_A = 12t \text{ m/s}^2$ and $a_C = 3 \text{ m/s}^2$. Here t is in seconds. The time when block B again comes to rest is

- a. 2 s b. 1 s
 c. $3/2 \text{ s}$ d. $1/2 \text{ s}$

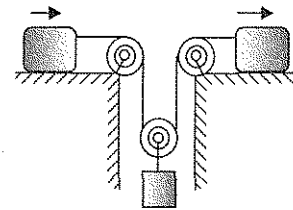


Fig. 7.328

34. In the given Fig. 7.329 the mass m_2 starts with velocity v_0 and moves with constant velocity on the surface. During motion the normal reaction between the horizontal surface and fixed triangle block m_1 is N . Then during motion

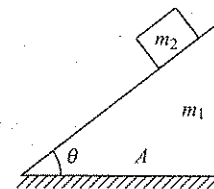


Fig. 7.329

- a. $N = (m_1 + m_2)g$
 b. $N = m_1g$
 c. $N < (m_1 + m_2)g$
 d. $N > (m_1 + m_2)g$

35. Figure 7.330 shows an arrangement in which three identical blocks are joined together with an inextensible string. All the surfaces are smooth and pulleys are massless. If a_A , a_B , and a_C are the respective accelerations of the blocks A , B , and C , then the value of a_B in terms of a_A and a_C is

- a. $a_A a_C$ b. $\frac{a_A - a_C}{2}$
 c. $\frac{a_A + a_C}{2}$ d. $a_A + a_C$

36. In the Fig. 7.331 shown, blocks A and B move with velocities v_1 and v_2 along horizontal direction. The ratio of $\frac{v_1}{v_2}$ is

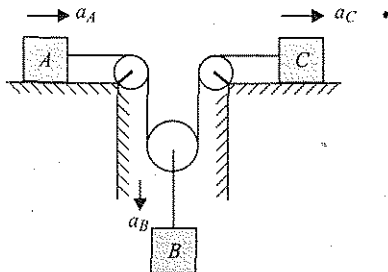


Fig. 7.330

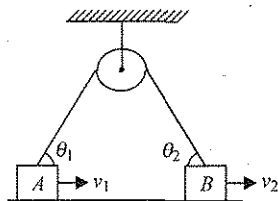


Fig. 7.331

a. $\frac{\sin \theta_1}{\sin \theta_2}$
c. $\frac{\cos \theta_2}{\cos \theta_1}$

b. $\frac{\sin \theta_2}{\sin \theta_1}$
d. $\frac{\cos \theta_1}{\cos \theta_2}$

37. Assuming that the block is always remains horizontal, hence the acceleration of B is

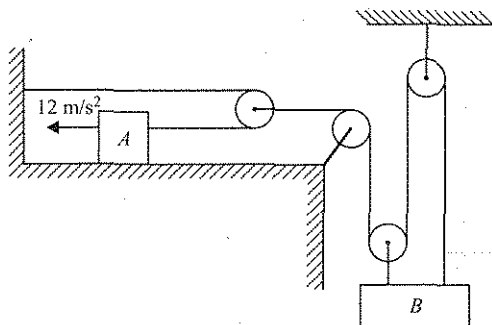


Fig. 7.332

a. 6 m/s^2
c. 4 m/s^2

b. 2 m/s^2
d. None of these

38. If the block B moves towards right with acceleration b , then the net acceleration of block A is

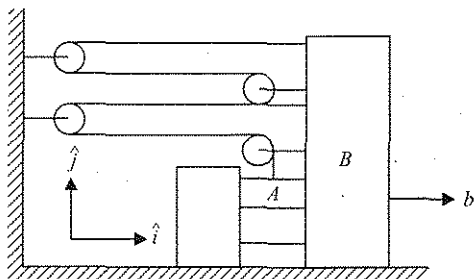


Fig. 7.333

a. $b\hat{i} + 4b\hat{j}$

b. $b\hat{i} + b\hat{j}$

c. $b\hat{i} + 2b\hat{j}$

d. None of these

39. If the blocks A and B are moving towards each other with acceleration a and b as shown in the Fig. 7.334. Find the net acceleration of block C.

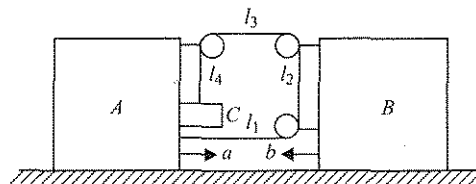


Fig. 7.334

a. $a\hat{i} - 2(a+b)\hat{j}$

b. $-(a+b)\hat{j}$

c. $a\hat{i} - (a+b)\hat{j}$

d. None of these

40. The small marble is projected with a velocity of 10 m/s in a direction 45° from the horizontal y -direction on the smooth inclined plane. Calculate the magnitude v of its velocity after 2 s .

a. $10\sqrt{2} \text{ m/s}$

b. 5 m/s

c. 10 m/s

d. $5\sqrt{2} \text{ m/s}$

41. Two masses each equal to m are lying on X -axis at $(-a, 0)$ and $(+a, 0)$, respectively, as shown in Fig. 7.335. They are connected by a light string. A force F is applied at the origin along vertical direction. As a result, the masses move towards each other without losing contact with ground. What is the acceleration of each mass? Assume the instantaneous position of the masses as $(-x, 0)$ and $(x, 0)$, respectively

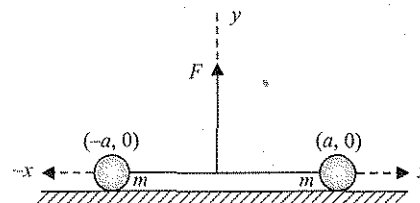


Fig. 7.335

a. $\frac{2F}{m} \frac{\sqrt{(a^2 - x^2)}}{x}$

b. $\frac{2F}{m} \frac{x}{\sqrt{(a^2 - x^2)}}$

c. $\frac{F}{2m} \frac{x}{\sqrt{(a^2 - x^2)}}$

d. $\frac{F}{m} \frac{x}{\sqrt{(a^2 - x^2)}}$

42. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $(g/8)$, then the ratio of masses is

a. $5 : 3$

b. $4 : 3$

c. $9 : 7$

d. $8 : 1$

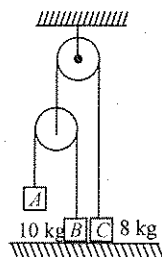


Fig. 7.338

55. As shown in the Fig. 7.339, if acceleration of M with respect to ground is 2 m/s^2 , then

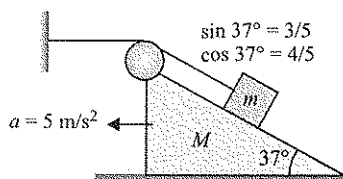


Fig. 7.339

- Acceleration of m with respect to M is 5 m/s^2 .
 - Acceleration of m with respect to ground is 5 m/s^2 .
 - Acceleration of m with respect to M is 2 m/s^2 .
 - Acceleration of m with respect to ground is 10 m/s^2 .
56. A force-time graph for the motion of a body is shown in the Fig. 7.340. The change in the momentum of the body between zero and 10 s is

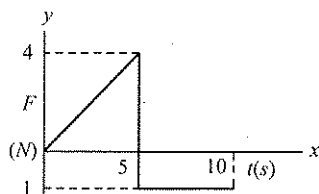


Fig. 7.340

- 15 kg m/s
 - 4 kg m/s
 - 3 kg m/s
 - 5 kg m/s
57. A block A has a velocity of 0.6 m/s to the right, determine the velocity of cylinder B.

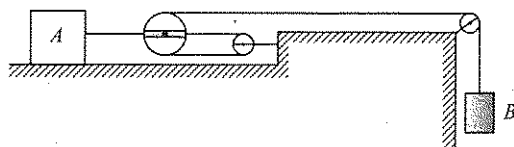


Fig. 7.341

- 1.2 m/s
 - 2.4 m/s
 - 1.8 m/s
 - 3.6 m/s
58. For the pulley system shown in Fig. 7.342, each of the cables at A and B is given a velocity of 2 m/s in the direction of the arrow. Determine the upward velocity v of the load m .
- 1.5 m/s
 - 3 m/s
 - 6 m/s
 - 4.5 m/s

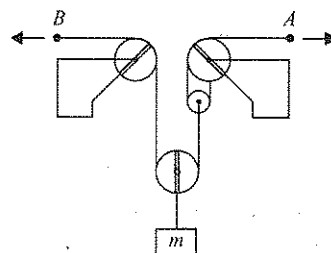


Fig. 7.342

59. A man pulls himself up the 30° incline by the method shown in Fig. 7.343. If the combined mass of the man and cart is 100 kg , determine the acceleration of the cart if the man exerts a pull of 250 N on the rope. Neglect all friction and the mass of the rope, pulleys and wheels.

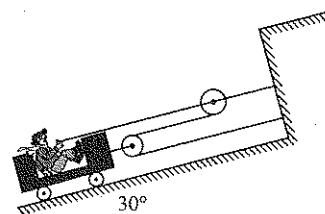


Fig. 7.343

- 4.5 m/s^2
 - 2.5 m/s^2
 - 3.5 m/s^2
 - 1.5 m/s^2
60. A painter of mass M stands on a platform of mass m and pulls himself up by two ropes which hang over pulley as shown in Fig. 7.344. He pulls each rope with force F and moves upward with a uniform acceleration a . Find a neglecting the fact that no one could do this for long time.

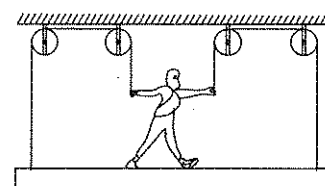


Fig. 7.344

- $\frac{4F + (2M + m)g}{M + 2m}$
 - $\frac{4F + (M + m)g}{M + 2m}$
 - $\frac{4F - (M + m)g}{M + m}$
 - $\frac{4F - (M + m)g}{2M + m}$
61. An object is resting at the bottom of the two strings which are inclined at an angle of 120° with each other. Each string can withstand a tension of 20 N . The maximum weight of the object that can be sustained without breaking the string is
- 10 N
 - 20 N
 - $20\sqrt{2} \text{ N}$
 - 40 N

- a. 0.1 b. 1 c. 10 d. 5

73. A block of mass 1 kg is at rest on a horizontal table. The coefficient of static friction between the block and the table is 0.50. If $g = 10 \text{ ms}^{-2}$, then the magnitude of a force acting upwards at an angle of 60° from the horizontal that will just start the block moving is:

- a. 5 N b. 5.36 N
c. 74.6 N d. 10 N

74. A heavy uniform chain lies on a horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain, that can hang over one edge of the table is

- a. 20 % b. 25 %
c. 35 % d. 15 %

75. In the Fig. 7.348, a block of weight 60 N is placed on a rough surface. The coefficient of friction between the block and the surfaces is 0.5. What should be the weight W such that the block does not slip on the surface?

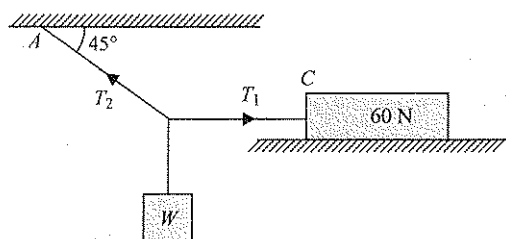


Fig. 7.348

- a. 60 N b. $\frac{60}{\sqrt{2}}$ N
c. 30 N d. $\frac{30}{\sqrt{2}}$ N

76. A suitcase is gently dropped on a conveyor belt moving at a velocity of 3 m/s. If the coefficient of friction between the belt and the suitcase is 0.5, find the displacement of the suitcase relative to conveyor belt before the slipping between the two is stopped ($g = 10 \text{ m/s}^2$)

- a. 2.7 m b. 1.8 m
c. 0.9 m d. 1.2 m

77. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2 (Fig. 7.349). The weight of the block is

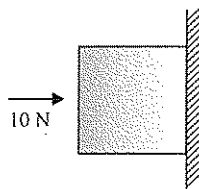


Fig. 7.349

- a. 2 N b. 20 N
c. 50 N d. 100 N

78. Two blocks of mass M_1 and M_2 are connected with a string which passes over a smooth pulley. The mass M_1 is placed on a rough incline plane as shown in the Fig. 7.350. The coefficient of friction between the block and the inclined plane is μ . What should be the minimum mass M_2 so that the block M_1 slides upwards?

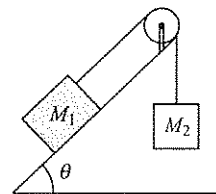


Fig. 7.350

- a. $M_2 = M_1(\sin \theta + \mu \cos \theta)$
b. $M_2 = M_1(\sin \theta - \mu \cos \theta)$
c. $M_2 = \frac{M_1}{\sin \theta + \mu \cos \theta}$
d. $M_2 = \frac{M_1}{\sin \theta - \mu \cos \theta}$

79. A box of mass 8 kg is placed on a rough inclined plane of inclination θ . Its downward motion can be prevented by applying an upward pull F and it can be made to slide upwards by applying a force $2F$. The coefficient of friction between the box and the inclined plane is

- a. $(\tan \theta)/3$ b. $3 \tan \theta$
c. $(\tan \theta)/2$ d. $2 \tan \theta$

80. A block of mass 15 kg is resting on a rough inclined plane as shown in Fig. 7.351. The block is tied by a horizontal string which has a tension of 50 N. The coefficient of friction between the surfaces of contact is

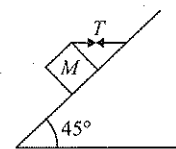


Fig. 7.351

- a. 1/2 b. 2/3 c. 3/4 d. 1/4

81. A horizontal force, just sufficient to move a body of mass 4 kg lying on a rough horizontal surface, is applied on it. The coefficient of static and kinetic friction between the body and the surface are 0.8 and 0.6, respectively. If the force continues to act even after the block has started moving, the acceleration of the block in m/s^2 is ($g = 10 \text{ m/s}^2$)

- a. 1/4 b. 1/2 c. 2 d. 4

82. Blocks A and B in the Fig. 7.352 are connected by a bar of negligible weight. Mass of each block is 170 kg and $\mu_A = 0.2$ and $\mu_B = 0.4$, where μ_A and μ_B are the coefficients of limiting friction between blocks and plane. Calculate the force developed in the bar ($g = 10 \text{ m/sec}^2$).

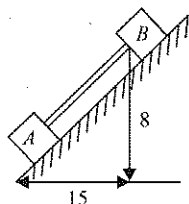


Fig. 7.352

- a. 150 N b. 75 N
c. 200 N d. 250 N

83. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by

- a. $2\tan \phi$ b. $\tan \phi$
c. $2\sin \phi$ d. $2\cos \phi$

84. A block of mass m is placed on another block of mass M which itself is lying on a horizontal surface (see Fig. 7.353). The coefficient of friction between two block is μ_1 and that between the block of block M and horizontal surface is μ_2 . What maximum horizontal force can be applied to the lower block so that the two blocks move without separation?

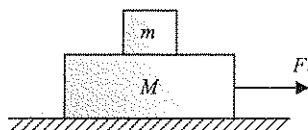


Fig. 7.353

- a. $(M + m)(\mu_2 - \mu_1)g$
b. $(M - m)(\mu_2 - \mu_1)g$
c. $(M - m)(\mu_2 + \mu_1)g$
d. $(M + m)(\mu_2 + \mu_1)g$

85. Two blocks A and B of masses 6 kg and 3 kg rest on a smooth horizontal surface as shown in the Fig. 7.354. If coefficient of friction between A and B is 0.4, the maximum horizontal force which can make them without separation is

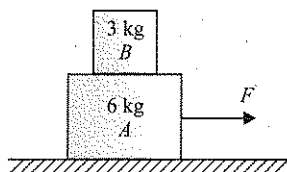


Fig. 7.354

- a. 72 N b. 40 N
c. 36 N d. 20 N

86. Two blocks of masses M_1 and M_2 are connected with a string passing over a pulley as shown in Fig. 7.355. The block M_1 lies on a horizontal surface. The coefficient of friction between the block M_1 and the horizontal surface is μ . The system accelerates. What additional mass m should be placed on the block M_1 so that the system does not accelerate?

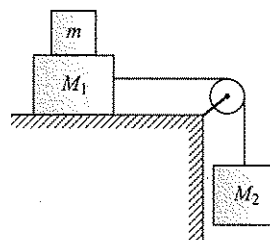


Fig. 7.355

- a. $\frac{M_2 - M_1}{\mu}$ b. $\frac{M_2}{\mu} - M_1$
c. $M_2 - \frac{M_1}{\mu}$ d. $(M_2 - M_1)\mu$

87. A block of mass m is placed on the top of another block of mass M as shown in the Fig. 7.356. The coefficient of friction between them is μ . What is the maximum acceleration with which the block M may move so that m also moves along with it?

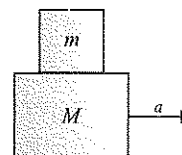


Fig. 7.356

- a. μg b. g/μ c. μ^2/g d. g/μ^2

88. The system is pushed by a force F as shown in Fig. 7.357. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ . Minimum value of F to prevent block B from down ward slipping is

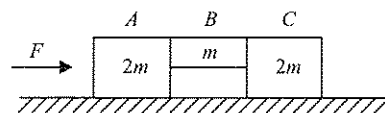


Fig. 7.357

- a. $\left(\frac{3}{2\mu}\right)mg$ b. $\left(\frac{5}{2\mu}\right)mg$
c. $\left(\frac{5}{2}\right)\mu mg$ d. $\left(\frac{3}{2}\right)\mu mg$

89. A body of mass M is resting on a rough horizontal plane surface, the coefficient of friction being equal to μ . At $t = 0$ a horizontal force $F = F_0 t$ starts acting on it, where F_0 is a constant. Find the time T at which the motion starts?

- a. $\mu Mg/F_0$ b. $Mg/\mu F_0$
 c. $\mu F_0/Mg$ d. None of these

90. The maximum value of mass of block C so that neither A nor B moves is (see Fig. 7.358) (Given that mass of A is 100 kg and that of B is 140 kg. Pulleys are smooth and friction coefficient between A and B and between B and horizontal surface is $\mu = 0.3$.) Take $g = 10 \text{ m/s}^2$.

- a. 210 kg b. 190 kg
 c. 185 kg d. 162 kg

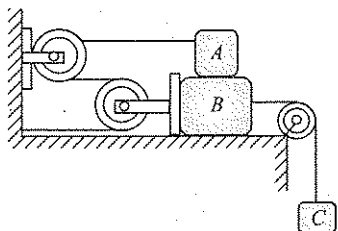


Fig. 7.358

91. A block A of mass 2 kg is placed over another block B of mass 4 kg which is placed over a smooth horizontal floor. The coefficient of friction between A and B is 0.4. When a horizontal force of magnitude 10 N is applied on A , the acceleration of blocks A and B are

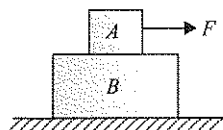


Fig. 7.359

- a. 1 ms^{-2} and 2 ms^{-2} , respectively.
 b. 5 ms^{-2} and 2.5 ms^{-2} , respectively.
 c. Both the blocks will move together with acceleration $1/3 \text{ ms}^{-2}$.
 d. Both the blocks will move together with acceleration $5/3 \text{ ms}^{-2}$.
92. Two blocks m and M tied together with an inextensible string are placed at rest on a rough horizontal surface with coefficient of friction μ . The block m is pulled with a variable force F at a varying angle θ with the horizontal. The value of θ at which the least value of F is required to move the blocks is given by

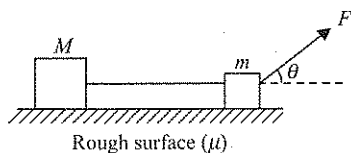


Fig. 7.360

- a. $\theta = \tan^{-1} \mu$ b. $\theta > \tan^{-1} \mu$
 c. $\theta < \tan^{-1} \mu$ d. Insufficient data
93. A trolley A has a simple pendulum suspended from a frame fixed to its desk. A block B is in contact on its vertical slide. The trolley is on horizontal rails and accelerates towards the right such that the block is just prevented from falling. The value of coefficient of friction between A and B is 0.5 (see Fig. 7.361). The inclination of the pendulum to the vertical is

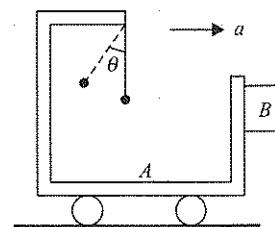


Fig. 7.361

- a. $\tan^{-1} \left(\frac{1}{2} \right)$ b. $\tan^{-1} (3)$
 c. $\tan^{-1} (\sqrt{2})$ d. $\tan^{-1} (2)$

94. Two block M and m are arranged as shown in the Fig. 7.362. The coefficient of friction between the blocks $\mu_1 = 0.25$ and between the ground and M be $\mu_2 = \frac{1}{3}$. If $M = 8 \text{ kg}$ then find the value of m so that the system will remain at rest.

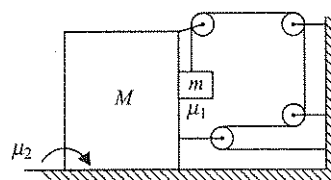


Fig. 7.362

- a. $\frac{4}{3} \text{ kg}$ b. $\frac{8}{9} \text{ kg}$
 c. 1 kg d. $\frac{8}{5} \text{ kg}$

95. Find the minimum force required to pull the lower block. If the coefficient of friction between the blocks is 0.1 and between the ground and 2 kg block is 0.2.
- a. 1 N b. 5 N c. 7 N d. 10 N
96. A body of mass m is launched up on a rough inclined plane making an angle 45° with horizontal. If the time of ascent is half of the time of descent, the frictional coefficient between plane and body is
- a. $\frac{2}{5}$ b. $\frac{3}{5}$ c. $\frac{3}{4}$ d. $\frac{4}{5}$
97. A wooden block of mass M resting on a rough horizontal floor is pulled with a force F at an angle ϕ with the horizontal. If μ is the coefficient of kinetic friction between the block and the surface, then acceleration of the block is

- a. $\frac{F}{M} (\cos \phi - \mu \sin \phi) - \mu g$
 b. $\frac{\mu F}{M} \cos \phi$
 c. $\frac{F}{M} (\cos \phi + \mu \sin \phi) - \mu g$
 d. $\frac{F}{M} \sin \phi$

98. A given object takes n times more time to slide down 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

a. $\sqrt{\frac{1}{1-n^2}}$ b. $\sqrt{1-\frac{1}{n^2}}$
 c. $1-\frac{1}{n^2}$ d. $\frac{1}{2-n^2}$

99. A passenger is travelling in a train which is moving at 40 m/s. His suitcase is kept on the berth. The driver of the train applies breaks such that the speed of the train decreases at a constant rate to 20 m/s in 5 s. What should be the minimum coefficient of friction between the suitcase and the berth if the suitcase is not to slide during retardation of the train?

a. 0.3 b. 0.5 c. 0.1 d. 0.2

100. Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide the same distance in the absence of friction. What is the coefficient of friction between the body and the inclined plane?

a. $\sqrt{3}/2$ b. $3/4$ c. $1/2$ d. $1/4$

101. The tension in rope (rope is light)

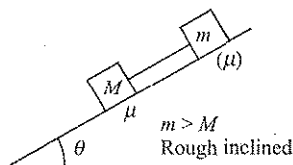


Fig. 7.363

- a. $(M+m)g \sin \theta$
 b. $(M+m)g \sin \theta - \mu mg \cos \theta$
 c. zero
 d. $(M+m)g \cos \theta$

102. There is a chain of length 6 m and coefficient of friction $\frac{1}{2}$. What will be the maximum length of chain which can be held outside of table without sliding

a. 2 m b. 4 m c. 3 m d. 1 m

103. A given object takes n times more time to slide down 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

a. $\frac{1}{2-n^2}$ b. $1-\frac{1}{n^2}$
 c. $\sqrt{1-\frac{1}{n^2}}$ d. $\sqrt{\frac{1}{1-n^2}}$

104. A block of mass m is at rest with respect to a rough incline kept in elevator moving up with acceleration a (Fig. 7.364). Which of following statement is correct?

- a. The contact force between block and incline is parallel to the incline.

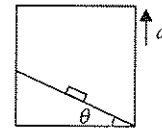


Fig. 7.364

- b. The contact force between block and incline is of magnitude $m(g+a)$.
 c. The contact force between block and incline is perpendicular to the incline.
 d. The contact force is of magnitude $mg \cos \theta$.

105. A block of mass 5 kg is at rest on a rough inclined plane as shown in the Fig. 7.365. The magnitude of net force exerted by the surface on the block will be

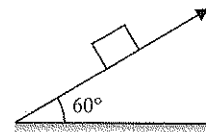


Fig. 7.365

- a. 25 N b. 50 N c. 10 N d. 30 N

106. A box of mass 8 kg is placed on a rough inclined plane of inclination 45° . Its downward motion can be prevented by applying an upward pull F and it can be made to slide upwards by applying a force $2F$. The coefficient of friction between the box and the inclined plane is

a. $\frac{1}{2}$ b. $\frac{1}{\sqrt{2}}$ c. $\frac{1}{2\sqrt{2}}$ d. $\frac{1}{3}$

107. A block of mass $m = 3$ kg is placed on the top of another block of mass $M = 5$ kg as shown in the Fig. 7.366. The coefficient of friction between them is $\mu = 0.4$. What is the maximum acceleration with which the block M may move so that m also moves along with it? (M is on frictionless surface.)

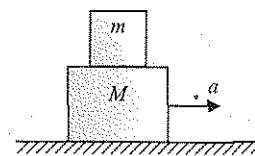


Fig. 7.366

- a. 2 m/s^2 b. 1 m/s^2
 c. 3 m/s^2 d. 4 m/s^2

108. Two blocks A (2 kg) and B (5 kg) rest one over the other on a smooth horizontal plane (see Fig. 7.367). The coefficient of static and dynamic friction between A and B is the same and is equal to 0.80. The maximum horizontal force that can be applied to B in order that both A and B do not have relative motion is

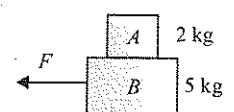


Fig. 7.367

- a. 1.2 N b. 42 N c. 4.2 N d. 56 N

109. The minimum acceleration that must be imparted to the cart in the Fig. 7.368 so that the block A will not fall (given $\mu = 0.5$ is the coefficient of friction between the surfaces of block and cart) is given by

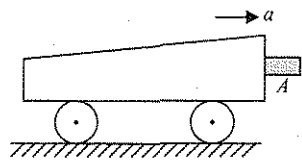


Fig. 7.368

- a. 2 m/s^2 b. 20 m/s^2
c. 5 m/s^2 d. 7.5 m/s^2
110. A block of mass m , lying on a horizontal plane, is acted upon by a horizontal force P and another force Q , inclined at an angle θ to the vertical. The block will remain in equilibrium, if the coefficient of friction between it and the surface is

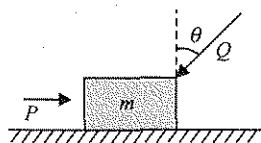


Fig. 7.369

- a. $(P \sin \theta - Q)/(mg - \cos \theta)$
b. $(P - Q \sin \theta)/(mg + Q \sin \theta)$
c. $(P \cos \theta + Q)/(mg - Q \sin \theta)$
d. $(P + Q \sin \theta)/(mg + Q \cos \theta)$
111. A horizontal force of 25 N is necessary to just hold a block stationary against a wall the coefficient of friction between the block and the wall is 0.4. The weight of the block is

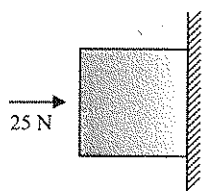


Fig. 7.370

- a. 2.5 N b. 20 N c. 10 N d. 5 N
112. A solid block of mass 2 kg is resting inside a cube as shown in Fig. 7.371. The cube is moving with a velocity $v = 5\hat{i} + 2\hat{j} \text{ m/s}$. If the coefficient of friction between the surface of cube and block is 0.2. Then the force of friction between the block and cube is

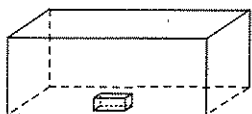


Fig. 7.371

- a. 10 N b. 4 N c. 14 N d. 0

113. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet releasing water at a rate of 1 kg/s and at a speed of 5 m/s. The initial acceleration of the block is

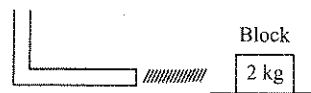


Fig. 7.372

- a. $\frac{5}{3} \text{ m/s}^2$ b. $\frac{25}{4} \text{ m/s}^2$
c. $\frac{25}{6} \text{ m/s}^2$ d. $\frac{5}{2} \text{ m/s}^2$
114. Springs of spring constant $K, 3K, 9K, 27K, \dots, \infty$ are connected in series. Equivalent spring constant of the combination is
- a. $\frac{3K}{2}$ b. $\frac{K}{2}$
c. $\frac{2K}{3}$ d. ∞
115. A block of mass m is lying on a wedge having inclination angle $\alpha = \tan^{-1}\left(\frac{1}{5}\right)$. Wedge is moving with a constant acceleration $a = 2\text{ m/s}^2$. The minimum value of coefficient of friction μ , so that m remains stationary w.r.t. to wedge is

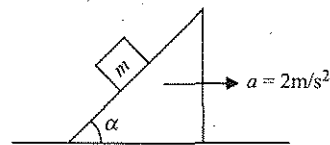


Fig. 7.373

- a. 2/9 b. 5/12
c. 1/5 d. 2/5
116. In the given Fig. 7.374 the blocks are at rest and a force of 10 N acts on the block of 4 kg mass. The coefficient of static friction and the coefficient of kinetic friction are $\mu_s = 0.2$ and $\mu_k = 0.15$ for both the surfaces in contact. The magnitude of friction force acting between the surface of contact between the 2 kg and 4 kg block in this situation is

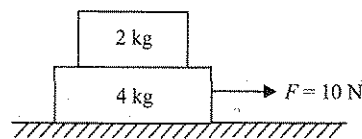


Fig. 7.374

- a. 3 N b. 4 N
c. 3.33 N d. 0
117. An ideal liquid of density ρ is pushed with velocity v through the central limb of the tube shown in the Fig. 7.375. What force does the liquid exert on the tube? The cross-sectional areas of the three limbs are equal to A each. Assume stream-line flow.

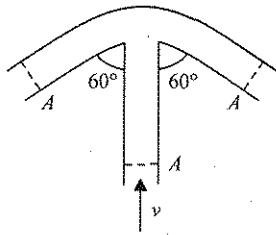


Fig. 7.375

- a. $\frac{9}{8}\rho Av^2$ b. $\frac{5}{4}\rho Av^2$
 c. $\frac{3}{2}\rho Av^2$ d. ρAv^2

118. Two uniform solid cylinders A and B each of mass 1 kg are connected by a spring of constant 200 N/m at their axles and are placed on a fixed wedge as shown in the Fig. 7.376. The coefficient of friction between the wedge and the cylinders is 0.2. The angle made by the line AB with the horizontal, in equilibrium, is

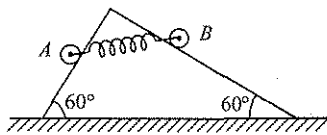


Fig. 7.376

- a. 0° b. 15°
 c. 30° d. None of these

119. Velocity of point A on the rod is 2 m/s (leftward) at the instant shown in the Fig. 7.377. The velocity of the point B on the rod at this instant is

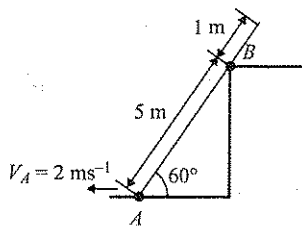


Fig. 7.377

- a. $\frac{2}{\sqrt{3}}$ m/s b. 1 m/s
 c. $\frac{1}{2\sqrt{3}}$ m/s d. $\frac{\sqrt{3}}{2}$ m/s

120. The masses of the blocks A and B are m and M . Between A and B there is a constant frictional force F , and B can slide frictionlessly on horizontal surface (see Fig. 7.378). A is set in motion with velocity while B is at rest. What is the distance moved by A relative to B before they move with the same velocity?

- a. $\frac{mM v_0^2}{F(m-M)}$ b. $\frac{mM v_0^2}{2F(m-M)}$
 c. $\frac{mM v_0^2}{F(m+M)}$ d. $\frac{mM v_0^2}{2F(m+M)}$

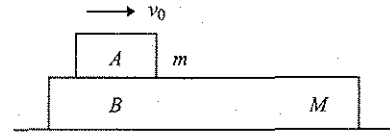


Fig. 7.378

121. Two Small rings O and O' are put on two vertical stationary rods AB and A'B', respectively (see Fig. 7.379). One end of an inextensible thread is tied at point A'. The thread passes through ring O' and its other end is tied to ring O. Assuming that ring O' moves downwards at a constant velocity v_2 of the ring O, when $\angle AOO' = \alpha$

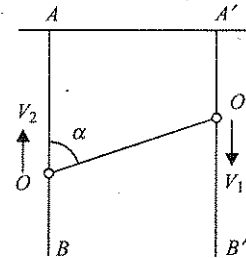


Fig. 7.379

- a. $v_1 \left[\frac{2 \sin^2 \alpha / 2}{\cos \alpha} \right]$ b. $v_1 \left[\frac{2 \cos^2 \alpha / 2}{\sin \alpha} \right]$
 c. $v_1 \left[\frac{3 \cos^2 \alpha / 2}{\sin \alpha} \right]$ d. None of these

122. A fixed U-shaped smooth wire has a semi-circular bending between A and B as shown in the Fig. 7.380. A bead of mass m moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B. The average force exerted by the bead on the part AB of the wire is

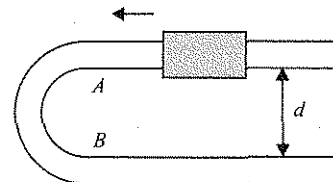


Fig. 7.380

- a. 0 b. $\frac{4mv^2}{\pi d}$
 c. $\frac{2mv^2}{\pi d}$ d. None of these

123. Find frictional force on block 30 kg (Fig. 7.381)

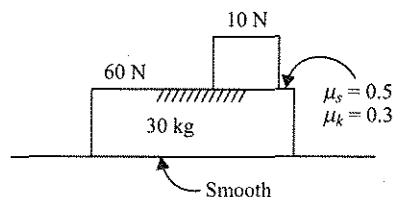


Fig. 7.381

- a. 20 N b. 30 N c. 40 N d. 50 N

124. Two identical particles A and B , each of mass m , are interconnected by a spring of stiffness k . If the particle B experiences a force F and the elongation of the spring is x , the acceleration of particle B relative to particle A is equal to

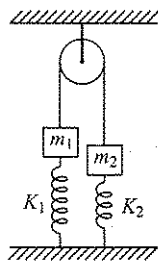


Fig. 7.382

- a. $\frac{F}{2m}$
 b. $\frac{F - kx}{m}$
 c. $\frac{F - 2kx}{m}$
 d. $\frac{kx}{m}$
125. The system shown in the Fig. 7.383 is in equilibrium. Masses m_1 and m_2 are 2 kg and 8 kg, respectively. Spring constants k_1 and k_2 are 50 N/m and 70 N/m, respectively. If the compression in second spring is 0.5 m. What is the compression in first spring? (Both springs have the same natural length.)

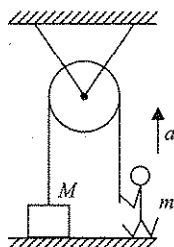


Fig. 7.383

- a. 1.3 m
 b. -0.5 m
 c. 0.5 m
 d. 0.9 m
126. In Fig. 7.384, the block of mass M is at rest on the floor. The acceleration with which a boy of mass m should climb along the rope of negligible mass so as to lift the block from the floor is

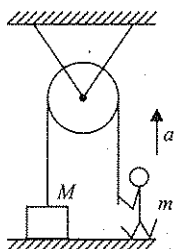


Fig. 7.384

- a. $\left(\frac{M}{m} - 1\right)g$
 b. $\left(\frac{M}{m} - 1\right)$
 c. $\frac{M}{m}g$
 d. $> \frac{M}{m}g$

127. Three blocks A , B , and C are of equal mass m and are placed one over other on a frictionless surface (table) as shown in the Fig. 7.385. Coefficient of friction between any blocks A , B and C is μ . The maximum value of mass of block M_D so that the block A , B , and C move without slipping over each other is

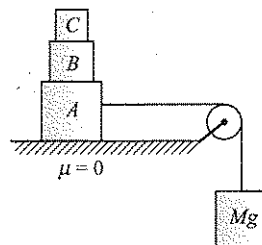


Fig. 7.385

- a. $\frac{3m\mu}{\mu + 1}$
 b. $\frac{3m(1 - \mu)}{\mu}$
 c. $\frac{3m(1 + \mu)}{\mu}$
 d. $\frac{3m\mu}{(1 - \mu)}$
128. Two blocks of masses 0.2 kg and 0.5 kg, which are placed 22 m apart on a rough horizontal surface ($\mu = 0.5$), are acted upon by two forces of magnitude 3 N each as shown in Fig. 7.386 at time $t = 0$. Then, the time t at which they collide each other is

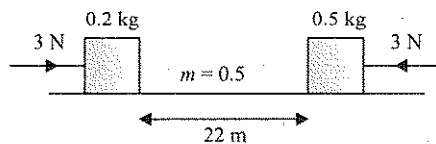


Fig. 7.386

- a. sec
 b. $\sqrt{2}$ sec
 c. 2 sec
 d. None
129. In an arrangement shown below in Fig. 7.387, the acceleration of block A and B are given

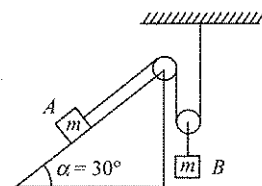


Fig. 7.387

- a. $g/3, g/6$
 b. $g/6, g/3$
 c. $g/2, g/2$
 d. 0, 0
130. A block of mass 1 kg lying on the floor is subjected to a horizontal force given by $f = 2 \sin \omega t$. The coefficient of friction between the block and the floor is 0.25.
- a. Acceleration of the block is positive and uniform.
 b. Acceleration of the block depend on value of ω .
 c. The block always remains at rest.
 d. Acceleration of the block is always zero.

131. A solid block of mass 2 kg is resting inside a cube as shown in Fig. 7.388. The cube is moving with a velocity $v = 5\hat{i} + 2\hat{j}$ m/s. If the coefficient of friction between the surface of cube and block is 0.2. Then the force of friction between the block and the cube is

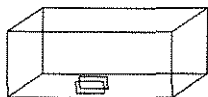


Fig. 7.388

- a. 10 N b. 4 N c. 14 N d. 0
132. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet releasing water at a rate of 1 kg/s and at a speed of 5 m/s. The initial acceleration of the block is

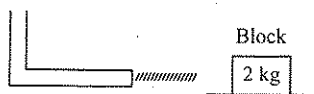


Fig. 7.389

- a. $\frac{5}{3}$ m/s² b. $\frac{25}{4}$ m/s²
 c. $\frac{25}{8}$ m/s² d. $\frac{5}{2}$ m/s²
133. All surfaces are smooth and pulleys are ideal. The string is pulled with force F , mass of $A = \text{mass of } B = m$.

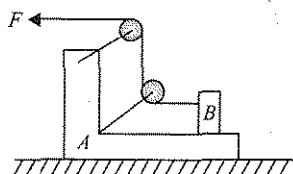


Fig. 7.390

- a. $a_A = a_B$
 b. $a_A = 0, a_B \neq 2a_B$
134. A block of mass m_1 lies on top of fixed wedge as shown in Fig. 7.391(a) and another block of mass m_2 lies on top of wedge which is free to move as shown in Fig. 7.391(b). At time $t = 0$, both the blocks are released from rest from a vertical height h above the respective horizontal surface on which the wedge is placed as shown. There is no friction between the block and the wedge in both the figures. Let T_1 and T_2 be the time taken by block in Fig. 7.391(a) and block in Fig. 7.391(b) respectively to just reach the horizontal surface, then

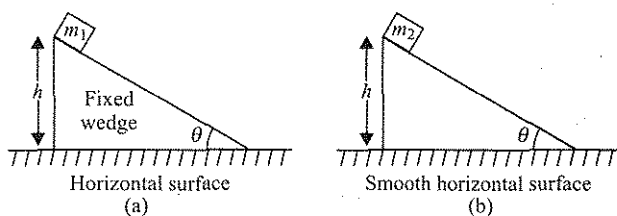
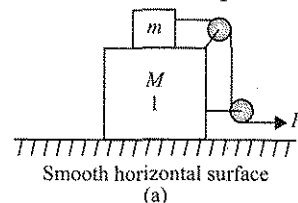


Fig. 7.391

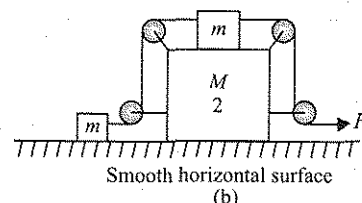
- a. $T_1 > T_2$
 b. $T_1 < T_2$

- c. $T_1 = T_2$
 d. Data insufficient

135. In the situation shown in Fig. 7.392 all the string are light and inextensible and pulleys are light. There is no friction at any surface and all block are of cuboidal shape. A horizontal force of magnitude F is applied to right most free end of string in both the cases of Fig. 7.392(a) and Fig. 7.392(b) as shown. At the instant shown, the tension in all strings are non zero. Let the magnitude of the acceleration of large blocks (of mass M) in Fig. 7.392(a) and Fig. 7.392(b) are a_1 and a_2 , respectively. Then



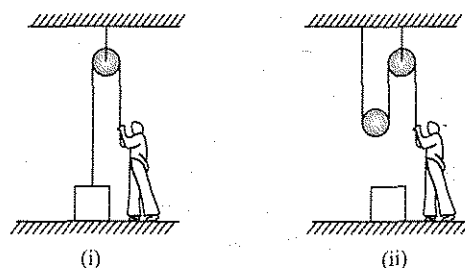
(a)



(b)

Fig. 7.392

- a. $a_1 = a_2 \neq 0$
 b. $a_1 = a_2 = 0$
 c. $a_1 > a_2$
 d. $a_1 < a_2$
136. In Fig. 7.393, a person wants to raise a block lying on the ground to a height h . In both the cases if time required is same then in which case he has to exert more force. Assume pulleys and strings lights.



(i)

(ii)

Fig. 7.393

- a. (i)
 b. (ii)
 c. Same in both
 d. Cannot be determined
137. A chain of length L is placed on a horizontal surface as shown in Fig. 7.394. At any instant x , the length of chain on rough surface and the remaining portion lies on smooth surface. Initially, $x = 0$. A horizontal force P is applied to the chain. In the duration x changes from $x = 0$ to $x = L$ for chain to move with a constant speed.

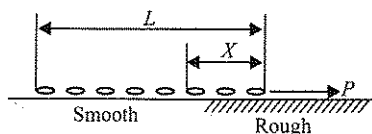


Fig. 7.394

- a. The magnitude of P should increase with time.
 b. The magnitude of P should decrease with time.
 c. The magnitude of P should increase first and then decrease with time.
 d. The magnitude of P should decrease first and then increase with time.
138. A 1.5 kg box is initially at rest on a horizontal surface when at $t = 0$ a horizontal force $\vec{F} = (1.8t) \hat{i}$ N (with t in seconds), is applied to the box. The acceleration of the box as a function of time t is given by
- $$\vec{a} = 0 \quad \text{for} \quad 0 \leq t \leq 2.85$$
- $$\vec{a} = (1.2t - 2.4) \hat{i} \text{ m/s}^2 \quad \text{for} \quad t > 2.85$$
- The coefficient of kinetic friction between the box and the surface is
- a. 0.12 b. 0.24
 c. 0.36 d. 0.48
139. A vehicle is moving with a velocity v on a curved road of width b and radius of curvature R . For counteracting the centrifugal force on the vehicle, the difference in elevation required in between the outer and inner edges of the road is

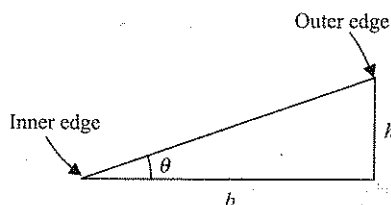


Fig. 7.395

- a. $v^2 b / Rg$ b. vb / Rg
 c. vb^2 / Rg d. $vb / R^2 g$
140. A circular road of radius 1,000 m has a banking angle of 45° . What will be the maximum safe speed (in m/s) of a car whose mass is 2,000 kg and the coefficient of friction between the tyre and the road is 0.5
- a. 172 b. 124 c. 99 d. 86
141. A circular table of radius 0.5 m has a smooth diametrical groove. A ball of mass 90 g is placed inside the groove along with a spring of spring constant 10^2 N/cm. One end of the spring is tied to the edge of the table and the other end to the ball. The ball is at a distance of 0.1 m from the centre when the table is at rest. On rotating the table with a constant angular frequency of 10^2 rad/s $^{-1}$, the ball moves away from the centre by a distance nearly equal to

- a. 10^{-1} m b. 10^{-2} m
 c. 10^{-3} m d. 2×10^{-1} m

142. A coin is placed at the edge of a horizontal disc rotating about a vertical axis through its axis with a uniform angular speed 2 rad/s. The radius of the disc is 50 cm. Find the minimum coefficient of friction between disc and coin so that the coin does not slip ($g = 10 \text{ ms}^{-2}$).

- a. 0.1 b. 0.2
 c. 0.3 d. 0.4

143. Mark out the incorrect statement

- a. Second law of motion is a local relation, i.e., if $\vec{a}$ at a point changes at any time t , then $\vec{F}$ has to change at the same point at same time t .
 b. In Newton's third law, action and reaction start acting at the same instant.
 c. If pseudo force acting on an object in a non-inertial frame is $\vec{F}$, then its reaction on inertial frame is $-\vec{F}$.

144. Friction force can be reduced to a great extent by

- a. Lubricating the two moving parts.
 b. Using ball bearings between two moving parts.
 c. Introducing a thin cushion of air maintained between two relatively moving surfaces.
 d. All of the above.

145. An interstellar spacecraft far away from the influence of any star or planet is moving at high speed under the influence of fusion rockets (due to thrust exerted by fusion rockets, the spacecraft is accelerating). Suddenly the engine malfunctions and stops. The spacecraft will

- a. immediately stops, throwing all of the occupants to the front
 b. begins slowing down and eventually comes to rest
 c. keep moving at constant speed for a while, and then begins to slow down
 d. keeps moving forever with constant speed

146. Three arrangement of a light spring balance are shown in the Fig. 7.396, 7.397, and 7.398 below. The readings of the spring scales in three arrangements are, respectively

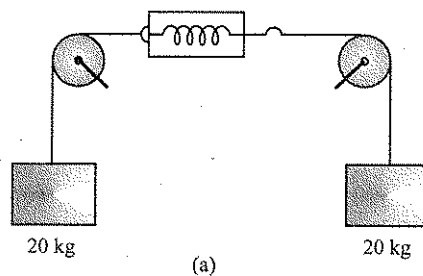
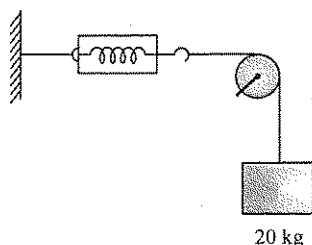


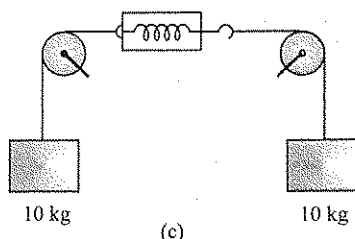
Fig. 7.396

- a. 20 g, 20 g, 10 g b. 20 g, 20 g, $\frac{40 \text{ g}}{3}$
 c. zero, 20 g, 10 g d. zero, 20 g, $\frac{40 \text{ g}}{3}$



(b)

Fig. 7.397



(c)

Fig. 7.398

147. Mark out the most appropriate statement.

- The normal force is the same thing as the weight.
- The normal force is different from the weight, but always has the same magnitude.
- The normal force is different from the weight, but the two form an action-reaction pair according to the Newton's third law.
- The normal force is different from the weight, but the two may have same magnitude in certain cases.

148. A system of two blocks, a light string and a light and frictionless pulley is arranged as shown in Fig. 7.399. The coefficient of friction between fixed incline and 10 kg block is given by $\mu_s = 0.25$ and $\mu_k = 0.20$. If the system is released from rest, then find the acceleration of 10 kg block?

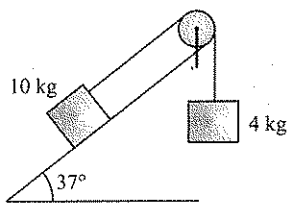


Fig. 7.399

- | | |
|--------------------------|--------------------------|
| a. 0 | b. 0.114 m/s^2 |
| c. 0.228 m/s^2 | d. 2.97 m/s^2 |

149. A wooden box is placed on a table. The normal force on the box from the table is N_1 . Now another identical box is kept on first box and the normal force on lower block due to upper block is N_2 and normal force on lower block by the table is N_3 . For this situation mark out the correct statement(s)

- | | |
|----------------------|----------------------|
| a. $N_1 = N_2 = N_3$ | b. $N_1 < N_2 = N_3$ |
|----------------------|----------------------|

c. $N_1 = N_2 < N_3$

d. $N_1 = N_2 > N_3$

150. Consider a 14-tyre truck, whose only rear 8 wheels are power driven (means only these 8 wheels can produce acceleration). These 8 wheels are supporting approximately half of the entire load. If coefficient of friction between rod and each of the tyres is 0.6, then what could be the maximum attainable acceleration by this truck?

- | | |
|----------------------|-----------------------|
| a. 6 m/s^2 | b. 24 m/s^2 |
| c. 3 m/s^2 | d. 10 m/s^2 |

151. A house is built on the top of a hill with 45° slope. Due to sliding of the material and the sand from the top to the bottom of the hill the slope angle has been reduced. If the coefficient of static friction between the sand particles is 0.75, what is the final angle attained by the hill? [$\tan^{-1}(0.75 \approx 37^\circ)$]

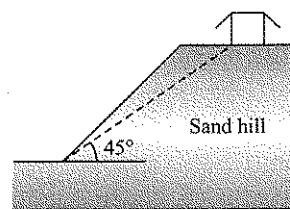


Fig. 7.400

- | | | | |
|--------------|---------------|---------------|---------------|
| a. 8° | b. 45° | c. 37° | d. 30° |
|--------------|---------------|---------------|---------------|

152. A block of mass 4 kg is pressed against the wall by a force of 80 N as shown in the Fig. 7.401. Determine the value of friction force and block's acceleration, (Take $\mu_s = 0.2$, $\mu_k = 0.15$).

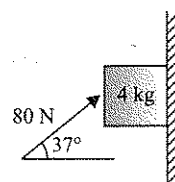


Fig. 7.401

- | | |
|---------------------------|----------------------------|
| a. 8 N, 0 m/s^2 | b. 32 N, 6 m/s^2 |
| c. 8 N, 6 m/s^2 | d. 32 N, 2 m/s^2 |

153. For the situation shown in the Fig. 7.402, the block is stationary w.r.t. incline fixed in an elevator. The elevator has an acceleration of $\sqrt{5}a_0$ whose components are shown in the figure. The surface is rough and coefficient of static friction between the incline and block is μ_s . Determine the magnitude of force exerted by incline on the block. [Take $a_0 = \frac{g}{2}$ and $\theta = 37^\circ$, $\mu_s = 0.6$]

- | | |
|--------------------------------------|----------------------------|
| a. $\frac{mg}{10}$ | b. $\frac{9mg}{25}$ |
| c. $\frac{3mg}{25} \times \sqrt{41}$ | d. $\frac{\sqrt{13}mg}{2}$ |

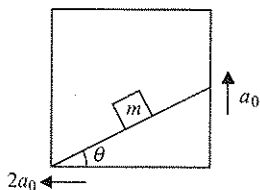


Fig. 7.402

154. A block of mass m is placed on a rough table, which is kept in a gravity free hall. Coefficient of friction between block and table is μ and a horizontal force F is applied to the block. The reaction force exerted by the table on the block is

- a. mg
 b. $mg\sqrt{1+\mu^2}$
 c. F
 d. $(\sqrt{1+\mu^2})F$

155. If coefficient of friction between all surfaces (see Fig. 7.403) is 0.4, then find the minimum force F to have equilibrium of the system.

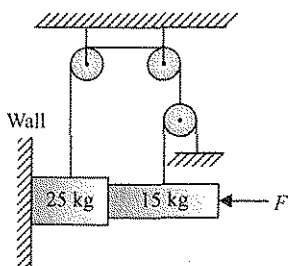


Fig. 7.403

- a. 62.5 N
 b. 150 N
 c. 135 N
 d. 50 N

156. In the Fig. 7.404 shown below, if acceleration of B is $\vec{a}$, then find the acceleration of A .

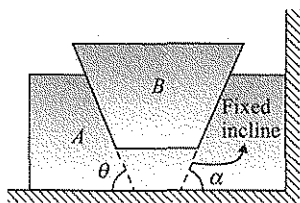


Fig. 7.404

- a. $a \sin \alpha$
 b. $a \cot \theta$
 c. $a \tan \theta$
 d. $a \sin \alpha \cot \theta$

157. If the acceleration of wedge in the Fig. 7.405 shown is $a \text{ m/s}^2$ towards left, then at this instant acceleration of the block (magnitude only) would be

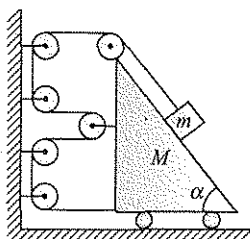


Fig. 7.405

- a. $4a \text{ m/s}^2$
 b. $a\sqrt{17-8\cos\alpha} \text{ m/s}^2$
 c. $\sqrt{17a} \text{ m/s}^2$
 d. $\sqrt{17}\cos\frac{\alpha}{2} \times a \text{ m/s}^2$

158. In the arrangement shown in the Fig. 7.406 below at a particular instant the roller is coming down with a speed of 12 m/s and C is moving up with 4 m/s . At the same instant it is also known that w.r.t. pulley P , block A is moving down with speed 3 m/s . Determine the motion of block B (velocity) w.r.t. ground.

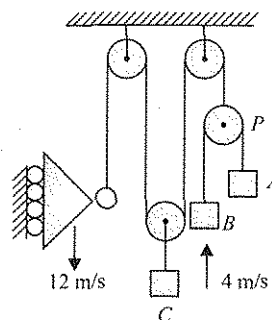


Fig. 7.406

- a. 4 m/s in downward direction
 b. 3 m/s in upward direction
 c. 7 m/s in downward direction
 d. 7 mm/s in upward direction

159. A professor holds an eraser against a vertical chalkboard by pushing horizontally on it. He pushes with a force that is much greater than it required to hold the eraser. The force of friction exerted by the board on the eraser increases if he

- a. pushes eraser with slightly greater force.
 b. pushes eraser with slightly less force.
 c. raises his elbow so that the force he exerts is slightly downward but has same magnitude.
 d. lowers his elbow so that the force he exerts is slightly upward but has the same magnitude.

160. A particle of mass 2 kg moves with an initial velocity of $(4\hat{i} + 2\hat{j}) \text{ m/s}$ on the x - y plane. A force $\vec{F} = (2\hat{i} - 8\hat{j}) \text{ N}$ acts on the particle. The initial position of the particle is $(2 \text{ m}, 3 \text{ m})$. Then for $y = 3 \text{ m}$

- a. The possible value of x is only $x = 2 \text{ m}$.
 b. The possible value of x is not only $x = 2 \text{ m}$, but there exists some other value of x also.
 c. Time taken is 2 s .
 d. All of the above.

161. Find the least horizontal force P to start motion of any part of the system of the three blocks resting upon one another as shown in Fig. 7.407. The weights of blocks are $A = 300 \text{ N}$, $B = 100 \text{ N}$ and $C = 200 \text{ N}$. Between A and B , coefficient of friction is 0.3 , between B and C is 0.2 and between C and the ground is 0.1 .

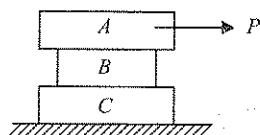


Fig. 7.407

- a. 60 N b. 90 N c. 80 N d. 70 N

162. A person is drawing himself up and a trolley on which he stands with some acceleration. Mass of the person is more than the mass of the trolley. As the person increases his force on the string, the normal reaction between person and the trolley will

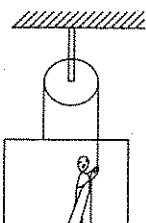


Fig. 7.408

- a. increase
b. decrease
c. remain same
d. cannot be predicted as data is insufficient

163. A block of mass m is attached with a mass less spring of force constant k . The block is placed over a rough inclined surface for which the coefficient of friction is 0.5. M is released from rest when the spring was unstretched (see Fig. 7.409). The minimum value of M required to move the block m up the plane is (neglect mass of string and pulley and friction in pulley)

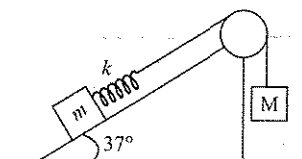


Fig. 7.409

- a. $m/2$ b. $m/3$
c. $m/4$ d. None of these

164. Figure 7.410 shows the variation of force acting on a body, with time. Assuming the body to start from rest, the variation of its momentum with time is best represented by which plot?

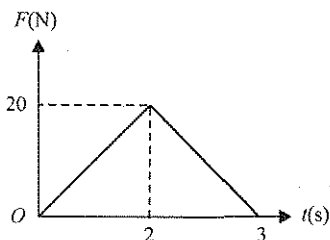
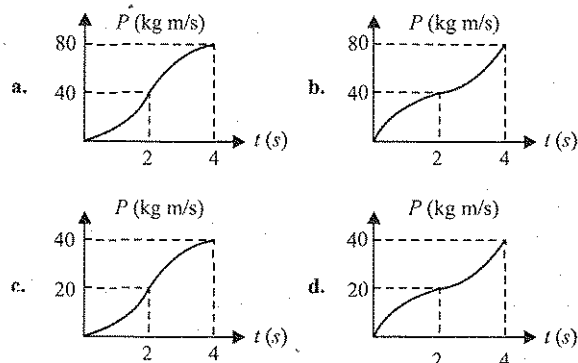


Fig. 7.410



165. The system shown in Fig. 7.411 is released from rest when the spring was in its normal length. On releasing, the spring starts elongating

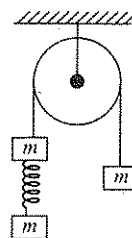


Fig. 7.411

- a. If $M > m$
b. If $M > 2m$
c. If $M < \frac{m}{2}$
d. For any value of M

166. In Fig. 7.412 string does not slip on pulley P , but pulley P is free to rotate about its own axis. Block A is displaced towards left, then pulley P

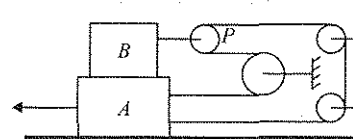


Fig. 7.412

- a. rotates clockwise and translates
b. rotates anticlockwise and translates
c. only translates
d. only rotates (clockwise or anticlockwise)

167. In the system in Fig. 7.413, the friction coefficient between and bigger block μ . There is no friction between both the blocks. The string connecting both the blocks is light; all three pulleys are light and frictionless. Then the minimum limiting value of μ so that the system remains in equilibrium is

- a. $\frac{1}{2}$ b. $\frac{3}{2}$
c. $\frac{2}{3}$ d. $\frac{3}{2}$

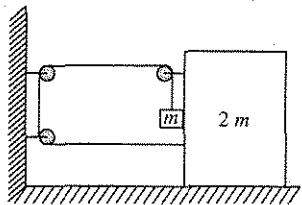


Fig. 7.413

168. Two blocks A of 6 kg and B of 4 kg are placed in contact with each other as shown in Fig. 7.414.

There is no friction between A and ground and between both the blocks. Coefficient of friction between B and ground is 0.5. A horizontal force F is applied on A. Find the minimum and maximum value of F which can be applied so that both blocks can move combinly without any relative motion between them.

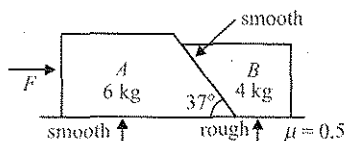


Fig. 7.414

- a. 10 N, 50 N b. 12 N, 50 N
c. 12 N, 75 N d. None of these
169. Two blocks are resting on ground with masses 5 kg and 7 kg. A string connects them which goes over a massless pulley A. There is no friction between pulley and string. A force $F = 124$ N is applied on pulley A. The acceleration of centre of mass of 7 kg block and 5 kg block in vertical direction is

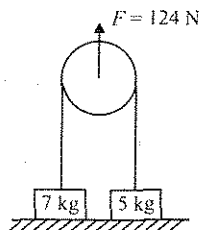


Fig. 7.415

- a. 0 b. 5 ms^{-2}
c. 2.5 ms^{-2} d. 1 ms^{-2}
170. Two masses $\sqrt{3}m$ and $\sqrt{2}m$ tied by a light string are placed on a wedge of mass 4 m. The wedge is placed on a smooth horizontal surface (see Fig. 7.416). Find out the value of θ so that the wedge does not move after the system is set free from the state of rest.

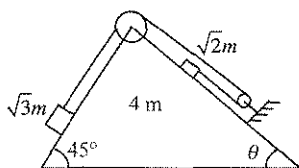


Fig. 7.416

- a. 30° b. 45°
c. 60° d. None of these

171. Two blocks A and B each of mass m are placed one over another on an incline as shown in Fig. 7.417. When the system is released from rest the block slides down with constant velocity while block B rests on top of A. If the coefficient of friction between A and B and between B and incline are same, then value of coefficient of friction would be

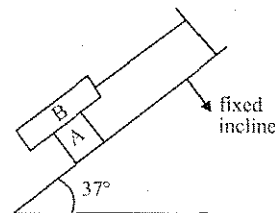


Fig. 7.417

- a. $\frac{1}{4}$ b. $\frac{3}{5}$
c. $\frac{4}{5}$ d. Information insufficient

172. Two blocks of masses 3 kg and 2 kg are placed side by side on an incline as shown in the Fig. 7.418. A force, $F = 20$ N is acting on 2 kg block along the incline. The coefficient of friction between the block and the incline is same and equal to 0.1. find the normal contact force exerted by 2 kg block on 3 kg block.

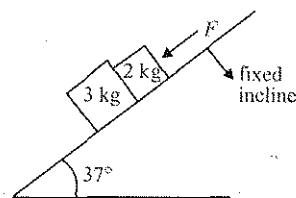


Fig. 7.418

- a. 18 N b. 30 N c. 12 N d. 27.6 N

173. Determine the time in which the smaller block reaches other end of bigger block in the Fig. 7.419

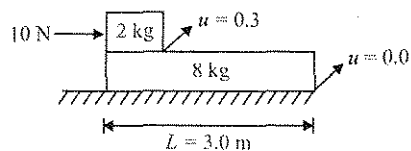


Fig. 7.419

- a. 4 s b. 8 c. 2.19 s d. 2.13 s

174. A body of mass m is held at rest at a height h on two smooth wedges of mass M each which are themselves at rest on a horizontal frictionless surface (see Fig. 7.420). When the mass m is released, it moves down, pushing aside the wedges. The velocity with which the wedges recede from each other, when m reaches the ground is

- a. $\sqrt{\frac{8mgh}{m+2M}}$ b. $\sqrt{\frac{40mgh \times 4}{5m+6M}}$

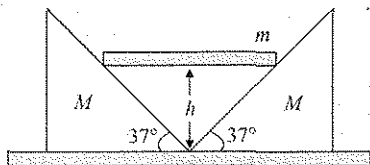


Fig. 7.420

c. $\sqrt{\frac{32mgh \times 4}{32M + 9m}}$

d. None of these

175. For the system shown in Fig. 7.421, $m_1 > m_2 > m_3 > m_4$. Initially, the system is at rest in equilibrium condition. If the string joining m_4 and ground is cut, then just after the string is cut:

 Statement I: m_1, m_2, m_3 remain stationary.

Statement II: the value of acceleration of all the 4 blocks can be determined.

 Statement III: Only m_4 remains stationary.

 Statement IV: Only m_4 accelerates.

Statement V: All the four blocks remain stationary.

Now, choose the correct options.

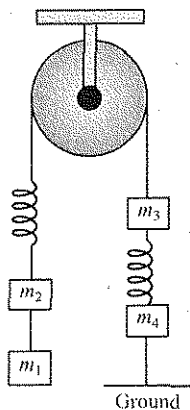


Fig. 7.421

- a. All the statement are correct.
 b. Only I, II and IV are correct.
 c. Only II and V are correct.
 d. Only II and IV are correct.

176. A particles is moving in x - y plane. At certain instant of time, the components of its velocity and acceleration are as follows: $v_x = 3 \text{ ms}^{-1}$, $v_y = 4 \text{ ms}^{-1}$, $a_x = 2 \text{ ms}^{-2}$, and $a_y = 1 \text{ ms}^{-2}$. The rate of change of speed at this moment is

- a. $\sqrt{10} \text{ ms}^{-2}$
 b. 4 ms^{-2}
 c. $\sqrt{5} \text{ ms}^{-2}$
 d. 2 ms^{-2}

177. If block A is moving horizontally with velocity v_A then find the velocity of block B at the instant as shown in the Fig. 7.422.

- a. $\frac{h v_A}{2\sqrt{x^2 + h^2}}$
 b. $\frac{x v_A}{\sqrt{x^2 + h^2}}$
 c. $\frac{x v_A}{2\sqrt{x^2 + h^2}}$
 d. $\frac{h v_A}{\sqrt{x^2 + h^2}}$

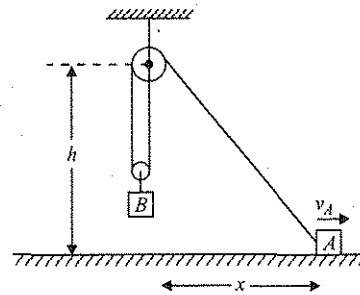


Fig. 7.422

178. Figure 7.423 shows two blocks, each of mass m . The system is released from rest. If accelerations of blocks A and B at any instant (not initially) are a_1 and a_2 , respectively, then

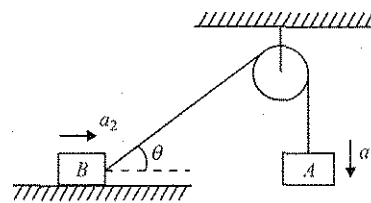


Fig. 7.423

- a. $a_1 = a_2 \cos \theta$
 b. $a_2 = a_1 \cos \theta$
 c. $a_1 = a_2$
 d. None of these

179. A small block of mass m rests on a smooth wedge of angle θ . With what horizontal acceleration a should the wedge be pulley, as shown in the Fig. 7.424, so that the block falls freely.

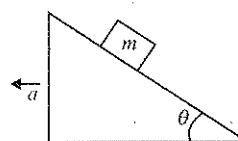


Fig. 7.424

- a. $g \cos \theta$
 b. $g \sin \theta$
 c. $g \cot \theta$
 d. $g \tan \theta$

180. In the given Fig. 7.425, man A is standing on a movable plank while man B is standing on a stationary platform. Both are pulling the string down such that the plank moves slowly up. As a result of this, the string slips through the hands of the men. Find the ratio of length of the string that slips through the hands of A to B.

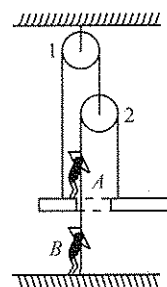


Fig. 7.425

- a. $3/2$ b. $3/4$ c. $4/3$ d. $2/3$

181. A uniform chain is placed at rest on a rough surface of base length l and height h on an irregular surface as shown in Fig. 7.426. Then, the minimum coefficient of friction between the chain and the surface must be equal to



Fig. 7.426

- a. $\mu = \frac{h}{2l}$ b. $\mu = \frac{h}{l}$
 c. $\mu = \frac{3h}{2l}$ d. $\mu = \frac{2h}{3l}$

182. In the Fig. 7.427, the block of mass m is at rest relative to the wedge of mass M and the wedge is at rest with respect to ground. This implies that

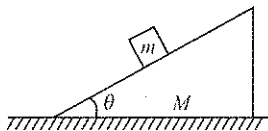


Fig. 7.427

- a. Net force applied by m on M is mg .
 b. Normal force applied by m on M is mg .
 c. Force of friction applied by m on M is mg .
 d. None of the above of the above.

183. A triangular prism of mass M with a block of mass m placed on it is released from rest on a smooth inclined plane of inclination θ . The block does not slip on the prism. Then

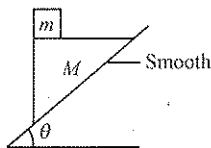


Fig. 7.428

- a. The acceleration of the prism is $g \cos \theta$.
 b. The acceleration of the prism is $g \tan \theta$.
 c. The minimum coefficient of friction between the block and the prism is $\mu_{\min} = \cot \theta$.
 d. The minimum coefficient of friction between the block and the prism is $\mu_{\min} = \tan \theta$.

184. Two trolleys 1 and 2 are moving with accelerations a_1 and a_2 , respectively, in the same direct. A block of mass m on trolley 1 is in equilibrium from the frame of observer stationary with respect to trolley 2. The magnitude of friction force on block due to trolley is (assume that no horizontal force other than friction force is acting on block)

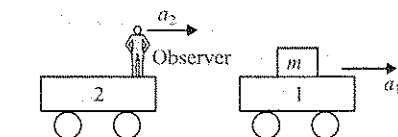


Fig. 7.429

- a. $m(a_1 - a_2)$ b. ma_2
 c. ma_1 d. Data insufficient

185. In Fig. 7.430 all the surfaces are frictionless while pulley and string are massless. Mass of block A is $2m$ and that of block B is m . Acceleration of block B immediately after system is released from rest is

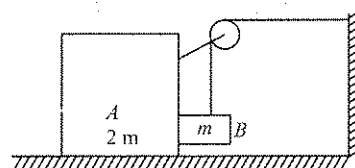


Fig. 7.430

- a. $g/2$ b. g
 c. $g/3$ d. None of these

186. In two pulley-particle Figs. 7.431 (i) and (ii), the acceleration and force imparted by the string on the pulley and tension in the strings are, (a_1, a_2) , (N_1, N_2) , and (T_1, T_2) respectively. Ignoring friction in all contacting surfaces Study the following statements

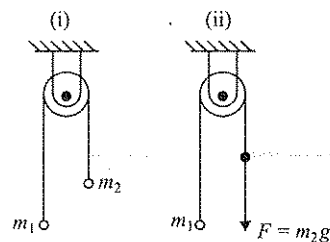


Fig. 7.431

- a. $\frac{a_1}{a_2} = 1$ b. $\frac{T_1}{T_2} < 1$
 c. $\frac{N_1}{N_2} > 1$ d. $\frac{a_1}{a_2} < 1$

Now mark correct answer

- (i) Relation (ii) and (iii) always follows.
 (ii) Relation (ii) and (iv) always follows.
 (iii) Relation (i) only always follows.
 (iv) Relation (iv) always follows.

187. A block of mass m is pressed against a vertical wall with a horizontal force $F = mg$ (see Fig. 7.432). Another force $F' = \frac{mg}{2}$ is acting vertically upon the block. If the coefficient of friction between the block and wall is $\frac{1}{2}$, the friction between them is

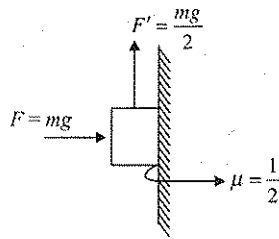


Fig. 7.432

- a. $\frac{mg}{2}$ up
b. $\frac{mg}{2}$ down
c. mg up
d. 0

188. A particle has initial velocity, $\vec{v} = 3\hat{i} + 4\hat{j}$ and a constant force $\vec{F} = 4\hat{i} - 3\hat{j}$ acts on the particle. The path of the particle is

- a. straight line
b. parabolic
c. circular
d. elliptical

189. In the Fig. 7.433 shown the acceleration of A is, $\vec{a}_A = 15\hat{i} + 15\hat{j}$ then the acceleration of B is (A remains in contact with B)

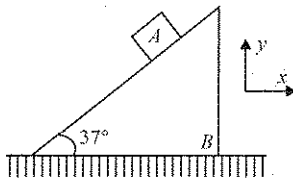


Fig. 7.433

- a. $6\hat{i}$
b. $-15\hat{i}$
c. $-10\hat{i}$
d. $-5\hat{i}$

190. Two blocks A and B of masses m and $2m$ (Fig. 7.434), respectively, are held at rest such that the spring is in natural length. Find out the accelerations of both the blocks just after release.

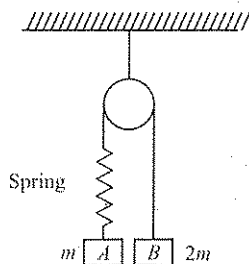


Fig. 7.434

- a. $g \downarrow, g \downarrow$
b. $\frac{g}{3} \downarrow, \frac{g}{3} \uparrow$
c. 0, 0
d. $g \downarrow, c$

191. A bob is hanging over a pulley inside a car through a string. The second end of the string is in the hand of a person standing in the car. The car is moving with constant acceleration a directed horizontally as shown in Fig. 7.435. Other end of the string is pulled with constant acceleration a vertically. The tension in the string is equal to

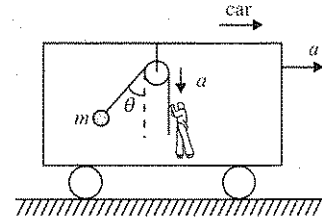


Fig. 7.435

- a. $m\sqrt{g^2 + a^2}$
b. $m\sqrt{g^2 + a^2} - ma$
c. $m\sqrt{g^2 + a^2} + ma$
d. $m(g + a)$

192. Inside a horizontally moving box, an experimenter finds that when an object is placed on a smooth horizontal table and is released, it moves with an acceleration of 10 m/s^2 . In this box of 1 kg body is suspended with a light string, the tension in the string in equilibrium position, (w.r.t. experimenter) will be (take $g = 10 \text{ m/s}^2$)

- a. 10 m/s^2
b. $10\sqrt{2} \text{ m/s}^2$
c. 20 m/s^2
d. zero

193. Two blocks A and B each of mass m are placed on a smooth horizontal surface. Two horizontal force F and $2F$ are applied on both the blocks A and B, respectively, as shown in Fig. 7.436. The block A does not slide on block B. Then the normal reaction acting between the two blocks is

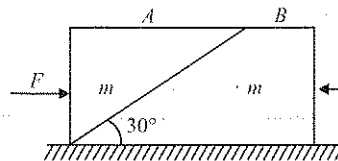


Fig. 7.436

- a. F
b. $F/2$
c. $\frac{F}{\sqrt{3}}$
d. $3F$

194. In the given Fig. 7.437, by what acceleration the boy must go up so that 100 kg block remains stationary on the wedge. The wedge is fixed and friction is absent everywhere (take $g = 10 \text{ m/s}^2$)

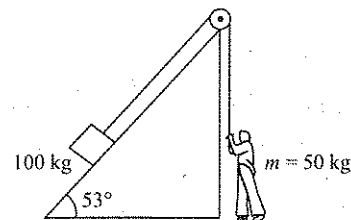


Fig. 7.437

- a. 2 m/s^2
b. 4 m/s^2
c. 6 m/s^2
d. 8 m/s^2

195. A system is shown in the Fig. 7.438. A man standing on the block is pulling the rope. Velocity of the point of string in contact with the hand of the man is 2 m/s downwards. The velocity of the block will be [assume that the block does not rotate]

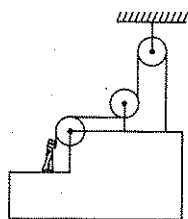


Fig. 7.438

- a. 3 m/s b. 2 m/s c. 1/2 m/s d. 1 m/s

196. In the Fig. 7.439 shown the velocity of lift is 2 m/s while string is winding on the motor shaft with velocity 2 m/s and block A is moving downwards with velocity of 2 m/s, then find out the velocity of block B.

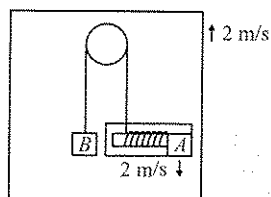


Fig. 7.439

- a. 2 m/s $\uparrow$ b. 2 m/s $\downarrow$
c. 4 m/s $\uparrow$ d. None of these

197. A system is shown in the Fig. 7.440. Assume that cylinder remains in contact with the two wedges hence the velocity of cylinder is

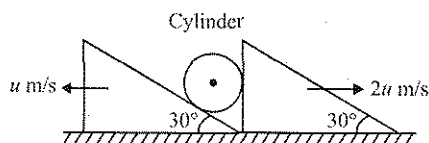


Fig. 7.440

- a. $\sqrt{19 - 4\sqrt{3}} \frac{u}{2}$ m/s b. $\frac{\sqrt{13}u}{2}$ m/s
c. $\sqrt{3}u$ m/s d. $\sqrt{7}u$ m/s

198. Two beads A and B move along a semicircular wire frame as shown in Fig. 7.441. The beads are connected by an inelastic string with always remains tight. At an instant the speed of A is u , $\angle BAC = 45^\circ$ and $\angle BOC = 75^\circ$, where O is the centre of the semicircular arc. The speed of bead B at that instant is

- a. $\sqrt{2}u$ b. u
c. $\frac{u}{2\sqrt{2}}$ d. $\sqrt{\frac{2}{3}}u$

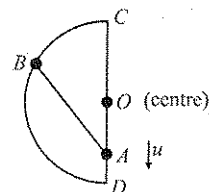


Fig. 7.441

199. A plank is held at an angle α to the horizontal (Fig. 7.442) on two fixed supports A and B. The plank can slide against the supports (without friction) because of the weight Mg . Acceleration and direction in which a man of mass m should move so that the plank does not move.

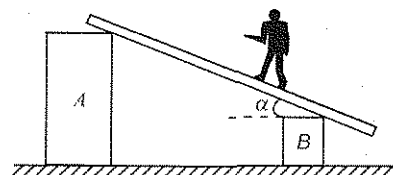


Fig. 7.442

- a. $g \sin \alpha \left(1 + \frac{m}{M}\right)$ down the incline
b. $g \sin \alpha \left(1 + \frac{M}{m}\right)$ down the incline
c. $g \sin \alpha \left(1 + \frac{m}{M}\right)$ up the incline
d. $g \sin \alpha \left(1 + \frac{M}{m}\right)$ up the incline

200. Object A and B each of mass m are connected by light inextensible cord. They are constrained to move on a frictionless ring to a vertical plane as shown in Fig. 7.443. The objects are released from rest at the positions shown. The tension in the cord just after release will be

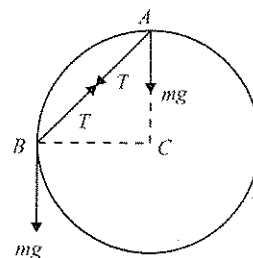


Fig. 7.443

- a. $mg\sqrt{2}$ b. $\frac{mg}{\sqrt{2}}$
c. $\frac{mg}{2}$ d. $\frac{mg}{4}$

201. A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is

- a. $\theta = \tan^{-1} \alpha$
b. $\theta = \tan^{-1} \left(\frac{a_0}{g}\right)$

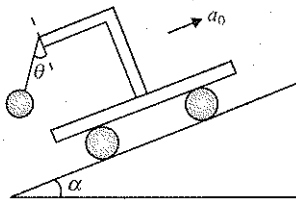


Fig. 7.444

- c. $\theta = \tan^{-1} \left(\frac{g}{a_0} \right)$
 d. $\theta = \tan^{-1} \left(\frac{a_0 + g \sin \alpha}{g \cos \alpha} \right)$

202. Find the acceleration of the block B in the Fig. 7.445, assuming that the surfaces and the pulleys P_1 and P_2 are all smooth.

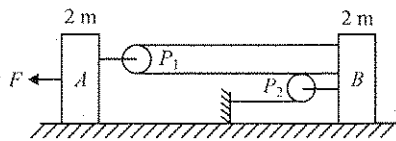


Fig. 7.445

- a. $\frac{F}{4m}$
 b. $\frac{F}{6m}$
 c. $\frac{F}{2m}$
 d. $\frac{3F}{17m}$

203. In the Fig. 7.446 shown all blocks are of equal mass m . All surfaces are smooth. The acceleration of the block A with respect to the ground is

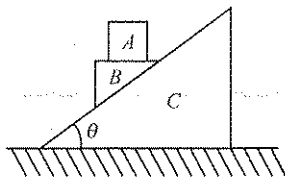


Fig. 7.446

- a. $\frac{4g \sin \theta}{1 + 3 \sin^2 \theta}$
 b. $\frac{4g \sin^2 \theta}{1 + 3 \sin^2 \theta}$
 c. $\frac{4g \sin^2 \theta}{\sqrt{1 + 3 \sin^2 \theta}}$
 d. None of these

204. In the question 203 the acceleration B w.r.t. ground is

- a. $\frac{2g \sin \theta}{1 + 3 \sin^2 \theta}$
 b. $\frac{4g \sin \theta}{1 + 3 \sin^2 \theta}$
 c. $\frac{2g \sin \theta}{\sqrt{1 + 3 \sin^2 \theta}}$
 d. $\frac{4g \sin \theta}{\sqrt{1 + 3 \sin^2 \theta}}$

205. In the question 203 the acceleration C w.r.t. ground is

- a. $\frac{2g \sin \theta \cos \theta}{1 + 3 \sin^2 \theta}$
 b. $\frac{g \sin \theta \cos \theta}{1 + 3 \sin^2 \theta}$
 c. $\frac{g \sin 2\theta}{\sqrt{1 + 3 \sin^2 \theta}}$
 d. $\frac{g \sin \theta \cos \theta}{\sqrt{1 + 3 \sin^2 \theta}}$

206. In the arrangement shown in the Fig. 7.447, the block of mass $m = 2$ kg lies on the wedge of mass $M = 8$ kg. The initial acceleration of the wedge if the surfaces are smooth

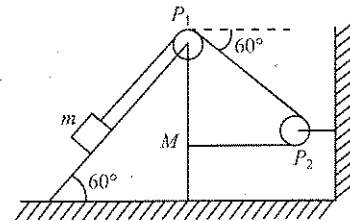


Fig. 7.447

- a. $\frac{\sqrt{3}g}{23} \text{ m/s}^2$
 b. $\frac{3\sqrt{3}g}{23} \text{ m/s}^2$
 c. $\frac{3g}{23} \text{ m/s}^2$
 d. $\frac{g}{23} \text{ m/s}^2$

207. An object moving with a constant acceleration in a non-inertial frame

- a. must have non-zero net force acting on it.
 b. may have zero net force acting on it.
 c. may have no force acting on it.
 d. this situation is practically impossible. (The pseudo force acting on the object has also to be considered)

208. An object moving with constant velocity in a non-inertial frame of reference

- a. must have non-zero net force acting on it.
 b. may have zero net force acting on it.
 c. must have zero net force acting on it.
 d. may have non-zero net force acting on it (Consider only the real forces).

209. In the Fig. 7.448 if f_1 , f_2 , and T be the frictional forces on 2 kg block, 3 kg block & tension in string, respectively, then their values are

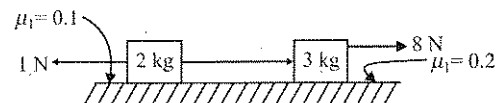


Fig. 7.448

- a. 2 N, 6 N, 3.2 N
 b. 2 N, 6 N, 0 N
 c. 1 N, 6 N, 2 N
 d. Data insufficient to calculate the required value

210. A rope of length 4 m having mass 1.5 kg/m lying on a horizontal frictionless surface is pulled at one end by a force of 12 N. What is the tension in the rope at a point 1.6 m from the other end?

- a. 5 N
 b. 4.8 N
 c. 7.2 N
 d. 6 N

211. A block of mass 5.0 kg slides down from the top of an inclined plane of length 3 m. The first 1 m of the plane is smooth and the next 2 m is rough. The block is released from rest and again comes to rest at the bottom of the plane. If the plane is inclined at 30° with the horizontal (Fig. 7.449), find the coefficient of friction on the rough portion.

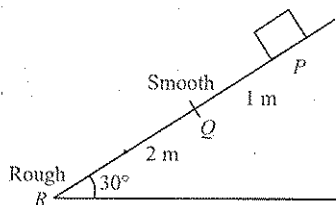


Fig. 7.449

- a. $\frac{2}{\sqrt{3}}$ b. $\frac{\sqrt{3}}{2}$
 c. $\frac{\sqrt{3}}{4}$ d. $\frac{\sqrt{3}}{5}$
212. A body of mass m starting from rest slides down a frictionless inclined surface of gradient $\tan \alpha$ fixed on the floor of a lift accelerating upward with acceleration a . Taking width of inclined plane as W , the time taken by body to slide from top to bottom of the plane is

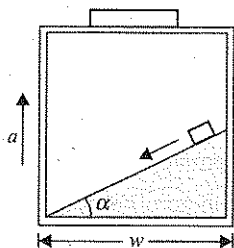


Fig. 7.450

- a. $\left(\frac{2W}{(g+a) \sin \alpha} \right)^{\frac{1}{2}}$
 b. $\left(\frac{4W}{(g-a) \sin \alpha} \right)^{\frac{1}{2}}$
 c. $\left(\frac{4W}{(g+a) \sin 2\alpha} \right)^{\frac{1}{2}}$
 d. $\left(\frac{W}{(g+a) \sin 2\alpha} \right)^{\frac{1}{2}}$
213. A rope is stretched between two boats at rest. A sailor in the first boat pulls the rope with a constant force of 100 N. First boat with the sailor has a mass of 250 kg whereas the mass of second boat is double of this mass (see Fig. 7.451). If the initial distance between the boats was 100 m, the time taken for two boats to meet each other is (neglect water resistance between boats and water)
- a. 13.8 s b. 18.3 s
 c. 3.18 s d. 31.8 s

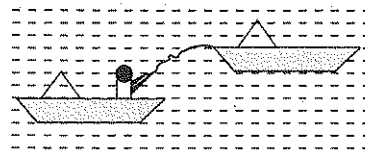


Fig. 7.451

Multiple Correct Answers Type

Solutions on page 7.150

- Which of the following are correct?
 - A parachute of weight W strikes the ground with his legs and comes to rest with an upward acceleration of magnitude $3g$. Force exerted on him by ground during landing is $4W$.
 - Two massless spring balances are hung vertically in series from a fixed point and a mass M kg is attached to the lower end of the lower spring balance. Each spring balance reads M kg -f.
 - A rough vertical board has an acceleration a along the horizontal direction so that a block of mass m passing against it does not fall. The coefficient of friction between the block and the board is greater than g/a .
 - A man is standing at a spring platform. If man jumps away from the platform the reading of the spring balance first increases and then decreases to zero.
- A man tries to remain in equilibrium by pushing with his hands and feet against two parallel walls. For equilibrium,

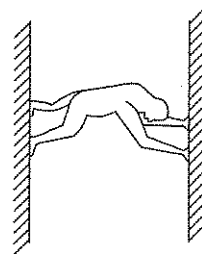


Fig. 7.452

- The forces of friction at the two walls must be equal.
 - Friction must be present on both walls.
 - The coefficient of friction must be the same between both walls and the man.
 - None of the above.
3. Two blocks of masses m_1 and m_2 are connected through a massless inextensible string. Block of mass m_1 is placed at the fixed rigid inclined surface while the block of mass m_2 hanging at the other end of the string, which is passing through a fixed massless frictionless pulley shown in Fig. 7.453. The coefficient of static friction between the block and the inclined plane is 0.8. The system of masses m_1 and m_2 is released from rest.
- The tension in the string is 20 N after releasing the system.

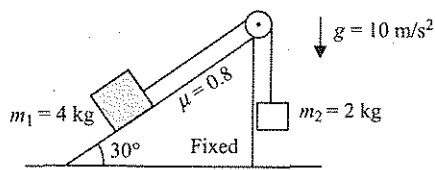


Fig. 7.453

- b. The contact force by the inclined surface on the block is along normal to the inclined surface.
 - c. The magnitude of contact force by the inclined surface on the block m_1 is $20\sqrt{3} \text{ N}$.
 - d. None of these.
4. Figure 7.454 shows two blocks, each of mass m . The system is released from rest at the position, shown in fig. If initial acceleration of blocks A and B are a_1 and a_2 , respectively and during the motion velocities of A and B are v_1 and v_2 respectively, then

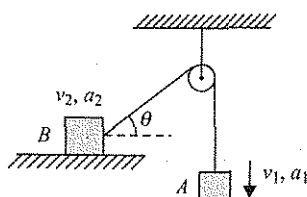


Fig. 7.454

- a. $a_1 = a_2 \cos \theta$
 - b. $a_2 = a_1 \cos \theta$
 - c. $v_1 = v_2 \cos \theta$
 - d. $v_2 = v_1 \cos \theta$
5. A body of mass 5 kg is suspended by the strings making angles 60° and 30° with the horizontal as shown in Fig. 7.455 ($g = 10 \text{ m/s}^2$)

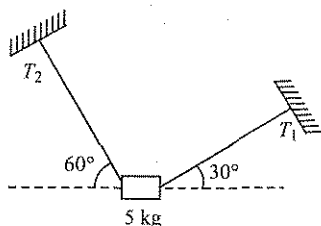


Fig. 7.455

- a. $T_1 = 25 \text{ N}$
 - b. $T_2 = 25 \text{ N}$
 - c. $T_1 = 25\sqrt{3} \text{ N}$
 - d. $T_2 = 25\sqrt{3} \text{ N}$
6. In Fig. 7.456, a man of true mass M is standing on a weighing machine placed in a cabin. The cabin is joined by a string with a body of mass m . Assuming no friction, and negligible mass of cabin and weighing machine, the measured mass of man is (normal force between the man and the machine is proportional to the mass)
- a. Measured mass of man is $\frac{Mm}{(M+m)}$.
 - b. Acceleration of man is $\frac{mg}{(M+m)}$.

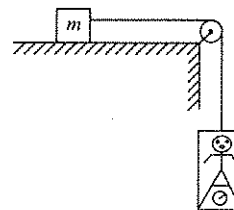


Fig. 7.456

- c. Acceleration of man is $\frac{Mg}{(M+m)}$.
 - d. Measured mass of man is M .
7. The string shown in the Fig. 7.457 is passing over small smooth pulley rigidly attached to trolley A. If speed of trolley is constant and equal to V_A . Speed and magnitude of acceleration of block B at the instant shown in figure is

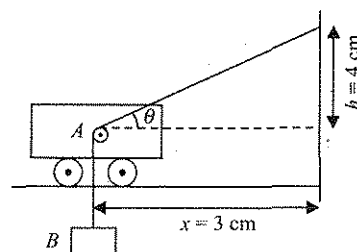


Fig. 7.457

- a. $v_B = v_A, a_B = 0$
 - b. $a_B = 0$
 - c. $v_B = \frac{3}{5} v_A$
 - d. $a_B = \frac{16v_A}{125}$
8. Figure 7.458 shows a block of mass m placed on a smooth wedge of mass M . Calculate the value of M' and tension in the string, so that the block of mass m will move vertically downward with acceleration 10 m/s^2

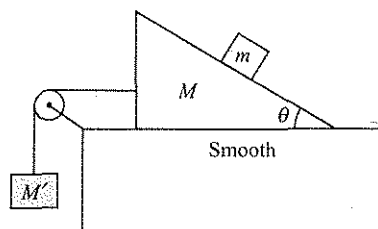


Fig. 7.458

- a. the value of M' is $\frac{M \cot \theta}{1 - \cot \theta}$
 - b. the value of M' is $\frac{M \tan \theta}{1 - \tan \theta}$
 - c. the value of tension in the string is $\frac{Mg}{\tan \theta}$
 - d. the value of tension is $\frac{\mu g}{\cot \theta}$
9. The ring shown in the Fig. 7.459 is given a constant horizontal acceleration ($a_0 = g/\sqrt{3}$). Maximum deflection of the string from the vertical is θ_0 , then

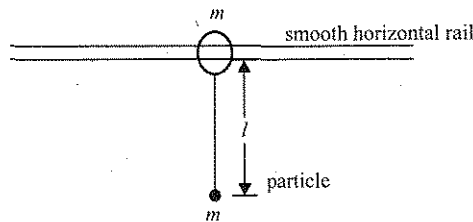


Fig. 7.459

- $\theta_0 = 30^\circ$
- $\theta_0 = 60^\circ$
- At maximum deflection, tension in string is equal to mg .
- At maximum deflection, tension in string is equal to $\frac{2mg}{\sqrt{3}}$.

10. In the Fig. 7.460 small block is kept on m then

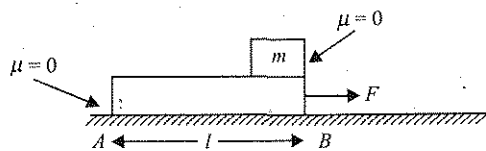


Fig. 7.460

- The acceleration of m w.r.t. ground is $\frac{F}{m}$.
- The acceleration of m w.r.t. ground is zero.
- The time taken by m to separate from M is $\sqrt{\frac{2lm}{F}}$.
- The time taken by m to separate from M is $\sqrt{\frac{2lM}{F}}$.

11. In the Fig. 7.461, if $F = 4$ N, $m = 2$ kg, $M = 4$ kg then

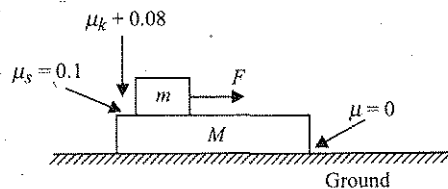


Fig. 7.461

- The acceleration of m w.r.t. ground is $\frac{2}{3} \text{ m/s}^2$.
- The acceleration of m w.r.t. ground is 1.2 m/s^2 .
- The acceleration of M is 0.4 m/s^2 .
- The acceleration of m w.r.t. ground is $\frac{2}{3} \text{ m/s}^2$.

12. A force F is applied vertically upward to the pulley and it is observed that the pulley in the Fig. 7.462 moves upward with a uniform velocity of 2 m/s . The possible value(s) of F is/are (in newtons)

- 150
- 120
- 75
- 400

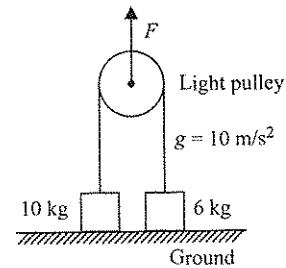


Fig. 7.462

13. A 20 kg block is placed on top of 50 kg block as shown in Fig. 7.463. A horizontal force F acting on A causes an acceleration of 3 m/s^2 to A and 2 m/s^2 to B as shown in Fig. 7.464. For this situation mark out the correct statement (s).

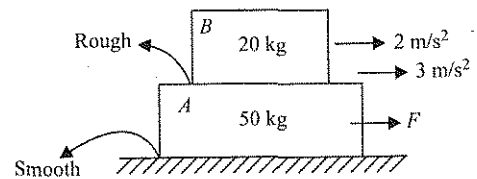


Fig. 7.463

- The friction force between A and B is 40 N .
- The net force acting on A is 150 N .
- The value of F is 190 N .
- The value of F is 150 N .

14. A block is pressed against a vertical wall as shown in Fig. 7.464.

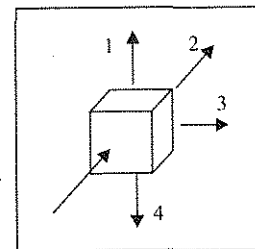


Fig. 7.464

- It is most easier to slide the block along 4.
- It is most difficult to slide the block along 1.
- It is equally easier or difficult to slide the block in any direction.
- It is most difficult to slide the block along 3.

15. Two blocks A and B masses m_A and m_B velocity v and $2v$, respectively, at a given instant (see Fig. 7.465). A horizontal force F acts on the block A . There is no friction between ground and block B and there is coefficient of friction between A and B is μ . The friction

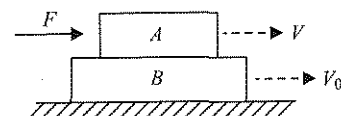


Fig. 7.465

- a. on A oppose its motion.
- b. on B oppose its motion relative to A .
- c. on A and B is $\mu m_{AB}g$.
- d. on the block B is $\frac{\mu m_B}{(m_A + m_B)}F$.

16. A block of mass m is placed in contact with one end of a smooth tube of mass M (see Fig. 7.466). A horizontal force F acts on the tube in each case (i) and (ii). Then,

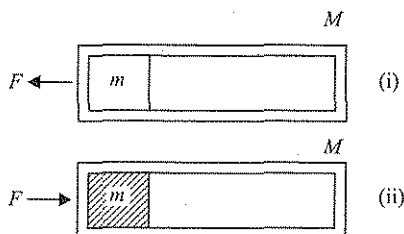


Fig. 7.466

- a. $a_m = 0$ and $a_M = \frac{F}{M}$ in (i)
- b. $a_m = a_M = \frac{F}{M + m}$ in (i)
- c. $a_m = a_M = \frac{F}{M + m}$ in (ii)
- d. Force on m is $\frac{mF}{M + m}$ in (ii)

17. Two rough block A and B , A placed over B , move with accelerations $\vec{a}_A$ and $\vec{a}_B$, velocities $\vec{v}_A$ and $\vec{v}_B$ by the action of horizontal forces $\vec{F}_A$ and $\vec{F}_B$, respectively (see Fig. 7.467). When no friction exists between the blocks A and B

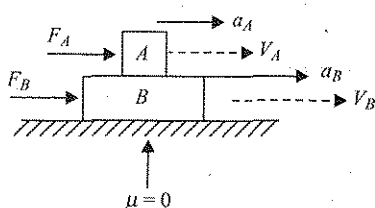


Fig. 7.467

- a. $v_A = v_B$ and $a_A = a_B$
- b. $a_A = a_B$
- c. both (a) and (b)
- d. $\frac{F_A}{m_A} = \frac{F_B}{m_B}$ and $v_A = v_B$

18. Which of the following statement(s) can be explained by Newton's second law of motion?

- a. To stop a heavy body (say truck), much greater force is needed than to stop a light body, (say motorcycle) in the same time, if they are moving with the same speed.
- b. For a given body, the greater the speed, the greater is the opposing force needed to stop the body in a certain time.

- c. To change the momentum (given), the force required is independent of time.
- d. The same force acting on two different bodies for the same time causes the same change in momentum for different bodies.

19. Some statements are given below, which are in one way or other can be explained by Newton's laws of motion. Mark the correct statement(s).

- a. In a tug of war, the team that pushes the ground harder, wins.
- b. In a tug of war, the team that pushes the ground harder (horizontally), wins.
- c. Observers win two different inertial frames will measure the same acceleration of a moving object then the velocity of the object wrt two observers would be also same.
- d. A horizontal force acts on a body that is free to move. Can it produce an acceleration if this force is equal to half of the weight of that body?

20. Suppose a body that is acted on by exactly two forces is accelerated. For this situation mark the incorrect statement(s).

- a. The body cannot move with constant speed.
- b. The velocity can never be zero.
- c. The sum of two forces cannot be zero
- d. The two forces must act in the same line.

21. Mark the correct statement(s) regarding friction.

- a. Friction force can be zero, even though the contact surface is rough.
- b. Even though there is no relative motion between surfaces, frictional force may exist between them.
- c. The expressions $f_L = \mu_s N$ or $f_K = \mu_k N$ are approximate expressions.
- d. Does the expression $f_L = \mu_s N$ tells that direction of f_L and N are the same.

22. A 3 kg block of wood is on a level surface where $\mu_s = 0.25$ and $\mu_k = 0.2$. A force of 7 N is being applied horizontally to the block. Mark the correct statement(s) regarding this situation.

- a. If the block is initially at rest, it will remain at rest and friction force will be about 7 N.
- b. If the block is initially moving, then it will continue its motion for forever if force applied is in direction of motion of the block.
- c. If the block is initially moving and direction of applied force is same as that of motion of block then block moves with an acceleration of $1/3 \text{ m/s}^2$ along its initial direction of motion.
- d. If the block is initially moving and direction of applied force is opposite to that of initial motion of block then block decelerates, comes to a stop and starts moving in the opposite direction.

23. Seven pulleys are connected with the help of three light strings as shown in the Fig. 7.468 given below. Consider P_3 , P_4 , and P_5 as light pulleys and pulleys P_6 and P_7 have masses m each. For this arrangement mark the correct statement(s).

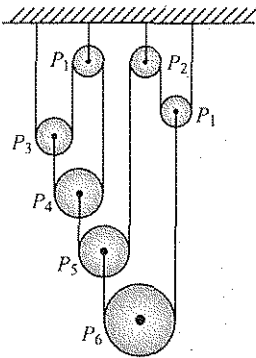


Fig. 7.468

- Tension in the string connecting P_1 , P_3 , and P_4 is zero.
 - Tension in the string connecting P_1 , P_3 , and P_4 is $mg/3$.
 - Tension in all the 3 strings is same and equal to zero.
 - Acceleration of P_6 is g downward and that of P_7 is g upward.
24. A force $\vec{F}$ (larger than the limiting friction force) is applied to the left to an object moving to the right on a rough horizontal surface. Then
- the object would be slowing down initially.
 - for some time $\vec{F}$ and friction force will act in same direction and for remaining time they act in opposite directions.
 - the object comes to rest for a moment and after that its motion is accelerating in the direction of $\vec{F}$.
 - the object slows down and finally comes to rest.
25. A block is resting over a rough horizontal floor. At $t = 0$, a time varying force starts acting on it, the force is described by equation $F = kt$, where k is constant and t is in seconds. Mark the correct statement(s) for this situation.

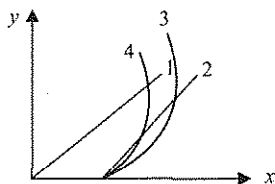


Fig. 7.469

- Curve 1 shows acceleration-time graph.
- Curve 2 shows acceleration-time graph.
- Curve 3 shows velocity-time graph.
- Curve 4 shows displacement-time graph.

26. A block of mass m is placed on a wedge. The wedge can be accelerated in four manners marked as (1), (2), (3), and (4) as shown in Figs. 7.470 and 7.471. If the normal reactions in situation (1), (2), (3), and (4) are N_1 , N_2 , N_3 , and N_4 , respectively, and acceleration with which the block slides on the wedge in situations are b_1 , b_2 , b_3 , and b_4 , respectively, then

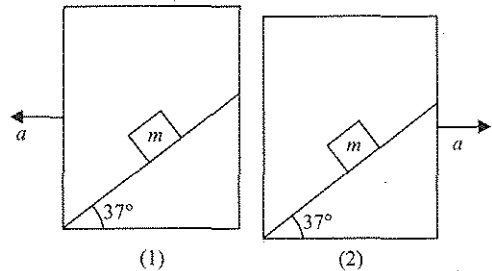


Fig. 7.470

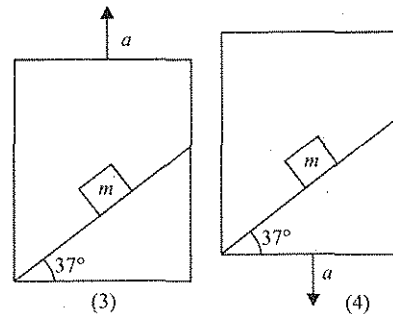


Fig. 7.471

- $N_3 > N_1 > N_2 > N_4$
 - $N_4 > N_3 > N_1 > N_2$
 - $b_2 > b_3 > b_4 > b_1$
 - $b_2 > b_3 > b_1 > b_4$
27. Two blocks A and B of mass 5 kg and 2 kg, respectively, connected by a spring of force constant = 100 N/m are placed on an inclined plane of inclination 30° as shown in Fig. 7.472. If the system is released from rest then

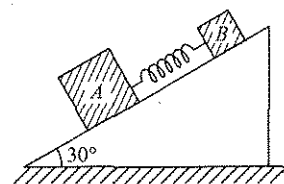


Fig. 7.472

- There will be no compression or elongation in the spring if all the surfaces are smooth.
- There will be elongation in the spring if A is rough and B is smooth.
- Maximum elongation in the spring is 35 cm if all surfaces are smooth.
- There will be elongation in the spring if A is smooth and B is rough.

28. Force exerted by the floor of a lift on the foot of a boy standing on it is more than the actual weight of the boy if the lift is moving
- down and speed is increasing
 - up and speed is increasing
 - up and speed is decreasing
 - down and speed is decreasing
29. The acceleration of a particle as observed from two different frames S_1 and S_2 have equal magnitudes of 2 ms^{-2} .
- The relative acceleration of the frame may either be 0 or 4 m/s^2 .
 - Their relative acceleration may have any value between 0 and 4 m/s^2 .
 - Both of the frames may be stationary with respect to earth.
 - The frames may be moving with same acceleration in same direction.
30. Coefficient of friction between the two blocks is 0.3. Whereas the surface AB is smooth

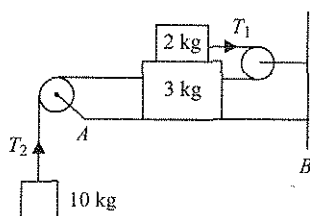


Fig. 7.473

- Acceleration of the system of masses is $88/15 \text{ m/s}^2$.
 - Net force acting on 3 kg mass is greater than that on 2 kg mass.
 - Tension $T_2 > T_1$.
 - Since 10 kg mass is accelerating downward, so net force acting on it should be greater than any of the two blocks shown in Fig. 7.473.
31. In the Fig. 7.474, the blocks A, B, and C of mass m each have acceleration a_1 , a_2 , and a_3 , respectively. F_1 and F_2 are external forces of magnitude $2mg$ and mg , respectively, then

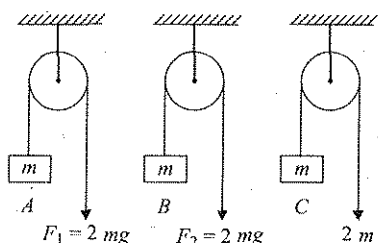


Fig. 7.474

- $a_1 \neq a_2 \neq a_3$
- $a_1 = a_2 \neq a_3$
- $a_1 > a_2 > a_3$
- $a_1 \neq a_2 = a_3$

32. Assume right side to be positive. The coefficient of friction between the blocks and ground is shown in Fig. 7.475. The correct options are

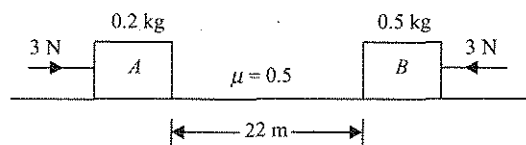


Fig. 7.475

- Acceleration of block A is 10 m/s^2 .
 - Acceleration of block B is -1 m/s^2 .
 - The time (t) at which they collide to each other is 2 s.
 - None of the above.
33. A 10 kg block is placed on the top of a 40 kg block as shown in Fig. 7.476. A horizontal force F acting on B causes an acceleration of 2 m/s^2 to B. For this situation mark out the correct statement (s).

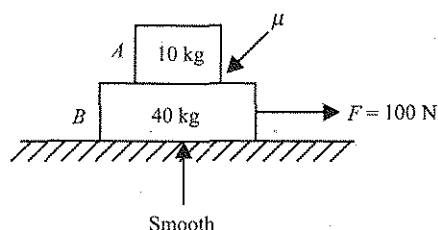


Fig. 7.476

- The acceleration of A may also be 2 m/s^2 .
- The acceleration of A must also be 2 m/s^2 .
- The coefficient of friction between the blocks may be 0.2.
- The coefficient of friction between the blocks must be 0.2 only.

Assertion-Reasoning Type

Solutions on page 7.155

Some questions (Assertion-Reason type) are given below. Each question contains Statement I (Assertion) and Statement II (Reason). Each question has 4 choices (a), (b), (c), and (d) out of which **only one** is correct. So select the correct choice.

- Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
- Statement I is True, Statement II is True; Statement II is NOT a correct explanation for Statement I.
- Statement I is True, Statement II is False.
- Statement I is False, Statement II is True.

1. **Statement I:** Rate of change of linear momentum is equal to external force.

Statement II: There is equal and opposite reaction to every action.

2. **Statement I:** A block of mass m is kept at rest on an inclined plane, the rest force applied by the surface to the block will be mg .
Statement II: Normal contact force is the resultant of normal contact force and friction force.
3. **Statement I:** The driver of a moving car sees a wall in front of him. To avoid collision, he should apply brakes rather than taking a turn away from the wall.
Statement II: Friction force is needed to stop the car or taking a turn on a horizontal road.
4. **Statement I:** A bird alights on a stretched wire depressing it slightly. The increase in tension of the wire is more than the weight of the bird.
Statement II: The tension must be more than the weight as it is required to balance weight.
5. **Statement I:** Newton's first law is merely a special case ($a = 0$) of the second law.
Statement II: Newton's first law defines the frame from where Newton's second law; $\vec{F} = m\vec{a}$, $\vec{F}$ representing the net real force acting on a body; is applicable.
6. **Statement I:** A uniform rope of mass m hangs freely from a ceiling. A monkey of mass M climbs up the rope with an acceleration a . The force exerted by the rope on the ceiling is $M(a + g) + mg$.
Statement II: Action and reaction force are acting on two different bodies.
7. **Statement I:** According to Newton's second law of motion action and reaction forces are equal and opposite.
Statement II: Action and reaction forces never cancel out each other because they are acting on different objects.
8. **Statement I:** A block of mass m is placed on a smooth inclined plane of inclination θ with the horizontal. The force exerted by the plane on the block has a magnitude $mg \cos \theta$.
Statement II: Normal reaction always acts perpendicular to the contact surface.
9. **Statement I:** A particle is found to be at rest when seen from a frame S_1 and moving with a constant velocity when seen from another frame S_2 . We can say both the frames are inertial.
Statement II: All frames moving uniformly with respect to an inertial frame are themselves inertial.
10. **Statement I:** Coefficient of friction can be greater than unity.
Statement II: Force of friction is dependent on normal reaction and ratio of force of friction and normal reaction cannot exceed unity.
11. **Statement I:** In high jump, it hurts less when an athlete lands on a heap of sand.
Statement II: Because of greater distance and hence greater time over which the motion of an athlete is stopped, the athlete experience less force when lands on heap of sand.
12. **Statement I:** For a boy it is difficult to run at high speed on a rainy day.
Statement II: Coefficient of friction μ is decreased due to rain.

13. **Statement I:** A body in equilibrium has to be at rest only.
Statement II: A body in equilibrium may be moving with a constant speed along a straight line path.
14. **Statement I:** Inertia is the property by virtue of which the body is unable to change by itself the state of rest.
Statement II: The bodies do not change their state unless acted upon by an un-balanced external force.
15. **Statement I:** Pulling (refer Fig. 7.477) is easier than pushing [refer Fig. 7.477(b)] on a rough surface.
Statement II: Normal reaction is less in pulling than is pushing.

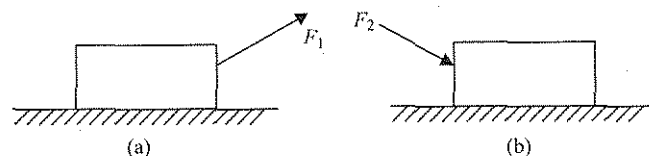


Fig. 7.477

16. **Statement I:** A block is lying stationary as on inclined plane and coefficient of friction is μ . Friction on block is $\mu mg \cos \theta$.
Statement II: Contact force on the block is mg (Fig. 7.478).

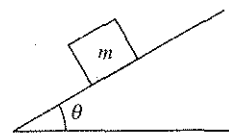


Fig. 7.478

17. **Statement I:** Static frictional force is always greater than the kinetic frictional force.
Statement II: (Coefficient of static friction) $\mu_s > \mu_k$ (coefficient of kinetic friction).
18. **Statement I:** Two particles are moving towards each other due to mutual gravitational attraction. The momentum of each particle will increase.
Statement II: Rate of change of momentum depends upon F_{ext} .
19. **Statement I:** A concept of pseudo forces is valid both for inertial as well as non-inertial frame of reference.
Statement II: A frame accelerated with respect to an inertial frame is a non-inertial frame.
20. **Statement I:** For all bodies the momentum always remains the same.
Statement II: If two bodies of different masses have same momentum the lighter body possesses greater velocity.
21. **Statement I:** In Fig. 7.479 the ground is smooth and the masses of both the blocks are different. Net force acting on each of the block is not same.
Statement II: Acceleration of the blocks both will be different.

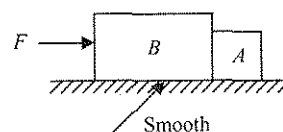


Fig. 7.479

22. **Statement I:** When static friction acts between two bodies, there is no loss of mechanical energy.

Statement II: When kinetic friction acts between two bodies, there is loss of mechanical energy.

23. **Statement I:** Greater is the rate of the change in the momentum vector, the greater is the force applied.

Statement II: Newton's second law is $\vec{F} = \frac{d\vec{p}}{dt}$.

24. **Statement I:** Frictional heat generated by the moving ski is the chief factor which promotes sliding in skiing while waxing the ski makes skiing more easy.

Statement II: Due to friction energy dissipates in the form of heat as a result it melts the snow below it. Wax is water repellent.

25. **Statement I:** The acceleration of a particle as seen from an inertial frame is zero.

Statement II: No force is acting on the particle.

26. **Statement I:** A reference frame attached to the earth is an inertial frame of reference.

Statement II: Newton's laws can be applied in this frame of reference.

27. **Statement I:** If a body is trying to slip over a surface then friction acting on the body is necessarily equal to the limiting friction.

Statement II: Static friction can be less than the limiting friction force.

28. **Statement I:** Blocks A is moving on horizontal surface towards right under action of force F . All surfaces are smooth. At the instant shown the force exerted by block A on block B is equal to net force on block B.

Statement II: From Newton's third law of motion, the force exerted by block A on B is equal in magnitude to force exerted by block B on A (Fig. 7.480).

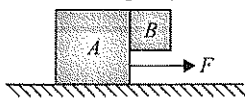


Fig. 7.480

29. **Statement I:** A man standing in a lift which is moving upwards, will feel his weight to be greater than when the lift was at rest.

Statement II: If the acceleration of the lift is a upward, then the man of mass m shall feel his weight to be equal to normal reaction (N) exerted by the lift given by $N = m(g + a)$ (where g is acceleration due to gravity).

30. **Statement I:** Greater is the mass, greater is the force required to change the state of body at rest or in uniform motion.

Statement II: The rate of change of momentum is the measure of the force.

rest by thread BC . Now thread BC is burnt. Answer the followings:

- Before burning the thread, what are the tensions in spring and thread BC , respectively?

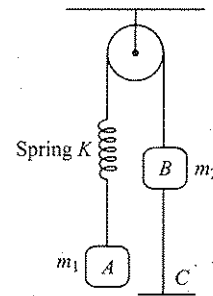


Fig. 7.481

- m_1g, m_2g
- $m_1g, m_1g - m_2g$
- m_2g, m_1g
- $m_1g, m_1g + m_2g$

- Just after burning the thread, what is the tension in the spring?

- m_1g
- m_2g
- 0
- can't say

- Just after burning the thread, what is the acceleration of m_2 ?

- $\left(\frac{m_2 - m_1}{m_2}\right)g$
- $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$
- 0
- $\left(\frac{m_1 - m_2}{m_2}\right)g$

For Problems 4–6

Three blocks A, B, and C have masses 1 kg, 2 kg, and 3 kg, respectively are arranged as shown in Fig. 7.482. The pulleys P and Q are light and frictionless. All the blocks are resting on a horizontal floor and the pulleys are held such that strings remain just taut. At moment $t = 0$ a force $F = 40t$ N starts acting on pulley P along vertically upward direction as shown in Fig. 7.482 (take $g = 10 \text{ m/s}^2$).

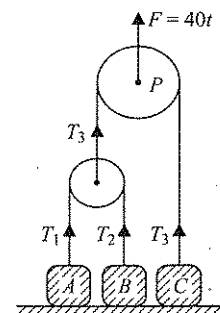


Fig. 7.482

- Regarding the time when the blocks lose contact with ground, which of the following is correct?

- A loses contact at $t = 2$ s

Comprehensive Type

Solutions on page 7.156

For Problems 1–3

In the system shown in Fig. 7.481, $m_1 > m_2$. System is held at

- b. C loses contact at $t = 1.5$ s
 c. A and B lose the contact at the same time
 d. All three blocks lose the contact at the same time
5. Find the velocity of A when B loses contact with ground.
 a. 5 m/s
 b. $5/4$ m/s
 c. 4 m/s
 d. $7/3$ m/s
6. What is the magnitude of relative acceleration of A with respect to B just after both have lost contact with ground?
 a. 15 m/s^2
 b. 5 m/s^2
 c. 20 m/s^2
 d. 10 m/s^2

For Problems 7–9

A body hangs from a spring balance supported from the roof of an elevator.

7. If the elevator has an upward acceleration of 2 m/s^2 and balance reads 240 N, what is the true weight of the body?
 a. 20 N
 b. 200 N
 c. 100 N
 d. 300 N
8. Under what acceleration will the balance read 160 N?
 a. 2 m/s^2 , up
 b. 2 m/s^2 , down
 c. 4 m/s^2 , up
 d. 4 m/s^2 , down
9. What will the balance read if the elevator cable breaks?
 a. 100 N
 b. 200 N
 c. zero
 d. 300 N

For Problems 10–13

Figure 7.483 shows a block of mass m_1 sliding on a block of mass m_2 , with $m_1 > m_2$. Friction is absent everywhere.

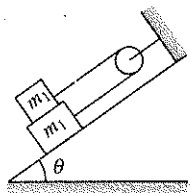


Fig. 7.483

10. Find the acceleration of each block.
 a. $\frac{(m_1 + m_2)g \sin \theta}{(m_1 - m_2)}$
 b. $\frac{(m_1 - m_2)g \sin \theta}{(m_1 + m_2)}$
 c. $\frac{(m_1 - m_2)g \sin \theta}{(m_1 + 2m_2)}$
 d. $\frac{(m_1 - m_2)g \sin \theta}{(2m_1 + m_2)}$
11. Find tension in the string.
 a. $\frac{2m_1 m_2 g \sin \theta}{(m_1 + 2m_2)}$
 b. $\frac{2m_1 m_2 g \sin \theta}{(2m_1 + m_2)}$
 c. $\frac{2m_1 m_2 g \sin \theta}{(m_1 + m_2)}$
 d. $\frac{m_1 m_2 g \sin \theta}{(m_1 + m_2)}$
12. Find force exerted by m_1 on m_2 .
 a. $m_1 g \tan \theta$
 b. $m_1 g \cos \theta$
 c. $m_2 g \tan \theta$
 d. $m_2 g \cos \theta$

13. Find force exerted by m_2 on the incline.

- a. $(m_1 + m_2)g \sin \theta$
 b. $(2m_1 + m_2)g \sin \theta$
 c. $(m_1 + 2m_2)g \cos \theta$
 d. $(m_1 + m_2)g \cos \theta$

For Problems 14–16

A ball of mass 200 gm is thrown with a speed 20 ms^{-1} . The ball strikes a bat and rebounds along the same line at a speed of 40 ms^{-1} . Variation in the interaction force, as long as the ball remains in contact with the bat, is as shown in Fig. 7.484.

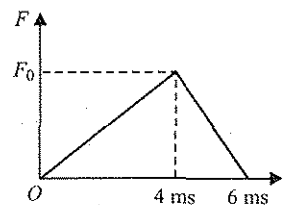


Fig. 7.484

14. Maximum force F_0 exerted by the bat on the ball is
 a. 4,000 N
 b. 5,000 N
 c. 3,000 N
 d. 2,500 N
15. Average force exerted by the bat on the ball is
 a. 5,000 N
 b. 2,000 N
 c. 2,500 N
 d. 6,000 N
16. What is the speed of the ball at the instant the force acting on it is maximum?
 a. 40 m/s
 b. 30 m/s
 c. 20 m/s
 d. 10 m/s

For Problem 17–20

In the system shown in Fig. 7.485, $m_A = 4$ m, $m_B = 3$ m, and $m_C = 8$ m. Friction is absent everywhere, and the string is inextensible and light. If the system is released from rest, then find the following.

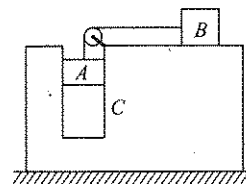


Fig. 7.485

17. The tension in the string is
 a. 1.5 mg
 b. 5.8 mg
 c. 4.7 mg
 d. 3.2 mg
18. The acceleration of Block C is
 a. $1/4 \text{ m/s}^2$
 b. $2/7 \text{ m/s}^2$
 c. $5/4 \text{ m/s}^2$
 d. $1/3 \text{ m/s}^2$
19. The acceleration of Block B is
 a. 20 m/s^2
 b. 5 m/s^2
 c. 15 m/s^2
 d. 10 m/s^2

20. The acceleration of Block A in horizontal direction is

- a. $5/4 \text{ m/s}^2$ b. $21/2 \text{ m/s}^2$
c. $11/7 \text{ m/s}^2$ d. $27/5 \text{ m/s}^2$

For Problems 21–23

Two blocks of mass $m = 10 \text{ kg}$ and $M = 20 \text{ kg}$ are connected by a string passing over a pulley B as shown in Fig. 7.486. Another string connects the centre of pulley B and passes over another pulley A as shown. An upward force F is applied at the centre of pulley A. Both the pulleys are massless. When F is 600 N then

21. Tension T in string over B connecting the blocks m and M is

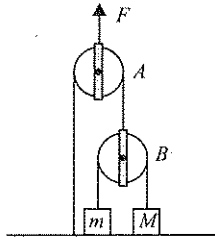


Fig. 7.486

- a. 25 N b. 50 N c. 75 N d. 150 N

22. The acceleration of mass m is

- a. 2.5 m/s^2 b. 5 m/s^2
c. 10 m/s^2 d. 0 m/s^2

23. The acceleration of mass M is

- a. 20 m/s^2 b. 1.5 m/s^2
c. 0 m/s^2 d. 5 m/s^2

For Problems 24–26

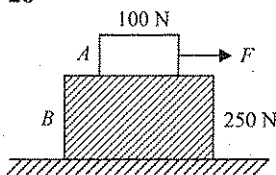


Fig. 7.487

Block B rests on a smooth surface (Fig. 7.487). If the coefficient of static friction between A and B is $\mu = 0.4$, determine the acceleration of each. When $F = 30 \text{ N}$ then

24. The acceleration of upper block is

- a. $3/2 \text{ m/s}^2$ b. $6/7 \text{ m/s}^2$
c. $4/3 \text{ m/s}^2$ d. $3/7 \text{ m/s}^2$

25. The acceleration of lower block is

- a. $3/7 \text{ m/s}^2$ b. $4/3 \text{ m/s}^2$
c. $6/7 \text{ m/s}^2$ d. $3/2 \text{ m/s}^2$

26. When $F = 250 \text{ N}$, the acceleration of lower block is

- a. $4/5 \text{ m/s}^2$ b. $3/2 \text{ m/s}^2$
c. $3/5 \text{ m/s}^2$ d. $8/5 \text{ m/s}^2$

For Problems 27–29

Study the following diagram (Fig. 7.488) and answer the following questions accordingly. Neglect all the friction and the masses of the pulleys.

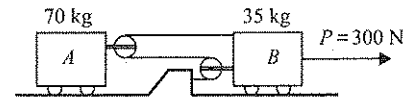


Fig. 7.488

27. What is the tension in the string?

- a. $\frac{700}{11} \text{ N}$ b. $\frac{450}{11} \text{ N}$
c. $\frac{500}{11} \text{ N}$ d. $\frac{900}{11} \text{ N}$

28. What is the acceleration of Block B?

- a. $30/77 \text{ m/s}^2$ b. $60/77 \text{ m/s}^2$
c. $80/77 \text{ m/s}^2$ d. $120/77 \text{ m/s}^2$

29. What is the acceleration of Block C?

- a. $60/77 \text{ m/s}^2$ b. $80/77 \text{ m/s}^2$
c. $180/77 \text{ m/s}^2$ d. $60/77 \text{ m/s}^2$

For Problems 30–33

A monkey of mass m clings to a rope slung over a fixed pulley (see Fig. 7.489). The opposite end of the rope is tied to a weight of mass M lying on a horizontal plate. The coefficient of friction between the weight and the plate is μ . Find the acceleration of the weight and the tension of the rope for two cases.

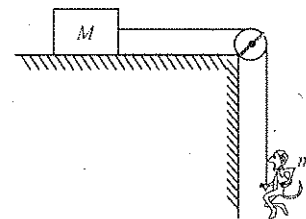


Fig. 7.489

Case 1

The monkey does not move with respect to the rope.

30. The acceleration of the weight is

- a. $\frac{(m - \mu M)}{m + M} g$ b. $\frac{(m - \mu M)}{m - M} g$
c. $\frac{(M - \mu m)}{M + m} g$ d. $\frac{(2M + \mu m)}{M + 2m} g$

31. The tension of rope is

- a. $\frac{mMg(3 + \mu)}{2M + m}$ b. $\frac{2mMg(2 + \mu)}{M + m}$
c. $\frac{mMg(1 - \mu)}{M - m}$ d. $\frac{mMg(1 + \mu)}{M + m}$

Case 2

The monkey moves downwards with respect to the rope with an acceleration b .

32. The acceleration of the weight is

- a. $\frac{2m(g + b) - \mu Mg}{M + 2m}$

- b. $\frac{m(g+b) - \mu Mg}{2(M+m)}$
 c. $\frac{m(g+b) - 3\mu Mg}{M+3m}$
 d. $\frac{m(g-b) - \mu Mg}{M+m}$

33. The tension of the rope is

- a. $\frac{Mmg(\mu+1+b)}{M-m}$
 b. $\frac{2Mmg(\mu-2+b)}{M+m}$
 c. $\frac{Mmg(\mu+1-b)}{M+m}$
 d. $\frac{2Mmg(\mu-1+b)}{M-m}$

For Problems 34–35

In Fig. 7.490 the weight w is 60.0 N.

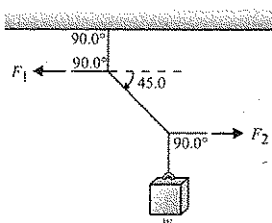


Fig. 7.490

34. The tension in the diagonal string is

- a. 60 N
 b. 90 N
 c. 85 N
 d. 100 N

35. Find the magnitudes of the horizontal forces F_1 and F_2 that must be applied to hold the system in the position shown.

- a. 75 N, 90 N, respectively
 b. 60 N, 60 N, respectively
 c. 90 N, 90 N, respectively
 d. 45 N, 90 N, respectively

For Problems 36–38

As shown in Fig. 7.491 Block A has a mass of 4.00 kg and block B has mass of 12.0 kg. The coefficient of kinetic friction between block B and the horizontal surface is 0.25.

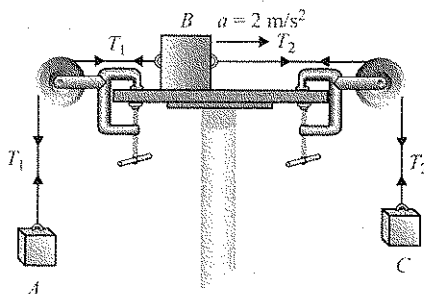


Fig. 7.491

36. What is the mass of block C (approximately) if block B is moving to the right and speeding up with an acceleration 2.00 m/s²?

- a. 20 kg
 b. 32 kg
 c. 25 kg
 d. 13 kg

37. When block B has this acceleration. What is the tension T_2 (approximately)?

- a. 102 N
 b. 200 N
 c. 48 N
 d. 60 N

38. What is the tension T_1 (approximately)?

- a. 102 N
 b. 200 N
 c. 48 N
 d. 60 N

For Problems 39–40

39. As shown in Fig. 7.492 Block A weighs 60.0 N. The coefficient of static friction between the block and the surface on which it rests is 0.25. The weight w is 12.0 N and the system is in equilibrium. Find the friction force exerted on block A.

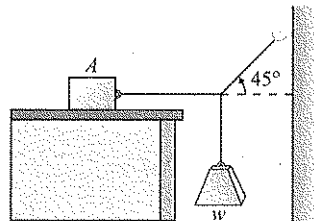


Fig. 7.492

- a. 5.7 N
 b. 7.5 N
 c. 9.0 N
 d. 12.0 N

40. Find the maximum weight w for which the system will remain in equilibrium.

- a. 5 N
 b. 7 N
 c. 15 N
 d. 17 N

For Problems 41–43

Block A weighs 4 N and block B weighs 8 N (Fig. 7.493). The coefficient of kinetic friction is 0.25 for all surfaces. Find the force F to slide B at a constant speed when

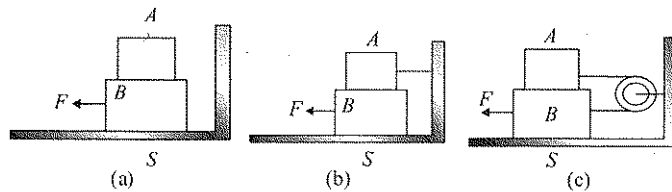


Fig. 7.493

41. A rests on B and moves with it.

- a. 2 N
 b. 3 N
 c. 4 N
 d. 5 N

42. A is held at rest.

- a. 2 N
 b. 3 N
 c. 4 N
 d. 5 N

43. A and B are connected by a light cord passing over a smooth pulleys.

- a. 2 N
 b. 3 N
 c. 4 N
 d. 5 N

For Problems 44–46

Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless inextensible string and a frictionless pulley as shown in Fig. 7.494. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between block A and the wedge is $2/3$ and that between block B and the wedge is $1/3$. If the system of A and B is released from rest, find the following.

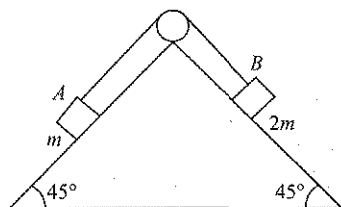


Fig. 7.494

44. The acceleration of A is

a. $\frac{g}{3\sqrt{2}}$

b. zero

c. $\frac{g}{\sqrt{7}}$

d. $\frac{g}{2\sqrt{3}}$

45. The tension in the string is

a. $\frac{3}{\sqrt{5}} mg$

b. $\frac{5}{3\sqrt{2}} mg$

c. $\frac{2\sqrt{2}mg}{3}$

d. $\frac{mg}{3}$

46. The magnitude and direction of the force of friction acting on A is

 a. mg , down the plane

 b. $\frac{mg}{2}$, up the plane

 c. $\frac{mg}{\sqrt{2}}$, up the plane

 d. $\frac{mg}{3\sqrt{2}}$, down the plane

For Problems 47–48

A block of mass 10 kg is kept on a rough floor. Coefficients of friction between floor and block are $\mu_s = 0.4$ and $\mu_k = 0.3$. Forces $F_1 = 5\text{ N}$ and $F_2 = 4\text{ N}$ are applied on the block as shown in Fig. 7.495.



Fig. 7.495

47. The magnitude of friction force is

a. $\sqrt{31}\text{ N}$

b. $\sqrt{26}\text{ N}$

c. $\sqrt{41}\text{ N}$

d. $\sqrt{36}\text{ N}$

48. If $F_1 = 5\text{ N}$ and $F_2 = a\text{ N}$, for what maximum value of a the motion of block impends?

a. $\sqrt{1575}\text{ N}$

b. $\sqrt{1225}\text{ N}$

c. $\sqrt{1664}\text{ N}$

d. $\sqrt{875}\text{ N}$

For Problems 49–50

Block A has a mass of 40 kg and block B has a mass of 15 kg , and F of 500 N is applied parallel to smooth inclined plane (see Fig. 7.496). The system is moving together.

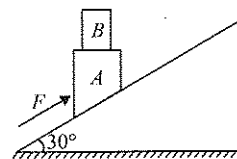


Fig. 7.496

49. The acceleration of the system is

a. $\frac{45}{11}\text{ m/s}^2$

b. $\frac{23}{11}\text{ m/s}^2$

c. $\frac{13}{7}\text{ m/s}^2$

d. $\frac{8}{3}\text{ m/s}^2$

50. The least coefficient of friction between A and B is

a. $\frac{5\sqrt{2}}{12}$

b. $\frac{9\sqrt{3}}{53}$

c. $\frac{9\sqrt{2}}{28}$

d. $\frac{5\sqrt{3}}{18}$

For Problems 51–52

A 10 kg block rests on a 5 kg bracket as shown in the Fig. 7.497. The 5 kg bracket rests on a horizontal frictionless surface. The coefficients of friction between the 10 kg block and the bracket on which it rests are $\mu_s = 0.40$ and $\mu_k = 0.30$.

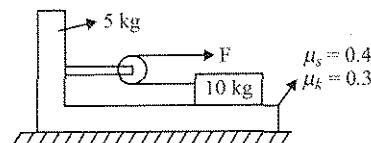


Fig. 7.497

51. The maximum force F that can be applied if the 10 kg block is not to slide on the bracket is

a. 32 N

b. 24 N

c. 18 N

d. 48 N

52. If 10 kg block does not slide on the bracket, the corresponding acceleration of the 5 kg bracket is

a. 1.6 m/s^2

b. 0.8 m/s^2

c. 1.2 m/s^2

d. 2.4 m/s^2

For Problems 53–56

Block B rests on a smooth surface (Fig. 7.498). If the coefficient of static friction between A and B is $\mu = 0.4$, determine the acceleration of each.

When $F = 30\text{ N}$

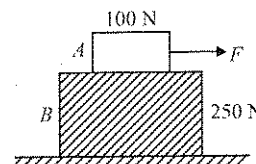


Fig. 7.498

53. The acceleration of upper the block is

- a. $3/7 \text{ m/s}^2$ b. $4/3 \text{ m/s}^2$
c. $6/7 \text{ m/s}^2$ d. $3/2 \text{ m/s}^2$

54. The acceleration of the lower block is

- a. $3/2 \text{ m/s}^2$ b. $6/7 \text{ m/s}^2$
c. $4/3 \text{ m/s}^2$ d. $3/7 \text{ m/s}^2$

55. When $F = 250 \text{ N}$ the acceleration of the upper block is

- a. 15 m/s^2 b. 10 m/s^2
c. 25 m/s^2 d. 21 m/s^2

56. The acceleration of the lower block is

- a. $8/5 \text{ m/s}^2$ b. $3/5 \text{ m/s}^2$
c. $3/2 \text{ m/s}^2$ d. $4/5 \text{ m/s}^2$

For Problem 57–59

A sufficiently long plank of mass 4 kg is placed on a smooth horizontal surface. A small block of mass 2 kg is placed over the plank and is being acted upon by a time varying horizontal force $F = (0.5t)$, where F is in N and t is in second as shown in Fig. 7.499(a). The coefficient of friction between the plank and the block is given as $\mu_s = \mu_k = \mu$. At time $t = 12 \text{ s}$, the relative slipping between the plank and the block is just likely to occur.

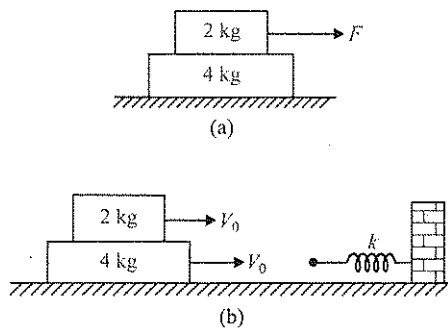


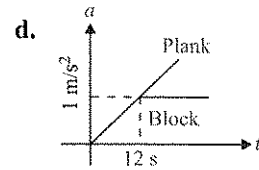
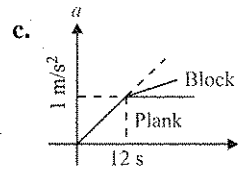
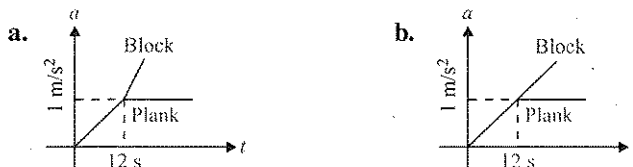
Fig. 7.499

If the force F acting on 2 kg block is removed and the system (plank + block) is given horizontal velocity V_0 , as shown in Fig. 7.499, this system strikes a massless spring of spring constant $k = 120 \text{ N/m}$ fixed at the end of the horizontal place and it is observed that during compression of the spring no relative slipping occurs between the plank and the block.

57. The coefficient of friction μ is equal to

- a. 0.10 b. 0.15 c. 0.20 d. 0.30

58. The acceleration (a) versus time (t) graph for the plank and the block as shown is correctly represented in



59. The average acceleration of the plank in the time interval 0 to 15 s in figure will be

- a. 0.20 m/s^2 b. 0.30 m/s^2
c. 0.40 m/s^2 d. 0.60 m/s^2

For Problems 60–62

Three blocks A, B and C of mass $3M$, $2M$, and M , respectively, are suspended vertically with the help of springs PQ and TU and a string RS as shown in Fig. 7.500. If acceleration of blocks A, B and C are a_1 , a_2 and a_3 , respectively.

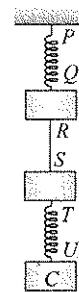


Fig. 7.500

60. The value of acceleration a_3 at the moment spring PQ is cut is

- a. g downward
b. g upward
c. more than g downward
d. zero

61. The value of acceleration a_1 at the moment string RS is cut is

- a. g downward
b. g upward
c. more than g downward
d. zero

62. The value of acceleration a_2 at the moment spring TU is cut is

- a. $g/5$ upward b. $g/5$ downward
c. $g/3$ upward d. zero

For Problems 63–65

In Fig. 7.501 all the pulleys and strings are massless and all the surfaces are frictionless. Small block of mass m is placed on fixed wedge (take $g = 10 \text{ ms}^{-2}$).

63. The tension in the string attached to m is

- a. 40 N b. 10 N c. 20 N d. 5 N

64. The acceleration of m is

- a. 4.5 m/s^2 down the incline

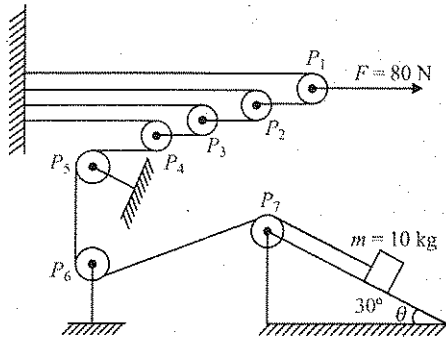


Fig. 7.501

- b. 4.5 m/s^2 up the incline
 c. 5 m/s^2 down the incline
 d. 5 m/s^2 up the incline
65. The acceleration of pulley p_4 is
- a. 2.25 m/s^2 towards left
 b. 2.25 m/s^2 towards right
 c. 9 m/s^2 towards left
 d. 9 m/s^2 towards right

For Problems 66–68

In Fig. 7.502 both the pulleys and the string are massless and all the surfaces are frictionless.

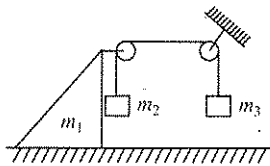


Fig. 7.502

Given $m_1 = 1 \text{ kg}$, $m_2 = 2 \text{ kg}$, $m_3 = 3 \text{ kg}$.

66. The tension in the string is
- a. $\frac{120}{7} \text{ N}$
 b. $\frac{240}{7} \text{ N}$
 c. $\frac{130}{7} \text{ N}$
 d. None of these
67. The acceleration of m_1 is
- a. $\frac{40}{7} \text{ m/s}^2$
 b. $\frac{30}{7} \text{ m/s}^2$
 c. $\frac{20}{7} \text{ m/s}^2$
 d. None of these
68. The acceleration of m_3 is
- a. $\frac{40}{7} \text{ m/s}^2$
 b. $\frac{30}{7} \text{ m/s}^2$
 c. $\frac{20}{7} \text{ m/s}^2$
 d. None of these

For Problems 69–70

A plank A of mass M rests on a smooth horizontal surface over which it can move without friction. A cube B of mass m lies on the plank at one edge. The coefficient of friction between

the plank and the cube is μ . The size of cube is very small in comparison to the plank (Fig. 7.503).

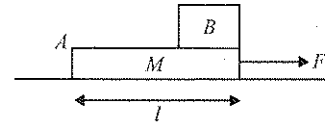


Fig. 7.503

69. At what force F applied to the plank in the horizontal direction will the cube begin to slide towards the other end of the plank?
- a. $F > \mu(m + M)g$
 b. $F > 0.5\mu(m + M)g$
 c. $F = 0.5\mu(m + M)g$
 d. $F = \mu(m + M)g$
70. In what time will the cube fall from the plank if the length of the latter is l .

- a. $\sqrt{\frac{Ml}{F - \mu g(M + m)}}$
 b. $\sqrt{\frac{2Ml}{F - \mu g(M + m)}}$
 c. $\sqrt{\frac{Ml}{F + \mu g(M + m)}}$
 d. $\sqrt{\frac{2Ml}{F + \mu g(M + m)}}$

For Problems 71–74

In the arrangement shown in Fig. 7.504, all pulleys are smooth and massless. When the system is released from the rest, acceleration of blocks 2 and 3 relative to 1 are 1 m/s^2 downwards and 5 m/s^2 downwards. Acceleration of block 3 relative to 4 is 0.

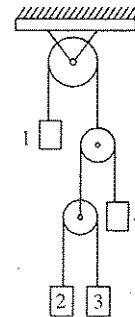


Fig. 7.504

71. The absolute acceleration of block 1 is
- a. 2 m/s^2 upward
 b. 1 m/s^2 downward
 c. 3 m/s^2 upward
 d. 1.5 m/s^2 downward
72. The absolute acceleration of block 2 is
- a. 2 m/s^2 downward
 b. 1 m/s^2 upward

- c. 3 m/s^2 upward
d. 1.5 m/s^2 downward

73. The absolute acceleration of block 3 is

- a. 2 m/s^2 upward
b. 1 m/s^2 downward
c. 3 m/s^2 downward
d. 1.5 m/s^2 upward

74. The absolute acceleration of block 4 is

- a. 2 m/s^2 upward
b. 1 m/s^2 downward
c. 3 m/s^2 downward
d. 1.5 m/s^2 upward

For Problems 75–76

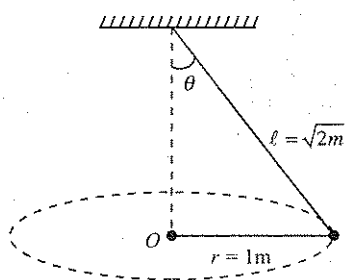


Fig. 7.505

A sphere of mass 500 gm is attached to a string of length $\sqrt{2} \text{ m}$, whose other end is fixed to a ceiling. The sphere is made to describe a circle of radius 1 m in a horizontal plane (Fig. 7.505).

75. The period of revolution for the sphere is

- a. $\pi\sqrt{10} \text{ s}$
b. $\pi\sqrt{5} \text{ s}$
c. $2\pi/\sqrt{10} \text{ s}$
d. $\pi/\sqrt{5} \text{ s}$

76. The tension in the string is

- a. $5\sqrt{2} \text{ N}$
b. $10\sqrt{2} \text{ N}$
c. $5\sqrt{3} \text{ N}$
d. $10\sqrt{3} \text{ N}$

For Problems 77–78

A ball of mass m is suspended from a rope of length L . It describes a horizontal circle of radius r with speed v . The rope makes an angle θ with vertical.

77. The tension in the rope is

- a. $\sqrt{(mg)^2 + \left(\frac{mv^2}{2r}\right)^2}$
b. $\sqrt{(mg)^2 - \left(\frac{mv^2}{r}\right)^2}$
c. $\sqrt{(mg)^2 - \left(\frac{mv^2}{2r}\right)^2}$
d. $\sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$

78. The speed of the ball is

- a. $\sqrt{2gr \tan \theta}$
b. $\sqrt{gr \tan \theta}$
c. $\sqrt{gr \cot \theta}$
d. $\sqrt{gr \cos \theta}$

For Problems 79–81

A small block of mass m is placed over a long plank of mass M . Coefficient of friction between them is μ . Ground is smooth. At $t = 0$, m is given a velocity v_1 and M a velocity v_2 ($> v_1$) as shown in Fig. 7.506. After this M is maintained at constant acceleration a ($< \mu g$).

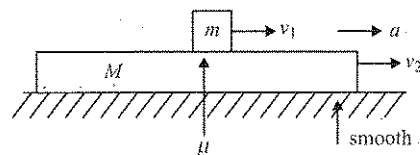


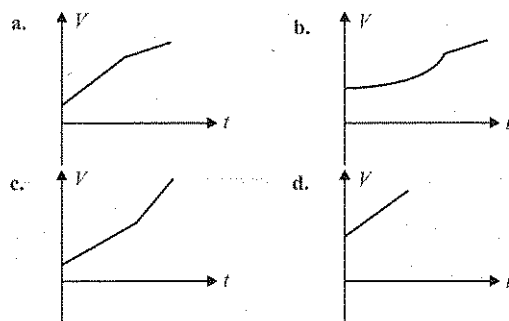
Fig. 7.506

Initially, there will be some relative motion between block and the plank, but after some time relative motion will cease and velocities of both will become same.

79. Find the time t_0 when velocities of both block and plank become same.

- a. $\frac{v_2 - v_1}{\mu g + a}$
b. $\frac{v_2 + v_1}{\mu g - a}$
c. $\frac{v_2 - v_1}{\mu g - a}$
d. $\frac{v_2 + v_1}{\mu g + a}$

80. The variation of velocity of block as a function of time is shown as



81. The forward force acting on the plank before and after t_0 respectively is

- a. $Ma, (M + m)a$
b. $\mu mg + Ma, (M + m)a$
c. $\mu Mg + ma, Ma$
d. $(M + m)a, \mu mg + Ma$

For Problems 82–84

Two blocks of masses m_1 and m_2 are connected with a light spring of force constant k and the whole system is kept on a frictionless horizontal surface. The masses are applied with forces F_1 and F_2 as shown in Fig. 7.507. At any time, the blocks have same acceleration a_0 but in opposite directions.

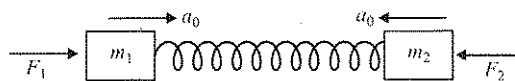


Fig. 7.507

Now answer the following questions.

82. The value of a_0 is

- a. $\frac{F_1 - F_2}{m_1 + m_2}$
 b. $\frac{F_1 - F_2}{m_1 - m_2}$
 c. $\frac{F_1 + F_2}{m_1 - m_2}$
 d. $\frac{F_1 + F_2}{m_1 + m_2}$

83. The value of spring force is

- a. $\frac{m_1 F_2 + F_1 m_2}{m_1 - m_2}$
 b. $\frac{m_1 F_2 - F_1 m_2}{m_1 + m_2}$
 c. $\frac{m_1 F_2 + F_1 m_2}{m_1 + m_2}$
 d. $\frac{m_1 F_2 - F_1 m_2}{m_1 - m_2}$

84. If F_2 is removed at this moment, then just after this acceleration of m_2 is

- a. $\frac{F_1}{m_2} - a_0$
 b. $a_0 + \frac{F_1}{m_2}$
 c. $\frac{F_2}{m_2} - a_0$
 d. $a_0 + \frac{F_2}{m_2}$

For Problems 85–88

A block of mass 4 kg is pressed against a rough wall by two perpendicular horizontal forces F_1 and F_2 as shown in Fig. 7.508 below. The coefficient of static friction between the block and floor is 0.6 and that of kinetic friction is 0.5.

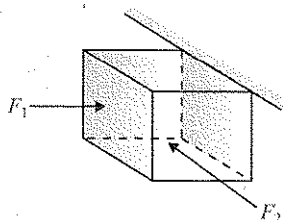


Fig. 7.508

85. For $F_1 = 300$ N and $f_2 = 100$ N, the direction and magnitude of friction force acting on the block are

- a. 180 N, vertically upwards.
 b. 40 N, vertically upwards.
 c. 107.7 N making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction.
 d. 91.6 N making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction.

86. For the data given in question 85, the acceleration of block is

- a. zero
 b. $\frac{140}{4}$ m/s², upward
 c. $\frac{180}{4}$ m/s², upward
 d. $\frac{107.7}{4}$ m/s² at an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction

87. For $F_1 = 150$ N and $F_2 = 100$ N, the direction and magnitude of friction force acting on block are

- a. 90 N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction
 b. 75 N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction
 c. 107.7 N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction
 d. zero

88. For data of question no. 85, find the magnitude of acceleration of block.

- a. zero
 b. 22.5 m/s²
 c. 26.925 m/s²
 d. 8.175 m/s²

For Problems 89–93

A system of two blocks and a light string are kept on two inclined faces (rough) as shown in Fig. 7.509. All the required data are mentioned in the diagram. Pulley is light and frictionless. (take $g = 10$ m/s², $\sin 37^\circ = \frac{3}{5}$).

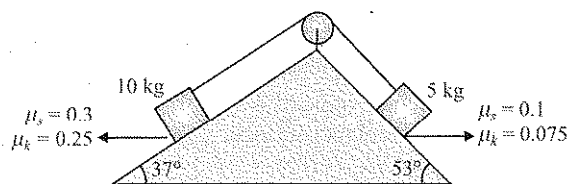


Fig. 7.509

89. If the system is released from rest, then the acceleration of the system is

- a. $\frac{7}{15}$ m/s²
 b. zero
 c. $\frac{47}{15}$ m/s²
 d. $\frac{2.25}{15}$ m/s²

90. A system is initially moving in such a way that block of 10 kg is coming down the incline with a speed of 2 m/s. Then how much time does the system take to come to a stop? [Assume the length of incline to be large enough.]

- a. 13.33 s
 b. 80 s
 c. Infinite
 d. Question is irrelevant

91. In the above question the motion of system would be best described by one of the following.

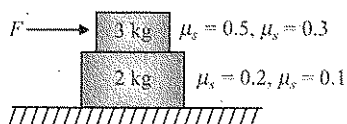
- a. The system first decelerates, comes to a stop and then continues to move in the opposite direction.
 b. The system will continuously move with constant speed.
 c. The system first decelerates and then comes to a stop.
 d. The system accelerates and its speed increases with time.

92. If the system is released from rest, the tension in the string would be
 a. $40 \leq T \leq 43$
 b. $40 \leq t \leq 60$
 c. $36 \leq T \leq 60$
 d. Cannot be determined
93. For above situation, the friction force between 10 kg block and the incline can be
 a. 24 N b. 18 N c. 21 N d. 15 N

For Problems 94–96

A system of two blocks is placed on a rough horizontal surface as shown in Fig. 7.510. The coefficients of static and kinetic friction at two surfaces are shown. A force F is horizontally applied on the upper block.

Let f_1 , f_2 represent the frictional forces between upper and lower surfaces of contact, respectively, and a_1 , a_2 represent the acceleration of 3 kg and 2 kg block, respectively.

**Fig. 7.510**

94. If F is a gradually increasing force then which of the following statement(s) would be true?
 a. For a particular value of F ($< F_0$) there is no motion at any of the contact surface.
 b. The value of F_0 is 10 N.
 c. As F increases beyond F_0 , f_1 increases and continues to increase until it acquires its limiting value.
 d. All of the above
95. For $F = 12$ N, mark the correct option.
 a. $f_1 = 7.8$ N, $f_2 = 7.8$ N, $a_1 = 1.4$ m/s², $a_2 = 0$ m/s²
 b. $f_1 = 7.8$ N, $f_2 = 10$ N, $a_1 = a_2 = 1.4$ m/s²
 c. $f_1 = 7.8$ N, $f_2 = 5$ N, $a_1 = a_2 = 1.4$ m/s²
 d. $f_1 = 7.4$ N, $f_2 = 5$ N, $a_1 = a_2 = 1.2$ m/s²
96. For relative motion to be there between two blocks, the minimum value of F should be
 a. 15 N b. 30 N c. 25 N d. 32 N

For Problems 97–99

A student performs two experiments to determine the coefficient of static and kinetic friction between a block of mass 100 kg and the horizontal floor.

Ist Experiment: He applies a gradual increasing force on the block and is just able to slide the block when force is 450 N.

IInd Experiment: He applies constant force of different magnitudes for the duration of 2 s and determine the distance travelled by the block in this duration.

Set Force Distance

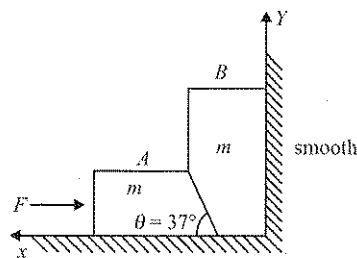
1. 300 N 0.5 m
 2. 600 N 2.0 m
 3. 750 N 3.0 m

Assume all the forces have been applied horizontally.

97. The coefficient of static friction between the block and the floor is
 a. 0.45 b. 0.5
 c. 0.3 d. 1.45
98. Which set of the readings of Experiment II is absolutely wrong?
 a. 1 b. 2
 c. 3 d. None of these
99. The speed of the block after 3 s (beginning from the starting of application of force) in set 2 for IInd experiment is
 a. 6 m/s
 b. 2 m/s
 c. 3 m/s
 d. Information is insufficient

For Problems 102–100

Two smooth blocks are placed at a smooth corner as shown in Fig. 7.511. Both the blocks are having mass m . We apply a force F on the small block m . Block A presses the block B in the normal direction, due to which the pressing force on vertical wall will increase, and the pressing force on the horizontal wall decreases, as we increases F ($\theta = 37^\circ$ with horizontal). As soon as the pressing force by block B on the horizontal wall becomes zero, it will lose the contact with the ground. If the value of F further increases, the block B will accelerate in upward direction and simultaneously the block A will move toward right.

**Fig. 7.511**

100. The minimum value of F , to lift block B from the ground is
 a. $\frac{25}{12}mg$ b. $\frac{5}{4}mg$
 c. $\frac{3}{4}mg$ d. $\frac{4}{3}mg$
101. If both the blocks are stationary, the force exerted by ground on block A is
 a. $mg + \frac{3F}{4}$
 b. $mg - \frac{3F}{4}$
 c. $mg + \frac{4F}{3}$
 d. $mg - \frac{4F}{3}$

102. If the acceleration of block A is from left to right, then the acceleration of block B will be

- a. $\frac{3a}{4}$ upwards
 b. $\frac{4a}{3}$ upwards
 c. $\frac{3a}{5}$ upwards
 d. $\frac{4a}{5}$ upwards

For Problems 103–104

Two containers of sand are arranged like the block as shown in Fig. 7.512. The containers alone have negligible mass; the sand in these containers has a total mass M_{tot} ; the sand in the hanging container H has mass m .

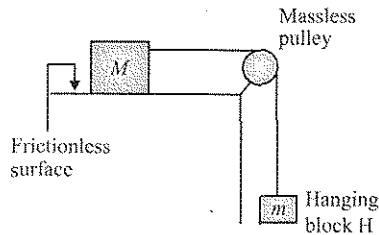


Fig. 7.512

To measure the magnitude a of the acceleration of the system, a large number of experiments have been carried out where m varies from experiment to experiment but M_{tot} does not; that is, sand is shifted between the containers before each trial.

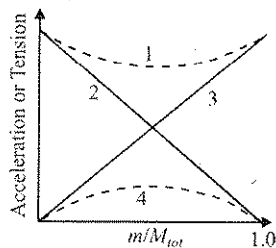


Fig. 7.513

103. Which of the curves in the graph correctly gives the acceleration magnitude as a function of the ratio m/M_{tot} (vertical axis is for acceleration)?

- a. 1 b. 2 c. 3 d. 4

104. Which of the curves gives the tension in the connecting cord (the vertical axis is for tension)?

- a. 1 b. 2 c. 3 d. 4

For Problems 105–107

Two bodies A and B of masses 10 kg and 5 kg are placed very slightly separated as shown in Fig. 7.514. The coefficients of friction between the floor and the blocks are as $\mu_s = 0.4$. Block A is pushed by an external force F . The value of F can be changed. When the welding between block A and ground breaks, block A will start pressing block B and when the welding of B also breaks, block B will start pressing the vertical wall.

105. If $F = 20$ N, with how much force does block A press the block B ?

- a. 10 N b. 20 N
 c. 30 N d. zero

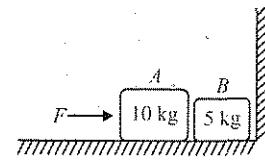
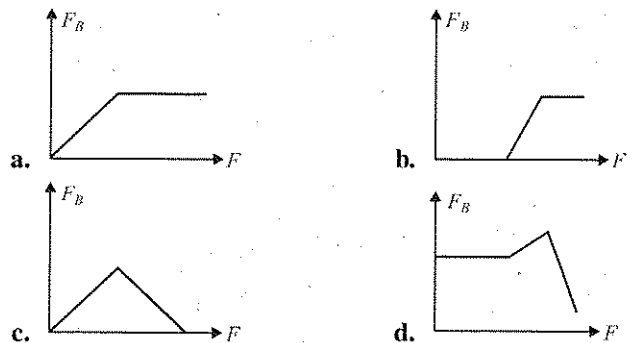


Fig. 7.514

106. If $F = 50$ N, the friction force acting between block B and ground will be

- a. 10 N b. 20 N
 c. 30 N d. None

107. The force of friction acting on B varies with the applied force F according to curve



For Problems 108–110

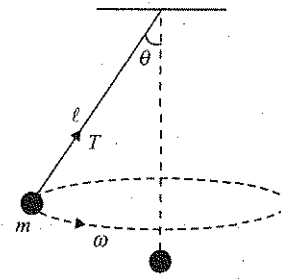


Fig. 7.515

A string of length l is fixed at one end and carries a mass m at the other end. The string makes $2/\pi$ rps around a vertical axis through the fixed end so that the mass moves in the horizontal circle (Fig. 7.515).

108. What is the tension in string?

- a. ml b. $16ml$
 c. $4ml$ d. $2ml$

109. What is the angle of inclination of the string with vertical?

- a. $\cos^{-1}\left(\frac{g}{l}\right)$ b. $\cos^{-1}\left(\frac{g}{16l}\right)$
 c. $\cos^{-1}\left(\frac{g}{4l}\right)$ d. $\cos^{-1}\left(\frac{g}{8l}\right)$

110. What is the linear speed of the mass?

- a. $4l\left[1 + \left(\frac{g}{16l}\right)^2\right]^{\frac{1}{2}}$ b. $4l\left[1 - \left(\frac{g}{16l}\right)^2\right]^{\frac{1}{2}}$

$$\text{c. } 4l \left[1 + \left(\frac{g}{4l} \right)^2 \right]^{\frac{1}{2}} \quad \text{d. } 4l \left[1 - \left(\frac{g}{4l} \right)^2 \right]^{\frac{1}{2}}$$

For Problems 111–113

A time varying force $F = 6t - 2t^2$ N at $t = 0$ starts acting on a body of mass 2 kg initially at rest, where t is in seconds. The force is withdrawn just at the instant when the body comes to rest again. We can see that at $t = 0$, the force $F = 0$. Now answer the following.

111. Find the duration for which the force acts on the body.

- a. 2 s b. 3 s c. 3.5 s d. 4.5 s

112. Find the time when the velocity attained by the body is maximum.

- a. 2 s b. 3 s c. 3.5 s d. 4.5 s

113. Mark the correct statement:

- The velocity of the body is maximum when the force acting on the body is maximum for the first time.
- The velocity of the body becomes maximum when the force acting on the body becomes zero again.
- When the force becomes zero again, the velocity of the body also becomes zero at that instant.
- All of the above.

For Problems 114–116

For the system shown in Fig. 7.516, there is no friction anywhere. Masses m_1 and m_2 can move up or down in the slots cut in mass M . Two non-zero horizontal forces F_1 and F_2 are applied as shown. The pulleys are massless and frictionless. Given $m_1 \neq m_2$.

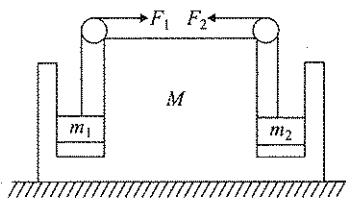


Fig. 7.516

114. According to the above passage, which is correct?

- It is not possible for the entire system to be in equilibrium.
- For some values of F_1 and F_2 , it is possible that entire system is in equilibrium.
- It is possible that F_1 and F_2 are applied in such a way that m_1 and m_2 remain in equilibrium but M does not.
- None of the above.

115. Let F_1 and F_2 are applied in such a way that m_1 and m_2 do not move w.r.t. M . Then what is the magnitude of acceleration of M ? Let $m_1 > m_2$.

- $\frac{(m_1 + m_2)g}{M + m_1 + m_2}$
- $\frac{(m_1 - m_2)g}{M}$
- $\frac{(m_1 - m_2)g}{M + m_1 + m_2}$
- $\frac{F_1 - F_2}{M}$

116. Let F_1 and F_2 are applied in such a way that accelerations of both masses m_1 and m_2 are same both in magnitude and direction. Then

- $\frac{F_1}{m_1} - \frac{m_2 g}{m_1} = \frac{F_2}{m_2} - \frac{m_1 g}{m_2}$
- $\frac{F_1}{m_2} = \frac{F_2}{m_1}$
- $\frac{F_1}{m_1} + \frac{m_2 g}{m_1} = \frac{F_2}{m_2} + \frac{m_1 g}{m_2}$
- $\frac{F_1}{m_1} = \frac{F_2}{m_2}$

For Problems 117–119

A mass M is suspended as shown in Fig. 7.517. The system is in equilibrium. Assume pulleys to be massless. K is the force constant of the spring.

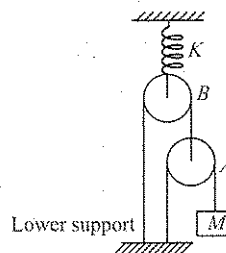


Fig. 7.517

117. The extension produced in the spring is given by

- $4Mg/K$
- Mg/K
- $2Mg/K$
- $3Mg/K$

118. The net tension force acting on the lower support is

- Mg
- $2Mg$
- $3Mg$
- $4Mg$

119. If each of the pulleys A and B has mass M , then the net tension force acting on the lower support (assume pulleys to be frictionless) is

- $2Mg$
- $6Mg$
- $3Mg$
- $4Mg$

For Problems 120–122

On a stationary block of mass 2 kg, a horizontal force F starts acting at $t = 0$ whose variation with time is shown in Fig. 7.518. The coefficient of friction between the block and ground is 0.5. Now answer the following questions.

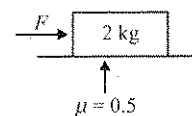


Fig. 7.518

120. The time when the acceleration of the block is zero is

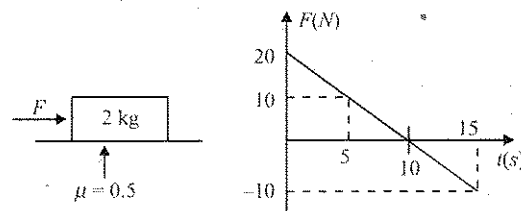


Fig. 7.519

- a. at 5 s only
 b. at 10 s only
 c. both at 5 s and 10 s
 d. at a time after $t = 10$ s only
121. The velocity of the block when first time its acceleration becomes zero is
 a. 12.5 m/s
 b. 25 m/s
 c. 10 m/s
 d. none of these
122. The velocity of the block at $t = 12$ s
 a. 20 m/s
 b. -12 m/s
 c. +6 m/s
 d. zero

For Problems 123–125

A long conveyer belt moves with a constant velocity of 8 m/s. Two blocks A and B each of mass 2 kg are placed gently on the belt with B on A. Initial velocity of both blocks is zero. Coefficient of friction between A and belt is 0.1. There is no friction between A and B. Length of A is 4 m.

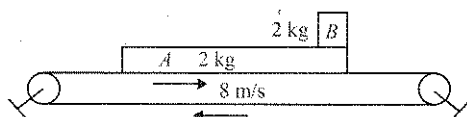


Fig. 7.520

123. The time when B falls off A. Initially B is on the right end of A. Ignore the dimensions of B is
 a. 1 s
 b. 3 s
 c. 2 s
 d. 4 s
124. The velocity of A when B falls off A is
 a. 2 m/s
 b. 4 m/s
 c. 6 m/s
 d. 8 m/s
125. If the coefficient of friction between the block B and belt is 0.4, then the separation between the two blocks when B comes to rest w.r.t. belt is
 a. 8 m
 b. 6 m
 c. 2 m
 d. None of these

Column I	Column II
i. String W breaks	a. $ \vec{a}_A = 0$
ii. Spring X breaks	b. $ \vec{a}_B = 0$
iii. String Y breaks	c. $ \vec{a}_C = 0$
iv. Spring Z breaks	d. $ \vec{a}_B = \vec{a}_C $

2. There is no friction anywhere in the system shown in Fig. 7.522. The pulley is light. The wedge is free to move on the frictionless surface. A horizontal force F is applied on the system in such a way that m does not slide on M or both move together with some common acceleration. Given $M > \sqrt{2}m$.

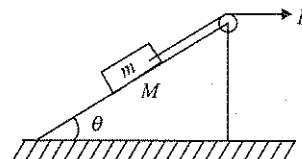


Fig. 7.522

Match the entries of Column I with that of Column II.

Column I	Column II
i. Pseudo-force acting on m as seen from the frame of M is	a. equal to $\frac{mF}{m+M}$
ii. Pseudo-force acting on M as seen from the frame of m is	b. greater than $\frac{mF}{m+M}$
iii. Normal force (for $\theta = 45^\circ$) between m and M is	c. less than $mg\sin\theta$
iv. Normal force between ground and M is	d. greater than $mg\sin\theta$

3. In the system shown in Fig. 7.523, masses of the blocks are such that when system is released, acceleration of pulley P_1 is a upwards and acceleration of block 1 is a_1 upwards. It is found that acceleration of block 3 is same as that of 1 both in magnitude and direction.

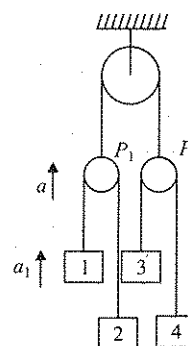


Fig. 7.523

Given that $a_1 > a > \frac{a_1}{2}$. Match the following

Matching Column Type

Solutions on page 7.165

1. In Fig. 7.521, strings, springs and the pulley are light and ideal. The system is in equilibrium with the strings taut. Masses are equal. Match Column I with Column II.

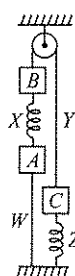


Fig. 7.521

Column I	Column II
i. Acceleration of 2	a. $2a + a_1$
ii. Acceleration of 4	b. $2a - a_1$
iii. Acceleration of 2 w.r.t. 3	c. upwards
iv. Acceleration of 2 w.r.t. 4	d. downwards

4. The coefficient of friction between the block and the surface is 0.4 in Fig. 7.524(i-iv). Match Column I with II

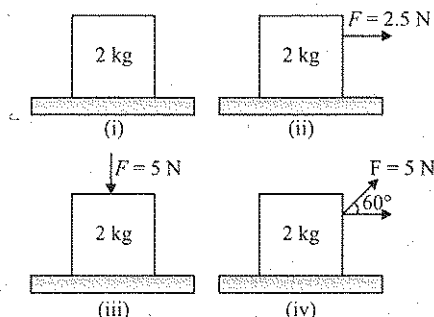


Fig. 7.524

Column I	Column II
i. Force of friction is zero in	a. Fig. (i)
ii. Force of friction is 2.5 N in	b. Fig. (ii)
iii. Acceleration of the block is zero in	c. Fig. (iii)
iv. Normal force is not equal to $2g$ in	d. Fig. (iv)

5. For the situation shown in Fig. 7.525, in Column I, the statements regarding friction forces are mentioned, while in Column II some information related to friction forces are given. Match the entries of Column I with the entries of Column II.

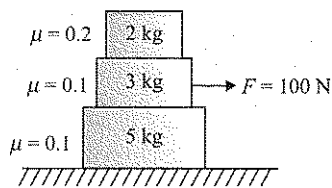


Fig. 7.525

Column I	Column II
i. Total friction force on 3 kg block is	a. towards right
ii. Total friction force on 5 kg block is	b. towards left
iii. Friction force on 2 kg block due to 3 kg block is	c. zero
iv. Friction force on 3 kg block due to 5 kg block is	d. non-zero

6. A horizontal force F pulls a ring of mass m_1 such that θ remains constant with time. The ring is constrained to move along a smooth rigid horizontal wire. A bob of mass m_2

hangs from m_1 by an inextensible light string. Then match the entries of Column I with that of Column II

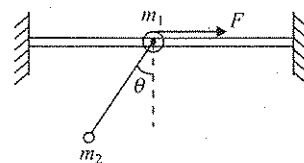


Fig. 7.526

Column I	Column II
i. F	a. $(m_1 + m_2)g$
ii. Force acting on m_2 is	b. $m_2 g \sec \theta$
iii. Tension in the string is	c. $m_2 \frac{F}{m_1 + m_2}$
iv. Force acting on m_1 by the wire is	d. $(m_1 + m_2)g \tan \theta$

7. Column I describes the motion of the object and one or more of the entries of Column II may be the cause of motions described in Column I. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. An object is moving towards east	a. Net force acting on the object must be towards east
ii. An object is moving towards east with constant acceleration	b. At least one force must act towards east
iii. An object is moving towards east with varying acceleration	c. No force may act towards east
iv. An object is moving towards east with constant velocity	d. No force may act on the object

8.

Column I	Column II
i. If friction force is less than applied force then friction may be	a. Static
ii. If friction force is equal to the force applied, then friction may be	b. Kinetic
iii. If object is moving, then friction may be	c. Limiting
iv. If object is at rest, then friction may be	d. No conclusion can be drawn

9. The system shown in Fig. 7.527 is initially in equilibrium.

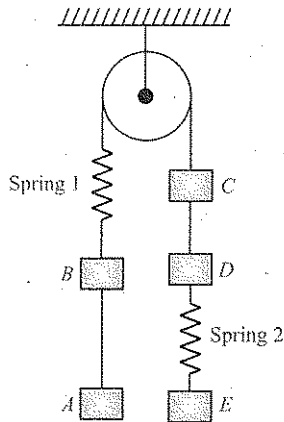


Fig. 7.527

Column I	Column II
i. Just after the Spring 2 is cut, the block D	a. Accelerates up
ii. Just after the Spring 2 is cut, the block C	b. Accelerates down
iii. Just after the Spring 2 is cut, the block A	c. Momentarily at rest
iv. Just after the Spring connecting A and B is cut, the block D	d. Moves up with acceleration g

10. For the situation shown in Fig. 7.528, match the entries of Column I with the entries of Column II.

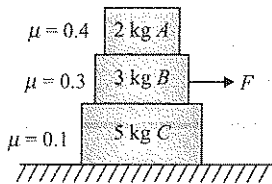


Fig. 7.528

Column I	Column II
i. If $F = 12$ N, then	a. There is relative motion between A and B
ii. If $F = 15$ N, then	b. There is relative motion between B and C
iii. If $F = 25$ N, then	c. There is relative motion between C and the ground
iv. If $F = 40$ N, then	d. There is relative motion is not there at any of the surface

11. Column I gives four different situations involving two blocks of mass m_1 and m_2 placed in different ways on smooth horizontal surface as shown. In each of the situations, horizontal forces F_1 and F_2 are applied on blocks of mass m_1 and m_2 , respectively and also $m_2 F_1 < m_1 F_2$. Match the statements in Column 1 with the corresponding results in Column II.

Column I	Column II
i. $F_1 \leftarrow m_1 \text{ --- } m_2 \rightarrow F_2$ Both the blocks are connected by the massless inelastic string. The magnitude of tension in the string is	a. $\frac{m_1 m_2}{m_1 + m_2} \left(\frac{F_1}{m_1} - \frac{F_2}{m_2} \right)$
ii. $F_1 \rightarrow m_1 \text{ --- } m_2 \rightarrow F_2$ Both the blocks are connected by the massless inelastic string. The magnitude of tension in the string is	b. $\frac{m_1 m_2}{m_1 + m_2} \left(\frac{F_1}{m_1} + \frac{F_2}{m_2} \right)$
iii. $F_1 \leftarrow m_1 \text{ --- } m_2 \leftarrow F_2$ The magnitude of normal reaction between the blocks is	c. $\frac{m_1 m_2}{m_1 + m_2} \left(\frac{F_2}{m_2} - \frac{F_1}{m_1} \right)$
iv. $F_1 \leftarrow m_1 \text{ --- } m_2 \leftarrow F_2$ The magnitude of normal reaction between the blocks is	d. $m_1 m_2 \left(\frac{F_1 + F_2}{m_1 + m_2} \right)$

12. The system shown in Fig. 7.529 is initially in equilibrium. Masses of the blocks A, B, C, D, and E are, respectively, 3 m, 3 m, 2 m, 2 m, and 2 m. Match the conditions in Column I with the effect in Column II.

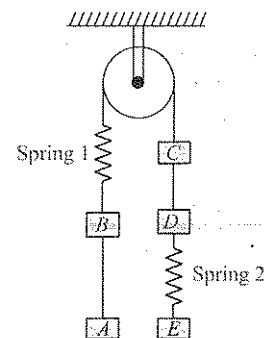


Fig. 7.529

Column I	Column II
i. After spring 2 is cut, tension in string AB	a. increases
ii. After spring 2 is cut, tension in string CD	b. decreases
iii. After string between C and pulley is cut, tension in string AB	c. remain constant
iv. After string between C and pulley is cut, tension in string CD	d. zero

13. In Fig. 7.530 block is attached to an unstretched vertical spring and released from rest. As a result, the block comes down due to its weight, stops momentarily and then bounces back. Finally the block starts oscillating up and down. Now match Column I with Column II.

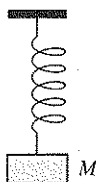


Fig. 7.530

Column I	Column II
i. When the block is at its maximum downward displacement position (may be known as extreme position)	a. Acceleration is in upward direction
ii. When the block is at its equilibrium position	b. Acceleration is in downward direction
iii. When the block is somewhere between equilibrium position and downward extreme position	c. Acceleration is zero
iv. When the block is above equilibrium position but below the initial unstretched position	d. Velocity may be in upward or in downward direction

14. In Fig. 7.531, a block of mass m is released from rest when spring was in its natural length. The pulley also has mass m but it is frictionless. Suppose the value of m is such that finally it is just able to lift the block M up after releasing it.

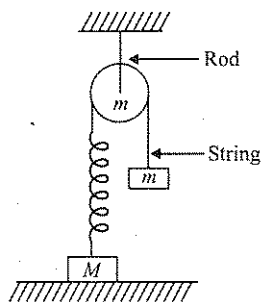


Fig. 7.531

Column I	Column II
i. The weight of m required to just lift M	a. $3\frac{M}{2}g$
ii. The tension in the rod, when m is in equilibrium	b. Mg
iii. The normal force acting on M when m is in equilibrium	c. $\frac{M}{2}g$
iv. The tension in the string when displacement of m is maximum possible	d. $2mg$

15. In Fig. 7.532, both the pulleys are massless and frictionless. A force F (of any possible magnitude) is applied in hor-

izontal direction. There is no friction between M and the ground. μ_1 and μ_2 are the coefficients of friction as shown between the blocks. Column I gives the different relations between μ_1 and μ_2 , and Column II is regarding the motion of M . Match the columns.

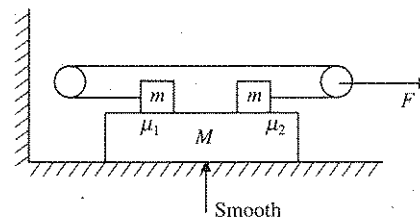


Fig. 7.532

Column I	Column II
i. if $\mu_1 = \mu_2 = 0$	a. may accelerate towards right
ii. if $\mu_1 = \mu_2 \neq 0$	b. may accelerate towards left
iii. if $\mu_1 > \mu_2$	c. does not accelerate
iv. if $\mu_1 < \mu_2$	d. may or may not accelerate

16. The coefficient of friction between the masses $2m$ and m is 0.5. All other surfaces are frictionless and pulleys are massless (see Fig. 7.533). Column I gives the different values of m_1 and Column II gives the possible acceleration of $2m$ and m . Match the columns.

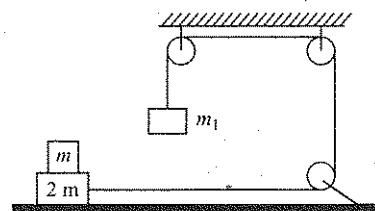


Fig. 7.533

Column I	Column II
i. $m_1 = 2m$	a. acceleration of $2m$ and m are same
ii. $m_1 = 3m$	b. acceleration of $2m$ and m are different
iii. $m_1 = 4m$	c. acceleration of $2m$ is greater than m .
iv. $m_1 = 6m$	d. acceleration of m is less than $0.6g$

17. When the system shown in Fig. 7.534 is released, A accelerates downward.

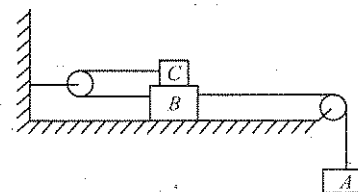


Fig. 7.534

Match the entries of Column I with that of Column II.

Column I	Column II
i. Acceleration of B	a. towards left
ii. Acceleration of C w.r.t. B	b. towards right
iii. Acceleration of A w.r.t. C	c. at some angle θ with horizontal, $0 < \theta < 90^\circ$
iv. Acceleration of B w.r.t. A	d. at some angle θ with vertical, $0 < \theta < 90^\circ$

18.

Column I	Column II
i. In equilibrium, the body must be stationary	a. True
ii. If a body is moving, then no force may be acting on it	b. False
iii. If a body is at rest, it means no net force is acting on it	c. Net force acting on the body is certainly zero
iv. If a body is accelerated, then direction of acceleration is always in the direction of net force	d. The body may have some acceleration also

19. For the situation shown in Fig. 7.535, in Column I, the statements regarding friction forces are mentioned, while in Column II some information related to friction forces is given.

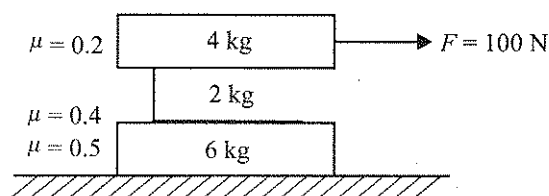


Fig. 7.535

Match the entries of Column I with the entries of Column II.

Column I	Column II
i. Total friction force on 4 kg block is	a. Towards right
ii. Total friction force on 2 kg block is	b. Towards left
iii. Friction force on 6 kg block due to 2 kg block is	c. Zero
iv. Total friction force on 6 kg block is	d. Non-zero

20. In Fig. 7.536, the whole system is in equilibrium. Tensions in different strings are shown. Match the followings

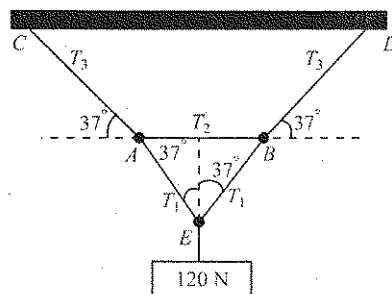


Fig. 7.536

Column I	Column II
i. $\vec{T}_1 + \vec{T}_2$ is	a. T_2
ii. $T_1 - T_2$ is	b. T_3
iii. $\vec{T}_1 + \vec{T}_3$ is	c. ≤ 40 N
iv. $T_1 + T_2$ is	d. ≥ 100 N

Archives

Solutions on page 7.169

Fill in the Blanks Type

- A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 m/s^2 , the frictional force acting on the block is _____ newtons. (IIT-JEE, 1984)
- A uniform rod of length L and density ρ is being pulled along a smooth floor with a horizontal acceleration a (see Fig. 7.537). The magnitude of the stress at the transverse cross-section through the mid-point of the rod is _____. (IIT-JEE, 1993)

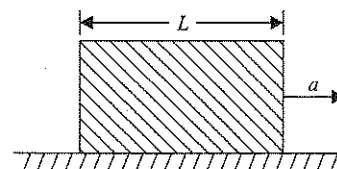


Fig. 7.537

True or False

- When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. (IIT-JEE, 1981)
- A simple pendulum with a bob of mass m swings with an angular amplitude of 40° . When its angular displacement is 20° , the tension in the string is greater than $mg \cos 20^\circ$. (IIT-JEE, 1984)
- The pulley arrangements of Fig. 7.538 (a) and (b) are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In (b), m is lifted up by pulling the other end of the rope with a constant downward force $F = 2mg$. The acceleration of m is the same in both cases (IIT-JEE, 1984)

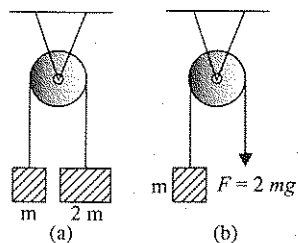


Fig. 7.538

4. Two particles of mass 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centre of mass has a velocity of 0.5 m/s. When the relative velocity of approach becomes 3 m/s, the velocity of the centre of mass is 0.75 m/s.

(IIT-JEE, 1989)

Multiple Choice Questions with One Correct Answer

1. A ship of mass 3×10^7 kg initially at rest, is pulled by a force of 5×10^4 N through a distance due to water is negligible, the speed of the ship. (IIT-JEE, 1980)

a. 1.5 m/s b. 60 m/s c. 0.1 m/s d. 5 m/s

2. A block of mass 2 kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is

a. 9.8 N b. $0.7 \times 9.8 \times \sqrt{3}$ N
c. $9.8 \times \sqrt{3}$ N d. 0.7×9.8 N

3. During the peddling of a bicycle, the force of friction exerted by the ground on the two wheels is such that it acts. (IIT-JEE, 1990)

a. in the backward direction on the front wheel and in the forward direction on the rear wheel
b. in the forward direction on the front wheel and in the backward direction on the rear wheel
c. in the backward direction on both the front and the rear wheels
d. in the forward direction on both the front and the rear wheels

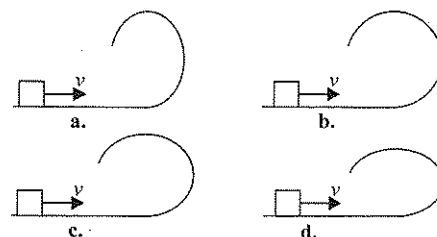
4. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod. (IIT-JEE, 1992)

a. zero b. 30° c. 45° d. 60°

5. A block of mass 0.1 is held against a wall applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the friction force acting on the block is (IIT-JEE, 1994)

a. 2.5 N b. 0.98 N c. 4.9 N d. 0.49 N

6. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in (IIT-JEE, 2001)



7. An insect crawls up a hemispherical surface very slowly (see Fig. 7.539). The coefficient of friction between the insect and the surface is $1/3$. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by (IIT-JEE, 2001)

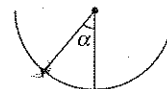


Fig. 7.539

a. $\cot \alpha = 3$ b. $\tan \alpha = 3$ c. $\sec \alpha = 3$ d. $\operatorname{cosec} \alpha = 3$

8. The pulleys and strings shown in the Fig. 7.540 are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be (IIT-JEE, 2001)

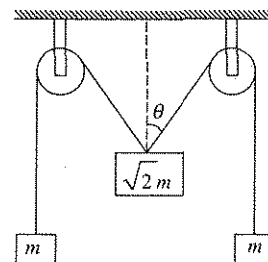


Fig. 7.540

a. 0° b. 30° c. 45° d. 60°

9. A string of negligible mass going over a clamped pulley of mass m supports a block of M as shown in the Fig. 7.541. The force on the pulley by the clamp is given by (IIT-JEE, 2001)

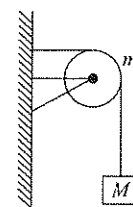


Fig. 7.541

a. $\sqrt{2}Mg$ b. $\sqrt{2}mg$
c. $\sqrt{(M+m)^2 + m^2}g$ d. $\sqrt{(M+m)^2 + M^2}g$

10. What is the maximum value of the force F such that the block shown in Fig. 7.542, does not move? (IIT-JEE, 2003)

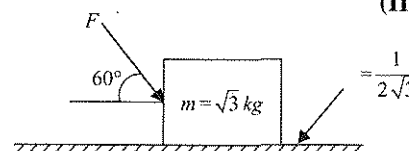


Fig. 7.542

a. 20 N b. 10 N c. 12 N d. 15 N

11. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a friction horizontal surface with the string held tight so that each mass is at a distance a from the centre P (as shown in the Fig. 7.782).

(IIT-JEE, 2004)

a. $\frac{F}{2m} \frac{a}{\sqrt{a^2 - x^2}}$ b. $\frac{F}{2m} \frac{x}{\sqrt{a^2 - x^2}}$
 c. $\frac{F}{2m} \frac{x}{a}$ d. $\frac{F}{2m} \frac{\sqrt{a^2 - x^2}}{x}$

12. A block of base $10 \text{ cm} \times 10 \text{ cm}$ and height 15 cm is kept on an inclined plane. The coefficient of friction between them is 3. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0° . Then

(IIT-JEE, 2009)

- a. at $\theta = 30^\circ$, the block will start sliding down the plane
 b. the block will remain at rest on the plane up to certain θ and then it will topple
 c. at $\theta = 60^\circ$, the block will start sliding down the plane and continue to do so at higher angles
 d. at $\theta = 60^\circ$, the block will start sliding down the plane and on further increasing θ , it will topple at certain θ

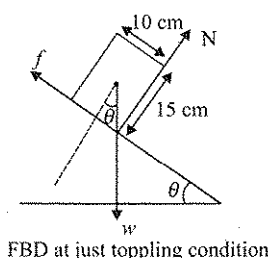


Fig. 7.543

Assertion and Reasoning

Mark your answer as

- a. If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
 b. If Statement I is true, Statement II is true; Statement II is not a correct explanation for statement I
 c. If Statement I is true; Statement II is false
 d. If statement I is false; Statement II is true

1. **Statement I:** A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table. (IIT-JEE, 2007)

Statement II: For every action there is an equal and opposite reaction.

2. **Statement I:** It is easier to pull a heavy object than to push it on a level ground. (IIT-JEE, 2008)

Statement II: The magnitude of frictional force depends on the nature of the two surfaces in contact.

Multiple Choice Questions with One, or More than One Correct Answer

1. In the arrangement shown in the Fig. 7.544, the ends P and Q of an unstretchable string move downwards with uniform speed U . Pulleys A and B are fixed.

(IIT-JEE, 1982)

Mass M moves upwards with a speed

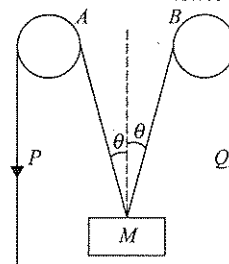


Fig. 7.544

- a. $2U \cos \theta$ b. $U / \cos \theta$
 c. $2U / \cos \theta$ d. $U \cos \theta$
2. A reference frame attached to the earth
- a. is an inertial frame by definition.
 b. cannot be an inertial frame because the earth is revolving round the sun.
 c. is an inertial frame because Newton's laws are applicable in this frame.
 d. cannot be an inertial frame because the earth is rotating about its own axis. (IIT-JEE, 1986)
3. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limit $-\phi$ and $+\phi$. For an angular displacement θ ($|\theta| < \phi$), the tension in the string and the velocity of the bob are T and V respectively. The following relations hold good under the above conditions (IIT JEE, 1986)
- a. $T \cos \theta = Mg$
 b. $T - Mg \cos \theta = \frac{MV^2}{L}$
 c. The magnitude of the tangential acceleration of the bob? $a_T = g \sin \theta$.
 d. $T = Mg \cos \theta$.
4. A particle P is sliding down a frictionless hemispherical bowl as shown in Fig. 7.545. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v . A bead Q of the same mass as P is ejected from A at $t = 0$ along the horizontal string AB , with the speed v . Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B . Then (IIT-JEE, 1993)

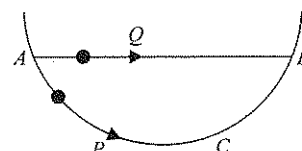


Fig. 7.545

- a. $t_P < t_Q$
 b. $t_P = t_Q$
 c. $t_P > t_Q$
 d. $\frac{t_P}{t_Q} = \frac{\text{length of arc } ACB}{\text{length of } AB}$

ANSWERS AND SOLUTIONS

Subjective Type

1. Denote the common magnitude of the maximum acceleration as a . For block A to remain at rest with respect to block B, $a < \mu_s g$.

The tension in the cord is then

$$T = (m_A + m_B)a + \mu_k g(m_A + m_B) \\ = (m_A + m_B)(a + \mu_k g).$$

This tension is related to the mass m_C by $T = m_C(g - a)$. Solving for a yields

$$a = g \frac{m_C - \mu_k(m_A + m_B)}{m_A + m_B + m_C} < \mu_s g$$

Solving the inequality for m_C yields

$$m_C < \frac{(m_A + m_B)(\mu_s + \mu_k)}{1 - \mu_s}$$

2. Draw free body diagram (Fig. 7.546) of block, $\sum F_y = 0$

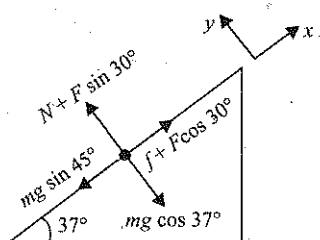


Fig. 7.546

$$\Rightarrow N + F \sin 30^\circ = mg \cos 37^\circ$$

$$\text{or, } N = mg \cos 37^\circ - F \sin 30^\circ \\ = (4)(10) \left(\frac{4}{5} \right) - (10) \left(\frac{1}{2} \right)$$

$$\text{or, } N = 27 \text{ N}$$

$$f_{\max} = \mu N = 0.5 \times 27 = 13.5 \text{ N}$$

$$mg \sin 45^\circ = (4)(10) \left(\frac{4}{5} \right) = 32 \text{ N}$$

$$\text{and } F \cos 30^\circ = (10) \left(\frac{\sqrt{3}}{2} \right) = 8.66 \text{ N}$$

Now since $mg \sin 37^\circ > f_{\max} + F \cos 30^\circ$

Therefore, the block will slide down and the friction will be maximum. Therefore, the net contact force is

$$F_C = \sqrt{N^2 + (f_{\max})^2} = \sqrt{(27)^2 + (13.5)^2}$$

$$F_C = 30.2 \text{ N}$$

3. Since $mg \sin 30^\circ > \mu mg \cos 30^\circ$ the block has a tendency to slip downwards.

Let F be the minimum force applied on it, so that it does not slip. Then,

$$N = F + mg \cos 30^\circ$$

$$\therefore mg \sin 30^\circ = \mu N = \mu(F + mg \cos 30^\circ)$$

$$\text{or, } F = \frac{mg \sin 30^\circ}{\mu} - mg \cos 30^\circ$$

$$= \frac{(2)(10)(1/2)}{0.5} - (2)(10) \left(\frac{\sqrt{3}}{2} \right)$$

$$\text{or, } F = 20 - 17.32 \text{ or } F = 2.68 \text{ N}$$

4. The friction force on block A is $\mu_k w_A = (0.30)(1.40 \text{ N}) = 0.420 \text{ N}$. This is the magnitude of the friction force that block A exerts on block B, as well as the tension in the string. The force F must then have the magnitude

$$F = \mu_k(w_B + w_A) + \mu_k w_A + T \\ = \mu_k(w_B + 3w_A) \\ = (0.30)(4.20 \text{ N} + 3(1.40 \text{ N})) = 2.52 \text{ N}$$

Note that the normal force exerted on block B by the table is the sum of the weights of the blocks.

5. Draw FBD of the block with respect to plane (Fig. 7.547). Acceleration of the block up the plane is

$$a = \frac{mg \cos 37^\circ - mg \sin 37^\circ}{m} = g \left(\frac{4}{5} - \frac{3}{5} \right) \\ = 2 \text{ m/s}^2$$

$$\text{Applying, } s = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1}{2}} = 1 \text{ s}$$

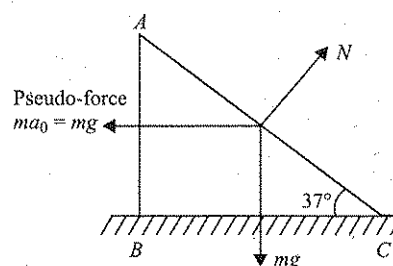


Fig. 7.547

6. Consider the forces on the person as shown in Fig. 7.548:

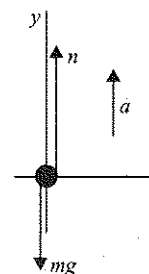


Fig. 7.548

$$\sum F_y = ma_y$$

$$n - mg = ma$$

$$n = 1.6mg \text{ so } a = 0.60g = 5.88 \text{ m/s}^2$$

$$y - y_0 = 3.0 \text{ m}, a_y = 5.88 \text{ m/s}^2,$$

$$v_{0y} = 0, v_y = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } v_y = 5.0 \text{ m/s}$$

7. Force of friction between the two will be maximum, i.e., μmg

$$\text{Retardation of A is } a_A = \frac{\mu mg}{m} = \mu g$$

$$\text{and acceleration of B is } a_B = \frac{\mu mg}{2m} = \frac{\mu g}{2}$$

$\therefore$ acceleration of B relative to A is

$$a_{BA} = a_A + a_B = \frac{3\mu g}{2}$$

$$\text{Substituting, } \mu = \frac{1}{2}; a_{BA} = \frac{3g}{4}$$

8. Let the tension in the cord attached to block A be T_A and the tension in the cord attached to block C be T_C . The equations of motion are then

$$T_A - m_A g = m_A a$$

$$T_C - \mu_k m_B g - T_A = m_B a$$

$$m_C g - T_C = m_C a$$

a. Adding the above given three equations to eliminate the tensions gives

$$a(m_A + m_B + m_C) = g(m_C - m_A - \mu_k m_B)$$

solving for m_C gives

$$m_C = \frac{m_A(a + g) + m_B(a + \mu_k g)}{g - a}$$

and substitution of numerical values gives

$$m_C = 12.9 \text{ kg}$$

$$\text{b. } T_A = m_A(g + a) = 47.2 \text{ N}$$

$$T_C = m_C(g - a) = 101 \text{ N}$$

9. a. The only horizontal force on the two-block combination is the horizontal component of $\vec{F}$, $F \cos \alpha$. The blocks will accelerate with

$$a = \frac{F \cos \alpha}{(m_1 + m_2)}$$

b. The normal force between the blocks is $m_1 g + F \sin \alpha$, for the blocks to move together, the product of this force and μ_s must be greater than the horizontal force that the lower block exerts on the upper block. That horizontal force is one of an action-reaction pair; the reaction to this force accelerates the lower block. Thus, for the blocks to stay together,

$$m_2 a \leq \mu_s(m_1 g + F \sin \alpha)$$

Using the result of part (a),

$$m_2 \frac{F \cos \alpha}{m_1 + m_2} \leq \mu_s(m_1 g + F \sin \alpha)$$

Solving the inequality for F gives the desired result.

10.

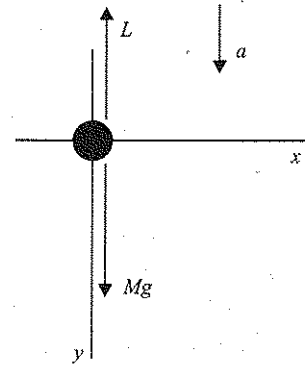


Fig. 7.549

a. L is the lift force

$$\text{b. } \sum F_y = ma_y$$

$$Mg - L = M(g/3)$$

$$L = 2Mg/3$$

$$\text{c. } L - mg = m(g/2),$$

where m is the mass remaining.

$$L = 2Mg/3$$

$$\text{so, } m = 4M/9$$

Mass $5M/9$ must be dropped overboard.

11. m = mass of one link

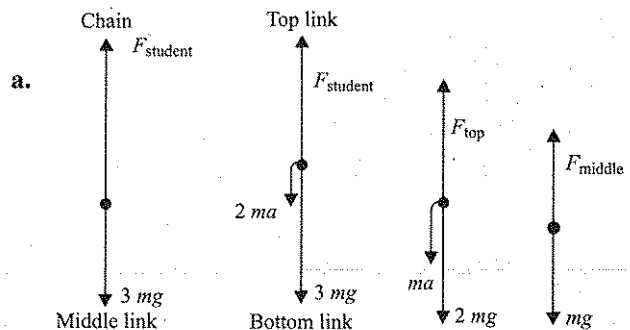


Fig. 7.550

The downward forces of magnitude $2ma$ and ma for the top and middle links are the reaction forces to the upward force needed to accelerate the links below.

b. i. The weight of each link is

$$mg = (0.300 \text{ kg})(9.80 \text{ m/s}^2) = 2.94 \text{ N}$$

$$a = \frac{F_{\text{net}}}{3m} = \frac{12 \text{ N} - 3(2.94 \text{ N})}{0.900 \text{ kg}} = \frac{3.18 \text{ N}}{0.900 \text{ kg}} = 3.53 \text{ m/s}^2 \text{ or } 3.5 \text{ m/s}^2$$

Using the free body diagram for the whole chain

ii. The second link also accelerates at 3.53 m/s^2 , so

$$F_{\text{net}} = F_{\text{top}} - ma - 2mg = ma$$

$$F_{\text{top}} = 2ma + 2mg$$

$$= 2(0.300 \text{ kg})(3.53 \text{ m/s}^2) + 2(2.94 \text{ N})$$

$$= 2.12 \text{ N} + 5.88 \text{ N} = 8.0 \text{ N}$$

12. a. Let a_1 and a_2 be the accelerations of the two men in the upward direction, and T the tension in the rope. Then,

$$T - Mg = Ma_1, \text{ and} \quad (i)$$

$$T - (M + m)g = (M + m)a_2 \quad (ii)$$

Subtracting equation (ii) from equation (i), we get,

$$\text{or, } a_2 = \left(\frac{M}{M + m} \right) a_1 - \frac{mg}{M + m}$$

$$a_2 < a_1$$

Hence, the lighter man will reach the pulley first.

- b. The lighter man ascends a distance h in time t with acceleration a_1 . Hence,

$$h = \frac{1}{2} a_1 t^2 \quad (iii)$$

Let s be the distance travelled by the heavier man in this time t , then

$$s = \frac{1}{2} a_2 t^2 = \frac{t^2}{2} \left[\frac{M}{M + m} a_1 - \frac{mg}{M + m} \right]$$

$$s = \frac{t^2}{2(M + m)} \left[M \left(\frac{2h}{t^2} \right) - mg \right]$$

$$= \frac{1}{2(M + m)} [2Mh - mgt^2]$$

The distance of the second man from the pulley = $h - s$

$$= h - \frac{1}{2(M + m)} [2Mh - mgt^2]$$

$$= \frac{1}{(M + m)} \left[Mh + mh - Mh + \frac{mgt^2}{2} \right]$$

$$= \frac{m}{(M + m)} \left[\frac{gt^2}{2} + h \right]$$

13.

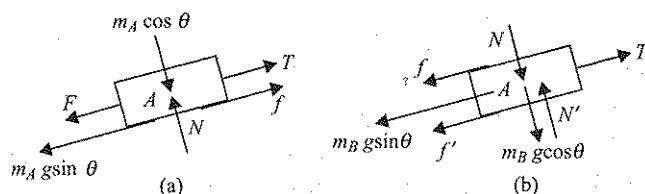


Fig. 7.551

$$m_A g \sin \theta + F - T - f = m_A a \quad (i)$$

$$f = \mu_k m_A g \cos \theta \quad (ii)$$

$$T - m_B g \sin \theta - f - f' = m_B a \quad (iii)$$

$$f' = \mu_k N' \quad (iv)$$

$$f' = \mu_k (m_A g \cos \theta + m_B g \cos \theta)$$

Solving above equations, we get

$$a_A = 5.2 \text{ m/s}^2$$

$$T = 215 \text{ N}$$

14. Let N be the reaction between man and platform. T_1 = tension in string passing over pulley 1, T_2 = tension in string passing

over pulley 2. M = mass of the man, m = mass of the platform (Fig. 7.552).

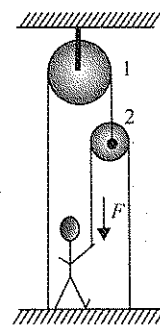


Fig. 7.552

The FBDs of man, platform and pulley 2 are shown in Figs. 7.553(a), (b), (c), and (d).

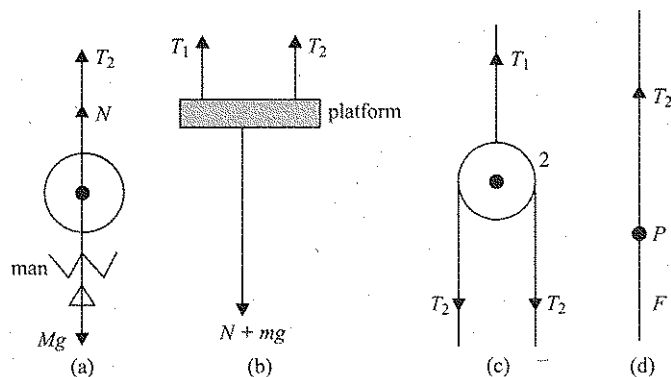


Fig. 7.553

Action exerted by the man on the platform = reaction exerted by the platform on the man = N

For equilibrium of the man

$$Mg = T_2 + N \quad (i)$$

For equilibrium of the platform

$$T_1 + T_2 = N + mg \quad (ii)$$

For equilibrium of pulley 2,

$$T_1 = 2T_2 \quad (iii)$$

For equilibrium of point P of the string

$$F = T_2 \quad (iv)$$

Adding equations (i) and (ii), we get

$$T_1 + T_2 + (T_2 + N) = N + mg + mg$$

$$T_1 + 2T_2 = (M + m)g$$

Using equation (iii), we get $2T_2 + 2T_2 = (M + m)g$

$$\Rightarrow T_2 = \frac{(M + m)}{4} g$$

$$\therefore F = T_2 = \frac{(M + m)}{4} g = \left(\frac{60 + 20}{4} \right) g \text{ N}$$

$$= 20 \text{ kg wt}$$

Now from equation (i), $N = Mg - T_2$

$$= Mg - F$$

$$= Mg - \frac{(M + m)}{4}g = \frac{(3M - m)}{4}g \quad (v)$$

$$= \left(\frac{3 \times 60 - 20}{4} \right)g$$

$$= 40 \text{ g N} = 40 \text{ kg wt}$$

From equation (v) $mg = 3Mg - 4N$

Clearly, mg is maximum when N is minimum

Minimum $N = 0$

$$\text{So } (mg)_{\max} = 3Mg = 3 \times 60g = 180 \text{ kg wt.}$$

15. Let T_1 be the tension in the rope and T_2 the tension in the tail of monkey A. Let force exerted by monkey A on the rope be F . Let M_A and M_B be masses of monkeys A and B, respectively. The free body diagrams of monkeys A and B and point P of the string are shown in Figs. 7.554(a), (b), and (c).

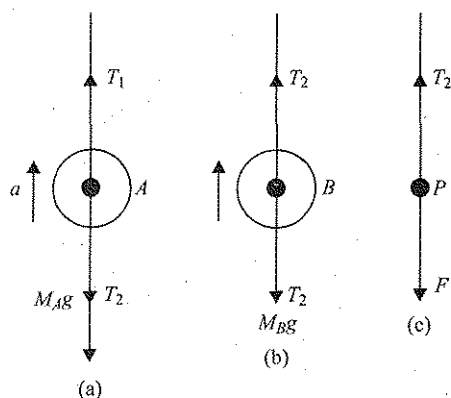


Fig. 7.554

The equations of motion of monkeys A and B are

$$T_1 - M_{Ag} - T_2 = M_A a \quad (i)$$

$$T_2 - M_B g = M_B a \quad (ii)$$

Equation of motion of point P is

$$T_1 - F = (\text{mass}) a$$

$$a = 0 \Rightarrow F = T_1 \quad (iii)$$

For maximum tension in tail

From equation (ii), we get

$$a = \frac{T_2 - M_B g}{M_B} = \frac{30 - 2 \times 10}{2} = 5 \text{ m/s}^2$$

then from equation (i), we get

$$T_1 = T_2 + M_A(g + a) = 30 + 5(10 + 5) = 105 \text{ N}$$

For minimum tension in tail $a = 0$

$\therefore$ from equation (ii), we get

$$T_2 = M_B g = 2 \times 10 = 20 \text{ N}$$

Then from equation (i), we get

$$T_1 = M_{Ag} + T_2 = 5 \times 10 + 20 = 70 \text{ N}$$

Therefore, the force exerted by monkey A on the rope $F = T_1$ must be between 70 N and 105 N.

16. Let M = mass of the painter = 10 kg

M = mass of the crate = 25 kg

Let F_A be the action force exerted by the painter on the crate, reaction force exerted by the crate on the man is

$$N = F_A = 450 \text{ N}$$

The free body diagram of painter is shown in Fig. 7.555(b)

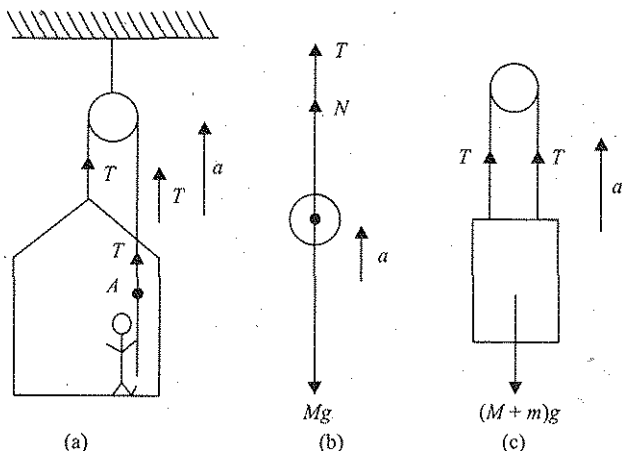


Fig. 7.555

$\therefore$ equation of motion of the painter is

$$N + T - Mg = Ma \quad (i)$$

The equation of motion of the whole system is

$$2T - (M + m)g = (M + m)a \quad (ii)$$

Multiplying equation (i) by (ii), we get

$$2N + 2T - 2Mg = 2ma \quad (iii)$$

Subtracting equation (ii) from equation (iii), we get

$$2N - 2Mg + (M + m)g = (2M - M + m)a$$

$$\text{Or } 2N - (M - m)g = (M + m)a$$

$$\begin{aligned} a &= \frac{2N - (M - m)g}{M + m} \\ &= \frac{2 \times 450 - (100 - 25) \times 10}{100 + 25} \\ &= \frac{900 - 750}{125} = 2 \text{ m/s}^2 \end{aligned}$$

From equation (ii), we get

$$\begin{aligned} \text{Tension } T &= \frac{(M + m)(g + a)}{2} \\ &= \frac{(100 + 25)(10 + 2)}{2} = 125 \times 6 = 750 \text{ N} \end{aligned}$$

17. a. When the ring slides from A to A' in time interval Δt , the block descends from C to C' as shown in Fig. 7.556(a). A'D is perpendicular dropped from A' on AB. For small displacement $A'B = DB$.

Now as the length of the string is constant.

$$AB + BC = A'B + BC'$$

$$\text{or } (AD + DB) + BC = A'B + BC'$$

$$AD = BC' - BC \quad (\because A'B = DB)$$

or $AA' \cos \theta = CC'$

$$\Rightarrow \frac{AA'}{\Delta t} \cos \theta = \frac{CC'}{\Delta t}$$

i.e., velocity of the ring $\times \cos \theta$ = velocity of the block

i.e., velocity of the block C , $v_C = v \cos \theta$ (i)

- b. If a is an initial acceleration of ring, then the acceleration of the block = $a \cos \theta$.

Let T be the tension in the string at this instant. Then the FBD of block is shown in Fig. 7.556(b).

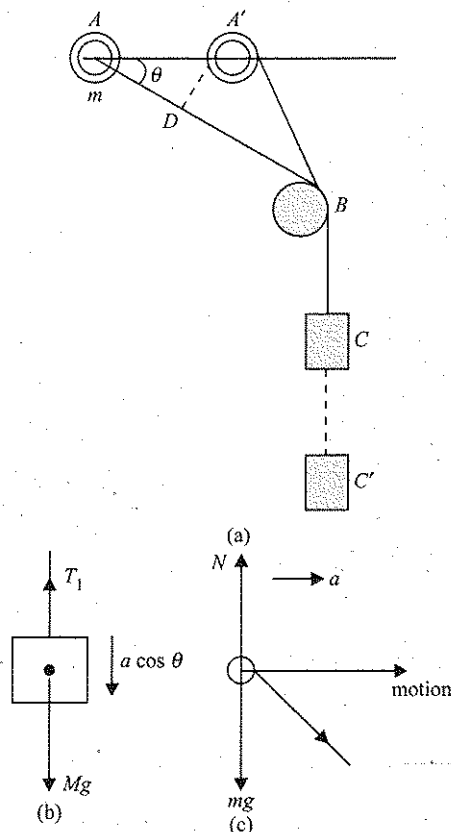


Fig. 7.556

- c. Equation of motion of block M is

$$Mg - T = -Ma \cos \theta \quad (ii)$$

The free body diagrams of ring is shown in Fig. 7.556(c).

Resolving T along horizontal and vertical, we have

$$\begin{aligned} T \cos \theta &= Ma \\ \Rightarrow T &= \frac{ma}{\cos \theta} \quad (iii) \end{aligned}$$

Equations (ii) and (iii) give

$$\begin{aligned} a &= \frac{Mg \cos \theta}{m + M \cos^2 \theta} = \frac{2mg \cos 30^\circ}{m + 2m \cos^2 30^\circ} \\ &= \frac{2g + \left(\sqrt{\frac{3}{2}}\right)}{1 + 2 \times \left(\frac{3}{4}\right)} = 6.78 \text{ m/s}^2 \end{aligned}$$

18.

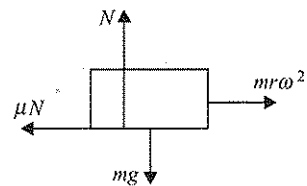


Fig. 7.557

$$N = mg$$

$$\mu N = mr \omega^2$$

$$\mu mg = m 2 \sin \theta \omega^2$$

$$\omega = \sqrt{\frac{\mu g}{2 \sin \theta}}$$

$$\omega = \sqrt{\frac{0.1 \times 10}{2 \times \sin 30}}$$

$$\omega = 1 \text{ rad/s.}$$

19. So long as the applied force does not produce the limiting frictional force, the two bodies will have no relative motion, that is, the two will behave like a compact body.

Here, the limiting frictional force = $0.25 \times 2 \times 10 = 5 \text{ N}$.

Let F be the force that produces limiting frictional force between the two. Then $F - 5 = 2a$ and $5 = 20a$, where a = common acceleration. Hence $F = 5.5 \text{ N}$. Here the applied force is only 2 N . Hence the two bodies will behave as one body and will have a common acceleration given by

$$2 = (20 + 2)a \text{ or } a = \frac{1}{11} = 0.09 \text{ ms}^{-2}$$

$$\text{Now } f_{fr} = 20 \times 0.09 = 1.8 \text{ N}$$

When $F = 20 \text{ N}$, the applied force exceeds the force required to produce the limiting frictional force. So there will be relative motion between the two. The frictional force = the limiting frictional force = 5 N .

From the free body diagram of the weight and the block,

$$20 - 5 = 2a_1 \text{ or } a_1 = 7.5 \text{ ms}^{-2}$$

$$5 = 20a_2 \quad a_2 = 0.25 \text{ ms}^{-2}.$$

20. Let X be the leftward displacement of M , and x and y be the leftward and downward displacement of m as shown in Fig. 7.558. Then by constraint relation

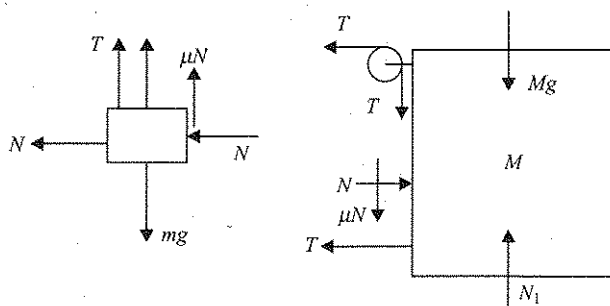


Fig. 7.558

$$x = X \Rightarrow \ddot{x} = \ddot{X} \Rightarrow a_x = A_x$$

and $l_1 - x + l_2 + l_3 - x + l_4 + y = l_1 + l_2 + l_3 + l_4$
where l_1, l_2, l_3, l_4 are the instantaneous lengths of the segments of the string.

$$\Rightarrow 2x = y \Rightarrow 2\ddot{x} = \ddot{y} \Rightarrow 2a_x = a_y$$

$$N = ma_x \text{ and } mg - \mu N - T = ma_y$$

$$\text{and } 2T - N = MA_x = Ma_x$$

Eliminating T , A and N , we get

$$a_x = \frac{2mg}{M + 5m + 2\mu m} \text{ and } a_y = \frac{4mg}{M + 5m + 2\mu m}$$

$$\therefore a = \sqrt{a_x^2 + a_y^2} = \frac{2\sqrt{5}mg}{m + 5m + 2\mu m}$$

21. Considering the free body diagram of m and M and keeping in mind the horizontal motion $f = ma$ and $F - f = Mb$, where f is the frictional force and a and b are the accelerations of m and M , respectively.

When there is no sliding, $a = b$. Hence $f = \frac{mF}{m + M}$

The mass m will begin to slide when $f = f_{\text{lim}} = kmg$

$$\text{or } kmg = \frac{mF}{m + M} \text{ or } F = k(m + M)g$$

When the force F is greater than $k(m + M)g$, the mass m slides over M with acceleration a given by

$$f_{\text{lim}} = ma \text{ or } kmg = ma \text{ or } a = kg$$

The acceleration of M is given by

$$F - f_{\text{lim}} = Mb \text{ or } b = \frac{F - kmg}{M}$$

The acceleration of M relative to m is

$$b_{\text{rel}} = b - a = \frac{F - kmg}{M} - kg = \frac{F - kg(M + m)}{M}$$

From the formula $S = ut + \frac{1}{2}ft^2$

$$l = 0.t + \frac{1}{2}b_{\text{rel}}t^2 = \frac{1}{2} \frac{F - kg(M + m)}{M} t^2$$

22. a. Let N_1 and N_2 be the reactions of the plane on 1 and 2 and F be the force of interaction between them. From the free body diagrams of 1 and 2, we get

$$m_1 g \sin \alpha - k_1 m_1 g \cos \alpha + F = m_1 a$$

$$m_2 g \sin \alpha - k_2 m_2 g \cos \alpha - F = m_2 a$$

Eliminating a between these two equations, we get

$$F = \frac{(k_1 - k_2)m_1 m_2 g \cos \alpha}{m_1 + m_2}$$

- b. When the bars just start sliding, $a = 0$

$$\therefore m_1 g \sin \alpha - k_1 m_1 g \cos \alpha + F = 0$$

$$\text{and } m_2 g \sin \alpha - k_2 m_2 g \cos \alpha - F = 0$$

$$\text{Adding } (m_1 + m_2)g \sin \alpha = (k_1 m_1 + k_2 m_2)g \cos \alpha$$

$$\text{or } \tan \alpha = \frac{k_1 m_1 + k_2 m_2}{m_1 + m_2}$$

23. a. Since A tends to slip down, frictional forces act on it from both sides up the plane

Let N be the reaction of the plane on A and N' be the mutual normal action - reaction between A and B .

From the FBD of A Fig. 7.559

$$N' + mg \cos \alpha = N$$

$$\text{and } mg \sin \alpha = (N + N')$$

From the FBD of B

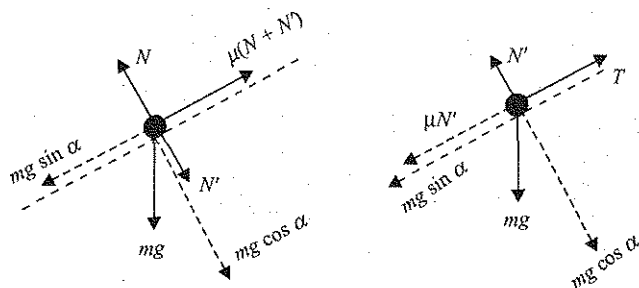


Fig. 7.559

$$N' = mg \cos \alpha$$

$$mg \sin \alpha + \mu N' = T$$

$$\therefore 2mg \cos \alpha = N$$

$$\text{and } mg \sin \alpha = (2mg \cos \alpha + mg \cos \alpha)$$

$$\text{or } \frac{1}{3} \tan \alpha$$

$$\therefore \frac{1}{3} \tan 37^\circ = \frac{1}{3} \times \frac{3}{4} = 0.25$$

24. a. When A is projected to the right, B falls freely under gravity. The string is tensioned when A and B travel the same distance. Let t be the time in which they cover the same distance. Then vt (distance described by A) = $\frac{1}{2}gt^2$ (distance covered by B)

$$\text{or } t = 2 \frac{u}{g}$$

Let v be the initial common velocity. When the string is tensioned, the two bodies experience the same impulsive force but in opposite directions.

$$\text{change in momentum of } A = 2mv_f - 2mu$$

$$= 2mv - 2mu$$

$$\therefore \text{change in momentum of } B = mv - mu_{\text{initial}}$$

$$\text{But } u_{\text{initial}} = gt = g \frac{2u}{g} = 2u$$

$$\text{Change in momentum of } B = mv - m \times 2u$$

Therefore, by Newton's laws of motion

$$(2mv - 2mu) = -(mv - 2mu)$$

$$\text{or } 3mv = 4mu \text{ or } v = \frac{4u}{3}$$

- b. Let a be the common acceleration.

$$mg - T = ma \quad T = 2ma$$

$$\therefore a = \frac{g}{3}$$

$$\therefore v^2 - v_0^2 = 2l' \frac{g}{3}$$

where l' is the distance covered by A or B with acceleration.

$$\text{Now } l = l' + ut = l' + u \frac{2u}{g} \Rightarrow l' = l - \frac{2u^2}{g}$$

$$\text{Here } v_0 = \frac{4u}{3}$$

$$v^2 = \frac{16u^2}{9} + \frac{2g}{3} \left(l - \frac{2u^2}{g} \right)$$

$$= \frac{16u^2}{9} + \frac{2gl}{3} - \frac{4u^2}{3} = \frac{4u^2 + 6gl}{9}$$

$$\therefore v = \frac{\sqrt{4u^2 + 6gl}}{3}$$

25. The same frictional force is effective on A and B. This force produces retardation on A and acceleration on B till they acquire a common velocity. $F = ma = Ma'$, where a = absolute retardation of m

a' = absolute acceleration of M

Relative retardation of $m = a - (-a') = a + a'$

Initial relative velocity = v_0

Final relative velocity = 0

$$\therefore v_0^2 = 2(a + a')s$$

where s = distance covered by m relative to M

$$\text{or } v_0^2 = 2 \left(\frac{F}{m} + \frac{F}{M} \right) s = \frac{2F(m+M)s}{mM}$$

$$\text{or } s = \frac{mMv_0^2}{2F(m+M)}$$

26. If a_0, a_1 , and a_2 are accelerations of M_0, M_1 , and M_2 , respectively, then

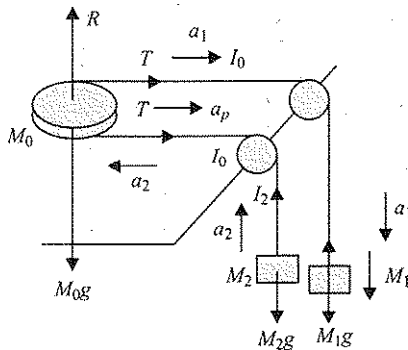


Fig. 7.560

$$a_0 = \frac{a_1 - a_2}{2}$$

$$2a_0 = a_1 - a_2 \quad (i)$$

For motion of M_0

$$2T = M_0 a_0 \quad (ii)$$

For motion of mass M_1 ,

$$M_1 g - T = M_1 a_1 \quad (iii)$$

For motion of mass M_2 ,

$$T - M_2 g = M_2 a_2 \quad (iv)$$

Substituting values of a_0, a_1 , and a_2 from equations (ii), (iii), and (iv) in equation (i), we get

$$2 \left(\frac{2T}{M_0} \right) = \left(g - \frac{T}{M_1} \right) - \left(\frac{T}{M_2} - g \right)$$

$$\frac{4T}{M_0} = 2g - T \left(\frac{1}{M_1} + \frac{1}{M_2} \right)$$

$$\text{i.e., } \left(\frac{4}{M_0} + \frac{1}{M_1} + \frac{1}{M_2} \right) T = 2g$$

$$\text{This gives } T = \frac{2M_0 M_1 M_2 g}{4M_1 M_2 + M_0(M_1 + M_2)} \quad (v)$$

$\therefore$ acceleration of pulley of A from equation (ii)

$$a_0 = \frac{2T}{M_0} = \frac{4M_1 M_2}{4M_1 M_2 + M_0(M_1 + M_2)} \quad (vi)$$

27. Evidently, the larger block of mass experiences more centrifugal force radially outwards, compared to the block of smaller block m [$M > m$ and $r_2 > r_1$].

Figure 7.561 shows their FBD

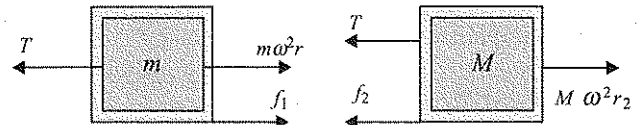


Fig. 7.561

Owing to the larger force experienced by block of mass M , it tends to fly off radially.

In the situation of limiting equilibrium, we have

$$T = m\omega^2 r_1 + f_1 \Rightarrow T + f_2 = M\omega^2 r_2$$

[where f_1 and f_2 are frictional forces for the two blocks and the surface.]

$$f_1 = \mu_1 mg \Rightarrow f_2 = \mu_2 Mg$$

The above two equations get reduced to

$$T = m\omega^2 r_1 + \mu_1 mg \quad (i)$$

$$T + \mu_2 Mg = M\omega^2 r_2 \quad (ii)$$

Subtracting equations (i) and (ii), we get

$$\mu_2 Mg = M\omega^2 r_2 - m\omega^2 r_1 - \mu_1 mg$$

$$\omega^2 = \frac{g[\mu_1 m + \mu_2 M]}{Mr_2 - mr_1} \Rightarrow \omega = \left[\frac{g(\mu_1 m + \mu_2 M)}{Mr_2 - mr_1} \right]^{1/2}$$

28. a. The tension in the cord must be $m_2 g$ in order that the hanging block moves at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so

$$m_2 g = (m_1 g \sin \alpha + \mu_k m_1 g \cos \alpha)$$

$$\text{and } m_2 = m_1 (\sin \alpha + \mu_k \cos \alpha)$$

- b. In this case, the frictional force acts in the same direction as the tension on the block of mass m_1 , so,

$$m_2 g = (m_1 g \sin \alpha - \mu_k m_1 g \cos \alpha)$$

$$\text{or, } m_2 = m_1 (\sin \alpha - \mu_k \cos \alpha)$$

- c. Similar to the analysis of parts (a) and (b), the largest m_2 could be

$$m_1 (\sin \alpha + \mu_s \cos \alpha)$$

and the smallest m_2 could be

$$m_1 (\sin \alpha - \mu_s \cos \alpha)$$

Objective Type

1. **c.** When a body is stationary its acceleration is zero. It means net force acting on the body is zero, i.e., $\sum \vec{F} = 0$. We can also say that all the forces acting balance each other.
2. **a.** The water jet striking the block at the rate of 1 kg/s at a speed of 5 m/s will exert a force on the blank

$$F = v \frac{dm}{dt} = 5 \times 1 = 5 \text{ N}$$

Under the action of this force of 5 N, the block of mass 2 kg will move with an acceleration given by

$$a = \frac{F}{m} = \frac{5}{2} = 2.5 \text{ m/s}^2$$

3. **d.** When a string is fixed horizontally as shown in Fig. 7.562 (by clamping its free ends) and loaded at the middle, then for the equilibrium of point P

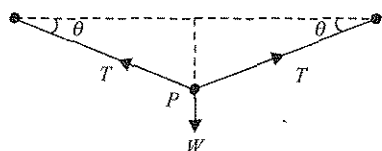


Fig. 7.562

$$2T \sin \theta = W$$

i.e.,
$$T = \frac{W}{2 \sin \theta}$$

Tension in the string will be maximum when $\sin \theta$ is minimum, i.e., $\theta = 0^\circ$ or $\sin \theta = 0$ and then $T = \infty$. However, as every string can bear a maximum finite tension (less than breaking strength). So this situation cannot be realized practically. We conclude that a string can never remain horizontal when loaded at the middle howsoever great mass be the tension applied.

4. **b.** For A : $T - 2g = 2a$ (i)
- For B : $T_1 + 2g - T = 2a$ (ii)
- For C : $2g - T_1 = 2a$ (iii)

Adding equations (i) and (ii), we get

$$T_1 = 4a \quad (\text{iv})$$

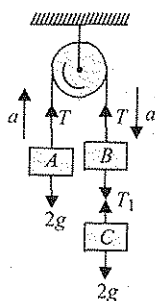


Fig. 7.563

From equations (iii) and (iv), we get

$$2g - 4a = 2a$$

or
$$a = g/3 \quad (\text{v})$$

From equations (iv) and (v), we get

$$T_1 = 4 \times \frac{g}{3} = 13 \text{ N}$$

5. **c.** Let a be the common acceleration of the system (Fig. 7.564)

Here $T = Ma$ (for block)
 $P - T = Ma$ (for rope)

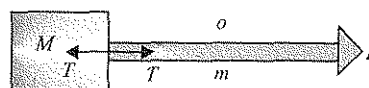


Fig. 7.564

$$\Rightarrow P - Ma = ma$$

or $P = (m + M)a$ or $a = P/(m + M)$

Now $T = Ma = \frac{MP}{M + m}$

6. **d.** Let A applies a force R on B,

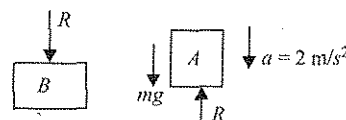


Fig. 7.565

Then B also applies an opposite force R on A as shown in Fig. 7.565.

For A : $mg - R = ma$

$$\Rightarrow R = m(g - a) = 0.5[10 - 2] = 4 \text{ N}$$

7. **b.** In Fig. 7.566, the point B is in equilibrium under the action of T, F, and Mg.

Here $T \sin \theta = F$
 or $T = F / \sin \theta$

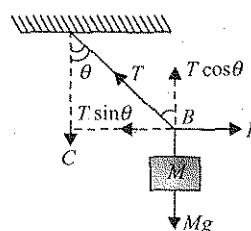


Fig. 7.566

8. **a.** $T = 60 \text{ N}$

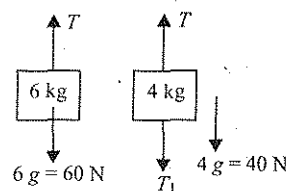


Fig. 7.567

$$T = T_1 + 40$$

$$60 = T_1 + 40$$

$$T_1 = 20 \text{ N}$$

9. d. Let the tension in the string AP_2 and P_2P_1 be T . Considering the force on pulley P_1 , we get

$$T = W_1$$

Further, let $\angle AP_2P_1 = 2\theta$

Resolving tensions in horizontal and vertical directions and considering the forces on pulley P_2 , we get

$$2T \cos \theta = W_2$$

$$\text{or } 2W_1 \cos \theta = W_2 \text{ or } \cos \theta = 1/2$$

$$\text{or } \theta = 60^\circ$$

$$\text{So } \angle AP_2P_1 = 2\theta = 120^\circ.$$

10. c. Let the tension in the rope be T . Let acceleration of the man and the chair is a upwards.

$$\text{For man: } T + 450 - 1000 = \frac{1000}{10}a$$

$$\text{or } T = 550 + 100a \quad (i)$$

$$\text{For chair: } T - 450 - 250 = \left(\frac{250}{10}\right)a$$

$$\text{or } T = 700 + 25a \quad (ii)$$

From equations (i) and (ii), we get $a = 2 \text{ ms}^{-2}$

11. c. The spring will exert maximum force when the ball is at its lowest position. If the ball has descended through a distance x to reach the position.

$$mgx = \frac{1}{2}Kx^2 \text{ or } x = 2mg/K \quad (i)$$

For the block B to leave contact spring force

$$Kx = Mg \quad (ii)$$

Comparing equations (i) and (ii), $m = M/2$

12. a. Acceleration of the skates will be in the ratio

$$\frac{F}{4} : \frac{F}{5} \text{ or } 5 : 4$$

Now according to the problem,

$$s = 0 + \frac{1}{2}at^2$$

$$\text{we get } \frac{s_1}{s_2} = \frac{a_1}{a_2} = \frac{5}{4}$$

$$13. \text{ a. } a = \frac{\sqrt{R_1^2 + R_2^2}}{m} = \frac{R_3}{m} \quad [\because R_3 = \sqrt{R_1^2 + R_2^2}]$$

$$14. \text{ b. Change in momentum of one ball} = 2mu, \text{ time taken} = 1 \text{ s}$$

$$F_{av} = \frac{\text{Total change in momentum}}{\text{Time taken}} \\ = \frac{n(2mu)}{1} = 2mnu$$

15. c. Area under force-time graph is impulse and impulse is change in momentum,

Area of graph = Change in momentum

$$\Rightarrow \frac{1}{2}TF_0 = 2mu \Rightarrow F_0 = \frac{4mu}{T}$$

16. d. From 0 to T , area is +ve and from T to $2T$, area is -ve. Net area is zero. Hence, no change in momentum.

17. b. The acceleration of the body perpendicular to OE is

$$a = \frac{F}{m} = \frac{4}{2} = 2 \text{ m/s}^2$$

Displacement along OE , $s_1 = vt = 3 \times 4 = 12 \text{ m}$

Displacement perpendicular to OE

$$s_2 = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times (4)^2 = 16 \text{ m}$$

The resultant displacement

$$s = \sqrt{s_1^2 + s_2^2} = \sqrt{144 + 256} = \sqrt{400} = 20 \text{ m}$$

18. c. Let T be the tension in the rope and a the acceleration of rope. The absolute acceleration of the man is therefore $\left(\frac{5g}{4} - a\right)$. Equations of motion for mass and man give

$$T - 100g = 100a \quad (i)$$

$$T - 60g = 60\left(\frac{5g}{4} - a\right) \quad (ii)$$

Solving equations (i) and (ii), we get,

$$T = 1218.75 \text{ N} \approx 1219 \text{ N}$$

19. b. $T \sin \theta - mg = \sin 30^\circ = ma$

$$\Rightarrow T \sin \theta = mg \sin 30^\circ + mg/2 \quad (i)$$

$$T \cos \theta = mg \cos 30^\circ \quad (ii)$$

Dividing equation (i) by (ii), we get

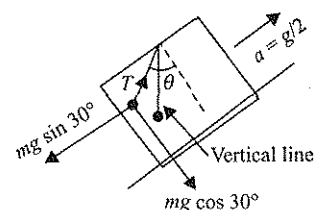


Fig. 7.568

$$\tan \theta = \frac{2}{\sqrt{3}}$$

20. c. $T_2 \cos \beta = mg \quad (i)$

$$T_2 \cos \beta = F = mg \quad (ii)$$

From equations (i) and (ii), $\beta = 45^\circ$

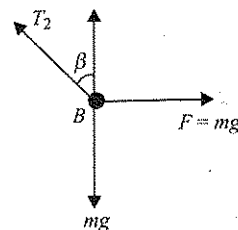


Fig. 7.569

$$T_2 \cos \beta + mg = T_1 \cos \alpha \quad (iii)$$

$$T_2 \sin \beta = T_1 \sin \alpha \quad (iv)$$

Dividing equation (iv) and (iii)

$$\tan \alpha = 1/2$$

$$\Rightarrow \sin \alpha = 1/\sqrt{5}$$

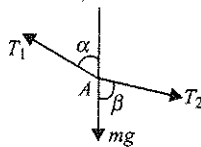


Fig. 7.570

From equation (iv), $T_2 \frac{1}{\sqrt{2}} = T_1 \frac{1}{\sqrt{5}}$

$$\Rightarrow \sqrt{5}T_2 = \sqrt{2}T_1.$$

21. a. Acceleration at $t = 1$ s, $a_1 = \frac{3.6}{2} = 1.8$ m/s.
 22. d. Tension will be the least during downward acceleration from $t = 10$ s to $t = 12$ s.
 23. a. $x = 0$, till $mg \sin \theta < \mu mg \cos \theta$ and gradually x will increase. At angle $\theta > \tan^{-1}(\mu)$

$$kx + \mu mg \cos \theta = mg \sin \theta$$

$$\text{or } x = \frac{mg \sin \theta - \mu mg \cos \theta}{k}$$

Here k = force constant of spring.

24. d. Net pulling force on the system is $Mg + mg - mg$ or simply Mg . Total mass being pulled is $M + 2m$. Hence, the acceleration of the system as shown in Fig. 7.571 is

$$a = \frac{Mg}{M + 2m}$$

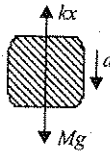


Fig. 7.571

Now since $a < g$, there should be an upward force on M so that its acceleration becomes less than g . Hence for any value of M , the spring will be elongated.

25. b. Suppose F = upthrust due to buoyancy
 Then while descending, we find

$$Mg - F = M\alpha$$

when ascending, we have

$$F - (M - m)g = (M - m)\alpha$$

Solving equations (i) and (ii), we get

$$m = \left[\frac{2\alpha}{\alpha + g} \right] M$$

26. a. Initial force = $2g = 20$ N

$$\text{Initial acceleration} = \frac{\text{Force}}{\text{Mass}} = \frac{20}{5 + 1} = \frac{20}{6} \text{ m/s}^2$$

$$\text{Final force} = (\text{load} + \text{mass of thread}) \times g = (2 + 1) \times 10 = 30 \text{ N}$$

$$\therefore \text{final acceleration} = \frac{30}{6} \text{ m/s}^2$$

27. a. Acceleration of the system:

$$a = \frac{P}{M + m} \quad (1)$$

The FBD of mass m is shown in Fig. 7.572.

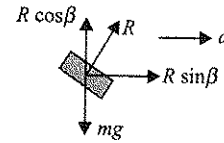


Fig. 7.572

$$R \sin \beta = ma \quad (2)$$

$$R \cos \beta = mg \quad (3)$$

From equations (2) and (3), we get

$$a = g \tan \beta$$

Putting the value of a in equation (1), we get

$$P = (M + m)g \tan \beta$$

28. d. Extension in the spring is

$$x = AB - R$$

$$= 2R \cos 30^\circ - R = (\sqrt{3} - 1)R$$

Spring force

$$F = kx = \frac{(\sqrt{3} + 1)mg}{R} \times (\sqrt{3} - 1)R = 2mg$$

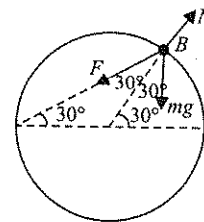


Fig. 7.573

From Fig. 7.573

$$N = (F + mg) \cos 30^\circ = \frac{3\sqrt{3}mg}{2}$$

29. b. Acceleration of the cylinder down the plane is

$$a = (g \sin 30^\circ)(\sin 30^\circ) = 10 \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) = 2.5 \text{ m/s}^2$$

$$\text{Time taken: } (t) = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 5}{2.5}} = 2 \text{ s}$$

30. b. 31. d. Same solution for both.

$$M_1 = 100 \text{ kg}, m_2 = 50 \text{ kg}, a = 5 \text{ m/s}^2$$

$$T + N - m_1g = m_1a$$

$$T - N = mg = m_2a$$

Solving these, we get

$$T = 1125 \text{ N and } N = 375 \text{ N}$$

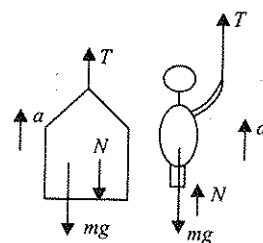


Fig. 7.574

32. d. As shown in Fig. 7.575 (a) and (b) $T \cos 45^\circ = ma$ (1)

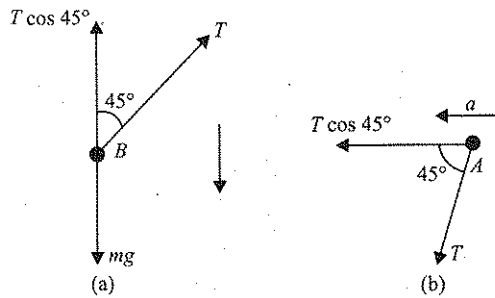


Fig. 7.575

$$Mg - T \cos 45^\circ = ma \quad (2)$$

From equations (1) and (2), we get

$$T = mg/\sqrt{2}$$

33. d. From constraint relations, we can see that the acceleration of block B in upward direction is

$$a_B = \left(\frac{a_C + a_A}{2} \right) \text{ with proper signs.}$$

$$\text{So, } a_B = \left(\frac{3 - 12t}{2} \right) = 1.5 - 6t$$

$$\text{or } \frac{dv_B}{dt} = 1.5 - 6t$$

$$\text{or } \int_0^{v_B} dv_B = \int_0^1 (1.5 - 6t) dt$$

$$\text{or } v_B = 1.5t - 3t^2$$

$$\text{or } v_B = 0 \text{ at } t = 1/2 \text{ s}$$

34. c. When mass m_2 moves downwards, the centre of mass of system $(m_1 + m_2)$ moves downwards. It means the acceleration is found in centre of mass in the downward direction during motion of m_2 . This is possible only when net downward force is greater than that of upward force.

$$\text{Mathematically, } m_1g + m_2g > N$$

$$N < m_1g + m_2g$$

35. b. Length of the string (Fig. 7.576) is

$$l = x_A + 2x_B + x_C$$

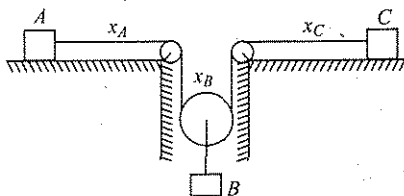


Fig. 7.576

Differentiating twice w.r.t. time, we get

$$0 = \frac{d^2x_A}{dt^2} + 2\frac{d^2x_B}{dt^2} + \frac{d^2x_C}{dt^2}$$

$$\text{or, } 0 = -a_A + 2a_B + a_C$$

$$\Rightarrow a_B = \frac{a_A - a_C}{2}$$

36. c. From Fig. 7.577

$$l_1 + l_2 = C$$

$$\frac{dl_1}{dt} + \frac{dl_2}{dt} = 0$$

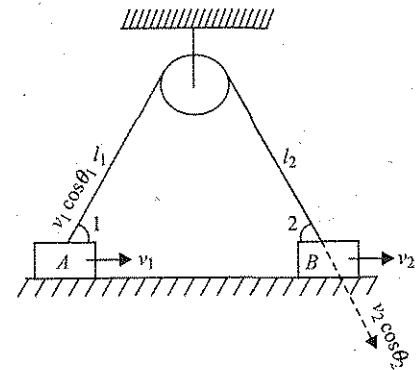


Fig. 7.577

$$-v_1 \cos \theta_1 + v_2 \cos \theta_2 = 0$$

$$\frac{v_1}{v_2} = \frac{\cos \theta_2}{\cos \theta_1}$$

37. b. $a_B = 2 \text{ m/s}^2$ ($\uparrow$)

For the following questions (Fig. 7.578) assume

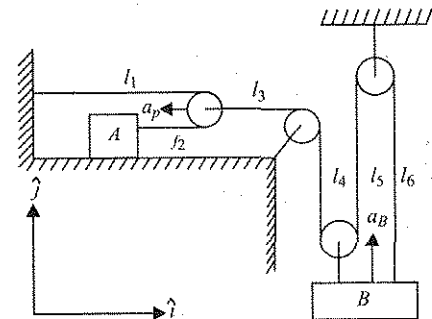


Fig. 7.578

$$l_1 + l_2 = C$$

$$l_3 + l_4 + l_5 + l_6 = C$$

$$l_1'' + l_2'' = 0$$

$$l_3'' + l_4'' + l_6'' = 0$$

$$-a_p - a_p + 12 = 0$$

$$a_p - a_B - a_B - a_B = 0$$

$$a_p = 6 \text{ m/s}^2$$

$$a_p = 3a_B \Rightarrow a_B = 2 \text{ m/s}^2$$

38. a. $-b\hat{i} - 4b\hat{j}$ (Fig. 7.579)

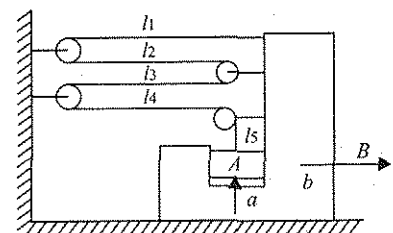


Fig. 7.579

Acceleration of A in horizontal direction = the acceleration of B = b in rightwards

Acceleration of A in vertical direction = the acceleration of A with respect to b in upwards direction = $a = 4b$.

Hence the net acceleration of A = $b\hat{i} + 4b\hat{j}$.

39. a. Acceleration of C in horizontal direction = acceleration of A = a (right direction) = $a\hat{i}$

Acceleration of C in vertical downward direction = $-2(a + b)\hat{j}$

Hence, net acceleration of C = $a\hat{i} - 2(a + b)\hat{j}$ (see Fig. 7.580)

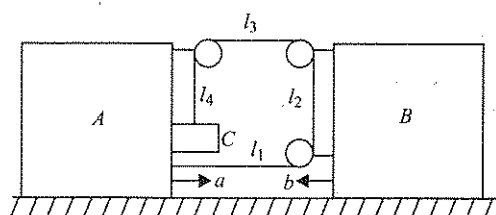


Fig. 7.580

From length constraint

$$l_1 + l_2 + l_3 + l_4 = C$$

$$l_1'' + l_2'' + l_3'' + l_4'' = 0$$

$$-a - b + 0 - a - b + c = 0$$

$$c = 2a + 2b$$

From wedge constraints, acceleration of C towards right side is a . Acceleration of C w.r.t. ground = $a\hat{i} - 2(a + b)\hat{j}$

40. c. In Fig. 7.581 given ($V = 10$ m/s)

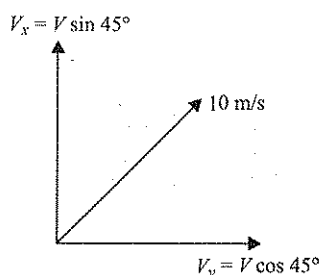


Fig. 7.581

$$V_x = \frac{V}{\sqrt{2}} - \frac{g}{\sqrt{2}} \times 2$$

$$\Rightarrow V_x = \frac{10}{\sqrt{2}} - \frac{10}{\sqrt{2}} \times 2$$

$$V_x = -\frac{10}{\sqrt{2}} \text{ m/s and } V_y = \frac{10}{\sqrt{2}} \text{ m/s}$$

$$V = \sqrt{\frac{100}{2} + \frac{100}{2}} = 10\sqrt{\frac{1}{2} + \frac{1}{2}} = 10 \text{ m/s}$$

41. c. From Fig. 7.582, $F = 2T \cos \theta$

$$\text{or } T = F/(2 \cos \theta)$$

The force responsible for motion of masses on X-axis is $T \sin \theta$.

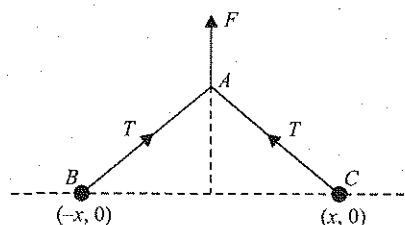


Fig. 7.582

$$\begin{aligned} \therefore ma &= T \sin \theta = \frac{F}{2 \cos \theta} \times \sin \theta \\ &= \frac{F}{2} \tan \theta = \frac{F}{2} \times \frac{OB}{OA} = \frac{F}{2} \times \frac{x}{\sqrt{(a^2 - x^2)}} \end{aligned}$$

$$\text{So, } a = \frac{F}{2m} \times \frac{x}{\sqrt{(a^2 - x^2)}}$$

42. c. In a given system,

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{g}{8}$$

$$\therefore \frac{m_1 - m_2}{m_1 + m_2} = \frac{1}{8}$$

$$8m_1 - 8m_2 = m_1 + m_2$$

$$7m_1 = 9m_2$$

$$\frac{m_1}{m_2} = \frac{9}{7}$$

43. b. $\vec{a}_{b,l} = \vec{a}_b - \vec{a}_l = (g - a) \downarrow$

$$\vec{a}_b = g \downarrow$$

44. c. $T = N \sin \theta$ and $N = mg \cos \theta$

$$T = mg \cos \theta \sin \theta = \frac{mg}{2} \sin 2\theta$$

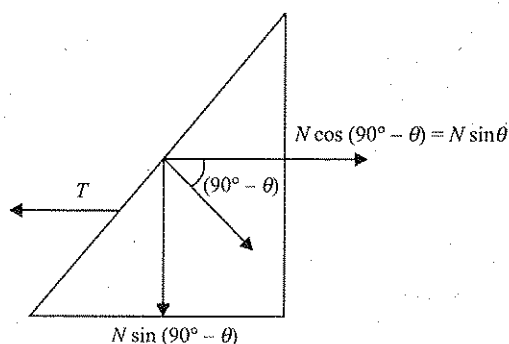


Fig. 7.583

45. d. $\vec{a} = \frac{\vec{F}}{m} = -10\hat{j} \text{ (m/s}^2\text{)}$

Displacement in y-direction

$$y = ut + \frac{1}{2}at^2$$

$$0 = 4 \times t - \frac{1}{2} \times 10 \times t^2$$

$$t = \frac{4}{5} \text{ s}$$

46. c. At equation $2T \cos \theta = Mg$

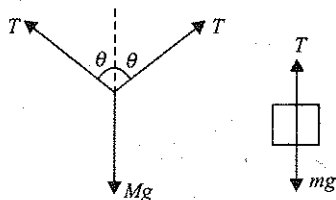


Fig. 7.584

$$\cos \theta = \frac{Mg}{2mg} = \frac{M}{2m}$$

$$\cos \theta < 1$$

$$\frac{M}{2m} < 1$$

$$\Rightarrow M < 2m$$

47. b. From Fig. 7.585

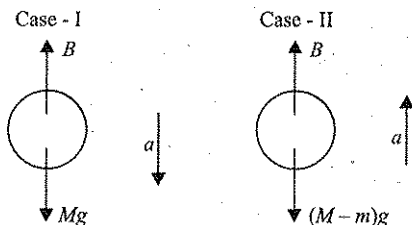


Fig. 7.585

$$Mg - B = Ma \quad (i)$$

$$B - (M - m)g = (M - m)a \quad (ii)$$

From equations (i) and (ii), we get

$$m = \frac{2Mg}{a}$$

48. b. Force exerted by block on inclined plane $= Mg \cos \theta$

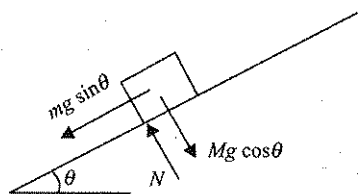


Fig. 7.586

49. a. As shown in Fig. 7.587

$$a = \frac{32 - 30}{3} = 4 \text{ m/s}^2$$

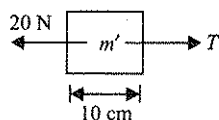


Fig. 7.587

$$m' = \frac{3}{30} \times 10 = 1 \text{ kg}$$

$$T = 20 = 1 \times 4 \Rightarrow T = 24 \text{ N}$$

50. b.

$$T = \frac{2mMg}{m+M}$$

$$= \frac{2mg}{1 + \frac{M}{m}}; 2mg$$

Hence the total downward force is $4mg$ (Fig. 7.588).

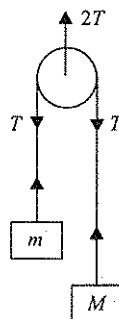


Fig. 7.588

51. b. In the given system,

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{g}{8}$$

$$\frac{m_1 - m_2}{m_1 + m_2} = \frac{1}{8}$$

$$8m_1 - 8m_2 = m_1 + m_2$$

$$7m_1 = 9m_2$$

$$\frac{m_1}{m_2} = \frac{9}{7}$$

52. c.

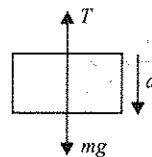


Fig. 7.589

$$\Rightarrow T = mg - ma$$

53. a. As F_2 and F_3 are mutually perpendicular, their resultant $\sqrt{F_2^2 + F_3^2}$.

As particle is stationary under F_1, F_2, F_3 ; therefore, $\sqrt{F_2^2 + F_3^2}$ must be equal and opposite to F_1 .

When F_1 is removed, resultant force is $\sqrt{F_2^2 + F_3^2}$.

Therefore, acceleration of the particle

$$= \frac{\sqrt{F_2^2 + F_3^2}}{m} = \frac{F_1}{m}$$

54. d. For a : (Fig. 7.590)

$$5g - T = 5(2C)$$

For C :

$$2T8g = 8C$$

$$\Rightarrow C = \frac{g}{14} = \frac{5}{7} \text{ ms}^{-2}$$

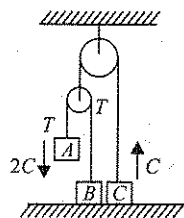


Fig. 7.590

55. c. Charge in the length of the string should be zero.

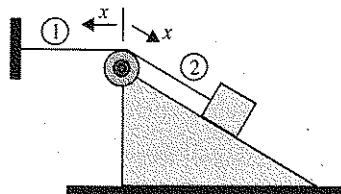


Fig. 7.591

Hence,

$$|X| = |x|$$

$|X|$ = displacement of M with respect to ground

$|x|$ = displacement of m with respect to M .

56. d. As $F = \frac{dP}{dt}$

$$\int dp = \int f dt$$

Δp = Area under $F-t$ graph

$$\Delta p = \frac{1}{2} \times 5 \times 4 - 1 \times 5 = 5 \text{ kg m/s}$$

57. c. T_x (Hanged part) = $3T'_x$ (Sliding part)

$$\therefore x = 3x' \Rightarrow 3 \times 0.6 = 1.8 \text{ m/s}$$

58. a. $V_A = 2 \text{ m/s}$ (towards right)

$$\therefore V_{P_1} = \frac{V_A}{2} = 1 \text{ m/s (upwards)}$$

$$V_B = 2 \text{ m/s (towards left)}$$

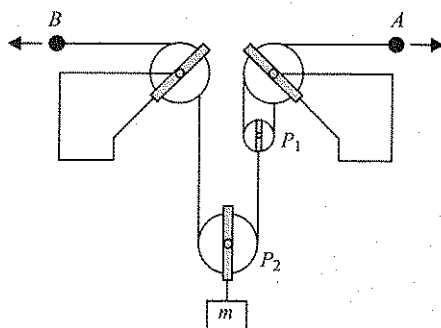


Fig. 7.592

$$\text{Now } 2V_{P_2} = V_B + V_{P_1}$$

$$\therefore V_{P_2} = \frac{V_B + V_{P_1}}{2} = \frac{2 + 1}{2} = 1.5 \text{ m/s}$$

59. b. $a = \frac{3T - mg \sin \theta}{m}$

$$\frac{3 \times 250 - 100 \times 10 \times \sin 30^\circ}{100} = 2.5 \text{ m/s}^2$$

60. c. From Fig. 7.593,

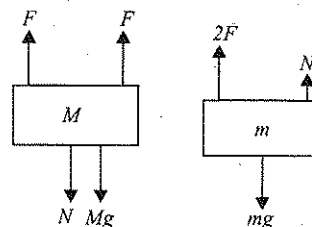


Fig. 7.593

$$2F - N - Mg = Ma$$

$$2F - mg + N = ma$$

$$4F - (M + m)g = (M + m)a$$

$$a = \frac{4F - (M + m)g}{M + m}$$

61. b. The situation before breaking the string $20 \cos 60^\circ + 20 \cos 60^\circ$ is shown in Fig. 7.594.

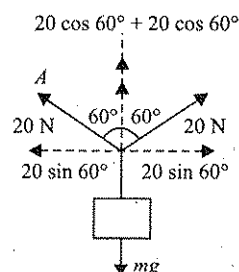


Fig. 7.594

From Fig. 7.594,

$$20 \cos 60^\circ + 20 \cos 60^\circ = mg$$

$$mg = 20 \text{ N}$$

62. d. The acceleration of block-rope system is

$$a = \frac{F}{(M + m)}$$

where M = mass of the block and m = mass of the rope
So, the tension in the middle of the rope will be

$$T = \{M + (m/2)\} a = \frac{M + (m/2) F}{M + m}$$

Given that $m = M/2$

$$\therefore T = \left[\frac{M + (M/4)}{M + (M/2)} \right] F = \frac{5F}{6}$$

63. d. The reading of the spring scale is the normal reaction between the man and the spring scale (see Fig. 7.595).

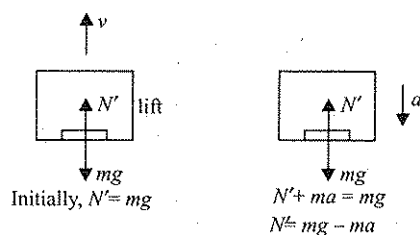


Fig. 7.595

As the reading decreases, it means that the normal reaction is also decreasing. Firstly, the lift must be moving upwards with a constant velocity and then decelerated to rest.

64. b.

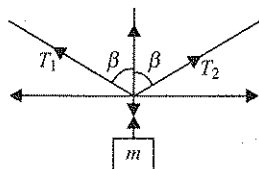


Fig. 7.596

$$\begin{aligned}\tan \beta &= \frac{12}{5} \quad \therefore \cos \beta = \frac{10}{13} \\ T_1 \cos \beta + T_2 \cos \beta &= mg \quad (i) \\ T_1 \sin \beta &= T_2 \sin \beta \quad (ii) \\ \therefore T_1 &= T_2 = T \\ \therefore 2T \cos \beta &= mg \\ T &= \frac{mg}{2 \cos \beta} \Rightarrow T = \frac{13}{10} mg\end{aligned}$$

65. b. To move up with an acceleration a the monkey will push the rope downwards with a force of $40a$

$$\begin{aligned}\therefore T_{\text{man}} &= mg + 40a_{\text{max}}; 600 = 400 + 40a \\ a_{\text{max}} &= \frac{200}{40} = 5 \text{ m/s}^2\end{aligned}$$

So, the rope will break if the monkey climbs up with an acceleration of 6 m/s^2 .

66. c. From the Fig. 7.597

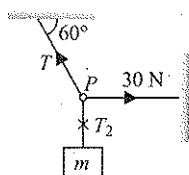


Fig. 7.597

$$\begin{aligned}T \cos 60^\circ &= 30 \text{ N} \Rightarrow T = 60 \text{ N} \\ T \sin 60^\circ &= T_2 = w \\ \Rightarrow w &= 60 \frac{\sqrt{3}}{2} = 30\sqrt{3} \text{ N}\end{aligned}$$

67. b. From Fig. 7.598

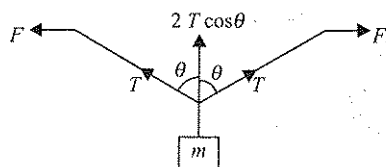


Fig. 7.598

$$\begin{aligned}T &= F \\ 2T \cos \theta &= Mg \\ \therefore F &= \frac{mg}{2 \cos \theta}\end{aligned}$$

To make the rope straight, $\theta = 90^\circ$
 F should be infinite.

68. d. Let upthrust = F

$$\begin{aligned}Mg - F &= Ma \\ \therefore F &= (Mg - Ma) \quad (ii)\end{aligned}$$

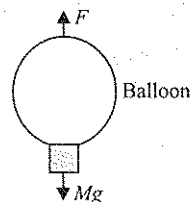


Fig. 7.599

Let m should be removed, then

$$F - (M - m)g = (M - m)a \quad (ii)$$

Solving equations (i) and (ii), we get

$$m = \frac{2Ma}{g + a}$$

69. c. By virtual work method:

$$\begin{aligned}2Yx_A &= T \times x_{BA} \\ 2x_A &= x_{BA} \\ x_{BA} &= 2 \times 5 = 10 \\ \vec{a}_B &= \vec{a}_{BA} + \vec{a}_A \\ \sqrt{10^2 + 5^2} &= \sqrt{100 + 25} = \sqrt{125}\end{aligned}$$

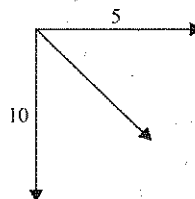


Fig. 7.600

70. c. From Fig. 7.601 $T_2 \cos \theta = mg$

$$T_2 \sin \theta = mg$$

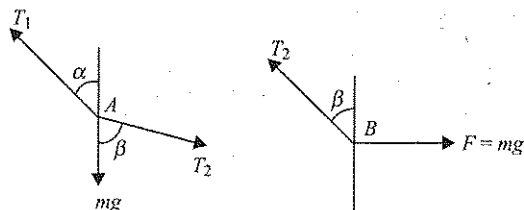


Fig. 7.601

$$\begin{aligned}T_1 \sin \alpha &= mg\sqrt{2} \sin 45^\circ \\ T_1 \sin \alpha &= mg \Rightarrow \frac{\cos \alpha}{\sin \alpha} = \frac{2mg}{mg} \\ \tan \alpha &= \frac{1}{2} \Rightarrow \cos \alpha = \frac{2}{\sqrt{5}} \\ T_1 \frac{2}{\sqrt{5}} &= 2mg \Rightarrow T_1 = mg\sqrt{5}\end{aligned}$$

$$\frac{T_1}{T_2} = \frac{\sqrt{5}}{\sqrt{2}} \Rightarrow \sqrt{2} T_1 = \sqrt{5} T_2$$

71. a. For vertical equilibrium of the block (Fig. 7.602)

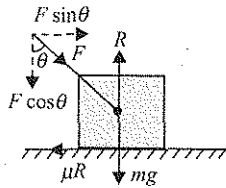


Fig. 7.602

$$R = F \cos \theta + mg$$

While for horizontal:

$$F \sin \theta = \mu R$$

From equations (i) and (ii), we get

$$F \sin \theta = \mu(F \cos \theta + mg)$$

$$\text{or } F = \frac{\mu mg}{(\sin \theta - \mu \cos \theta)}$$

72. a. Force of friction, $F = \mu R = \mu mg$

$$\text{Retardation due to friction: } \frac{F}{m} = \frac{\mu mg}{m} = \mu g$$

$$\text{Given } u = 10 \text{ m/s, } s = 50 \text{ m, } v = 0, a = -\mu g$$

$$\text{Now } v^2 = u^2 + 2as$$

$$\text{or } 0^2 = 10^2 + 2(-10\mu)(50)$$

$$\text{or } \mu = 0.1$$

73. b. $R + P \sin 60^\circ = Mg$

$$R = Mg - P \sin 60^\circ$$

Frictional force

$$F = \mu R$$

$$= \mu[Mg - P \sin 60^\circ]$$

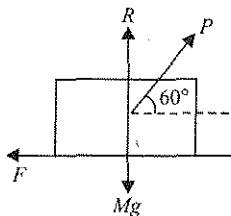


Fig. 7.603

The body just moves when

$$P \cos 60^\circ = F$$

$$\text{or } P \cos 60^\circ = \mu[Mg - P \sin 60^\circ]$$

$$\text{or } \frac{P}{2} = \frac{1}{2} \left[1 \times 10 - \frac{P\sqrt{3}}{2} \right]$$

$$\text{or } P = 5.36 \text{ N}$$

74. a. Let the total mass of the chain is M and the mass of hanging part is m_1 . Then, the mass of the part placed on the table will be $m_2 = M - m_1$.

Here, weight of the hanging part will be balanced by the frictional force acting on the upper part, i.e.,

$$m_1 g = \mu m_2 g$$

Solve to get $(m_1/M) \times 100 = 20\%$.

75. c. Frictional force:

$$F = \mu R = 0.5 \times mg = 0.5 \times 60 = 30 \text{ N}$$

$$\text{Now } F = T_1 = T_2 \cos 45^\circ$$

$$\text{or } 30 = T_2 \cos 45^\circ$$

$$\text{and } W = T_2 \sin 45^\circ$$

Solving them we get $W = 30 \text{ N}$.

- (i) 76. c. Acceleration of the suitcase till the slipping continues is

$$a = \frac{f_{\max}}{m}$$

$$(ii) \quad a = \frac{\mu mg}{m} = \mu g = 0.5 \times 10 = 5 \text{ m/s}^2$$

Slipping will continue till its velocity also becomes 3 m/s.

$$\therefore v = u + at$$

$$\text{or } 3 = 0 + 5t \text{ or } t = 0.6 \text{ s}$$

In this time, the displacement of the suitcase

$$s_1 = \frac{1}{2} at^2 = \frac{1}{2} \times 5 \times (0.6)^2 = 0.9 \text{ m}$$

and the displacement of the belt,

$$s_2 = vt = 3 \times 0.6 = 1.8 \text{ m}$$

Displacement of the suitcase with respect to the belt

$$s_1 - s_2 = 0.9 \text{ m}$$

77. a. Normal reaction on the block from the wall:

$$R = F = 10 \text{ N}$$

Weight of the block will be balanced by the frictional force

$$W = \text{Frictional force} = \mu R = 0.2 \times 10 = 2 \text{ N}$$

78. a. For upward acceleration of M_1

$$M_2 g \geq M_1 g \sin \theta + \mu M_1 g \cos \theta$$

$$\Rightarrow (M_2)_{\min} = M_1 (\sin \theta + \mu \cos \theta)$$

For downward acceleration:

$$(M_1 g \sin \theta - \mu M_1 g \cos \theta) \geq M_2 g$$

$$\Rightarrow (M_2)_{\max} = M_1 (\sin \theta - \mu \cos \theta)$$

79. a. During downward motion:

$$F = mg \sin \theta - \mu mg \cos \theta$$

During upward motion:

$$2F = mg \sin \theta + \mu mg \cos \theta$$

Solving above two equations, we get

$$\mu = (\tan \theta)/3$$

80. a. The string is under tension, hence there is limiting friction between the block and the plane (Fig. 7.604).

$$\sum F_x = 0$$

$$\Rightarrow \mu N + 50 \cos 45^\circ = 150 \sin 45^\circ \quad (i)$$

$$\sum F_y = 0$$

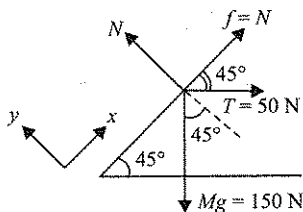


Fig. 7.604

$$\Rightarrow N = 50 \sin 45^\circ + 150 \cos 45^\circ \quad (\text{ii})$$

Solving equations (i) and (ii), we get $\mu = 1/2$

81. c. The minimum force required to just move a body will be $f_1 = \mu_s mg$. After the motion is started, the friction will become kinetic. So the force which is responsible for the increase in the velocity of the block is

$$\begin{aligned} F &= (\mu_s - \mu_k)mg \\ &= (0.8 - 0.6) \times 4 \times 10 = 8 \text{ N} \end{aligned}$$

So $a = \frac{F}{m} = \frac{8}{4} = 2 \text{ m/s}^2$

82. a. If the plane makes an angle θ with horizontal, then $\tan \theta = 8/15$. If R is the normal reaction

$$R = 170g \cos \theta = 170 \times 10 \times \left(\frac{15}{17}\right) = 1500 \text{ N}$$

$$\text{Force of friction on A} = 1500 \times 0.2 = 300 \text{ N}$$

$$\text{Force of friction on B} = 1500 \times 0.4 = 600 \text{ N}$$

Considering the two blocks as a system, the net force parallel to the plane is

$$= 2 \times 170g \sin \theta - 300 - 600$$

$$= 1600 - 900 = 700 \text{ N}$$

$$\therefore \text{acceleration} = \frac{700}{340} = \frac{35}{17} \text{ m/s}^2$$

Consider the motion of A alone.

$$170g \sin \theta - 300 - P = 170 \times \frac{35}{17}$$

(where P is pull on the bar).

$$P = 500 - 350 = 150 \text{ N}$$

83. a. For first half, acceleration = $g \sin \phi$
 $\therefore$ velocity after travelling half distance:
 $v^2 = 2(g \sin \phi)l$ (i)

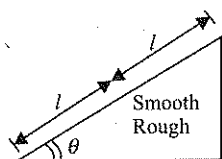


Fig. 7.605

For second half, acceleration = $g(\sin \phi - \mu_k \cos \phi)$

$$\text{So } 0^2 = v^2 + 2g(\sin \phi - \mu_k \cos \phi)l \quad (\text{ii})$$

Solving equations (i) and (ii), we get $\mu_k = 2 \tan \phi$.

84. d. Here, the force applied should be such that frictional force acting on the upper block of m should not be more than the limiting friction ($= \mu_1 mg$). Let the system moves with ac-

celeration a . Then for whole system:

$$F - \mu_2(M + m)g = (M + m)a \quad (\text{i})$$

For block of mass m :

$$f_1 = ma \text{ or } \mu_1 mg = ma$$

$$\text{or } a = \mu_1 g \quad (\text{ii})$$

From equations (i) and (ii), we get

$$F = (M + m)g(\mu_1 + \mu_2)$$

85. c. Maximum frictional force on block B

$$= \mu m_B g = 0.4 \times 3 \times 10 = 12 \text{ N}$$

$$\text{Hence, maximum acceleration} = \frac{12}{3} = 4 \text{ ms}^{-2}$$

Hence, maximum force

$$F = (m_A + m_B)a = (6 + 3) \times 4 = 36 \text{ N}$$

Aliter: We can also apply the formula discussed in the previous problem by putting $\mu_2 = 0$ and $\mu_1 = 0.4$.

86. b. For the equilibrium of block of mass M_1 :

Frictional force, f = tension in the string, T

$$\text{where } T = f = \mu(m + M_1)g \quad (\text{i})$$

For the equilibrium of block of mass M_2 :

$$T = M_2 g \quad (\text{ii})$$

From equations (i) and (ii), we get

$$\mu(m + M_1)g = M_2 g$$

$$m = \frac{M_2}{\mu} - M_1$$

87. a. Maximum frictional force between the blocks is

$$f_{\max} = \mu mg$$

So maximum acceleration

$$= a_{\max} = f_{\max}/m = \mu g$$

88. b. Horizontal acceleration of the system is

$$a = \frac{F}{2m + m + 2m} = \frac{F}{5m}$$

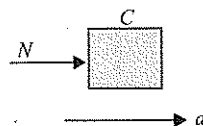


Fig. 7.606

Let N be the normal reaction between B and C. Free body diagram of C (Fig. 7.606) gives

$$N = 2ma = \frac{2}{5}F$$

Now B will not slide downward if $\mu N \geq m_B g$

$$\text{Or } \mu \left(\frac{2}{5}F\right) \geq mg \text{ or } F \geq \frac{5}{2\mu}mg$$

$$\text{or } F_{\min} = \frac{5}{2\mu}mg$$

89. a. $f_l = \mu Mg$, $f = F = F_0 t$

Motion will start when $f = f_l$

$$\Rightarrow F_0 T = \mu Mg \Rightarrow T = \frac{\mu Mg}{F_0}$$

90. d. Maximum friction that can be obtained between A and B is $f_1 = \mu m_A g = (0.3)(100)(10) = 300 \text{ N}$ and maximum friction between B and ground is $f_2 = \mu(m_A + m_B)g = (0.3)(100 + 140)(10) = 720 \text{ N}$ Drawing free body diagrams of A, B, and C in limiting case

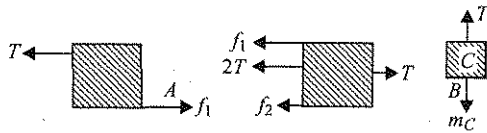


Fig. 7.607

Equilibrium of A gives

$$T_1 = f_1 = 300 \text{ N} \quad (1)$$

Equilibrium of B gives

$$2T_1 + f_1 + f_2 = T_2$$

$$\text{or} \quad T_2 = 2(300) + 300 + 720 = 1620 \text{ N} \quad (2)$$

and equilibrium of C gives $m_C g = T_2$

$$\text{or} \quad 10m_C = 1620$$

$$\text{or} \quad m_C = 162 \text{ kg}$$

91. d. If the blocks move together,

$$a = \frac{F}{m_A + m_B} = \frac{10}{6} = \frac{5}{3} \text{ ms}^{-2}$$

f_B (frictional force on B)

$$= \frac{m_B F}{m_A + m_B} = \frac{20}{3} \text{ N}$$

$$f_{B \max} = \mu m_A g = 0.4 \times 2 \times 10 = 8 \text{ N}$$

As $f_{B \max} > f_B$, Hence the blocks will not be separated

92. c. Minimum effort is required by pulling a block at the angle of friction.

93. d. In the free body diagram of B [Fig. 7.608(a)]

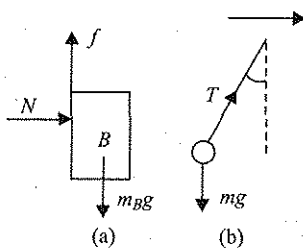


Fig. 7.608

$$N = m_B a$$

$$f = m_B g$$

$$\mu N = m_B g$$

From equations (i) and (ii), $a = \frac{g}{\mu} = 20 \text{ m/s}^2$

In the free body diagram of the bob [Fig. 7.608(b)]

$$T \sin \theta = ma$$

$$T \cos \theta = mg$$

From equations (iii) and (iv), we get

$$\tan \theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}(2).$$

94. c. In the free body diagram of m [Fig. 7.609(a)]

$$T = mg \quad (i)$$

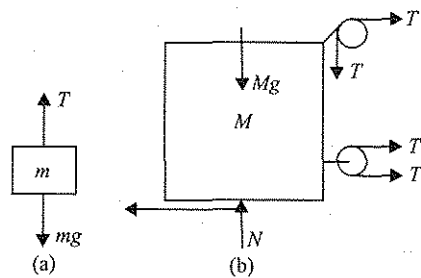


Fig. 7.609

In the free body diagram of M [Fig. 7.609(b)]:

$$\mu_2 N = 3T \quad (ii)$$

$$\text{and} \quad T + Mg = N \quad (iii)$$

$$\text{From equation (iii)} \quad N = (m + M)g$$

$$\text{From equation (ii)} \quad \mu_2(m + M)g = 3mg$$

$$m = \frac{\mu_2 M}{(3 - \mu_2)} = \frac{1/3 \times 8}{(3 - 1/3)} = 1 \text{ kg}$$

95. c. As shown in the free body diagram of 1 kg and 2 kg blocks (Fig. 7.610)

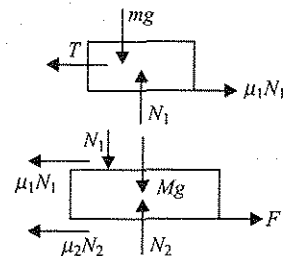


Fig. 7.610

Minimum force required to pull block M

$$F = \mu_1 N_1 + \mu_2 N_2 = \mu_1 mg + \mu_2 (Mg + N_1)$$

$$= \mu_1 mg + \mu_2 (Mg + mg)$$

$$= 0.1 \times 10 + 0.2(20 + 10) = 7 \text{ N}$$

$$96. \text{ b. } t_A = \sqrt{\frac{2s}{a_A}} \Rightarrow t_D = \sqrt{\frac{2s}{a_D}}$$

$$t_A = \frac{1}{2} t_D$$

$$\sqrt{\frac{2s}{g \sin \theta + \mu g \cos \theta}} = \frac{1}{2} \sqrt{\frac{2s}{g \sin \theta - \mu g \cos \theta}}$$

$$4g \sin \theta - 4\mu g \cos \theta = g \sin \theta + \mu g \cos \theta$$

$$3g \sin \theta = 5\mu g \cos \theta$$

$$\mu = \frac{3}{5} \tan \theta = \frac{3}{5}$$

97. c. $N = Mg - F \sin \phi$

From Fig. 7.611

$$a = \frac{F \cos \phi - \mu (Mg - F \sin \phi)}{M}$$

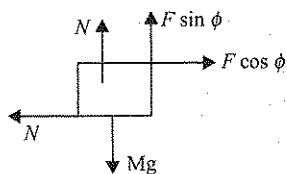


Fig. 7.611

98. c. From $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$, $t = \sqrt{\frac{2s}{a}}$

For smooth plane $a = g \sin \theta$

For rough plane, $a' = g(\sin \theta - \mu \cos \theta)$

$$\therefore t' = \sqrt{\frac{2s}{g(\sin \theta - \mu \cos \theta)}} = nt = n\sqrt{\frac{2s}{g \sin \theta}}$$

$$\therefore n^2 g (\sin \theta - \mu \cos \theta) = g \sin \theta$$

When $\theta = 45^\circ$, $\sin \theta = \cos \theta = 1/\sqrt{2}$

$$\text{Solving, we get } \mu = \left(1 - \frac{1}{n^2}\right)$$

99. b. Retardation of train $= 20/4 = 5 \text{ m/s}^2$.

It acts in the backward direction. Fictitious force on suitcase $= 5 \text{ m Newton}$, where m is the mass of suitcase.

It acts in the forward direction. Due to this force, the suitcase has a tendency to slide forward. If suitcase is not to slide, then $5 \text{ m} = \text{force } f \text{ of friction}$

$$\text{or } 5m = \mu mg \text{ or } \mu = \frac{5}{10} = 0.5$$

100. b. For a frictionless surface,

$$a = g \sin 45^\circ = g/\sqrt{2}$$

$$\therefore l = \frac{1}{2} \times \frac{g}{\sqrt{2}} \times t_1^2 \quad (1)$$

In the presence of friction,

$$a = \frac{g}{\sqrt{2}} - \frac{\mu g}{\sqrt{2}} = \frac{g}{\sqrt{2}} (1 - \mu)$$

$$\therefore l = \frac{1}{2} \times \left[\frac{g}{\sqrt{2}} (1 - \mu) \right] \times t_2^2 \quad (2)$$

Dividing eq. (2) by eq. (1), we get

$$1 = (1 - \mu) \frac{t_1^2}{t_2^2} \therefore \mu = 3/4$$

101. c. $N_1 = mg \cos \theta$ and $f_1 = \mu mg \cos \theta$

From Fig. 7.612

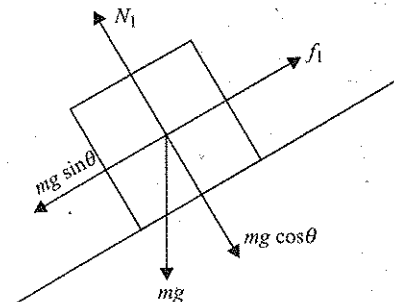


Fig. 7.612

$$N_2 = Mg \cos \theta \text{ and } f_2 = \mu Mg \cos \theta$$

From Fig. 7.613

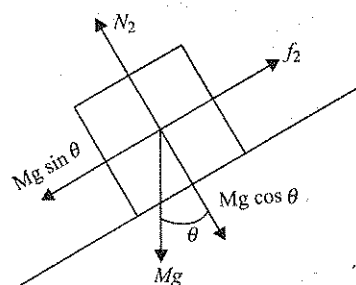


Fig. 7.613

Equations of motion are

$$T - f_1 + mg \sin \theta = ma \quad (1)$$

$$Mg \sin \theta - T - f_2 = Ma \quad (2)$$

Solving equations (1) and (2), we get $T = 0$
options (a), (b), and (d) are wrong.

102. a. Mass per unit length of chain $= \frac{M}{6}$

$$\text{Mass of the suspended part} = \frac{M}{6} y$$

For equilibrium of chain, the weight of suspended part must be balanced by force of friction on the portion on the table.

$$\mu \frac{M(6-y)g}{6} = \frac{M}{6} y g$$

$$\frac{1}{2}(6-y) = y \Rightarrow (6-y) = 2y$$

$$3y = 6 \Rightarrow y = 2 \text{ m}$$

103. b. From $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$, $t = \sqrt{\frac{2s}{a}}$

For smooth plane, $a = g \sin \theta$

For rough plane, $a' = g(\sin \theta - \mu \cos \theta)$

$$\therefore t' = \sqrt{\frac{2s}{g(\sin \theta - \mu \cos \theta)}}$$

$$= nt = n\sqrt{\frac{2s}{g \sin \theta}}$$

$$\therefore n^2 g (\sin \theta - \mu \cos \theta) = g \sin \theta$$

When $\theta = 45^\circ$, $\sin \theta = \cos \theta = 1/\sqrt{2}$

$$\text{Solving, we get } \mu = \left(1 - \frac{1}{n^2}\right)$$

104. b. Fig. 7.614

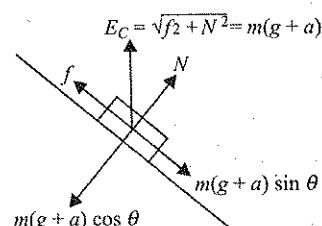


Fig. 7.614

105. b. Net force applied by block on the inclined plane is equal to the weight of the body.

106. d. During downward motion:

$$F = mg \sin \theta - \mu mg \cos \theta$$

During upward motion:

$$2F = mg \sin \theta + \mu mg \cos \theta$$

Solving above two equations, we get:

$$\mu = \frac{1}{3} \tan \theta = \frac{1}{3}$$

107. d. $f_s = \mu mg$ (frictional force on mass m)
So, maximum acceleration

$$= \frac{f_s}{m} = \frac{\mu mg}{m} = \mu g$$

108. d. Suppose due to the force R on B , both blocks A and B move together.

In this case,

$$F = (m_A + m_B)a = (2 + 5)a \text{ or } a = F/7$$

Now force on $A = ma = (2F/7)$

For no relative motion between A and B , $2F/7$ must not exceed the limiting force of friction between A and B . The limiting force of friction between A and B is given by

$$\mu m_{Ag} = 0.8 \times 2 \times g$$

$$\frac{2F}{7} = 0.8 \times 2 \times g \text{ or } F = 56 \text{ N.}$$

109. b. The frictional force on the block A that represents the cart back is given by ma .

The upward frictional force, $F = \mu ma$

For block A to be stationary,

$$\mu ma \geq mg \text{ or } a \geq \frac{g}{\mu} \Rightarrow a_{\min} = \frac{g}{\mu} = 20 \text{ m/s}^2$$

110. b. Frictional force $= \mu R = \mu(mg + Q \cos \theta)$ and horizontal push $= P - Q \sin \theta$

For equilibrium we have,

$$\mu(mg + Q \cos \theta) = P - Q \sin \theta$$

$$\mu = \frac{P - Q \sin \theta}{mg + Q \cos \theta}$$

111. c. Given horizontal force $F = 25 \text{ N}$ and the coefficient of friction between block and wall (μ) $= 0.1$.

We know that at equilibrium horizontal force provides the normal reaction to the block against the wall. Therefore, normal reaction to the block (R) $= F = 25 \text{ N}$.

We also know that weight of the block (W) = Frictional force $= \mu R = 0.4 \times 25 = 10 \text{ N}$.

112. d. As there is no relative slipping between the block and cube, the friction force is zero.

113. d. Force $= V \frac{dm}{dt}$

114. c. $\frac{1}{K'} = \frac{1}{K} + \frac{1}{3K} + \frac{1}{9K} + \dots \infty$

$$\frac{1}{K'} = \frac{1}{K} \frac{1}{(1 - 1/3)} \\ \Rightarrow K' = \frac{2K}{3}$$

115. b. The free body diagram of m in frame of wedge,

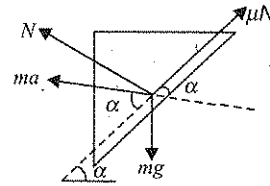


Fig. 7.615

$$N = mg \cos \alpha - ma \sin \alpha$$

$$\text{Now } f = \mu N = ma \cos \alpha + mg \sin \alpha$$

$$\mu = \frac{a \cos \alpha + g \sin \alpha}{g \cos \alpha - a \sin \alpha}$$

$$\mu = \frac{a + g \tan \alpha}{g - a \tan \alpha} = \frac{5}{12}$$

116. d. The minimum value of F required to be applied on the blocks to move is $2 \times (2 + 4) \times 10 = 12 \text{ N}$. Since the applied force is less than minimum value of force required to move the blocks together, the blocks will remain stationary.

117. b. Velocity of liquid through inclined limbs $= \frac{v}{2}$
Rate of change of momentum of the liquid

$$= \rho A v^2 + 2 \left[\rho A \left(\frac{v}{2} \right)^2 \cos 60^\circ \right] = \frac{5}{4} \rho A v^2$$

118. c. In equilibrium, there will not be any friction between the cylinder and the wedge. If θ be the required angle

Cylinder A : $mg \sin 60^\circ$

$$= kx \cos (60^\circ - \theta) (= mg \cos 30^\circ)$$

Cylinder B : $mg \sin 30^\circ$

$$= kx \cos (30^\circ + \theta) = kx \sin (60^\circ - \theta)$$

$$\therefore \cot 30^\circ = \cot (60^\circ - \theta)$$

$$\Rightarrow \theta = 30^\circ$$

119. b. $x^2 + y^2 = l^2$

On differentiating, we get

$$x V_x + y V_y = 0$$

$\Rightarrow V_y = y$ component of velocity of B which is along the rod, i.e., BA (Fig. 7.616)

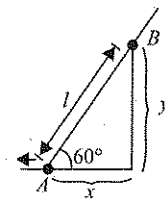


Fig. 7.616

$$|V_y| = \left| V_x \frac{y}{x} \right| = \frac{2}{\sqrt{3}} \text{ m/s}$$

$$\text{Hence, } V_B \cos 30^\circ = \frac{2}{\sqrt{3}} \text{ m/s}$$

$$\Rightarrow V_B = 1 \text{ m/s}$$

120. d. Free body diagrams (Fig. 7.617):

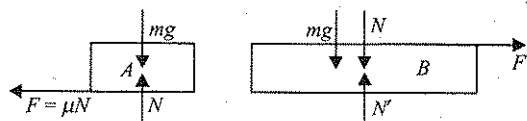


Fig. 7.617

Equations of motion:

$$a_B = \frac{F}{M} \text{ (in } +x\text{-direction)}$$

$$a_A = \frac{F}{m} \text{ (in } -x\text{-direction)}$$

Relative acceleration of A w.r.t. B:

$$\vec{a}_{A,B} = \vec{a}_A - \vec{a}_B = -\frac{F}{m} - \frac{F}{M} = -F \left(\frac{m+M}{mM} \right)$$

(along $-x$ -direction)

Initial relative velocity of A w.r.t. B

$$u_{A,B} = v_0$$

Final relative velocity of A w.r.t. B = 0

$$\text{Using } v^2 = u^2 + 2as$$

$$0 = v_0^2 - 2 \frac{F(m+M)}{mM} S$$

$$S = \frac{Mmv_0^2}{2F(m+M)}$$

121. a. $v_2 \cos \alpha + v_1 \cos \alpha = v_1$

$$\Rightarrow v_2 = v_1 \left[\frac{2 \sin^2(\alpha/2)}{\cos \alpha} \right]$$

122. b. Choosing the positive X - Y axis as shown in Fig. 7.618, the momentum of the bead at A is $\vec{p}_1 = +m\vec{v}$. The momentum of the bead at B is $\vec{p}_f = -m\vec{v}$.

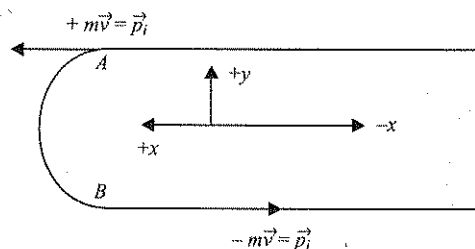


Fig. 7.618

Therefore, the magnitude of the change in momentum between A and B is

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_1 = -2m\vec{v}$$

i.e. $\Delta p = 2mv$ along positive X -axis.

The time interval taken by the bead to travel from A to B is

$$\Delta t = \frac{\pi d/2}{v} = \frac{\pi d}{2v}$$

Therefore, the average force exerted by the bead on the wire is

$$F_{av} = \frac{\Delta p}{\Delta t} = \left(2mv / \frac{\pi d}{2v} \right) = \frac{4mv^2}{\pi d}$$

123. b. The free body diagram of 10 kg

$$N' = 10 \text{ kg}$$

$\therefore$ block 10 kg will slip,

$$\mu N' = 0.3 \times 10 \times 10 = 30 \text{ N}$$

Friction = 30 N

124. c. Equation of motion for A (Fig. 7.619)

$$Kx = ma \Rightarrow a = \frac{kx}{m}$$

For B, $F - T = ma'$

$$\Rightarrow a' = \frac{F - kx}{m}$$

$$\Rightarrow \text{The relative acceleration} = a_r = |a' - a| = \frac{F - 2kx}{m}$$

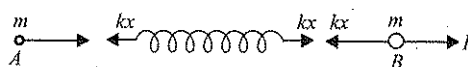


Fig. 7.619

125. b. As the springs are fixed to the horizontal and have the same natural length. Hence, if one spring is compressed, the other will be expanded. Hence, the compression will be negative

The free body diagram of m_2 [Fig. 7.620(a)]

$$T + F_2 = 80 \text{ N and } F_2 = 70 \times 0.5 = 35 \text{ N}$$

$$T = 80 - 35 = 45 \text{ N}$$

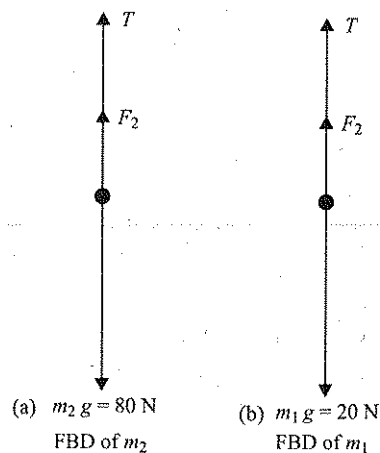


Fig. 7.620

FBD of m_1 Fig. 7.620

$$T + F_1 + mg$$

$$\text{or } 45 = F_1 + 20$$

$$F_1 = 25 \text{ N}$$

$$\therefore X_1 = \frac{25}{k_1} = \frac{25}{50} = 0.5 \text{ m}$$

$\therefore$ compression in first spring = -0.5 m

126. b. Equation of motion for M :

Since M is stationary

$$T - Mg = 0$$

$$\Rightarrow T = Mg$$

Since the boy moves up with an acceleration a Fig. 7.621.

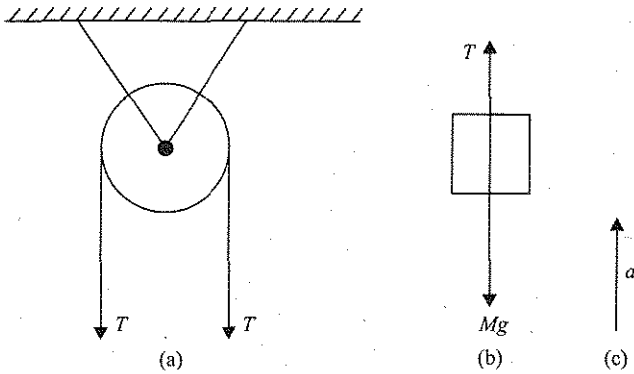


Fig. 7.621

$$\begin{aligned}
 T - mg &= ma \\
 \Rightarrow T &= m(g + a) \\
 \text{Equating eqs. (i) and (ii), we obtain} \\
 Mg &= m(g + a) \\
 \Rightarrow a &= \left(\frac{M}{m - 1} \right) g, \text{ the block } M \text{ can be lifted}
 \end{aligned}$$

127. d. Figure 7.622

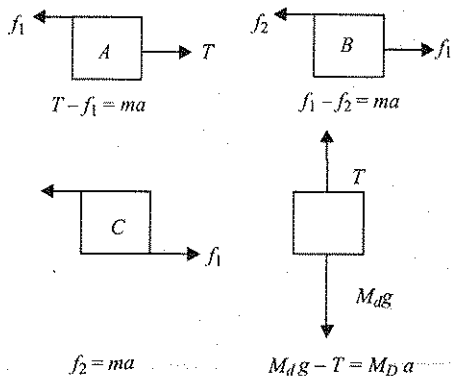


Fig. 7.622

$$\begin{aligned}
 \text{Solving we get,} \\
 \Rightarrow f_1 &= 2ma; M_D g = (M_D + 3m)a \\
 \text{and } f_1 &= \mu N = \mu(2mg) \\
 \Rightarrow M_D g &= (M_D + 3m)\mu g
 \end{aligned}$$

$$M_D = \frac{3m\mu}{1 - \mu}$$

$$128. \text{ c. } a_1 = \frac{F - f_1}{m_1} = \frac{F - \mu m_1 g}{m_1} = 10 \text{ m/s}^2$$

$$a_2 = \frac{-F + \mu m_2 g}{m_2} = -1 \text{ m/s}^2$$

$$\therefore s = \frac{1}{2} a_{\text{real}} t^2 = \frac{1}{2} [10 - (-1)] t^2$$

$$\Rightarrow t = 2 \text{ s}$$

129. d. For B, (see Fig. 7.623)

$$mg - 2T = ma$$

For A,

$$T = \frac{mg}{2} = 2ma$$

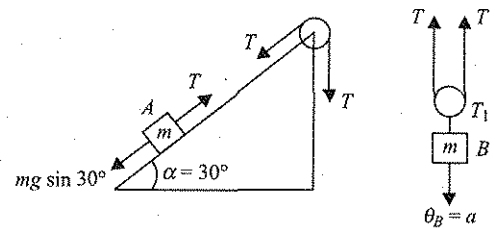
 Solving $a = 0$


Fig. 7.623

$$\begin{aligned}
 130. \text{ c. } f_{\text{net}} &= 2 \sin \omega t - f \\
 f_{\text{max}} &= 0.25 \times 10 \times 1 = 2.5 \text{ N}
 \end{aligned}$$

As maximum value of the force applied on the block is less than f_{max} , hence, the block will not move.

131. d. As there is no relative slipping between the block and cube, the friction force is zero.

$$132. \text{ d. Force} = V \frac{dm}{dt}$$

133. b.

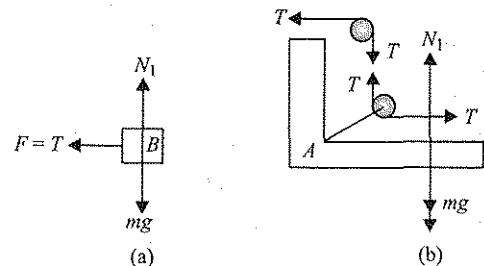


Fig. 7.624

134. a. We draw axes for each block along the incline and the normal to the incline. The component of the acceleration for each block are as shown in Figs. 7.625 and 7.626, where a is acceleration of wedge.

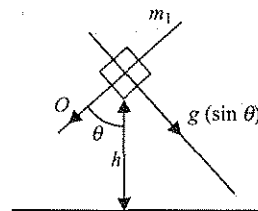


Fig. 7.625

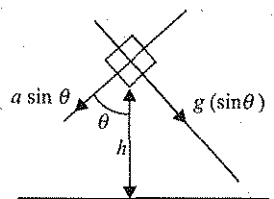


Fig. 7.626

It is obvious that vertical component of acceleration is larger for block in Fig. 7.626.

$$\therefore T_1 > T_2$$

135. b. The free body diagrams of two large blocks are given in Fig. 7.627 and Fig. 7.628.

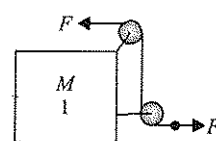


Fig. 7.627

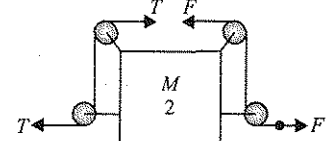


Fig. 7.628

(i)

(ii)

From FBD it is obvious that the net force on each block is zero in the horizontal direction.

$$\therefore a_1 = a_2 = 0$$

136. a. Since, $h = \frac{1}{2}at^2 \Rightarrow a$ should be same in both the cases, because h and t are same in both the cases as given.

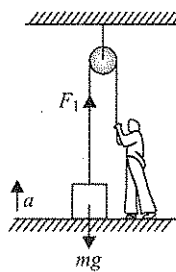


Fig. 7.629

In Fig. 7.629 $F_1 - mg = ma$

$$\Rightarrow F_1 = mg + ma$$

In Fig. 7.630 $2F_2 - mg = ma$

$$\Rightarrow F_2 = \frac{mg + ma}{2}$$

$$\therefore F_1 > F_2$$

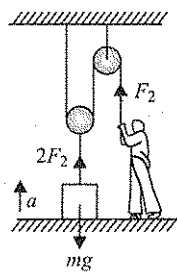


Fig. 7.630

137. a. For a chain to move with a constant speed P needs to be equal to the frictional force on the chain. As the length of the chain on the rough surface increases, the friction force $f_k = \mu_k N$ also increases.

138. b. $1.8t - \mu_k 15 = 1.5(1.2t - 2.4)$

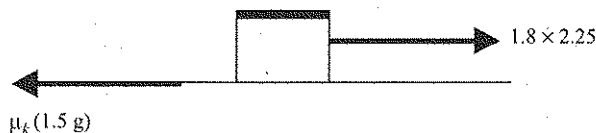


Fig. 7.631

For $T = 2.85$ s

$$\mu_k = 0.24$$

139. a. $\tan \theta = v^2/Rg \Rightarrow \frac{h}{b} = v^2/Rg$

$$\Rightarrow h = \frac{v^2 b}{Rg}$$

140. a. $V_{\max} = \sqrt{\frac{Rg[\tan \theta + \mu]}{1 - \mu \tan \theta}}$

141. b. $K = 10^2 \text{ N/cm} = 10^4 \text{ N/m}$. Let the ball moves a distance x away from the centre as shown in Fig. 7.632.

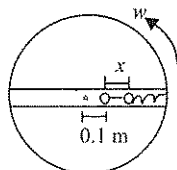


Fig. 7.632

$$kx = m\omega^2(0.1 + x)$$

$$\Rightarrow 10^4 x = \frac{90}{1000} \times (10^2)^2 \times (0.1 + x)$$

Solve to get: $x \approx 10^{-2} \text{ m}$

142. b. $f_l = \mu mg$, friction will provide the necessary centripetal force (Fig. 7.633). $f = m\omega^2 r$

$$f \leq f_l \Rightarrow m\omega^2 r \leq \mu mg$$

$$\Rightarrow \mu \geq \frac{\omega^2 r}{g} = \frac{2^2 \times 50/100}{10}$$

$$\Rightarrow \mu \geq 0.2$$

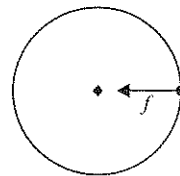


Fig. 7.633

143. c. (b) and (d) are the standard statements related to Newton's second and third law, respectively.

For option (c), pseudo force is an imaginary force, Newton's third law tells us about the material interaction forces.

For option (d), although Newton's first and second laws are valid in inertial frames only as they are concerned with the motion of body (directly or indirectly) but Newton's third law is valid in all frames.

144. d.

145. d. Due to the malfunctioning of engine, the process of rocket fusion stops and hence net force experienced by the spacecraft becomes zero. Afterwards the spacecraft continues to move with a constant speed.

146. d. Spring balance reads the tension in the string connected to its hook side. As the spring balance is light, the tension in the string on its either side is the same. Now the only thing that remains to be found is the tension in the string which could be found easily by using Newton's second law.

147. d. If we consider the situation shown (a) in Fig. 7.634 the FBD would be as shown in Fig. 7.634(b).

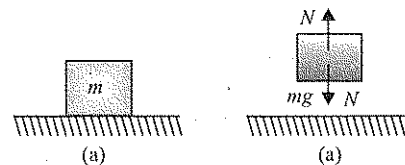


Fig. 7.634

In this situation, the option (b) seems to be correct, but in other case like if this system is in a life moving with some acceleration, then $N \neq mg$. So, (d) is the correct option.

148. a. Let us first assume that the 4 kg block is moving down, then different forces acting on the two blocks would be like as shown in the Fig. 7.635. [Normal to the incline forces are not shown in the Fig. 7.635.]

To have the motion, the frictional force f should be equal to the limiting value.

$$\text{i.e., } f_L = \mu_s mg \cos 37$$

$$= 0.27 \times 10 \text{ g} \times \frac{4}{5} = 2.16 \text{ g}$$

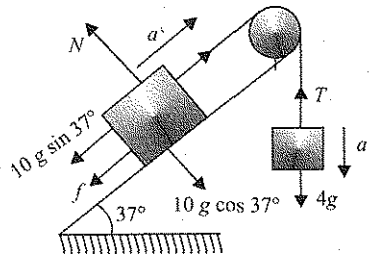


Fig. 7.635

Here, the 4 kg block is not able to pull the 10 kg block up the incline as $4g < 10g \sin 37^\circ + f_L$, so the system won't move in the direction that we assumed. So, if there is a chance of motion of system it can only move down the incline, and the system will move only if the net pulling force down the incline is greater than zero. For down the incline motion, the FBD is as shown in the Fig. 7.636.

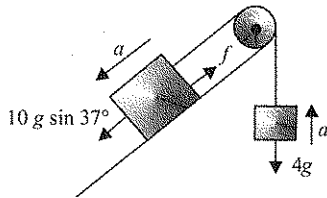


Fig. 7.636

For a to be non-zero, i.e., +ve

$$10g \sin 37^\circ > f_L + 4g$$

which is not, so the system is neither moving down the incline, nor up the incline and so the system remains at rest.

149. c. Let the weight of each block be W (Fig. 7.637).

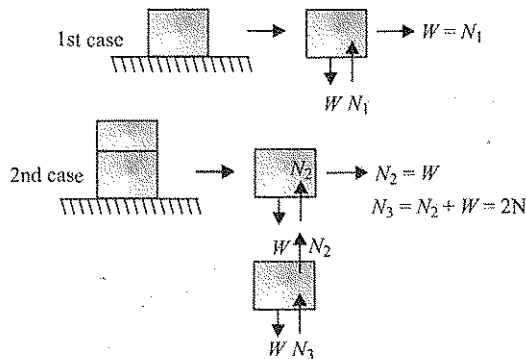


Fig. 7.637

$$\text{So, } N_1 = N_2 = \frac{N_3}{2}$$

150. c. Here the frictional force would be responsible to cause the acceleration of truck. Here the maximum frictional force can be $f = \mu \times \frac{Mg}{2}$ where $M \rightarrow$ Mass of entire truck.

This is the net force acting on tyre, so $Ma = \frac{\mu Mg}{2}$

$$\Rightarrow a = \frac{0.6 \times 10}{2} = 3 \text{ m/s}^2$$

151. c. As the sand particles are sliding down the slope of the hill gets reduced. The sand particles stop coming down when

component of gravity force along the hill is balanced by limiting the frictional force.

$$mg \sin \theta = \mu_s mg \cos \theta$$

$\Rightarrow \theta = \tan^{-1}(\mu_s) \approx 37^\circ$ where θ is the new slope angle of hill.

152. a. The FBD of the block is shown in the Fig. 7.638
 $N = 80 \cos 37^\circ = 64 \text{ N}$

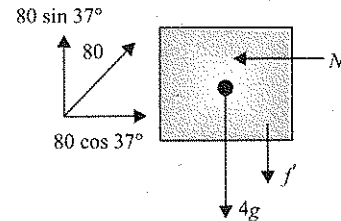


Fig. 7.638

So, $f_L = 0.2 \times 64 = 32 \text{ N}$

As $4g < 80 \sin 37^\circ$, so the frictional force will act downwards.

Net applied force in upward direction (excluding friction force) is

$$80 \sin 37^\circ - 40 = 48 - 40 = 8 \text{ N}$$

As F_{applied} in vertical direction is $< f_L$ so block won't move in vertical direction and value of static friction force is, $f = 8 \text{ N}$.

153. d. The FBD of block from the lift frame is shown in Fig. 7.639. From the given data, as $m(g + a_0) \sin \theta > 2ma_0 \cos \theta$.

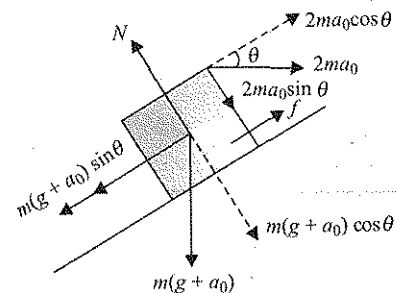


Fig. 7.639

So, the friction force acts upwards.

$$f = m(g + a_0) \sin \theta - 2ma_0 \cos \theta$$

$$= \frac{9g}{10} - \frac{4mg}{5} = \frac{mg}{10}$$

$$N = m(g + a_0) \cos \theta - 2ma_0 \sin \theta = \frac{9mg}{5}$$

As $f_L = \mu_s N = \frac{18mg}{50} = \frac{9mg}{25} > f$
 so the static friction.

Reaction force,

$$R = \sqrt{f^2 + N^2} = \frac{mg}{5} \sqrt{\frac{1}{4} + 9^2} = \frac{mg\sqrt{13}}{2}$$

154. c. As the block is in equilibrium, a gravity free hall, its FBD would be as shown in Fig. 7.640.



Fig. 7.640

As the block is at rest, the friction is static in nature and its value is equal to the applied force (F), i.e., $f = F$.

As gravity is not present, and no other force or component of any force is acting on the block in the vertical direction hence normal contact force between the table and the block would be zero.

$$\text{Reaction force, } R = \sqrt{f^2 + N^2} = \sqrt{F^2 + 0^2} = F$$

155. a. Free body diagram of various components are shown in Fig. 7.641.

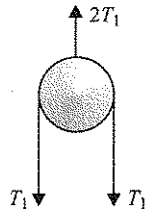


Fig. 7.641

The adjacent for horizontal equilibrium, $F = N_1$ of 15 kg block and $N_1 = N_2$ for 25 kg block as shown in Fig. 7.642.

$$\text{i.e., } N_1 = N_2 = F$$

$$f_{L1} (\text{limiting friction force}) = \mu N_1 = \mu F$$

$$f_{L2} = \mu N_2 = \mu F$$

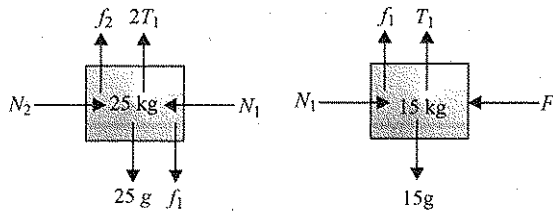


Fig. 7.642

156. d. The direction of acceleration of B is along the fixed incline, and that of A is along horizontal towards the left.

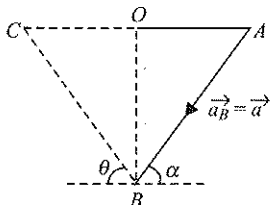


Fig. 7.643

From Fig. 7.643, acceleration of B is represented by $\vec{AB}$ while its horizontal and vertical components are shown by $\vec{AO}$ and $\vec{OB}$, respectively.

Acceleration of A is represented by $\vec{OC}$,

$$\vec{OC} = a \sin \alpha \cot \theta$$

157. b. If the wedge moves leftward by x , then the block moves down the wedge by $4x$, i.e., w.r.t. wedge the block comes down by $4x$.

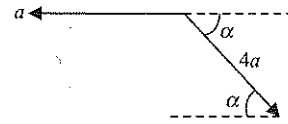


Fig. 7.644

So the acceleration of block w.r.t. wedge = $4a$ along the incline plane of wedge (Fig. 7.644).

Acceleration of wedge with respect to the ground is a , along left. So acceleration of block with respect to the ground is vector sum of the two vectors shown in the figure.

$$\begin{aligned} \text{i.e., } |\vec{a}_{BG}| &= \sqrt{a^2 + (4a)^2 + 2 \times a \times 4a \times \cos(\pi - \alpha)} \\ &= (\sqrt{17 - 8 \cos \alpha}) a \text{ m/s}^2 \end{aligned}$$

158. d. Using the constraint theory (Fig. 7.645)

$$l_1 + 2l_2 + l_3 = \text{constant.}$$

$\Rightarrow$

$$v_1 + 2v_2 + v_3 = 0$$

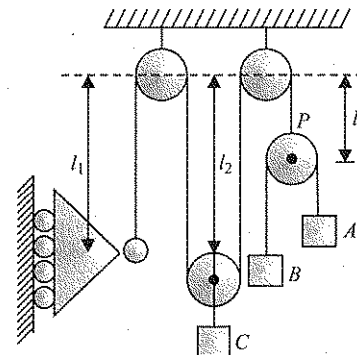


Fig. 7.645

Take downward as +ve and upward as -ve.

$$\text{So, } +12 + 2(-4) + v_3 = 0$$

$$\begin{aligned} v_3 &= \text{velocity of pulley } P = -4 \text{ m/s} \\ &= 4 \text{ m/s in upward direction} \end{aligned}$$

$$\vec{v}_{AP} = -\vec{v}_{BP}$$

$$v_B = v_P - v_{AP} = -4 - (-3) = -7 \text{ m/s}$$

i.e., block B is moving up with a speed of 7 m/s.

159. c. As the eraser is at rest w.r.t. board, friction between two is static in nature.

For Figs. 7.646 (a) and (b), the friction force is same as that of gravity force as shown in Fig. 7.646(a).

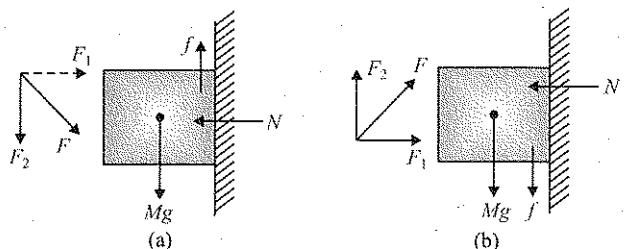


Fig. 7.646

For (c), $f = F_2 + Mg > Mg$ [shown in Fig. 7.646(b)]

For (d), $f = F_2 - Mg$ as angle by which arm is tilted is very small, so F_2 would be small.

160. b. $\vec{u} = 4\hat{i} + 2\hat{j}$, $\vec{a} = \frac{\vec{F}}{m} = \hat{i} - 4\hat{j}$

Let at any time, the coordinates are (x, y)

$$x - 2 = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x - 2 = 4t + \frac{1}{2} t^2 \text{ and } y - 3 = 2t - \frac{1}{2} 4t^2$$

$$\Rightarrow y - 3 = 2t - 2t^2$$

When $y = 3$ m, $t = 0, 1$ s

$$\text{at } t = 1 \text{ s, } x - 2 = 4 \times 1 + \frac{1}{2}$$

$$\Rightarrow x = 6.5 \text{ m}$$

161. a. $fl_1 = \mu_1 m_A g = 0.3 \times 300 = 90 \text{ N}$

$$fl_2 = \mu_2 (m_A + m_B) g = 0.2(300 + 100) = 80 \text{ N}$$

$$fl_3 = \mu_3 (m_A + m_B + m_C) g$$

$$= 0.1(300 + 100 + 200) = 60 \text{ N}$$

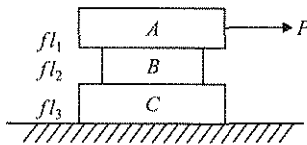


Fig. 7.647

162. a. $2T - (M + m)g = (M + m)a$ (1)

$$T - mg + N = ma$$
 (2)

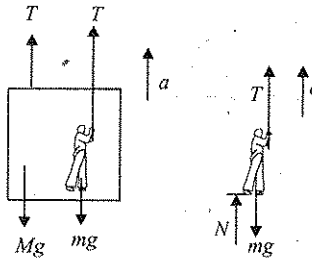


Fig. 7.648

From equations (1) and (2), we get:

$$N = \left(\frac{m - M}{2} \right) (g + a) > 0$$

As $m > M$, thus if T increases, a increases and if a increases then N increases (see Fig. 7.648).

163. c. From 0 to 2 s: at any time t , $F = 10t$

$$\Rightarrow a = F/m = 10t/m$$

$$\Rightarrow \int_0^v dv = \int_0^t \frac{10t}{m} dt \Rightarrow v = \frac{5t^2}{m}$$

$$\text{Momentum: } P = mv = 5t^2$$

$$\text{at } t = 2 \text{ s, } P = 5(2)^2 = 20 \text{ kg m/s, } v = 20/m$$

$$\text{from 2 to 4 s: } F = 40 - 10t$$

$$\int_{20/m}^v dv = \int_2^t \frac{40 - 10t}{m} dt$$

$$\Rightarrow v = \frac{1}{m} [40t - 40 - 5t^2]$$

$$P = mv = 40t - 40 - 5t^2$$

Hence the correct graph is (c).

164. a. Let m starts moving down and the extension produced in spring is x at any time as shown in Fig. 7.649. Value of x required to move the block m is:

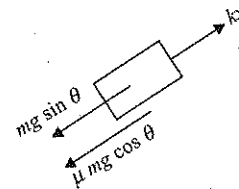


Fig. 7.649

$$Kx = \mu mg \cos \theta + mg \sin \theta$$

$$\Rightarrow kx = 0.5mg \frac{4}{5} + mg \frac{3}{5} = mg$$

For minimum M , it will stop after producing extension in the spring x .

$$Mgx = \frac{1}{2} kx^2 \Rightarrow Mg = \frac{1}{2} kx$$

$$\Rightarrow Mg = \frac{1}{2} mg \Rightarrow M = \frac{m}{2}$$

165. d. Acceleration of both m will be zero after releasing. But M will accelerate down. So the spring will get elongated for any value of M .

166. c. If we take two points 1 and 2 on a string near pulley P as shown in Fig. 7.650, then velocities of both points 1 and 2 will be same. Hence P does not rotate but only translates.

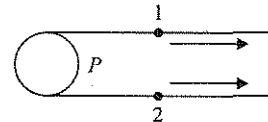


Fig. 7.650

167. c. In equilibrium (Fig. 7.651)

$$T = mg$$

$$N = 3mg$$

$$f = 2T = 2mg$$

In limiting case $f < f_{\max}$

$$2mg < \mu N$$

$$2mg \leq 3\mu mg$$

$$\mu \geq \frac{2}{3}$$

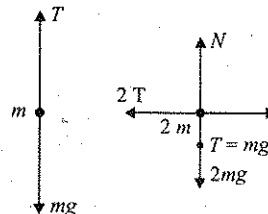


Fig. 7.651

168. c. $F - N \sin 37^\circ = 6a \Rightarrow F - \frac{3N}{5} = 6a$ (Fig. 7.652) (1)

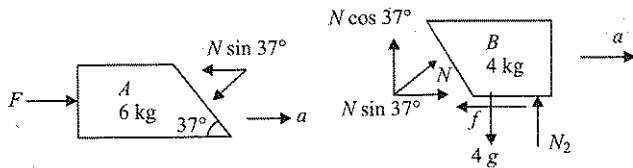


Fig. 7.652

$$N_2 = 4g - N \cos 37^\circ = 40 - \frac{4N}{5} \quad (2)$$

$$N \sin 37^\circ - f = 4a \quad (3)$$

From equations (1) and (3): $F - f = 10a$

$$\Rightarrow F - \mu N_2 = 10a \quad (4)$$

$$\Rightarrow F - \mu \left[40 - \frac{4N}{5} \right] = 10a \quad (5)$$

Put the value of N from equation (1) in equation (5) and also put the value of μ to get: $a = \frac{5F - 60}{42}$.

Now to start the motion:

$$a > 0$$

$$\Rightarrow F > 12 \text{ N}$$

So the minimum force F to just start the motion is 12 N.

Now the maximum F will be when N_2 just becomes 0.

Then from equation (2): $N = 50 \text{ N}$.

From equations (1) and (4) we get $F = 75 \text{ N}$.

(by putting $N_2 = 0$)

If we apply $F > 75 \text{ N}$, then B will start sliding up on A , but we do not want this.

169. d. $T = F/2 = 62 \text{ N}$. This tension is not sufficient to lift 7 kg block as in Fig. 7.653. Hence its acceleration $a_2 = 0$.

For 5 kg block: $T - 5g = 5a_1$

$$\Rightarrow a_1 = 2.4 \text{ m/s}^2$$

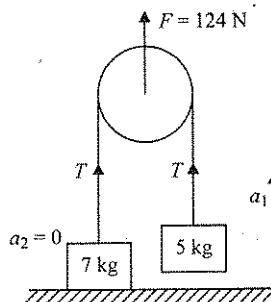


Fig. 7.653

$$a_{\text{cm}} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{5 \times 2.4 + 7 \times 0}{5 + 7} = 1 \text{ m/s}^2$$

170. a. Centre of mass will not shift in the horizontal direction. Let $\sqrt{3} \text{ m}$ moves a distance x on wedge in the downward direction. $\sqrt{2} \text{ m}$ will also move on the other side in downward direction by a distance of x . Then $m_1 x_1 = m_2 x_2$

$$\sqrt{3} m x \cos 45^\circ = \sqrt{2} m x \cos \theta$$

$$\Rightarrow \cos \theta = \sqrt{3}/2 \Rightarrow \theta = 30^\circ$$

171. a. The free body diagrams of two blocks are as shown in Fig. 7.654.

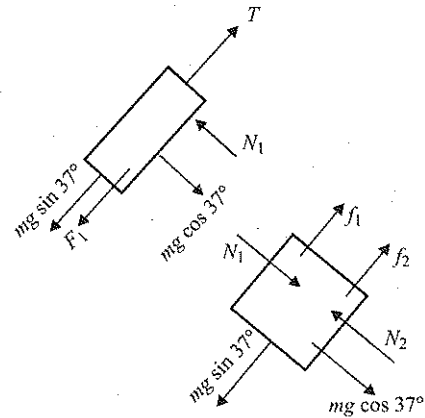


Fig. 7.654

From free body diagram of B

$$N_1 = mg \cos 37^\circ$$

$$T = mg \sin 37^\circ + f_1$$

$$f_1 = \mu N_1 \quad (\text{where } \mu \text{ is coefficient of friction})$$

From the free body diagram of A

$$N_2 = N_1 + mg \cos 37^\circ = 2mg \cos 37^\circ$$

$$mg \sin 37^\circ = f_1 + f_2$$

$$f_2 = \mu N_2 \Rightarrow \mu = \frac{1}{4}$$

172. c. The free body diagrams of two blocks are shown in Fig. 7.655. Under the assumption that both the blocks are moving together,

$$F + 2g \sin 37^\circ + 3g \sin 37^\circ - f_1 - f_2 = 5a$$

$$\text{where } f_1 = \mu \times 3g \cos 37^\circ$$

$$\text{and } f_2 = \mu \times 2g \cos 37^\circ$$

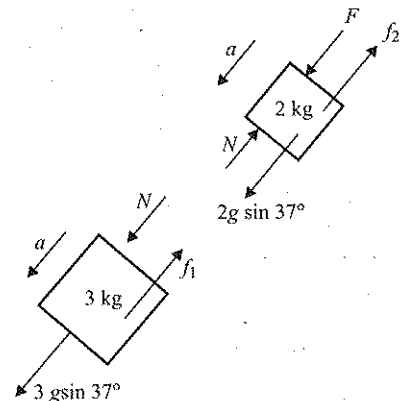


Fig. 7.655

$$\Rightarrow a = \frac{46}{5} \text{ m/s}^2$$

$$\text{For 3 kg block, } N + 3g \sin 37^\circ - f_1 = 3a$$

$$\Rightarrow N = 12 \text{ N}$$

173. c. Friction between 2 kg and 8 kg blocks is kinetic in nature (see Fig. 7.656), so $F = \mu \times 2g = 0.3 \times 2 \times 10 = 6 \text{ N}$

$$\text{For 2 kg block, } 10 - 6 = 2a_1$$

$$\text{For 8 kg block, } 6 = 8a_2$$

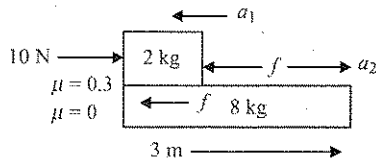


Fig. 7.656

$$\Rightarrow a_1 = 2 \text{ m/s}^2, a_2 = \frac{3}{4} \text{ m/s}^2$$

Acceleration of 2 kg block relative to 8 kg block is,

$$a_{\text{rel}} = a_1 - a_2 = \frac{5}{4} \text{ m/s}^2$$

$$\text{Using the equation of motion, } 3 = \frac{1}{2} \times \frac{5}{4} t^2$$

$$\Rightarrow t = 2.19 \text{ s}$$

174. c. At any instant, the velocity of two wedges would be of same magnitude but in the opposite directions. This can be concluded from the conservation of momentum or by symmetry.

From constraint theory, $v_M = \frac{4}{3} v_m$

From energy conservation,

$$\frac{M v_M^2}{2} \times 2 + \frac{m v_m^2}{2} - 0 = mgh$$

$$\Rightarrow v_M = \sqrt{\frac{32mgh}{32M + 9m}}$$

So the velocity with which wedges recedes away from each other is $2v_M = \sqrt{\frac{32mgh \times 4}{32M + 9m}}$

175. d. Before cutting the string the tension in string joining m_4 and ground is,

$$T = (m_1 + m_2 - m_3 - m_4)g$$

and the spring force in the spring joining m_3 and m_4 is $T + m_4g$.

As the string is cut, the spring forces doesn't change instantly, so just after cutting the string the equilibrium of m_1 , m_2 , and m_3 would be maintained but m_4 accelerates in upward direction with acceleration given by, $a = \frac{T + m_4g - m_4g}{m_4}$.

176. d. Speed: $v = \sqrt{v_x^2 + v_y^2}$

$$\text{Rate of change of speed: } \frac{dv}{dt} = \frac{2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt}}{2\sqrt{v_x^2 + v_y^2}}$$

$$= \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}} = \frac{3 \times 2 + 4 \times 1}{\sqrt{3^2 + 4^2}} = 2 \text{ m/s}^2$$

177. c. From Fig. 7.657

$$L = 2h - 2y + \sqrt{x^2 + h^2}$$

Differentiating the equation

$$\frac{dy}{dt} = \frac{x}{2\sqrt{h^2 + x^2}} \frac{dx}{dt}$$

$$\Rightarrow v_B = \frac{x v_A}{2\sqrt{h^2 + x^2}}$$

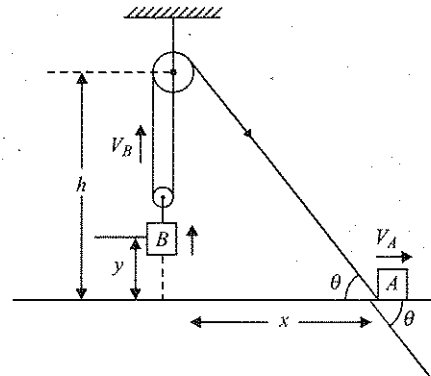


Fig. 7.657

178. d. Let at any time, their velocities are v_1 and v_2 respectively, then $v_1 = v_2 \cos \theta$

$$\text{Differentiating: } a_1 = a_2 \cos \theta - v_2 \sin \theta \frac{d\theta}{dt}$$

Hence none of them is correct.

Note: Option (a) is correct initially, because initially $v_2 = 0$

179. c. $\tan \theta = \frac{g}{a} \Rightarrow a = g \cot \theta$

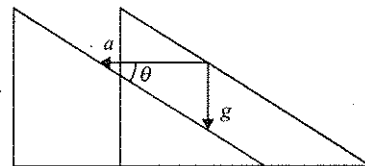


Fig. 7.658

180. c. Let a plank move up by x , then pulley 2 will move down by x as shown in the Fig. 7.659. Let end of the string C moves down by a distance y .

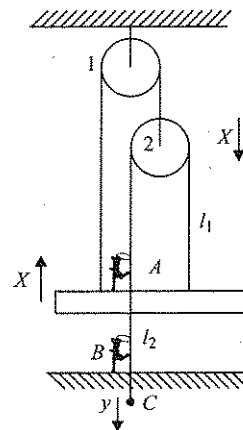


Fig. 7.659

Let Initial length of string passing over pulley 2:

$$\lambda_1 + \lambda_2$$

(1)

After displacements x and y , mentioned above, the lengths become

$$(l_1 - 2x) + (l_2 + y - x) \quad (2)$$

Equating (1) and (2): $y = 3x$

length of string that slips through A

$$y + x = 4x$$

and through B: $y = 3x$

$$\text{Required ratio} = \frac{4x}{3x} = \frac{4}{3}$$

181. b. For equilibrium

$$\int \mu dm g \cos \theta \geq \int dm g \sin \theta$$

$$\text{or } \int \mu \lambda dl g \cos \theta \geq \int \lambda dl g \sin \theta$$

$$\mu \int dl \cos \theta \geq \int dl \sin \theta$$

$$\left(\because \sin \theta = \frac{dy}{dl}, \cos \theta = \frac{dx}{dl} \right)$$

$$\text{or } \mu \int dx \geq \int dy \text{ or } \mu l \geq h$$

182. a. From Fig. 7.660

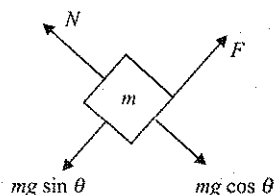


Fig. 7.660

$$N = mg \cos \theta, f = mg \sin \theta$$

Net force applied by M on m (or m on M)

$$F = \sqrt{N^2 + f^2} = \sqrt{(mg \cos \theta)^2 + (mg \sin \theta)^2} = mg$$

183. d. As the block does not slip on prism (Fig. 7.661), the combined acceleration of the prism is $a = g \sin \theta$.

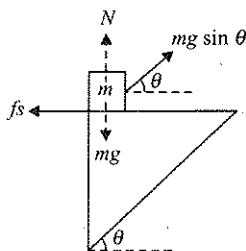


Fig. 7.661

$mg \sin \theta$ is the pseudo force on m .

$$N + mg \sin \theta \sin \theta = mg \text{ or } N = mg \cos^2 \theta$$

And for no slipping: $mg \sin \theta \cos \theta \leq \mu N$

$$mg \sin \theta \cos \theta \leq \mu mg \cos^2 \theta \text{ or } \mu \geq \tan \theta$$

184. b. Since m is in equilibrium w.r.t. the observer, so the acceleration of m should also be a_2 . So the net friction force (as there is no other horizontal force on m) acting on m should be = mass $\times$ acceleration = ma_2 .

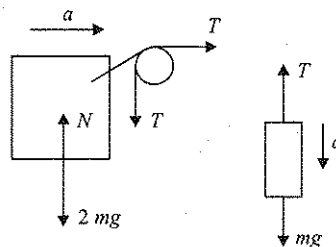


Fig. 7.662

$$mg - T = ma$$

$$a = g/3$$

$$186. \text{ d. } a_1 = \frac{(m_2 - m_1)g}{(m_1 + m_2)} \text{ and } a_2 = \frac{(m_2 - m_1)g}{m_1}$$

$$\text{Hence } \frac{a_1}{a_2} = \frac{m_1}{(m_1 + m_2)} = \frac{1}{\left(1 + \frac{m_2}{m_1}\right)}$$

$$\frac{a_1}{a_2} < 1$$

$$\text{As } T_1 = \frac{2m_1 m_2 g}{(m_1 + m_2)} \text{ and } T_2 = m_2 g$$

$$\text{Hence } \frac{T_1}{T_2} = \frac{2m_1}{(m_1 + m_2)} = \frac{2}{\left(1 + \frac{m_2}{m_1}\right)}$$

Hence, $\frac{T_1}{T_2}$ will depend upon the values of m_1 and m_2 .

$$\therefore N_2 = 2T_1, \quad N_2 = 2T_2$$

So the relation of N_1/N_2 will be same as $\frac{T_1}{T_2}$.

187. a. Figure 7.663

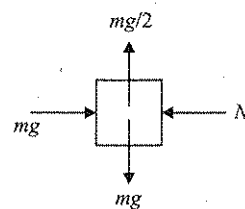


Fig. 7.663

$$\text{Driving force} = mg - \frac{mg}{2} = \frac{mg}{2} \text{ (down)}$$

$$\text{Maximum resting force} = \mu mg = \frac{mg}{2} \text{ (up)}$$

188. b. For constant acceleration if the initial velocity makes an angle with acceleration then the path will be parabolic.

189. d. From constraint, the velocity of both the block and the wedge should be same in a direction perpendicular to the inclined plane as shown in Fig. 7.664.

$$(a_A)_\perp = (a_B)_\perp, a_{AX} = 15, a_{AY} = 15$$

$$a_{AX} \cos 53^\circ - a_{AY} \cos 37^\circ = a_B \cos 53^\circ$$

$$\text{or } a_B = -5 \text{ m/s or } \vec{a}_B = -5\hat{i}$$

190. a. In this case, the spring force is initially 0
FBD of A and B (Fig. 7.665)

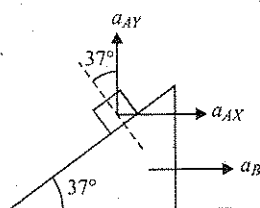


Fig. 7.664

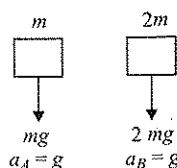


Fig. 7.665

Tension in the string and the spring will be zero just after the release.

191. c.

Sol. (Check Fig. 7.666 in the frame of the car).

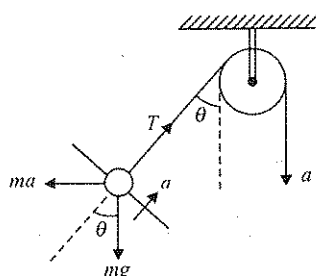


Fig. 7.666

Applying Newton's law perpendicular to string

$$mg \sin \theta = ma \cos \theta \Rightarrow \tan \theta = \frac{a}{g}$$

Applying Newton's law along string

$$T - mg \cos \theta - ma \sin \theta = ma$$

$$\Rightarrow T = m \sqrt{g^2 + a^2} + ma$$

192. b. Acceleration of the box = 10 m/s²

Inside the box forces are acting on the bob (see Fig. 7.667).

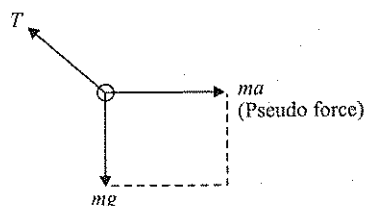


Fig. 7.667

$$T = \sqrt{(mg)^2 + (ma)^2} = 10\sqrt{2} \text{ N}$$

193. d.

Sol. Acceleration of two mass systems is

$$a = \frac{F}{2m} \text{ leftward. FBD of block A (Fig. 7.668)}$$

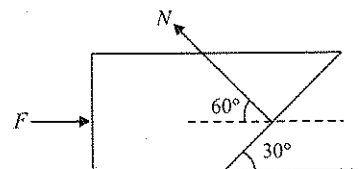


Fig. 7.668

$$N \cos 60^\circ - F = ma = \frac{mF}{2m}$$

Solving, we get $N = 3F$

194. c. For a block to be stationary $T = 800 \text{ N}$ (Fig. 7.669)

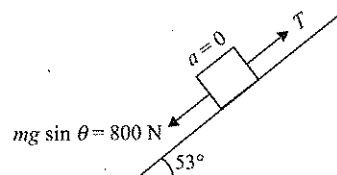


Fig. 7.669

If a man moves up by acceleration a .

$$T - mg = ma$$

$$8000 - 5000 = 50a \Rightarrow a = 6 \text{ m/s}^2$$

195. b. As in Fig. 7.670

$$l_1 + l_2 + l_3 = C$$

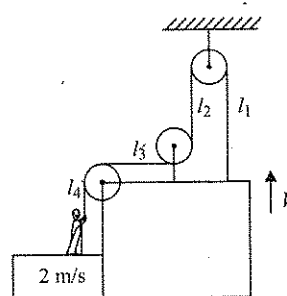


Fig. 7.670

$$\frac{dl_1}{dt} + \frac{dl_2}{dt} + \frac{dl_3}{dt} + \frac{dl_4}{dt} = 0$$

$$-v - v + 0 + v + 2 = 0$$

$$v = 2 \text{ m/s}$$

196. d. As in Fig. 7.671

$$\vec{V}_{B,l} = 4 \text{ m/s} \uparrow \quad \vec{V}_{B,l} = \vec{V}_{Bg} - \vec{V}_{lg}$$

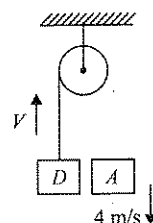


Fig. 7.671

$$4 \text{ m/s} = \vec{V}_{Bg} - 2 \text{ m/s}, \vec{V}_{Bg} = 6 \text{ m/s} \uparrow$$

197. d. Method-1

As the cylinder will remain in contact with wedge A Fig. 7.672:

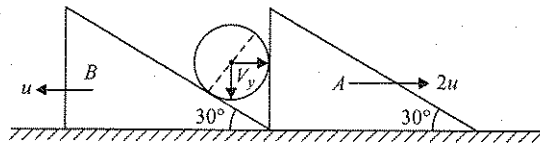


Fig. 7.672

As it also remain in contact with wedge B

$$u \sin 30^\circ = V_y \cos 30^\circ - V_x \sin 30^\circ$$

$$V_y = V_x \frac{\sin 30^\circ}{\cos 30^\circ} + \frac{u \sin 30^\circ}{\cos 30^\circ}$$

$$V_y = 3u \tan 30^\circ = \sqrt{3}u$$

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u$$

Method-2

In the frame of A (Fig. 7.673)

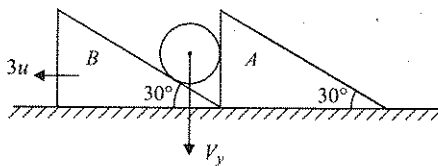


Fig. 7.673

$$3u \sin 30^\circ = V_y \cos 30^\circ$$

$$V_y = 3u \tan 30^\circ = \sqrt{3}u$$

$$\text{and } V_x = 2u \Rightarrow V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u$$

198. a. As in Fig. 7.674

$$u \cos 45^\circ = v \cos 60^\circ$$

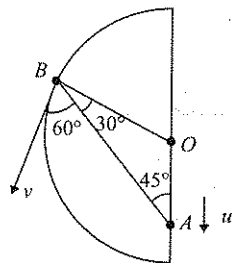


Fig. 7.674

$$\text{or } v = \sqrt{2}u$$

199. b. Free body diagram of man and plank is given below (Fig. 7.675)

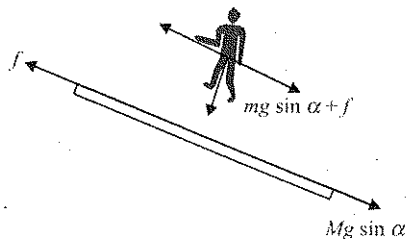


Fig. 7.675

For the plank to be at rest, applying Newton's second law to the plank along the incline will be

$$Mg \sin \alpha = f \quad (i)$$

By applying Newton's second law to the man along the incline, will be

$$Mg \sin \alpha + f = ma \quad (ii)$$

$$a = g \sin \alpha \left(1 + \frac{M}{m} \right) \text{ down the incline}$$

200. b. From length constraint on AB (Fig. 7.676)

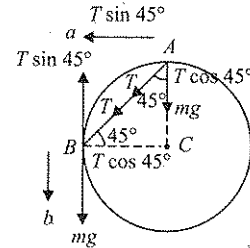


Fig. 7.676

$$A \cos 45^\circ = b \cos 45^\circ$$

$$T \sin 45^\circ = m(a)$$

$$mg - T \sin 45^\circ = mb$$

$$2ma = mga = \frac{g}{2}$$

$$\frac{T}{\sqrt{2}} = \frac{mg}{2} \quad \text{or} \quad T = \frac{mg}{\sqrt{2}}$$

201. d. As in Fig. 7.677

$$T \sin \theta = ma_0 + mg \sin \alpha$$

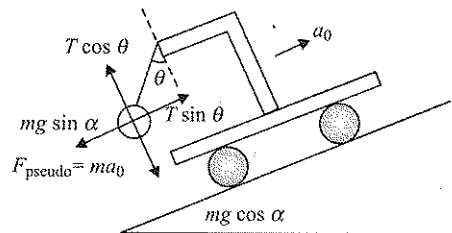


Fig. 7.677

$$T \cos \theta = mg \cos \alpha$$

$$\tan \theta = \frac{a_0 + g \sin \alpha}{g \cos \alpha}$$

202. d. As in Fig. 7.678

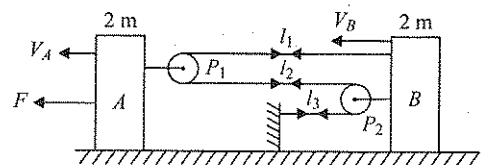


Fig. 7.678

$$l_1 + l_2 + l_3 = C$$

$$l'_1 + l'_2 + l'_3 = 0$$

$$-V_B + V_A - V_B + V_A - V_B = 0$$

$$3V_B = 2V_A \quad 3a_B = 2a_A$$

Applying Newton's law on A and B

$$F - 2T = 2ma_A$$

$$3F = (6a_A + 8a_B)m$$

$$3F = (9 + 8)a_B m \quad a_B = \frac{3F}{17m}$$

203. a. a : Acceleration of the block A downwards w.r.t ground (Fig. 7.679).

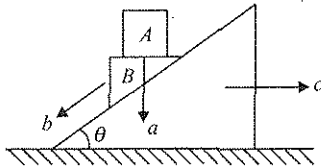


Fig. 7.679

- b : Acceleration of the block B w.r.t. inclined plane.
 c : Acceleration of the block C w.r.t. ground right side.
 $\vec{b} + \vec{c}$: acceleration of B w.r.t. ground.

Applying Newton's law on a system along the horizontal direction

$$mc + m(c - b \cos \theta) = 0 \quad (i)$$

Applying Newton's law on $(A + B)$ along the inclined plane.

$$2mg \sin \theta = m(b - c \cos \theta) + ma \sin \theta$$

$$2g \sin \theta = b - c \cos \theta + a \sin \theta \quad (ii)$$

From the wedge constraint between A and B

$$a = b \sin \theta \quad (iii)$$

From equations (i), (ii), and (iii)

$$b = \frac{4g \sin \theta}{1 + 3 \sin^2 \theta}$$

204. c. $a_{B,g} = \sqrt{b^2 + c^2 + 2bc \cos(180 - \theta)}$

$$= \sqrt{b^2 + \left(\frac{b \cos \theta}{2}\right)^2 + bb \cos \theta (-\cos \theta)}$$

$$= b \sqrt{1 + \frac{\cos^2 \theta}{4} - \cos^2 \theta} = \frac{b}{2} \sqrt{1 + 3 \sin^2 \theta}$$

$$= \frac{2g \sin \theta}{\sqrt{1 + 3 \sin^2 \theta}}$$

205. a. From equation (i) (Question 203) $c = \frac{b \cos \theta}{2}$

206. b. If the initial acceleration of M towards right is A then we can show that the acceleration of m w.r.t. M down the incline is

$$a = A(1 + \cos \theta) = \frac{3A}{2}$$

FBD of block m (w.r.t. M) (Fig. 7.680)

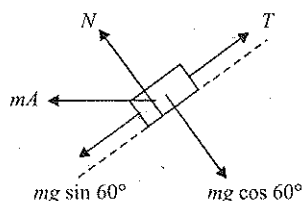


Fig. 7.680

FBD of M (Fig. 7.681)

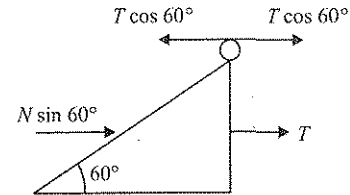


Fig. 7.681

Equation of motion:

$$\text{For } m: mg \frac{\sqrt{3}}{2} + mA \times \frac{1}{2} - T = m \frac{3}{2} A \quad (i)$$

$$N + mA \frac{\sqrt{3}}{2} = mg \frac{1}{2} \quad (ii)$$

$$\text{For } M: T + N \frac{\sqrt{3}}{2} = MA \quad (iii)$$

$$\text{From equations (i), (ii), and (iii) } A = \frac{3\sqrt{3}g}{23} \text{ m/s}^2$$

207. a. Let $\vec{a}_0$ be the acceleration of chosen non-inertial frame of reference w.r.t. some inertial frame of reference and $\vec{a}_1$ be the acceleration of the object in a non-inertial frame (Fig. 7.682).

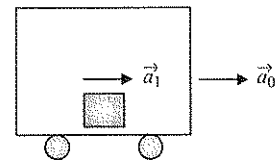


Fig. 7.682

For $\vec{a}_1$ to be non-zero, the net force acting on the object (including pseudo force) must be non-zero.

208. a. Velocity of object w.r.t. non-inertial frame is constant and hence w.r.t. some inertial frame of reference it changes, hence it is accelerating. So the net force acting on the object must be non-zero.

209. c. Net force without friction on system is 7 N in right side so first maximum friction will come on 3 kg block (Fig. 7.683).

$$\text{So } f_2 = 1 \text{ N, } f_3 = 6 \text{ N, } T = 2 \text{ N}$$

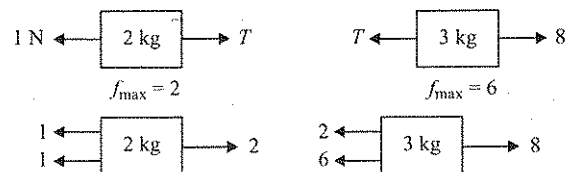


Fig. 7.683

210. b. As in Fig. 7.684 the mass of the rope: $m = 4 \times 1.5 = 6 \text{ kg}$
 Acceleration: $a = 12/6 = 2 \text{ m/s}^2$

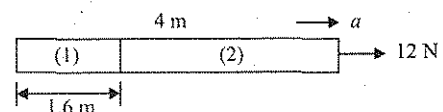


Fig. 7.684

$$\text{Mass of part 1 as in Fig. 7.685: } m_1 = 1.6 \times 1.5 = 2.4 \text{ kg}$$

$$T = m_1 a = 2.4 \times 2 = 4.8 \text{ N}$$

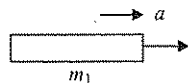


Fig. 7.685

211. b. Force Method:

From P to Q : $v^2 = 0^2 + 2a_1s_1$

where $a_1 = g \sin 30^\circ$, $s_1 = 1$ m

and $0^2 = v^2 + 2a_2s_2$

where $a_2 = g \sin 30^\circ - \mu g \cos 30^\circ$, $s_2 = 2$ m

solve to get $\mu = \sqrt{3}/2$

Work-Energy Method: (Fig. 7.686)

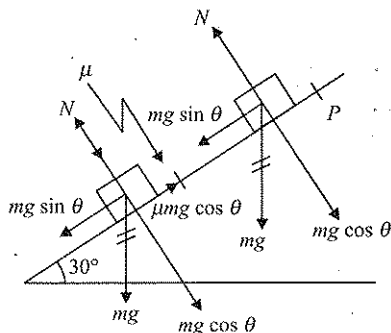


Fig. 7.686

In terms of energy considerations you can summarize the whole process as a loss in the gravitational potential energy of the block which is equal to work done against friction, or

$$(mg) \times (3 \sin 30^\circ) = (\mu mg \cos \theta) \times 2$$

$$\Rightarrow 3 \times \frac{1}{2} \Rightarrow \times \frac{\sqrt{3}}{2} \times 2 \Rightarrow \mu = \frac{\sqrt{3}}{2}$$

212. c. As in Fig. 7.687

$$\cos \alpha = \frac{W}{OA} \quad \text{or} \quad OA = \frac{W}{\cos \alpha}$$

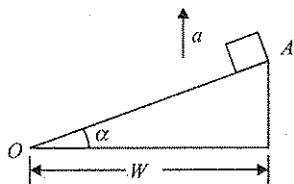


Fig. 7.687

Acceleration of body relative to the incline: $(g + a) \sin \alpha$

$$\text{So } OA = \frac{1}{2}(g + a) \sin \alpha t^2 \quad \left(\text{from } S = ut + \frac{1}{2}at^2 \right)$$

$$\text{or} \quad \frac{W}{\cos \alpha} = \frac{1}{2}(g + a) \sin \alpha t^2$$

$$t = \left[\frac{2W}{\cos \alpha (g + a) \sin \alpha} \right]^{1/2}$$

$$= \left[\frac{4W}{(2 \cos \alpha \sin \alpha)(g + a)} \right]^{1/2} = \left[\frac{4W}{(g + a) \sin 2\alpha} \right]^{1/2}$$

213. b. The force of 100 N acts on both the boats

$$250a_1 = 100 \quad \text{and} \quad 500a_2 = 100$$

$$\text{or } a_1 = 0.4 \text{ ms}^{-2} \quad \text{and} \quad a_2 = 0.2 \text{ ms}^{-2}$$

$$\text{Relative acceleration: } a = a_1 + a_2 = 0.6 \text{ ms}^{-2}$$

$$\text{Using } S = ut + \frac{1}{2}at^2, \text{ we get}$$

$$100 = (1/2) \times 0.6 \times t^2 \quad \text{or} \quad t = 18.3 \text{ s}$$

Multiple Correct Answers Type**1. a., b., c., d.**

a. As in Fig. 7.688

$$g' = g + a$$

$$= g + 3g = 4g$$

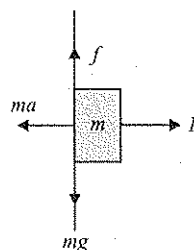


Fig. 7.688

$$mg' = 4mg = 4W$$

b. Think of Newton's third law of motion.

$$\text{c. } mg < f_{ms}$$

$$\text{or } mg < \mu_s R$$

$$\text{or } mg < \mu_s ma$$

$$\text{or } g < \mu_s a$$

$$\text{or } \mu_s a > g \text{ or } \mu_s > \frac{g}{a}$$

replace μ_s by μ

d. The jumping away of the man involves upward acceleration.

2. b., d. The horizontal forces on the man must balance, i.e., the forces exerted by the two walls on him must be equal.

The vertical forces can balance even if the forces of friction on the two walls are unequal. The torques due to the forces of friction about his centre of mass must balance. This requires friction on both the walls.

3. a., b., c. If the tendency of relative motion along the common tangent does not exist, then the component of contact force along the common tangent will be zero.

4. a., c. Using the constrained equation, $v_2 \cos \theta = v_1$.
On differentiation $a_2 \cos \theta = a_1$

5. a., d. As in Fig. 7.689

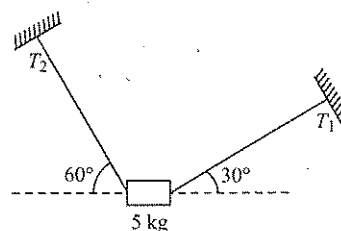


Fig. 7.689

$$T_2 \cos 60^\circ = T_1 \cos 30^\circ, \text{ and}$$

(i)

$$T_2 \sin 60^\circ + T_1 \sin 30^\circ = 5g \quad (\text{ii})$$

From equations (i) and (ii)

$$T_1 = 25 \text{ N and } T_2 = 25\sqrt{3} \text{ N}$$

6. a., c.

$$Mg - T = Ma$$

$$T = ma$$

Solving equations (i) and (ii)

$$a = \frac{Mg}{(M + m)}$$



Fig. 7.690

FBD of man (Fig. 7.690)

$$Mg - N = Ma$$

$$N = \frac{Mmg}{(M + m)}$$

7. c., d. Figure 7.691

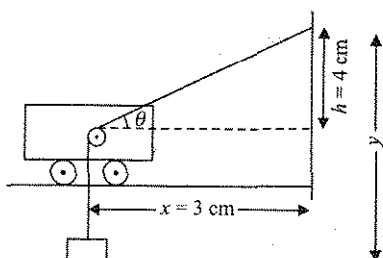


Fig. 7.691

$$(y - h) + \sqrt{x^2 + h^2} = l$$

$$\frac{dy}{dt} + \frac{x}{\sqrt{x^2 + h^2}} \frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{\sqrt{x^2 + h^2}} \frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{3}{5} v_A$$

$$|u_B| = \frac{3}{5} v_A$$

$$\frac{d^2y}{dt^2} = v_A \frac{h^2}{(x^2 + h^2)^{3/2}}$$

$$a_B = v_A \frac{16}{(5)^3}$$

$$a_B = \frac{16}{125} v_A$$

8. a., d. Figures 7.692 and 7.693

$$M'g - T = M'a$$

$$T = Ma$$

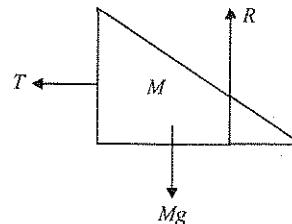


Fig. 7.692

$$M'g = a(M + M')$$

$$a = \frac{M'g}{(M + M')}$$

$$ma \sin \theta = mg \cos \theta$$

$$a = g \cot \theta$$

$$g \cot \theta = \frac{M'g}{(M + M')}$$

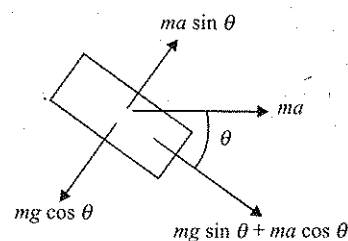


Fig. 7.693

$$\cot \theta M + \cot \theta M' = M'$$

$$M' = \frac{M \cot \theta}{(1 - \cot \theta)}$$

$$T = Ma = Mg \cot \theta$$

$$T = \frac{Mg}{\tan \theta}$$

9. a., d. Figure 7.694

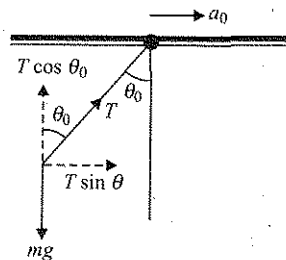


Fig. 7.694

$$T \cos \theta_0 = mg$$

$$T \sin \theta_0 = ma_0$$

Dividing equation (ii) by (i)

$$\tan \theta_0 = \frac{a}{g}$$

$$\theta_0 = 30^\circ$$

$$T = \frac{mg}{\cos 30^\circ} = \frac{2mg}{\sqrt{3}}$$

(i)

(ii)

(i)

(ii)

(i)

(ii)

10. b., d. Acceleration of M , $a = \left(\frac{F}{M}\right)$

$$l = \frac{1}{2} \frac{F}{M} t^2$$

$$t = \sqrt{\frac{2Ml}{F}}$$

11. b., c. Here $F > \mu_s mg \left(1 + \frac{m}{M}\right)$

For m

$$F - \mu_k mg = ma$$

For M

$$\mu_k mg = MA$$

$$A = 0.4 \text{ m/s}^2$$

12. a., b. For the two values of F , i.e., $F = 150 \text{ N}$ and $F = 120 \text{ N}$; the tensions in the string are $T = \frac{F}{2} = 75 \text{ N}$ and 60 N . In the first case, the accelerations of the two masses are equal and opposite, while in the second the accelerations is zero.
13. a., b., c. As the acceleration of A and B are different, it means that there is relative motion between A and B . The free body diagram of A and B can be drawn as (Fig. 7.695)

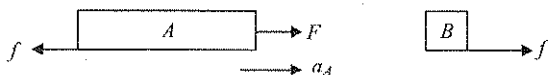


Fig. 7.695

$$\text{For } A, F - f = Ma_A = 50 \times 3$$

$$\text{For } B, f = ma_B = 20 \times 2$$

$$\Rightarrow f = 40 \text{ N}, F = 190 \text{ N}$$

14. a., b. Because mg acts downwards which makes sliding along 4 to be easiest and along 1 to be the most difficult.
15. a., b., c. From the FBD of A it is clear that friction opposes its motion. Also friction always opposes the relative motion. Since velocities of A and B are different, hence there is relative motion between them. So there is kinetic friction between the two blocks which is μm_{AB}
16. a., c., d. In the first case, m will remain at rest.

$$a_m = \frac{F}{M}$$

In the second case, both will accelerate:

$$a_m = a_M = \frac{F}{M + m}$$

$$\text{In the second case, force on } m = ma_m = \frac{mF}{M + m}$$

17. a., b., c., d. When friction between the blocks becomes zero, the relative sliding between the blocks will be stopped hence $v_A = v_B$.

Also when the friction becomes zero, only force to move the blocks are F_A and F_B

$$\text{Hence } a_A = a_B \text{ also } \frac{F_A}{m_A} = \frac{F_B}{m_B}$$

18. a., b., d.

Newton's second law is, $\vec{F} = \frac{d\vec{p}}{dt}$, which itself explains the validity of the given statements.

19. b., d. In a tug of war, the FBD of the teams are as shown in the Fig. 7.696. From the FBD, it is clear that the team wins on which horizontal force exerted by ground is more.

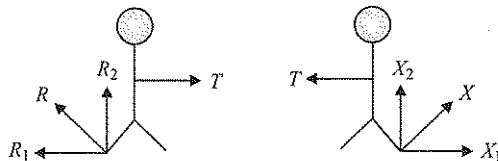


Fig. 7.696

20. a., b., d. Under the action of two forces if the body is accelerating, it means a net force is acting on body and it can never attain a constant velocity or speed. [Provided initial velocity is either zero or its direction is same as that of acceleration]. If the two forces are equal then for the present situation they can't act along the same line, but if forces are unequal then they may act along the same line.
21. a., b., c. For (i): Consider a block at rest on a rough surface and no force (horizontal) is acting on it (Fig. 7.697). Now friction force on it would be zero.

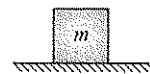


Fig. 7.697

For (ii): Consider a heavy block, under the application of small force F which is not sufficient to cause its motion, so friction force is static in nature and block doesn't move.

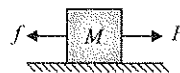


Fig. 7.698

For (iii): Refer to Concepts and Formulae.

For (iv): Friction force and normal force always act perpendicular to each other.

22. a., b., c. If the block is at rest, then the force applied has to be greater than the limiting frictional force for its motion to begin (Fig. 7.699).

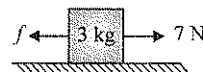


Fig. 7.699

$$f_L = \mu_s mg = 0.25 \times 3g = 7.5 \text{ N} < F_{\text{applied}}$$

So, the friction is static in nature and its value would be equal to the applied force, i.e., 7 N . If the body is initially moving, then the kinetic friction is present ($f_k = \mu_k mg$)

$= 6 \text{ N}$), acting opposite to the direction of motion. If the applied force is along direction of motion, then the situation would be as shown in Fig. 7.700.

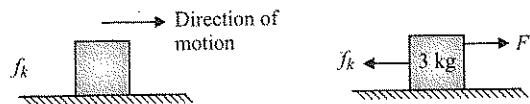


Fig. 7.700

As $F > f_k$, the block is accelerated with an acceleration of $a = \left[\frac{F - f_k}{m} \right] = \frac{1}{3} \text{ m/s}^2$ and hence its speed is continuously increasing.

If the applied force is opposite to the direction of motion then the block is under deceleration of $a = -\left[\frac{F + f_k}{m} \right] = -\frac{13}{3} \text{ m/s}^2$ and hence after some time the block stop and kinetic friction vanishes but applied force continues to act (Fig. 7.701).

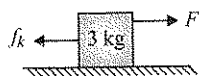


Fig. 7.701

But as $F < f_L$, the block remains at rest and the frictional force acquires the value equal to the applied force, i.e., friction is static in nature.

23. a., c. First of all draw FBD of P_3 . Let the tension in three strings be T_1 , T_2 , and T_3 , respectively, (Fig. 7.702).

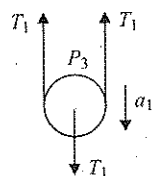


Fig. 7.702

$$2T_1 - T_1 = 0 \times a \Rightarrow T_1 = 0$$

Now draw FBD of P_4 and P_5 (Fig. 7.703)

$$2T_1 - T_2 = 0 \Rightarrow T_2 = 0$$

$$2T_2 - T_3 = 0 \Rightarrow T_2 = T_3 = 0$$

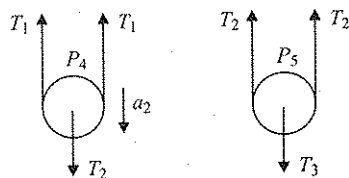


Fig. 7.703

Similarly for the acceleration draw the FBD of P_6 and P_7 and get the values of acceleration.

24. a., b., c. From the Fig. 7.704 it is clear that the object slows down and comes to rest. At this instant F is still acting in the same direction and is greater than f_L so the block starts accelerating in the opposite direction of its initial motion.
25. b., c., d. For some time, the block won't move due to the frictional force. When $F > f_L$, the motion of block starts.

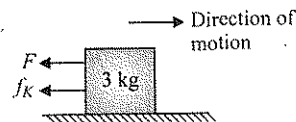


Fig. 7.704

$$a = \frac{F - f_k}{m} = \frac{kt - f_k}{m}$$

$$\frac{dv}{dt} = a = \frac{kt - f_k}{m}$$

$$v = \frac{kt^2}{2} - \frac{f_k t}{m}$$

$$\frac{ds}{dt} = v = \frac{kt^2}{2} - \frac{f_k t}{m} \Rightarrow s = \frac{kt^3}{6} - \frac{f_k t^2}{2}$$

26. a., c. Figure 7.705

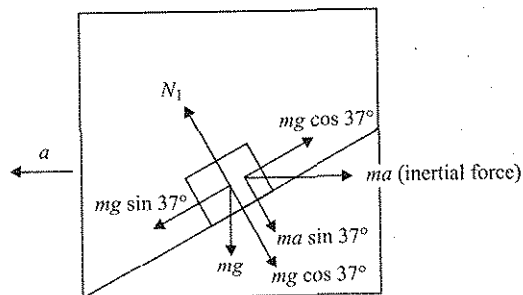


Fig. 7.705

Balancing forces perpendicular to the incline

$$N_1 = mg \cos 37^\circ + ma \sin 37^\circ$$

$$N_1 = \frac{4}{5}mg + \frac{3}{5}ma$$

and along the incline, $mg \sin 37^\circ - ma \cos 37^\circ = mb_1$

$$b_1 = \frac{3}{5}g - \frac{4}{5}a$$

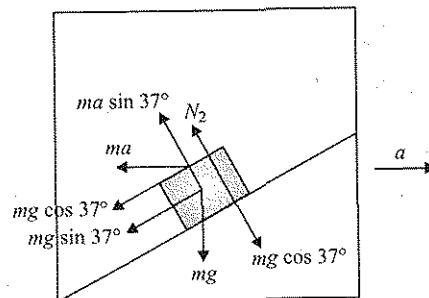


Fig. 7.706

Similarly for this case (Fig. 7.706) get $N_2 = \frac{4}{5}mg - \frac{3}{5}ma$

$$\text{and } b_2 = \frac{3}{5}g + \frac{4}{5}a$$

$$N_2 = \frac{4}{5}mg - \frac{3}{5}ma$$

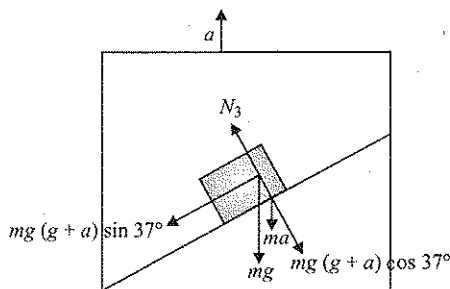


Fig. 7.707

Similarly for this case (Fig. 7.707) get

$$N_3 = \frac{4}{5}mg + \frac{4}{5}ma$$

$$b_3 = \frac{3}{5}g + \frac{3}{5}a$$

Similarly for this case (Fig. 7.708) get

$$N_4 = \frac{4}{5}mg - \frac{4}{5}ma$$

and $b_4 = \frac{3}{5}g - \frac{3}{5}a$

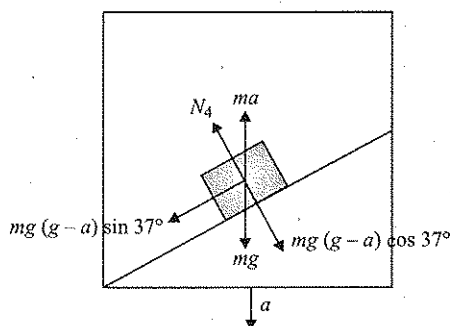


Fig. 7.708

27. a., d. If initially the acceleration of A is greater than B, then there will be an extension and if that of B is greater than A then there will be compression in the spring. Otherwise the length of spring will remain same.

28. b., d. Since the apparent weight is increasing, hence acceleration of the lift should be upwards. This is possible in case of (b) and (d).

29. b., c., d. Acceleration of particle w.r.t. frame S_1 :

$$\vec{a}_p - \vec{a}_{s_1} = 2\hat{n}$$

Acceleration of particle w.r.t. frame S_2 :

$$\vec{a}_p - \vec{a}_{s_2} = 2\hat{m}$$

where $\hat{m}$ and $\hat{n}$ are unit vectors in any directions. Now the relative acceleration of frames:

$$\vec{a}_{s_2} - \vec{a}_{s_1} = 2(\hat{n} - \hat{m}).$$

Its magnitude can have any value between 0 to 4 m/s², depending upon the directions of $\hat{m}$ and $\hat{n}$.

30. a., b., c.

a. Let the acceleration of each block be a .

$$10g - T_2 = 10a, T_2 - T_1 - f = 3a$$

$$T_1 - f = 2a, \text{ where } f = 0.3 \times 2g = 6 \text{ N}$$

From the above equations:

$$10g - 2f = 15a \Rightarrow 10 \times 10 - 2 \times 6 = 15a \Rightarrow a = 88/15 \text{ m/s}^2$$

$$T_2 = 10g - 10a = 10 \times 10 - 10 \times \frac{88}{15} = 41.3 \text{ N}$$

$$T_1 = f + 2a = 6 + 2 \times \frac{88}{15} = 17.7 \text{ N}$$

Clearly $T_2 > T_1$

b. This is correct because of the greater mass of 3 kg. Since acceleration is same for both.

c. This is incorrect because the net force acting on 10 kg mass is greater due to its larger mass, not due to its acceleration downward.

31. a., c.

$$a_1 = \frac{2mg - mg}{m} = g$$

$$a_2 = \frac{mg + mg - mg}{2m} = g/2$$

$$a_3 = \frac{2mg - mg}{3m} = g/3$$

clearly $a_1 > a_2 > a_3$

32. a., b., c. Figure 7.709

$$a_1 = \frac{3 - f_1}{0.2} = \frac{3 - 0.5 \times 0.2g}{0.2} = 10 \text{ m/s}^2$$

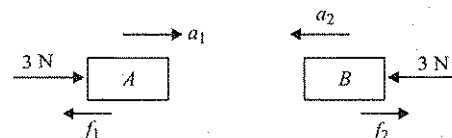


Fig. 7.709

$$a_2 = \frac{3 - f_2}{0.5} = \frac{3 - 0.5 \times 0.5g}{0.5} = 1 \text{ m/s}^2$$

$$\text{relative acceleration} = 10 + 1 = 11 \text{ m/s}^2$$

$$22 = \frac{1}{2} \times 11 \times t^2 \Rightarrow t = 2 \text{ s}$$

33. b., c. Figure 7.710

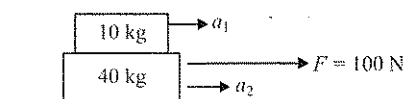


Fig. 7.710

$$F = 10a_1 + 40a_2$$

$$\Rightarrow 100 = 10a_1 + 40 \times 2 \Rightarrow a_1 = 2 \text{ m/s}^2$$

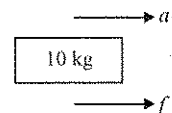


Fig. 7.711

So acceleration of A must be 2 m/s² for the given conditions to be satisfied (Fig. 7.711).

$$F = 10a_1 = 10 \times 2 = 20 \text{ N}$$

$$f < f_l \Rightarrow 20 < \mu m_{AG}$$

$$\Rightarrow 20 < \mu \times 10g \Rightarrow \mu \leq 0.2$$

Hence μ can be greater or equal to 0.2

Assertion-Reasoning Type

1. b.
 - i. $F = \frac{dp}{dt}$ (Newton's second law)
 - ii. Newton's third law.
2. a. Figure 7.712

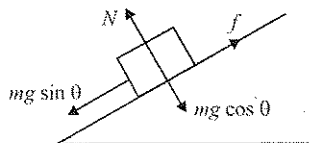


Fig. 7.712

$$f = mg \sin \theta$$

$$M = mg \cos \theta$$

$$R = \sqrt{N^2 + f^2} = mg$$

3. b. Force needed when the breaks are applied

$$f_1 = ma = \frac{mv^2}{d}$$

(v : initial speed, d : distance from wall) when the turn is taken

$$f_2 = ma = \frac{mv^2}{d}$$

$\therefore$ brakes must be applied.

4. a. Figure 7.713

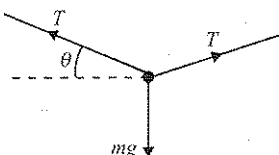


Fig. 7.713

$$2T \sin \theta = mg$$

$$T = \frac{mg}{2 \sin \theta} > mg \text{ for } \sin \theta < \frac{1}{2}, \text{ i.e., } \theta < 30^\circ.$$

5. d. Here the assertion is based on the idea that if you substitute $\vec{F} = \vec{O}$ in $\vec{F} = m\vec{a}$ (because there happens to be a particle on which net force $\vec{F} = \vec{O}$)

You get $\vec{a} = \vec{O} \Rightarrow \vec{v} = \text{constant} \Rightarrow$ such a particle will move with constant speed along a fixed direction which is Newton's first law. But the point is, you cannot employ $\vec{F} = m\vec{a}$, without first ascertaining that it is valid in this form (i.e. without pseudo forces). And that can be done only by applying Newton's first law and checking the behavior of your frame against the description laid down in the first law.

6. b. Figure 7.714

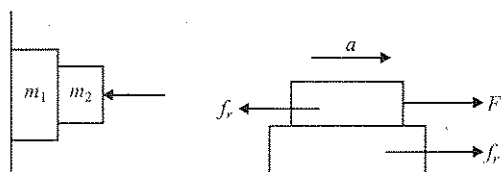


Fig. 7.714

7. d. According to Newton's third law of motion action and reaction are equal and opposite.
8. a. In the direction of normal reaction net acceleration is zero. Hence the forces in this direction will be balanced. Hence $N = mg \cos \theta$.
9. b. By the definition of inertial and non-inertial frame.
10. c. Coefficient of friction $\mu = \tan(\theta)$. The value of $\tan \theta$ may exceed unity.
11. a. $F = \frac{\Delta P}{\Delta t}$, If Δt is more, then F will be less.
12. d. $a_1 = g - \frac{F}{M}$
 $a_2 = g - \frac{F}{m} \Rightarrow a_1 > a_2$
13. d. While a running boy pushes the ground in backward direction and the available friction pushed him in forward direction.
14. a. In equilibrium, net force on the body is 0, therefore, its acceleration (a) is 0. If the body is at rest it will remain at rest. If the body is moving with a constant speed along a straight line, it will continue to do so.
15. d. Inertia is the property by virtue of which the body is unable to change by itself not only the state of rest but also the state of motion.
16. d. Due to change in normal reaction pulling is easier.
17. a. Contact force is sum of friction and normal reaction.
18. b. Static frictional force is self adjusting.
19. d. Figure 7.715



Fig. 7.715

FBD of A (Fig. 7.716)

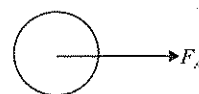


Fig. 7.716

20. For pulling condition (Fig. 7.717):

$$N + F \sin \theta = mg$$

$$N = Mg - F \sin \theta \quad (i)$$

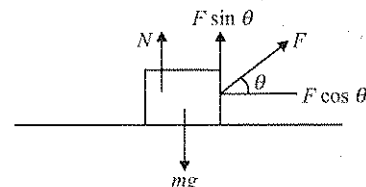


Fig. 7.717

For pushing condition Fig. 7.718:

$$N = F \sin \theta + Mg \quad (ii)$$

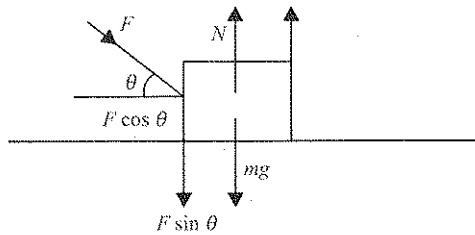


Fig. 7.718

21. c. Here the acceleration of both will be same, but their masses are different. Hence, the net force acting on each of them will not be same.
22. b. During the static friction there is no slipping between the two bodies. But during kinetic friction bodies slip due to which heat is produced at the cost of mechanical energy.
23. a. Statement II is correct, as it represents Newton's second law as $\vec{F} = \frac{d\vec{p}}{dt}$, from this only we can say that for greater value of $\frac{d\vec{p}}{dt}$, force applied has to be more.
24. a. Once the ski is in motion it melts the snow below it and hence skiing can be performed. To make skiing easier, wax has been put on bottom surface of ski as wax is water repellent and hence reduces the friction between the ski and film of water.
25. b. The acceleration of a particle as seen from an inertial frame is zero if the net force acting on the particle is zero.
26. d. Reference frame attached to earth is not an inertial frame of reference because earth is revolving about the sun, as well as it is rotating about its own axis.
27. d. When a body is at rest the static friction may be less than the limiting friction.
28. d. The FBD of block A in Fig. 7.719 is

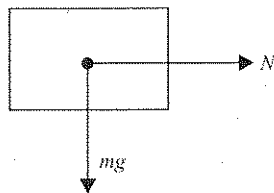


Fig. 7.719

The force exerted by B on A is N (normal reaction). The forces acting on A are N (horizontal) and mg (weight downwards).

Hence statement I is false.

29. d. If the lift is retarding while it moves upward, the man shall feel lesser weight as compared to when lift was at rest. Hence statement-I is false and statement-II is true.
30. d. Even a small force will change the state, hence statement-I is wrong. Statement-II is the statement of second law of motion.

Comprehensive Type

For Problems 1-3

1. b., 2. a., 3. d.

Sol. Before burning BC, the free-body diagrams are shown in Fig. 7.720.

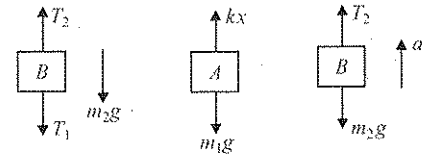


Fig. 7.720

$$T_2 = T_1 + m_2g \quad (1)$$

$$kx = T_2 = m_1g \quad (2)$$

where x is the extension in the spring. Just after burning, T_1 will become zero, but T_2 will remain same.

$$T_2 - m_2g = m_2a$$

$$\Rightarrow a = \frac{(m_1 - m_2)g}{m_2}$$

As T_2 remains same, acceleration of block A will still remain zero.

For Problems 4-6

4. b., 5. a., 6. d.

Sol. $T_3 = 20t$, $T_1 = T_2 = 10t$

For A to lose contact: $10t = 1g \Rightarrow t = 1$ s

For B to lose contact: $10t = 2g \Rightarrow t = 2$ s

For C to lose contact: $20t = 3g \Rightarrow t = 1.5$ s

$a_A = \frac{T_1 - 1g}{1}$, Velocity of A when B loses contact.

$$V_1 = \int_1^2 a_A dt = \int_1^2 (10t - g) dt = 5 \text{ m/s}$$

$$\text{at } t = 2 \text{ s, } a_B = 0, a_A = \frac{10 \times 2 - 10}{1} = 10 \text{ m/s}^2$$

$$a_{A/B} = a_A - a_B = 10 - 0 = 10 \text{ m/s}^2$$

For Problems 7-9

7. b., 8. b., 9. c.

Sol.

$$7. \text{ b. } W' = m(g + a) \Rightarrow 240 = m(10 + 2)$$

$$\Rightarrow m = 20 \text{ kg.}$$

$$\text{True weight} = mg = 20 \times 10 = 200 \text{ N}$$

$$8. \text{ b. } W' = m(g - a) \Rightarrow 160 = 20(10 - a)$$

$$\Rightarrow a = 2 \text{ m/s}^2$$

$$9. \text{ c. zero, } a = g \text{ in freefall.}$$

For Problems 10-13

10. b., 11. c., 12. b., 13. d.

Sol.

$$m_1g \sin \theta - T = m_1a \quad (1)$$

$$T - m_2g \sin \theta = m_2a \quad (2)$$

Solve to get a and T .

$$N_1 = m_1g \cos \theta$$

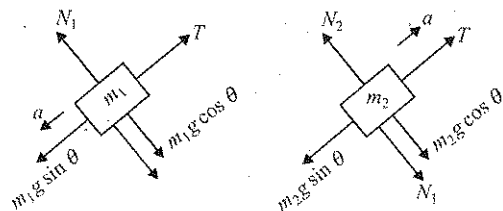


Fig. 7.721

$$N_2 = N_1 + m_2 g \cos \theta = (m_1 + m_2) g \cos \theta$$

For Problems 14–16

14. a., 15. b., 16. c.

Sol.

14. a. Area under F - t graph = change in momentum

$$\Rightarrow \frac{1}{2} F_0 (6 \times 10^{-3}) = \frac{200}{1000} [40 + 20]$$

$$\Rightarrow F_0 = 4,000 \text{ N}$$

15. b. $F_{av} = \frac{\text{Total change in momentum}}{\text{time taken}}$

16. c. Area under F - t graph = change in momentum

$$\Rightarrow \frac{1}{2} (F_0) (4 \times 10^{-3}) = \frac{200}{1000} [v + 20]$$

$$\Rightarrow v = 20 \text{ m/s}$$

For Problems 17–20

17. a., 18. c., 19. b., 20. a.

Sol. Let, acceleration of block C is a_1 (rightwards) and the acceleration of block B is a_2 (leftwards).

Then, acceleration of A will be $(a_1 + a_2)$ downwards and a_1 rightwards.

Free body diagram of A is shown in Fig. 7.722.

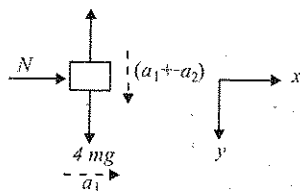


Fig. 7.722

Using $\sum F_x = ma_x$ and $\sum F_y = ma_y$, we get

$$N = 4m(a_1), \text{ and} \quad (1)$$

$$4mg - T = 4m(a_1 + a_2) \quad (ii)$$

Free body diagram of B (showing horizontal forces only) is shown in Fig. 7.723.

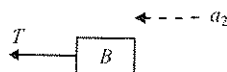


Fig. 7.723

Using $\sum F_x = ma_x$, we get

$$T = 3ma_2 \quad (iii)$$

Free body diagram of C (showing horizontal forces only) is shown in Fig. 7.724.

Using $\sum F_x = ma_x$, we get

$$T - N = 8ma_1 \quad (iv)$$

We have four unknowns T , N , a_1 , and a_2 . Solving these four equations, we get

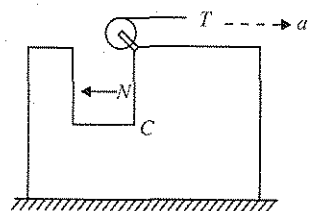


Fig. 7.724

$$a_1 = \frac{g}{8} \text{ and } a_2 = \frac{g}{2}$$

$$a_1 + a_2 = \frac{5}{8}g$$

Thus, the acceleration of A is $\frac{g}{8}$ in horizontal direction and $\frac{5g}{8}$ in the vertical direction.

Acceleration of B is $\frac{g}{2}$ in the horizontal direction (leftwards) and acceleration of C is $\frac{g}{8}$ in the horizontal direction (rightwards).

For Problems 21–23

21. d., 22. b., 23. c.

Sol. Figure 7.725

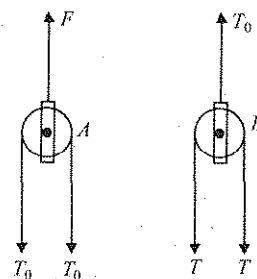


Fig. 7.725

Let

T_0 = tension in the string passing over A

T = tension in the string passing over B

$$2T_0 = F \text{ and } 2T = T_0$$

$$\Rightarrow T = F/4$$

When $F = 600 \text{ N}$

$$T = F/4 = 150 \text{ N}$$

As $T < Mg$ and $T > mg$, M will remain stationary on the floor, whereas m will move.

Acceleration of m ,

$$a = \frac{T - mg}{m} = \frac{150 - 100}{10} = 5 \text{ m/s}^2$$

For Problems 24–26

24. b., 25. c., 26. d.,

Sol. For upper block

$$a_{\max} = \mu g = 4 \text{ m/s}^2 \text{ and } f_{\max} = 40 \text{ N}$$

(1) When $F = 30 \text{ N}$

As $F < f_{\max}$

So both blocks will move together.

$$\therefore a = \frac{F}{M + m} = \frac{30}{35} = \frac{6}{7} \text{ m/s}^2$$

(2) When $F = 250 \text{ N}$

For upper block

$$250 - 40 = 10a$$

$$\Rightarrow 210 = 10a \Rightarrow a = 21 \text{ m/s}^2$$

$$\therefore a = \frac{40}{25} = \frac{8}{5} \text{ m/s}^2$$

For Problems 27–29

27. d., 28. d., 29. d.

Sol. From the constraint relations we can see that

$$3TX_B = 2TX_A$$

$$X_A = \frac{3}{2}X_B \Rightarrow a_A = \frac{3}{2}a_B$$

So let $a_B = a$ then $a_A = 1.5a$

Writing equation of motion:

From block A,

$$2T = 70a_A = 105a = 3 \times 35a$$

$$35a = \frac{2}{3}T$$

From block B,

$$300 - 3T = 35a_B = 35a$$

Solving eqs. (i) and (ii) we get

$$300 - 3T = \frac{2T}{3}$$

$$\Rightarrow 900 - 9T = 2T \Rightarrow 900 = 11T$$

$$T = \frac{900}{11} \text{ N}$$

$$a_A = \frac{180}{77} \text{ m/s}^2 \text{ and } a_B = \frac{120}{77} \text{ m/s}^2$$

For Problems 30–33

30. a., 31. d., 32. d., 33. c.

Sol. Case I

As the monkey doesn't move with respect to the rope, it means that the acceleration of the block or the rope and the monkey is same. So equations of motion are

$$T - \mu Mg = Ma \quad (i)$$

$$mg - T = m(a - b) \quad (ii)$$

Putting the value T from eqs. (i) in (ii)

$$mg - \mu Mg = Ma + ma$$

$$\Rightarrow \frac{(m - \mu M)g}{(M + m)} = a$$

$$T = \mu Mg + M \left(\frac{(m - \mu M)g}{(M + m)} \right)$$

$$= \mu Mg + \frac{Mmg - \mu M^2g}{M + m}$$

$$= \frac{\mu M^2g + \mu Mmg + Mmg - \mu M^2g}{M + m}$$

$$= \frac{Mmg(\mu + 1)}{M + m}$$

Case II The monkey moves downward with respect to the rope with an acceleration b , therefore, its absolute acceleration is $a + b$ where a is the acceleration of the rope. Therefore, the equations of motion are

$$mg - T = m(a + b) \quad (i)$$

$$T - \mu Mg = Ma \quad (ii)$$

Putting the value of T from eq. (ii) into (i)

$$(m - \mu M)g = (M + m)a + mb$$

$$\Rightarrow \frac{m(g - b) - \mu Mg}{(M + m)} = a$$

$$T = \mu Mg + \frac{Mmg - Mmb - \mu M^2g}{M + m}$$

$$= \frac{\mu M^2g + \mu Mmg + Mmg - Mmb - \mu M^2g}{M + m}$$

$$T = \frac{Mmg(u + 1 - b)}{M + m}$$

For Problems 34–35

34. c., 35. b.,

Sol. a $W = 60 \text{ N}$, $T \sin \theta = W$,

so, $T = (60 \text{ N}) / \sin 45^\circ$, or, $T = 85 \text{ N}$

b. $F_1 = F_2 = 85 \text{ N} \cos 45^\circ = 60 \text{ N}$

For Problems (36–38)

36. d., 37. a., 38. c.

Sol. Let the tension in the cord attached to block A be T_1 and the tension in the cord attached to block C be T_C . The equations of motion are then

$$T_1 - m_Ag = m_Aa; \quad T_2 - \mu_k m_Bg - T_1 = m_Ba$$

$$m_Cg - T_2 = m_Ca$$

a. Adding these three equations to eliminate the tensions give

$$a(m_A + m_B + m_C) = g(m_C - m_A - \mu_k m_B)$$

solving for m_C gives

$$m_C = \frac{m_A(a + g) + m_B(a + \mu_k g)}{g - a}$$

and substitution of numerical values gives

$$m_C = 12.9 \text{ kg} = 13$$

b. $T_1 = m_A(g + a) = 48 \text{ N}$

$$T_2 = m_C(g - a) = 102 \text{ N}$$

For Problems 39–40

39. d., 40. c.

Sol. For an angle of 45.0° , the tensions in the horizontal and the vertical wires will be the same.

a. The tension in the vertical wire will be equal to the weight $w = 12.0 \text{ N}$; this must be the tension in the horizontal wire, and hence the friction force on block A is also 12.0 N .

b. The maximum frictional force is

$$\mu_s w_A = (0.25)(60.0 \text{ N}) = 15 \text{ N}$$

this will be tension in both horizontal and vertical parts of the wire, so maximum weight is 15 N .

For Problems 41–43

41. b., 42. c., 43. d., 44. b., 45. c., 46. d.

Sol.

$$41. \text{ b. } f_{11} = 0.25 \times 4 = 1 \text{ N}, \quad f_{12} = 0.25 \times (4 + 8) = 3 \text{ N}$$

$F = f_{12} = 3 \text{ N}$ for constant velocity

42. c. $F = f_{11} + f_{12} = 1 + 3 = 4 \text{ N}$

43. d. $F = f_{11} + f_{12} + T, T = f_{11}$

$$\Rightarrow F = 2f_{11} + f_{12} = 5 \text{ N}$$

For Problems 44-46

44. d., 45. c., 46. d.

Sol.

44. b. As in Fig. 7.726

$$T - mg \sin 45^\circ - f_{11} = ma,$$

$$2mg \sin 45^\circ - T - f_{12} = 2ma$$

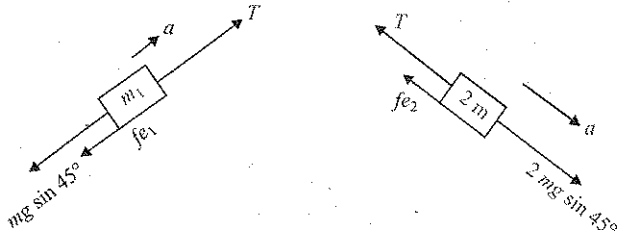


Fig. 7.726

where $f_{11} = \mu_1 mg \cos 45^\circ = 2mg \cos 45^\circ / 3$

and $f_{12} = \mu_2 2mg \cos 45^\circ$

Now we get: $a = -\frac{g}{9\sqrt{2}}$

 This is negative which is not possible. Hence $a = 0$

 45. c. $f_{12} < 2mg \sin 45^\circ$, hence friction only will not be able to prevent slipping of 2 m mass. So on 2 m mass friction will be maximum i.e. f_{12} .

$$T + f_{12} = 2mg \sin 45^\circ \Rightarrow T = \frac{2\sqrt{2}}{3}mg$$

46. d. $T = mg \sin 45^\circ + f_1 \Rightarrow f_1 = \frac{mg}{3\sqrt{2}}$

For Problems 47-48

47. c., 48. a.

Sol.

47. c. Net force $F = \sqrt{F_1^2 + F_2^2} = \sqrt{41} \text{ N}$

$f_l = 0.4 \times 10g = 40 \text{ N}, f_k = 0.3 \times 10g = 30 \text{ N}$

 Net force is less than f_l , Hence

Required friction force = applied force = $\sqrt{41} \text{ N}$

 48. a. Now applied force will be equal to maximum friction force, i.e., $\sqrt{5^2 + a^2} = f_l = 40$

$$\Rightarrow a = \sqrt{1575} \text{ N}$$

For Problems 49-50

49. a., 50. b.

Sol. Figure 7.727

$$500 - 55g \sin 30^\circ = 55a \Rightarrow a = \frac{45}{11} \text{ m/s}^2$$

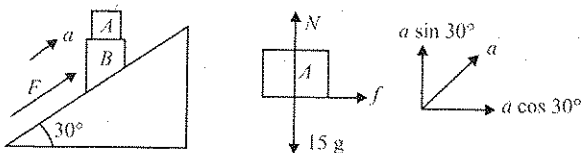


Fig. 7.727

$$N - 15g = 15a \sin 30^\circ$$

$$N = 15 \left[10 + \frac{45}{11} \times \frac{1}{2} \right] = \frac{265 \times 15}{22}$$

$$f = 15a \cos 30^\circ = 15 \times \frac{45}{11} \sqrt{\frac{3}{2}}$$

 For A not to slide on B: $f \leq f_l$

$$\Rightarrow \frac{15 \times 45}{11} \times \sqrt{\frac{3}{2}} \leq \mu \left(\frac{265 \times 15}{22} \right)$$

$$\Rightarrow \mu \geq \frac{9\sqrt{3}}{53} = 0.294$$

For Problems 51-52

51. b., 52. a.

 Sol. Assuming that the system moves together and there is no sliding, therefore, acceleration of the system $a = \frac{F}{(5 + 10)}$

$$a = \frac{F}{15}$$

(i)

FBD of m (Fig. 7.728)

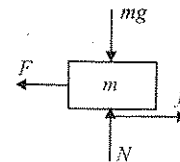


Fig. 7.728

FBD of M: $f - F = ma = 10 \left(\frac{F}{15} \right)$

$$f = F \left(1 + \frac{10}{15} \right)$$

(ii)

 If there is no sliding $F \leq \mu_s N$

$$F \left[\frac{5}{3} \right] \leq 0.4 \times 10 \times 10 \Rightarrow F = 24 \text{ N}$$

From equation (i), $a = \frac{F}{15} = \frac{24}{15} = 1.6 \text{ m/s}^2$

For Problems 53-56

53. c., 54. b., 55. d., 56. a.

Sol. For upper block

$a_{\max} = \mu g = 4 \text{ m/s}^2 \text{ and } f_{\max} = 40 \text{ N}$

 (1) When $F = 30 \text{ N}$

 As $F < f_{\max}$

So both the blocks will move together.

$$\therefore a = \frac{F}{M + m} = \frac{30}{35} = \frac{6}{7} \text{ m/s}^2$$

 (2) When $F = 250 \text{ N}$

For the upper block

$$250 - 40 = 10a$$

$$\Rightarrow 210 = 10a \Rightarrow a = 21 \text{ m/s}^2$$

$$\therefore a = \frac{40}{25} = \frac{8}{5} \text{ m/s}^2$$

For Problems 57-59

57. c., 58. a., 59. d.

Sol.

57. c. $0.5t = \mu mg \left(1 + \frac{m}{M} \right)$

$$\mu = 0.2$$

58. a. Use concept of function force for drawing the graph.

59. d. $\frac{t}{2} = (M+m) \frac{dv}{dt}$

$$\int_0^v dv = \int_0^{12} \frac{t}{2(M+m)} dt$$

$$v = 6 \text{ m/s}$$

After 12 second, up to 15 seconds, for 3 seconds the acceleration of plank will be

$$a = \frac{0.2 \times 2 \times 10}{4} = 1 \text{ m/s}^2$$

$$v' = 6 + 3$$

$$v' = 9 \text{ m/s}$$

$$a_{av} = \frac{v' - v}{15} = \frac{3}{15} = 0.6 \text{ m/s}^2$$

For Problems 60–62

60. d., 61. b., 62. a.

Sol. Figure 7.729

Force in spring can't change abruptly whereas the tension in the string can change.

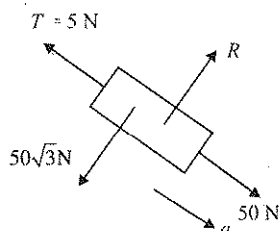


Fig. 7.729

For Problems 63–65

63. d., 64. a., 65. a.

Sol.

$$mg \sin 30^\circ - T = ma$$

$$50 - 5 = 10a$$

$$a = 4.5 \text{ m/s}^2$$

$$\frac{0 + 45}{2} = 2.25 \text{ m/s}$$

For Problems 66–68

66. b., 67. c., 68. d.

Sol. Free body diagrams Fig. 7.730:

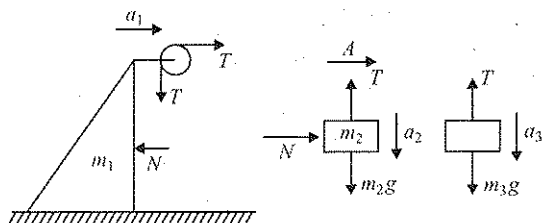


Fig. 7.730

Constraint relations:

$$x_1 = x_2 + x_3$$

$$a_1 = a_2 + a_3$$

Equation of motion

$$T - N = m_1 a_1 \quad (i)$$

$$N = m_2 a_1 \quad (ii)$$

$$m_2 g - T = m_2 a_2 \quad (iii)$$

$$m_3 g - T = m_3 a_3 \quad (iv)$$

Using the above equation we can calculate the values.

For Problems 69–70

69. d., 70. b.

Sol. Free body diagram Fig. 7.731:

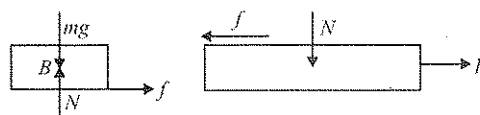


Fig. 7.731

Let both the blocks moves together

$$\text{Acceleration of the blocks, } a = \frac{F}{(m+M)}$$

$$f = m \left(\frac{F}{m+M} \right)$$

If both the blocks moves together,

$$f \leq \mu mg$$

$$\frac{mF}{(m+M)} \leq \mu mg$$

$$F \leq \mu(m+M)g$$

If the cube begins to slide then,

$$F = \mu(m+M)g$$

$$a_m = \frac{f}{m} = \mu g \quad (\text{towards } +x \text{ direction})$$

$$a_M = \frac{F - \mu mg}{M} \quad (\text{towards } +x \text{ direction})$$

$$\begin{aligned} \vec{a}_{m,M} &= \vec{a}_m - \vec{a}_M = \mu g - \left(\frac{F - \mu mg}{M} \right) \\ &= \frac{\mu(m+M)g - F}{M} \end{aligned}$$

If the cube falls from the plank it will cover a distance l

$$-l = \frac{1}{2} a_{mM} t^2 \Rightarrow t = \sqrt{\frac{2l}{a_{m,M}}}$$

$$t = \sqrt{\frac{2lM}{F - \mu(m+M)g}}$$

For Problems 71–74

71. a., 72. b., 73. c., 74. c.

Sol. Let the acceleration of block 1 and the pulley P_2 relative to ground is x in the directions shown in Fig. 7.732. Similarly, the acceleration of block 4 and pulley P_3 relative to the pulley P_2 is y and the acceleration of block 2 and block 3 relative to the pulley P_3 is z .

Then,

absolute acceleration of 1 is $a_2 = x$

absolute acceleration of 2 is $a_2 = y + z - x$
 absolute acceleration of 3 is $a_3 = -y + z - x$
 and absolute acceleration of 4 is $a_4 = -x - y$

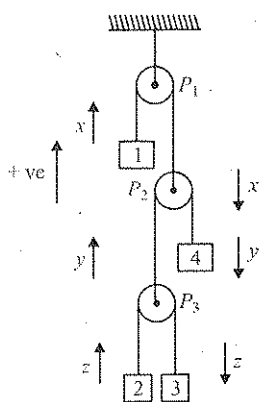


Fig. 7.732

Now given that

$$a_{21} = a_2 - a_1 = -1 \text{ m/s}^2$$

or, $y + z - 2x = -1$

$$a_{31} = a_3 - a_1 = -5 \text{ m/s}^2$$

or, $y - z - 2x = -5$

$$a_{34} = a_3 - a_4 = 0$$

or, $2y - z = 0$

Solving these three equations, we find

$$x = 2 \text{ m/s}^2 \quad y = 1 \text{ m/s}^2, \text{ and } z = 2 \text{ m/s}^2$$

or, $a_1 = 2 \text{ m/s}^2$ (upwards)

$$a_2 = 1 \text{ m/s}^2$$
 (upwards)

$$a_3 = -3 \text{ m/s}^2$$
 (downwards)

$$a_4 = -3 \text{ m/s}^2$$
 (downwards)

For Problems 75–76

75. c., 76. a.

Sol.

$$75. \text{ c. } h = \sqrt{l^2 - r^2} = 1 \text{ m,}$$

$$\text{Time period: } T_0 = 2\pi \sqrt{\frac{h}{g}} = 2\pi \sqrt{\frac{1}{10}} = \frac{2\pi}{\sqrt{10}} \text{ s}$$

$$76. \text{ a. } \omega^2 = g/h$$

$$T = m\omega^2 l = \frac{500}{1000} \times \frac{g}{h} \times \sqrt{2} = 5\sqrt{2} \text{ N}$$

For Problems 77–78

77. d., 78. b.

Sol.

$$77. \text{ d. } T \cos \theta = mg, T \sin \theta = mv^2/r$$

Squaring and adding both, we get the answer.

78. b. Dividing the above equations:

$$v = \sqrt{rg \tan \theta}$$

For Problems 79–81

79. c., 80. a., 81. b.

Sol.

79. c. Figure 7.733

$$\text{For } t < t_0 : f = \mu mg, a_1 = \frac{f}{m} = \mu g$$

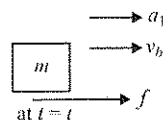


Fig. 7.733

Velocity of block at any time: $v_b = v_1 + a_1 t$

$$\Rightarrow v_b = v_1 + \mu g t$$

Velocity of plank at any time: $v_p = v_2 + at$

At $t = t_0$, both the velocities are same:

$$v_1 + \mu g t_0 = v_2 + at_0 \Rightarrow t_0 = \frac{v_2 - v_1}{\mu g - a}$$

80. a. For $t < t_0$:

Velocity of block at any time: $v_b = v_1 + \mu g t$

slope of graph is μg

Velocity of block at $t = t_0$: $v_{b0} = v_1 + \mu g t_0$

For $t > t_0$ (Fig. 7.719):

Velocity of block at any time: $v_b = v_{b0} + a(t - t_0)$

$$\Rightarrow v_b = v_1 + (\mu g - a)t_0 + at$$

slope of graph is a . So slope will decrease.

Hence the correct graph is (a)

81. b. For $t < t_0$: (see Fig. 7.734)

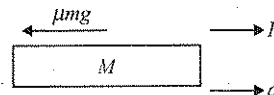


Fig. 7.734

Let F be the forward force, then

$$F - \mu mg = Ma \Rightarrow F = \mu mg + Ma$$

For $t > t_0$:

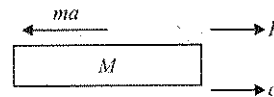


Fig. 7.735

Let F be the forward force, then

$$F - ma = Ma \Rightarrow F = (m + M)a$$

For Problems 82–84

82. b., 83. d. 84. c.

Sol. Let x is the compression in spring at any time Fig. 7.736



Fig. 7.736

$$F_1 - kx = m_1 a_0, F_2 - kx = m_2 a_0$$

$$\text{Solve to get: } a_0 = \frac{F_1 - F_2}{m_1 - m_2}$$

$$\text{and } kx = \frac{m_1 F_2 - F_1 m_2}{m_1 - m_2}$$

84. c. Just after m_2 is removed:

$$a_2 = \frac{kx}{m_2} = \frac{F_2 - m_2 a_0}{m_2} = \frac{F_2}{m_2} - a_0$$

For Problems 85–88

85. c., 86. a., 87. b., 88. d.

Sol.

85. c. The forces acting on the block are F_1 , F_2 , mg , normal contact force, and frictional force. Here the frictional force won't act along the vertical direction as the component of the resultant force along the surface acting on the body is not along vertical direction and the direction of the friction force is either opposite to the motion of block (direction of acceleration of block) if it is moving or opposite to component of resultant force along the surface if it is not moving.

$$N_1 = 300 \text{ N}$$

$$\text{So, } f_L = \mu N_1 = 0.6 \times 300 = 180 \text{ N}$$

Resultant of $4g$ and F_2 is 107.7 N making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal. As the force applied along the surface is f_L , so the block doesn't move and the friction is static in nature.

$F = 107.7 \text{ N}$ making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction.

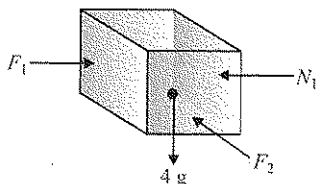


Fig. 7.737

86. a. As $F_{\text{applied}} < f_L$, so the block remains at rest.

87. b. For $F_1 = 150 \text{ N}$, $f_L = 0.6 \times 150 = 90 \text{ N}$

As the component of resultant force along the surface is 107.7 N and it is greater than f_L , so the kinetic friction comes into existence, i.e., friction force acquires the value $f = \mu_k N_1 = 0.5 \times 150 = 75 \text{ N}$. Its direction is opposite to component of resultant force along the surface.

88. d. Acceleration of block = $\frac{107.7 - 75}{4} = 8.175 \text{ m/s}^2$

For Problems 89–93

89. b., 90. a., 91. c., 92. a., 93. b.

Sol.

89. b. As acceleration is coming -ve for both the possible direction of motion, so it means that the net applied force is not enough to cause the motion of system or to overcome the limiting friction force.

90. a. As it is given that the system is moving initially in a specific direction, so it means that the direction of friction is known and it is kinetic in nature (Fig. 7.738).

$$a = \frac{10g \sin 37^\circ - 5g \sin 53^\circ - f_1 - f_2}{15}$$

$$f_1 = \mu_{k1} \times 10g \cos 37^\circ = 20.0$$

$$f_2 = \mu_{k2} \times 5g \cos 53^\circ = 2.25$$

$$a = -\frac{2.25}{15} \text{ m/s}^2$$

So, the required time,

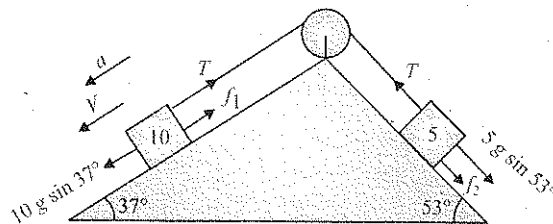


Fig. 7.738

$$t_0 = \frac{2}{2.25/15} = \frac{30}{2.25} = 13.33 \text{ s}$$

91. c. Once the block comes to rest, kinetic friction disappears and static friction comes into the existence and then situation would be identical to that of question no 40.

92. a.

93. b. Sol. $f_{L1} = 24 \text{ N}$ [Limiting friction force between 10 kg block and incline Fig. 7.739]

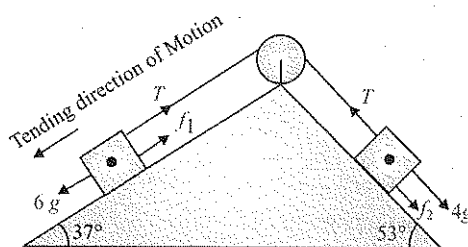


Fig. 7.739

$F_{L2} = 3 \text{ N}$ (Limiting friction force between 5 g block and incline) If we ignore the friction for the time being, then the system has the tendency to move down the incline as shown in Fig. 7.739 above. So we can say that the friction force is acting opposite to direction of this tending motion.

As the system is not moving, f_1 and f_2 are static in nature.

For equilibrium of both the blocks, $6g = -T + f_1$ and $4g + f_2 = T$.

Other condition are $f_1 < 24 \text{ N}$ and $f_2 < 3 \text{ N}$.

From hit and trial (better substitute f_2 first) we can draw some conclusions.

If $f_2 = 0$, $T = 40 \text{ N}$, $f_1 = 20 \text{ N}$

If $f_2 = 3 \text{ N}$, $T = 43 \text{ N}$, $f_1 = 17 \text{ N}$

So, f_2 should lie between 0 to 3 N

f_1 should lie between 20 to 17 N

T should lie between 40 to 43 N

For Problems 94–96

94. d. 95. c., 96. b.

Sol. From the data given we can find the limiting friction force for the two surfaces Fig. 7.740.

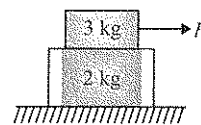


Fig. 7.740

$$f_{L1} = 0.5 \times 3 \times 10 = 15 \text{ N}$$

$$f_{L2} = 0.2 \times 5 \times 10 = 10 \text{ N}$$

For $F < f_{L2}$

Both the blocks remain at rest and $f_1 = F$, $f_2 = f_1$, and $a_1 = a_2 = 0$.

For $F > f_{L2}$ and $F < a$ certain value say F_1 , the motion starts at a lower surface but both the blocks continue to move with the same acceleration. The friction on lower surface becomes kinetic in nature.

$$\text{Here, } a = a_1 = a_2 = \frac{F - f_{k2}}{5} \text{ m/s}^2$$

$$F - f_1 = 3a \text{ and } f_1 - f_k = 2a$$

$$\text{All these equations give } f_1 = \frac{2F + 3f_{k2}}{5}$$

For relative motion to start between two bodies, $f_1 \geq f_{L1}$
 $F \geq 30 \text{ N}$. So, minimum value of F to cause relative motion between blocks is 30 N.

For Problems 97–99

97. a., 98. a., 99. c.

Sol.

97. a. From exp. 1, $f_L = 450 \text{ N}$

$$\mu_s N = 450 \Rightarrow \mu_s = 0.45$$

$$[\therefore N = mg = 1000 \text{ N}]$$

98. a. In 1st set the force applied is less than f_L , so the block can't move.

99. c. From 2nd and 3rd set of exp. 2.

$$2.0 m = \frac{1}{2} \left[\frac{600 - f_k}{m} \right] t^2$$

$$\text{and } 3.0 m = \frac{1}{2} \left[\frac{750 - f_k}{m} \right] t^2$$

$$f_k = 300 \text{ N} \Rightarrow \mu_k = 0.3$$

$$\text{For 2nd set, } a = \frac{600 - 300}{100} = 3 \text{ m/s}^2$$

For Problems 100–102

100. a., 101. c., 102. a.

Sol.

100. a. For equilibrium of block (A) (Fig. 7.741)

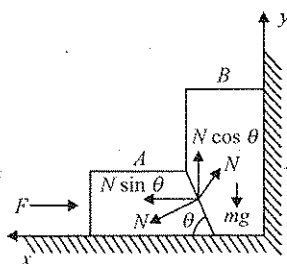


Fig. 7.741

$$F = N \sin \theta$$

$$N = F / \sin \theta$$

To lift block B from the ground

$$N \cos \theta \geq mg \Rightarrow \frac{F}{\sin \theta} \cos \theta \geq mg$$

$$F \geq mg \tan \theta = mg \left(\frac{3}{4} \right)$$

So,

$$F_{\min} = \frac{3}{4} mg$$

101. c. If both the blocks are stationary (Fig. 7.742).

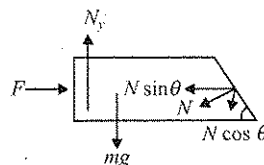


Fig. 7.742

Balancing the forces along x-direction

$$F = N \sin \theta \Rightarrow N = F / \sin \theta$$

Balancing the forces along y-direction

$$N_y = mg + N \cos \theta$$

$$= mg + \left(\frac{F}{\sin \theta} \right) \cos \theta = mg + F \cot \theta$$

$$N_y = mg + \frac{4F}{5}$$

102. a. To keep a regular contact see Fig. 7.743

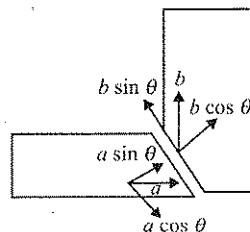


Fig. 7.743

$$a \sin \theta = b \cos \theta$$

$$b = a \tan \theta = \frac{3}{4} a$$

For Problems 103–104

103. a., 104. d.

Sol.

103. a. Figure 7.744

$$a = \left(\frac{m}{M_{\text{tot}}} \right) g$$

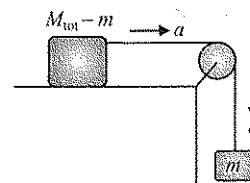


Fig. 7.744

$$104. \text{ d. } T = \frac{2 M_1 M_2 g}{M_1 + M_2} = \frac{2m (M_{\text{tot}} - m)}{M_{\text{tot}}} g$$

$$T = 2 \left(\frac{m M_{\text{tot}} - m^2}{M_{\text{tot}}} \right) g$$

For Problems 105–107

105. d., 106. a., 107. b.

Sol.

105. d. If $F = 20$ N, 10 kg block will not move and it will not press 5 kg block. So $N = 0$. (Fig. 7.745)

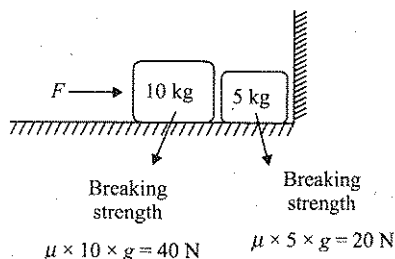


Fig. 7.745

106. a. If $F = 50$ N, force on 5 kg block = 10 N (Fig. 7.746)

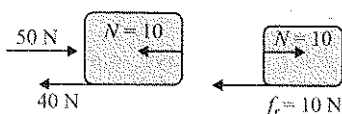


Fig. 7.746

So friction force = 10 N

107. b. Until the 10 kg block is stuck with ground ($F = 40$ N), no force will be felt by 5 kg block. After $F = 40$ N, the friction force on 5 kg increases, till $F = 60$ N, and after that, the kinetic friction starts acting on 5 kg block, which will be constant (20 N).

For Problems 108–110

108. b., 109. b., 110. b.

Sol.

108. b. $\omega = \frac{2}{\pi} 2\pi = 4$ rad/s

$$T = m\omega^2 l = m(4)^2 l = 16 ml$$

109. b. $\cos \theta = \frac{g}{\omega^2 l} = \frac{g}{16l} \Rightarrow \theta = \cos^{-1} \left[\frac{g}{16l} \right]$

110. b. $v = \omega r = \omega l \sin \theta$
 $= \omega r \sqrt{1 - \cos^2 \theta} = 4l \sqrt{1 - \left(\frac{g}{16l} \right)^2}$

For Problems 111–113

111. d., 112. b., 113. b.

Sol.

111. d. Initial velocity: $u = 0$

$$I = \int F dt \Rightarrow m(v - u) = \int_0^t (6t - 2t^2) dt$$

$$\Rightarrow 2v = 3t^2 - \frac{2}{3}t^3$$

Put $v = 0$, we get $t = 4.5$ s

112. b. If we draw $F-t$ graph (Fig. 7.747).

We can see that upto $t = 3$ s, force is +ve, so velocity increases upto 3 s and after that velocity starts decreasing. So at 3 s velocity is maximum.

113. b. We can see from the figure above that when force becomes maximum (at $t = 1.5$ s) for the first time, velocity is not maximum at that instant. Hence (a) is incorrect. (b) is correct as explained in the above solution. (c) is incorrect because when force becomes zero, velocity does not become zero.

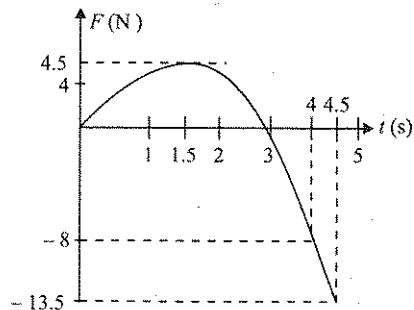


Fig. 7.747

For Problems 114–116

114. a., 115. c., 116. d.

Sol.

114. a. Let m_1 and m_2 do not accelerate up or down, then $F_1 = m_1 g$, $F_2 = m_2 g$. But $m_1 \neq m_2$ so $F_1 \neq F_2$.

Hence net horizontal force on M is $F_1 - F_2$. So M cannot be in equilibrium. If M accelerates horizontally, then m_1 and m_2 also accelerate horizontally.

115. c.

$$a = \frac{\text{Net force}}{\text{net mass}} = \frac{F_1 - F_2}{M + m_1 + m_2} = \frac{m_1 g - m_2 g}{M + m_1 + m_2}$$

116. d. In the horizontal direction, both m_1 and m_2 will have the same acceleration for any case. In vertical direction also they should have same acceleration. Let it be a upwards for both, then

$$F_1 - m_1 g = m_1 a \quad (1)$$

$$F_2 - m_2 g = m_2 a \quad (2)$$

From equations (1) and (2) $\frac{F_1}{m_1} = \frac{F_2}{m_2}$

For Problems 117–119

117. a., 118. c., 119. d.

Sol.

117. a. Figure 7.748

$$T = Mg$$

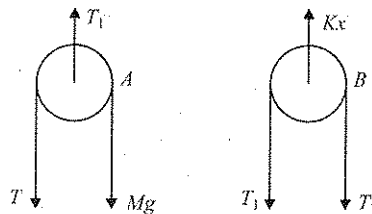


Fig. 7.748

$$T_1 = T + Mg = 2Mg$$

$$Kx = 2T_1 \quad \text{or} \quad x = \frac{2T_1}{K} = \frac{4Mg}{K}$$

118. c. Total tension on lower support: (Fig. 7.749)

$$T_1 + T = 2Mg + Mg = 3Mg$$

119. d. $T_1 = 3Mg$

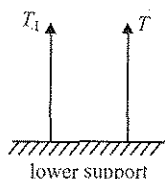


Fig. 7.749

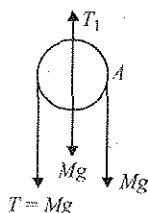


Fig. 7.750

Net tension at lower support (Fig. 7.749)

$$T + T_1 = Mg + 3Mg = 4Mg$$

For Problems 120–122

120. c., 121. a., 122. d.

120. c. Figure 7.749

$$f_l = f_k = 0.5 \times 2g = 10 \text{ N}$$



Fig. 7.751

Initially, $F = 20 \text{ N} > f_l$, so the block will start accelerating immediately.

At any time t : $F = 20 - 2t$

$$\text{Acceleration: } a = \frac{20 - 2t - f_k}{m} = \frac{10 - 2t}{m} \quad (1)$$

$$\text{For } a = 0, \frac{10 - 2t}{2} = 0 \Rightarrow t = 5 \text{ s}$$

$$\text{From equation (1)} \frac{dv}{dt} = \frac{10 - 2t}{2} = 5 - t \quad (2)$$

$$\int_0^v dv = \int_0^t (5 - t) dt \Rightarrow v = 5t - \frac{t^2}{2}$$

Let us see when the velocity becomes zero. For this:

$$5t - \frac{t^2}{2} = 0 \Rightarrow t = 10 \text{ s}$$

We see that at $t = 10 \text{ s}$, also $F = 0$. So the block has no tendency to move. Hence the acceleration is zero at this time. Now the block will not move from $t = 10 \text{ s}$ to 15 s because for this magnitude of $F < 10 \text{ N}$. So block will remain at rest from $t = 10 \text{ s}$ to 15 s or acceleration is zero from 10 s to 15 s .

121. a. From equation (2), first time acceleration becomes zero at $t = 5 \text{ s}$. Velocity at this time:

$$v = 5t - \frac{t^2}{2} = 5 \times 5 - \frac{5^2}{2} = 12.5 \text{ m/s}$$

122. d. From the above discussion clearly, velocity is zero at $t = 12 \text{ s}$.

For Problems 123–125

123. c., 124. b., 125. c.

Sol.

123. c. Since there is no friction between A and B, so B will remain at rest (Fig. 7.752).



Fig. 7.752

$$f_k = \mu(2 + 2)g = 0.1(2 + 2)g = 4 \text{ N}$$

$$\text{Acceleration of A: } a = \frac{f_k}{m} = \frac{4}{2} = 2 \text{ m/s}^2$$

$$\text{For B to fall off A: } S = ut + \frac{1}{2}at^2$$

$$\Rightarrow 4 = 0 \times t + \frac{1}{2}2t^2 \Rightarrow t = 2 \text{ s}$$

$$124. \text{ b. } v_A = u + at = 0 + 2 \times 2 = 4 \text{ m/s}$$

125. c. Just when B falls of A, take this instant to be $t = 0$ (Fig. 7.753)

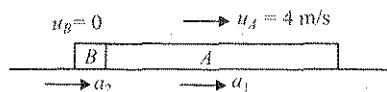


Fig. 7.753

at $t = 0$:

velocity of A: $u_A = 4 \text{ m/s}$, velocity of B: $u_B = 0$

acc. of A: $a_1 = \mu_1 g = 0.1 \times 10 = 1 \text{ m/s}^2$

acc. of B: $a_2 = \mu_2 g = 0.4 \times 10 = 4 \text{ m/s}^2$

Let us see when A comes to rest (w.r.t. belt):

For this: $v_A = u_A + a_1 t$

$$\Rightarrow 8 = 4 + 1 \times t \Rightarrow t = 4 \text{ s}$$

Let us see when B comes to rest (w.r.t. belt):

For this: $v_B = u_B + a_2 t$

$$\Rightarrow 8 = 0 + 4t \Rightarrow t = 2 \text{ s}$$

So B comes to rest earlier, and till that A continues to move with acceleration 1 m/s^2 .

So we have to find separation between the blocks at $t = 2 \text{ s}$. At $t = 2 \text{ s}$:

$$S_A = 4 \times 2 + \frac{1}{2} \times 1 \times (2)^2 = 10 \text{ m}$$

$$S_B = 0 \times 2 + \frac{1}{2} \times 4 \times (2)^2 = 8 \text{ m}$$

$$\text{separation} = S_A - S_B = 10 - 8 = 2 \text{ m}$$

Matching Column Type

1. a. $\rightarrow$ q., r., s.; b. $\rightarrow$ s; c. $\rightarrow$ p.s; d. $\rightarrow$ p., s.

2. i. $\rightarrow$ a., c., ii. $\rightarrow$ b., c.,

iii. $\rightarrow$ b., c., iv. $\rightarrow$ b., d.

Acceleration of the whole system towards right:

$$a = \frac{F}{M + m}$$

$$F - mg \sin \theta = ma \cos \theta$$

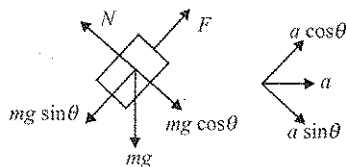


Fig. 7.754

$$\Rightarrow F - mg \sin \theta = m \frac{F}{M+m} \cos \theta$$

$$\Rightarrow F = \frac{(M+m)mg \sin \theta}{M+m-m \cos \theta}$$

pseudo force on m as seen from the frame of M :

$$\begin{aligned} F_{s1} = ma &= \frac{mF}{m+M} \\ &= mg \sin \theta \left(\frac{m}{M+m(1-\cos \theta)} \right) < mg \sin \theta \end{aligned}$$

Hence (a) $\rightarrow$ (p, r)

pseudo force on M as seen from the frame of m :

$$\begin{aligned} F_{s2} = Ma &= \frac{MF}{m+M} \left(> \frac{mF}{m+M} \right) \\ &= mg \sin \theta \left(\frac{M}{M+m(1-\cos \theta)} \right) < mg \sin \theta \end{aligned}$$

Hence (b) $\rightarrow$ (q, r)

$$\text{Now } mg \cos \theta - N = ma \sin \theta$$

$$\Rightarrow N = mg \cos \theta - ma \sin \theta$$

Hence N is less than $mg \cos \theta$. Hence it will also be less than $mg \sin \theta$, because $\theta = 45^\circ$.

Applying equation on m in horizontal direction:

$$F \cos \theta - N \sin \theta = ma$$

$$\Rightarrow F \cos \theta - N \sin \theta = m \frac{F}{M+m}$$

$$\Rightarrow N = \frac{mF}{M+m} \left(\frac{(M+m) \cos \theta - m}{m \sin \theta} \right)$$

put $\theta = 45^\circ$

$$\Rightarrow N = \frac{mF}{M+m} \left(\frac{M+m-\sqrt{2}m}{m} \right) > \frac{mF}{m+M}$$

Hence (c) $\rightarrow$ (q, r)

Normal force between ground and M will be $(M+m)g$.

It is greater than $mg \sin \theta$. It is also greater than $\frac{mF}{M+m}$,

because $\frac{mF}{M+m}$ is less than $mg \sin \theta$.

Hence (d) $\rightarrow$ (q, s)

3. i. $\rightarrow$ b., c., ii. $\rightarrow$ a., d.,

iii. $\rightarrow$ d., iv. $\rightarrow$ c.

Let accelerations of various blocks are as shown in Fig. 7.755. Pulley P_2 will have downward acceleration a .

$$\text{Now } a = \frac{a_1 + a_2}{2} \Rightarrow a_2 = 2a - a_1 > 0$$

So acceleration of 2 is upwards

Hence (i) $\rightarrow$ (b, c)

$$\text{and } a = \frac{-a_1 + a_4}{2} \Rightarrow a_4 = 2a + a_1 > 0$$

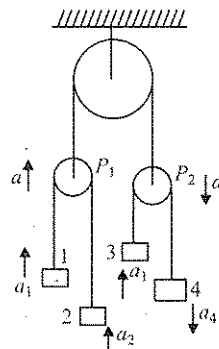


Fig. 7.755

So acceleration of 4 is downwards

Hence (ii) $\rightarrow$ (a, d)

Acceleration of 2 w.r.t. 3:

$$a_{2/3} = a_2 - a_3 = a_2 - a_1 = 2(a - a_1) < 0$$

This is downwards.

Hence (iii) $\rightarrow$ (d)

Acceleration of 2 w.r.t. 4:

$$a_{2/4} = a_2 - (-a_4) = 4a > 0$$

This is upwards.

Hence (iv) $\rightarrow$ (c)

4. i. $\rightarrow$ a., c. ii. $\rightarrow$ b., d.

iii. $\rightarrow$ a., b., c., d. iv. $\rightarrow$ c., d.

(i) Force of friction is zero in (a) and (c) because the block has no tendency to move.

(ii) Force of friction is 2.5 N in (b) and (d) because the applied force in horizontal direction in both is 2.5 N.

(iii) Acceleration is zero in all the cases.

(iv) Normal force is not equal to $2g$ in (c) and (d) because some extra vertical force is also acting.

5. i. $\rightarrow$ b., d., ii. $\rightarrow$ c.

iii. $\rightarrow$ a., d., iv. $\rightarrow$ b., d.

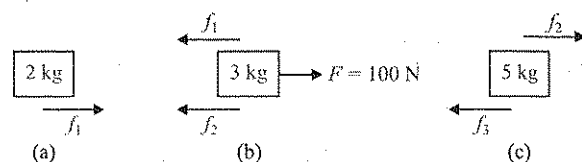


Fig. 7.756

$$f_{l1} = 0.2 \times 2g = 4 \text{ N}$$

$$f_{l2} = 0.1 \times 5g = 5 \text{ N}$$

$$f_{l3} = 0.1 \times 10g = 10 \text{ N}$$

Friction on 3 kg block is towards left and non-zero.

Hence (i) $\rightarrow$ b, d

$f_{l2} < f_{l3}$ hence 5 kg block will not move. So the net friction on 5 kg will be zero.

Hence (ii) $\rightarrow$ c

6. i. $\rightarrow$ d., ii. $\rightarrow$ c.

iii. $\rightarrow$ b iv $\rightarrow$ a.

From Fig. 7.757

$$F = (m_1 + m_2) a$$

$$T \sin \theta = m_2 a$$

(i)

(ii)

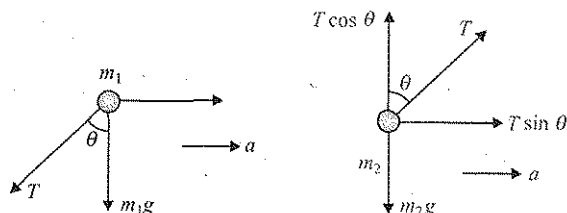


Fig. 7.757

$$T = m_2 g \sec \theta \quad (\text{iii})$$

From equations (ii) and (iii) $-a = a = g \tan \theta$

Put in 1 $-F = (m_1 + m_2) g \tan \theta$

$$\text{Net force acting on } m_2 = m_2 a = \frac{m_2 F}{m_1 + m_2}$$

Force acting on m_1 by wire:

$$m_1 g + T \cos \theta = m_1 g + m_2 g$$

7. i. $\rightarrow$ c., d., ii. $\rightarrow$ a., c.

iii. $\rightarrow$ a., c., iv. $\rightarrow$ c., d.

For (i), it is not mentioned whether the object is accelerated or moving with a constant velocity. So nothing can be predicted with surety.

If no net force is acting along east; then also it can move with a constant velocity, and if no force is acting at all, then also it can move with a constant velocity.

For (ii) and (iii): As the object is accelerated (whether uniform or non-uniform) a force must act on the object in such a manner that a component or whole of the force would be along east, and also the net force must be towards east.

For (iv): It is moving with a constant velocity, so the net force must be zero that implies no force may act on the object.

8. i. $\rightarrow$ b., ii. $\rightarrow$ a., c.

iii. $\rightarrow$ b., iv. $\rightarrow$ a., c.

For (i) and (iii): If $F > f$ it means object is moving and friction is kinetic in nature.

For (ii) and (iv): If $F > f$, then object is at rest and friction is static in nature. Another possibility is that $F = f_L$.

9. i. $\rightarrow$ a., ii. $\rightarrow$ c.

iii. $\rightarrow$ c., iv. $\rightarrow$ c.

Just after spring 2 is cut.

The net force acting on the block D changes and it is acting in upward direction and hence D accelerates. As there is no change in the elongation of spring 1, the equilibrium of A and B wouldn't be disturbed.

Similarly, we can find reasons for D.

10. i. $\rightarrow$ c., ii. $\rightarrow$ c.

iii. $\rightarrow$ b., c. iv. $\rightarrow$ a., b., c.

Let f_1, f_2, f_3 represent the friction forces between 3 contact surfaces A - B, B - C, and C - Ground, respectively. Limiting values of friction forces at 3 surfaces are 8 N, 15 N, and 10 N, respectively (Fig. 7.758).

For relative motion between C and Ground, the minimum force needed is $F = 10$ N.

For $F = 12$ N, all the 3 blocks move together with same acceleration (Fig. 7.759), i.e., $a_1 = a_2 = a_3 = a$.

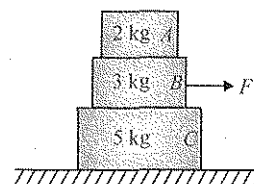


Fig. 7.758

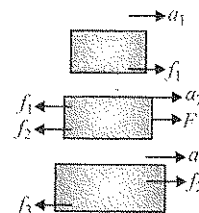


Fig. 7.759

$$F - f_3 = (2 + 3 + 5) a$$

$$a = \frac{12 - 10}{10} = \frac{2}{10} \text{ m/s}^2$$

$$f_1 = 2a = \frac{2}{5} \text{ N}$$

$$f_2 = 12 - f_1 - 3a = 11 \text{ N}$$

$$F_3 = 10 \text{ N}$$

For $F = 15$ N, the situation is similar.

For relative motion to start between B and C, $f_2 \geq f_{L2}$

$$F - f_3 = 10 a \text{ and } F - f_2 = 5 a$$

$$f_2 = F - 5 a = F - 5 \left[\frac{F - f_3}{10} \right] = \frac{F + f_3}{2}$$

$$\frac{F + 10}{2} > 15 \Rightarrow F > 20 \text{ N}$$

[Condition for relative motion to start between B and C]

For relative motion to start between A and B,

$$f_1 \geq f_{L1} = 8 \text{ N}$$

$$F - f_1 - f_2 = 3 a \text{ and } f_1 = 2 a$$

$$f_1 = 2 \left[\frac{F - 15}{5} \right] > 8$$

$$F > 35 \text{ N}$$

[condition for relative motion between A and B]

11. i. $\rightarrow$ b., ii. $\rightarrow$ c. iii. $\rightarrow$ b., iv. $\rightarrow$ c.

Sol. (i) Let a be the acceleration of two block systems towards

right (Fig. 7.760), then $a = \frac{F_2 - F_1}{m_1 + m_2}$

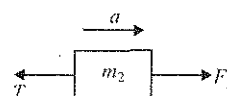


Fig. 7.760

$$F_2 - T = m_2 \cdot a$$

$$\text{Solving } T = \frac{m_1 m_2}{m_1 + m_2} \left(\frac{F_2}{m_2} + \frac{F_1}{m_1} \right)$$

(ii) Replace F_1 by $-F_1$ is result of A

$$T = \frac{m_1 m_2}{m_1 + m_2} \left(\frac{F_2}{m_2} - \frac{F_1}{m_1} \right)$$

(iii) Let a be the acceleration of two block system towards left, then $a = \frac{F_2 - F_1}{m_1 + m_2}$

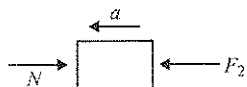


Fig. 7.761

$$F_2 - N = m_2 a$$

Solving, $N = \frac{m_1 m_2}{m_1 + m_2} \left(\frac{F_1}{m_1} + \frac{F_2}{m_2} \right)$

(iv) Replace F_1 by $-F_1$ in result of C

$$N = \frac{m_1 m_2}{m_1 + m_2} \left(\frac{F_2}{m_2} - \frac{F_1}{m_1} \right)$$

12. i. $\rightarrow$ c., ii. $\rightarrow$ b. iii. $\rightarrow$ d., iv. $\rightarrow$ b.

(A), (B) After spring 2 is cut, tension in string AB will not change.

$$(T_{CD})_i = 4mg$$

$$(T_{CD})_f = m_D g + m_D \cdot \frac{m_A + m_B - m_C - m_D}{m_A + m_B + m_C + m_D} \cdot g$$

$$= 2mg \left(1 + \frac{1}{5} \right) = 2.4mg$$

Hence T_{CD} decreases.

(C), (D) After string between C and pulley is cut tension in string AB will become zero.

$$(T_{CD})_i = (m_D + m_E)g = 4mg$$

Acceleration of C and D blocks is

$$(m_C + m_D)g + m_E g = (m_C + m_D)a$$

$$a = \frac{6mg}{4mg} = \frac{3}{2}g$$

$$(T_{CD})_i + m_C g = m_C a$$

$$(T_{CD})_f = 2m \frac{3}{2}g - 2mg = mg$$

The tension decrease.

13. i. $\rightarrow$ a., ii. $\rightarrow$ c., d. iii. $\rightarrow$ a., d., iv. $\rightarrow$ b., d.

In Fig. 7.762, 3 is the equilibrium position where velocity is maximum and acceleration is zero. 1 and 2 are the extreme positions where velocity is zero and acceleration is maximum. I is the unstretched position.

When the block is at position '3', then $mg = kx$. So net force is zero, hence acceleration is zero. But velocity may be either in upward or downward direction.

Hence (ii) - c., d.

When the block is between position '3' and '2', then $kx > mg$. So the net force is in upward direction, hence acceleration is in upward direction. But the velocity may be either in upward or downward direction.

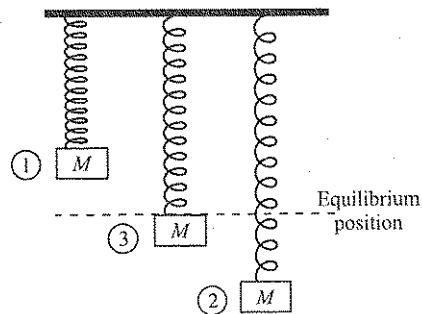


Fig. 7.762

Hence (iii) - a., d.

But if the block is at position '2', then the velocity is zero and the acceleration is in upward direction.

Hence (i) - a.

When the block is between position '3' and '1', then $mg > kx$. So the net force is in downward direction, hence acceleration is in downward direction. But velocity may be either in upward or downward direction.

Hence (iv) - b., d.

14. i. $\rightarrow$ c. ii. $\rightarrow$ c. iii. $\rightarrow$ c. iv. $\rightarrow$ b., d.

Let the maximum downward displacement of m is x_0 , then

$$\frac{1}{2} k x_0^2 = mg x_0 \Rightarrow x_0 = 2mg/k$$

To lift the block (M): $kx_0 = Mg$

$$\Rightarrow 2mg = Mg$$

$$\Rightarrow mg = Mg/2 \text{ Hence (i) - (c)}$$

(ii) When m is in equilibrium (Fig. 7.763)

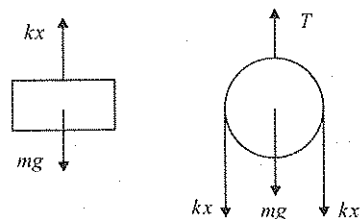


Fig. 7.763

$$kx = mg, T = 2kx + mg = 3mg = \frac{3M}{2}g$$

Hence (ii) $\rightarrow$ (a).

(iii) Figure 7.764

$$N + kx = Mg$$

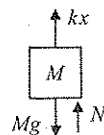


Fig. 7.764

$$N = Mg - kx = Mg - \frac{M}{2}g = \frac{M}{2}g$$

Hence (iii) $\rightarrow$ c.

(iv) Tension $= kx_0 = Mg = 2mg$

Hence (iv) $\rightarrow$ (b., d.)

15. i. $\rightarrow$ c., ii. $\rightarrow$ c. iii. $\rightarrow$ b., d., iv. $\rightarrow$ a., d.

(i) If $\mu_1 = \mu_2 = 0$, then there is no force on M in horizontal direction. So M does not accelerate

Hence (i) $\rightarrow$ (c)

(ii) If $\mu_1 = \mu_2 \neq 0$

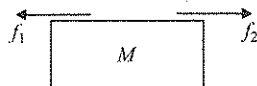


Fig. 7.765

$$f_1 < \mu_1 mg, \quad f_2 < \mu_2 mg$$

f_1 and f_2 will be of same magnitude at any time whether the blocks slip on larger block or not, so the net force on M is zero (Fig. 7.765). Hence M does not accelerate. so (ii) $\rightarrow$ (c)

(iii) $\mu_1 > \mu_2$, here $f_1 > f_2$, hence (iii) $\rightarrow$ (b, d)

(iv) $\mu_1 < \mu_2$ here $f_1 < f_2$, hence (iv) $\rightarrow$ (a, d)

16. i. $\rightarrow$ a., d., ii. $\rightarrow$ a., d. iii. $\rightarrow$ b., c., d., iv. $\rightarrow$ b., c., d.

Maximum possible accelerate of m : $a_0 = \mu g = 0.5 g$

So (d) matches with all (i), (ii), (iii) and (iv)

Let us assume that m and $2m$ move together with acceleration a : $a = \frac{m_1 g}{3m + m_1}$

$$\text{if } a = a_0 \Rightarrow \frac{m_1 g}{3m + m_1} = 0.5 g$$

$$\Rightarrow m_1 = 3m$$

So $3m$ is the maximum value of m_1 so that both move together.

(i) $m_1 = 2m < 3m$ hence (i) $\rightarrow$ (a), (d)

(ii) $m_1 = 3m$ hence (ii) $\rightarrow$ (a), (d)

(iii) $m_1 = 4m > 3m$ hence (iii) $\rightarrow$ (b), (c), (d)

(iv) $m_1 = 6m > 3m$ hence (iv) $\rightarrow$ (b), (c), (d)

17. i. $\rightarrow$ b., ii. $\rightarrow$ a. iii. $\rightarrow$ c., d. iv. $\rightarrow$ c., d.

The direction of accelerations of various blocks are as shown in Fig. 7.766.



Fig. 7.766

acc. of B is towards right, Hence (i) $\rightarrow$ b

acc. of C w.r.t. B is towards left, Hence (ii) $\rightarrow$ a

acc. of A w.r.t. C : $\vec{a}_{A/C} = \vec{a}_A - \vec{a}_C$
 $= -a\hat{j} - (-a\hat{i}) = a\hat{i} - a\hat{j}$ as shown below

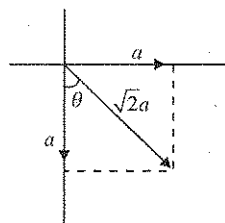


Fig. 7.767

Hence (iii) $\rightarrow$ (c,d), Similarly (iv) $\rightarrow$ (c,d)

18. i. $\rightarrow$ b., c., ii. $\rightarrow$ a., d. iii. $\rightarrow$ b., d., iv. $\rightarrow$ a.

19. i. $\rightarrow$ b., d., ii. $\rightarrow$ c. iii. $\rightarrow$ a., d., iv. $\rightarrow$ c.

From Fig. 7.768

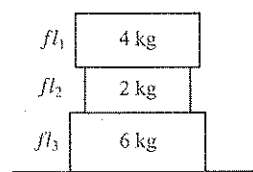


Fig. 7.768

$$f_{l1} = 0.2 \times 4g = 8 \text{ N}$$

$$f_{l2} = 0.4 \times 6g = 24 \text{ N}$$

$$f_{l3} = 0.5 \times 12g = 60 \text{ N}$$

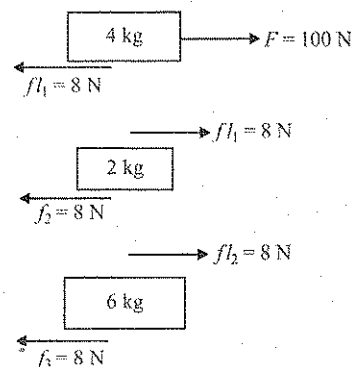


Fig. 7.769

Here only 4 kg will accelerate, 2 kg and 6 kg will remain at rest (Fig. 7.769).

20. i. $\rightarrow$ b., d., ii. $\rightarrow$ c., iii. $\rightarrow$ a., c. iv. $\rightarrow$ d.

From Fig. 7.770

$$2T_1 \cos 37^\circ = 120$$

$$\Rightarrow 2T_1 \times \frac{4}{5} = 120 \Rightarrow T_1 = 75 \text{ N}$$

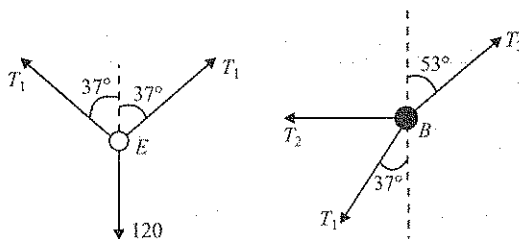


Fig. 7.770

$$T_1 \cos 37^\circ = T_3 \cos 53^\circ$$

$$\Rightarrow 75 \times \frac{4}{5} = T_3 \times \frac{3}{5} \Rightarrow T_3 = 100 \text{ N}$$

$$T_2 + T_1 \sin 37^\circ = T_3 \sin 53^\circ$$

$$\Rightarrow T_2 + 75 \times \frac{3}{5} = 100 \times \frac{4}{5}$$

$$\Rightarrow T_2 = 35 \text{ N}$$

Archives

Fill in the Blanks Type

1. Method I The maximum frictional force that can act on the mass of one kg will be

$$= \mu N = mg$$

$$= 0.6 \times 1 \times 9.8 = 5.88 \text{ N}$$

The truck is accelerating at 5 m/s^2 .

The pseudo force acting on the mass Fig. 7.771 as seen by the observer on the truck is

$$m \times a = 1 \times 5 = 5 \text{ N}$$

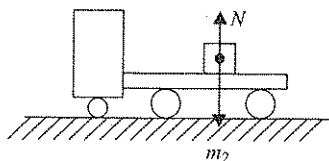


Fig. 7.771

The frictional force will try to oppose the movement of the mass by this force. Therefore the frictional force acting will be 5 N .

Method II As seen by the observer on the ground, the frictional force is responsible to move the mass with an acceleration of 5 m/s^2 . Therefore the frictional force would be

$$= m \times a = 1 \times 5 = 5 \text{ N}$$

2. Let A be the area of cross-section of the rod Fig. 7.772. Therefore

$$\rho = \frac{m}{AL} \Rightarrow m = \rho AL = \text{mass of rod}$$

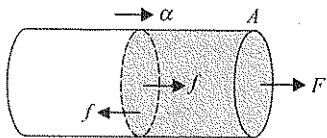


Fig. 7.772

The rod is moving with an acceleration α under the action of F

$$\therefore F = \rho AL\alpha \quad (i)$$

Consider the half rod of length $\frac{L}{2}$. Let the force acting on the mid portion be f . Applying $F_{\text{net}} = ma$ for the dotted position

$$F - f = \frac{\rho AL}{2}\alpha \quad (ii)$$

From equations (i) and (ii),

$$\rho AL\alpha - f = \rho A \frac{L}{2}\alpha \Rightarrow f = \frac{\rho AL}{2}\alpha$$

$$\Rightarrow \text{Stress at mid point } \frac{f}{A} = \frac{\rho L\alpha}{2}$$

B True or False

1. False

Concept: Friction force opposes the relative motion of the surfaces of contact. When a person walks on a rough surface, the foot is the surface of contact Fig. 7.773. When he pushes the foot backward, the motion of surface of contact is backward. Therefore, the frictional force will act forward (in the direction of motion of the person).

2. False

When the angular displacement is 20° , the mass is at extreme end (Fig. 7.774)

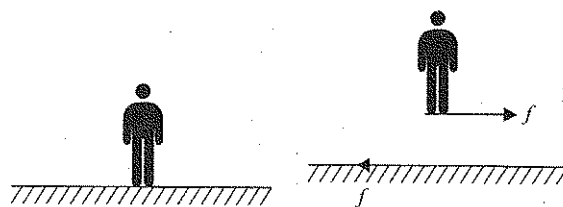


Fig. 7.773

$$T - mg \cos \theta = \frac{mv^2}{r}$$

$$\text{At extreme end } v = 0 \quad \therefore T = mg \cos \theta$$

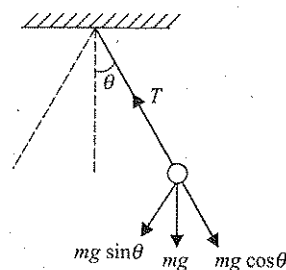


Fig. 7.774

3. False

For mass m

$$T - mg = ma$$

For mass $2m$ (Fig. 7.775)

(i)

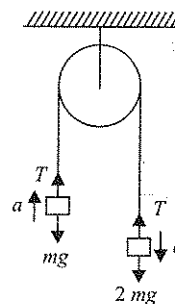


Fig. 7.775

$$2mg - T = 2ma \quad (ii)$$

From equations (i) and (ii)

$$a = g/3$$

$$T - mg = ma'$$

$$2mg - mg = ma'$$

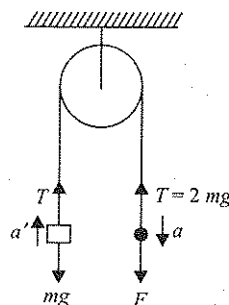


Fig. 7.776

$$\therefore a' = g$$

4. Since no external force is acting on the two particle system,

$$\therefore a_{cm} = 0$$

$$\Rightarrow V_{cm} = \text{Constant.}$$

The statement is false.

Multiple Choice Questions with One Correct Answer

1. c. $a = \frac{F}{m} = \frac{5 \times 10^4}{3 \times 10^7} = \frac{5}{3} \times 10^{-3} \text{ m/s}^2$

$$v = \sqrt{2as} = \sqrt{2 \times \frac{5}{3} \times 10^{-3} \times 3} = 0.1 \text{ m/s}$$

2. Since, $\mu mg \cos \theta > mg \sin \theta$

$$\therefore \text{force of friction is } f = mg \sin \theta$$

3. a. We give power to rear wheel, so friction is rear wheel acts in forward direction. Front wheel is a free wheel. On this friction acts in backward direction. Net friction is in forward direction, due to which cycle accelerates.

4.

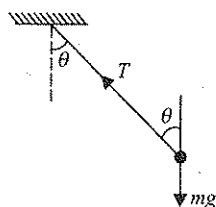


Fig. 7.777

FBD of bob (Fig. 7.777) is $T \sin \theta = \frac{mv^2}{R}$

and $T \cos \theta = mg$

$$\therefore \tan \theta = \frac{v^2}{Rg} = \frac{(10)^2}{(10)(10)}$$

$$\tan \theta = 1$$

or

$$\theta = 45^\circ$$

5. The magnitude of the frictional force f has to balance the weight 0.98 N acting downwards (Fig. 7.778).

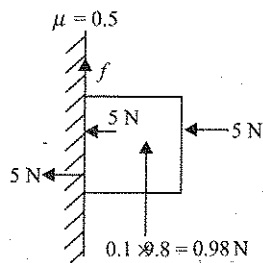


Fig. 7.778

Therefore, the frictional force is 0.98 N . Hence option (b) is the correct option.

6. a. Since the body presses the surface with a force N hence according to Newton's third law the surface presses the body with a force N (Fig. 7.779). The other force acting on the body is its weight mg .

For circular motion to take place, a centripetal force is required which is provided by $(mg + N)$.

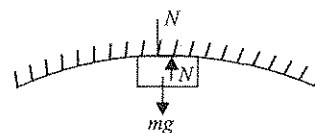


Fig. 7.779

$$\therefore mg + N = \frac{mv^2}{r}$$

If the surface is smooth then on applying conservation of mechanical energy, the velocity of the body is always same at the top most point. Hence N and r have inverse relationship. From the figure it is clear that r is min for first figure therefore N will be maximum.

7. a. The two forces acting at the insect are mg and N . Let us resolve mg into two components (Fig. 7.780)

$$mg \sin \alpha \text{ balances } N$$

$$mg \sin \alpha \text{ is balanced by the frictional force.}$$

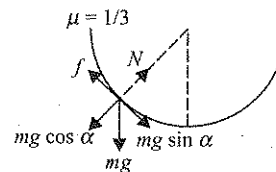


Fig. 7.780

$$\therefore N = mg \cos \alpha$$

$$f = mg \sin \alpha$$

$$\text{But } f = \mu N = \mu mg \cos \alpha$$

$$\therefore \mu mg \cos \alpha = mg \sin \alpha$$

$$\Rightarrow \cot \alpha = \frac{1}{\mu} \Rightarrow \cot \alpha = 3$$

8. For equilibrium in vertical direction for body B we have (Fig. 7.781)

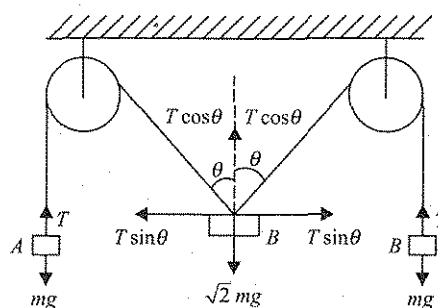


Fig. 7.781

$$\sqrt{2}mg = 2T \cos \theta$$

$$\therefore \sqrt{2}mg = 2(mg) \cos \theta$$

$$\therefore T = mg \text{ (at equilibrium)}$$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

9. d. Forces on the pulley are (Fig. 7.782)

$$F = \sqrt{F_1^2 + F_2^2}$$

$$F = [\sqrt{(m + M)^2 + M^2}]g$$

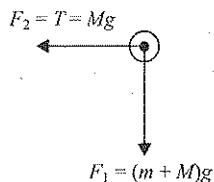


Fig. 7.782

10. a. The forces acting on the block are shown in Fig. 7.783. Since the block is not moving forward for the maximum force F applied, therefore

$$F \cos 60^\circ = f = \mu N, \text{ and} \quad (i)$$

$$F \sin 60^\circ + mg = N \quad (ii)$$

From equations (i) and (ii)

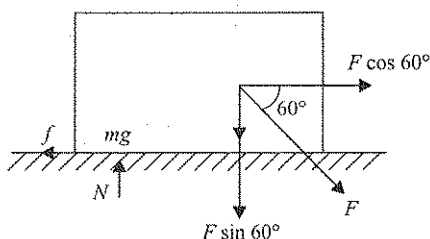


Fig. 7.783

$$F \cos 60^\circ = \mu [F \sin 60^\circ + mg]$$

$$\Rightarrow F = \frac{\mu mg}{\cos 60^\circ - \mu \sin 60^\circ} = \frac{\frac{1}{2\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{2}} = \frac{5}{\frac{1}{4}} = 20 \text{ N}$$

11. b. $2T \cos \theta = F$

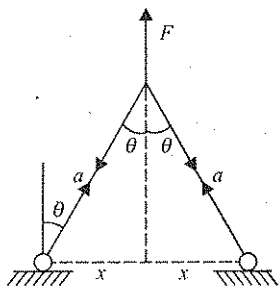


Fig. 7.784

$$\therefore T = \frac{F}{2} \sec \theta$$

Acceleration of particle (Fig. 7.784)

$$\begin{aligned} &= \frac{T \sin \theta}{m} = \frac{F \tan \theta}{2m} \\ &= \frac{F}{2m} \frac{x}{\sqrt{a^2 - x^2}} \end{aligned}$$

12. b. For sliding $\tan \theta \geq \sqrt{3}$ ($= 1.732$) For toppling $\tan \theta \geq \frac{2}{3}$ ($= 0.67$)

Assertion and Reasoning

1. b. The cloth can be pulled out without dislodging the dishes from the table due to the law of inertia, which is Newton's first law. While, the statement II is true, but it is Newton's third law.

2. b. and c. Both statements are correct. But statement II does not explain correctly statement I.

Correct explanation is : There is increase in normal reaction when the object is pushed and there is decrease in normal reaction when the object is pulled (but strictly not horizontally).

Multiple Choice with More than One Correct Answer

1. b.

Sol. From $\triangle MNO$, using pythagorous theorem (Fig. 7.785)

$$MN^2 + NO^2 = MO^2$$

$$\therefore x^2 + z^2 = l^2$$

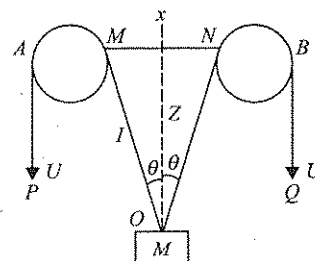


Fig. 7.785

Here x is a constant. Differentiating the above equation by t

$$0 + 2z \frac{dz}{dt} = 2l \frac{dl}{dt}$$

$$\Rightarrow zv_M = lU$$

$$\Rightarrow v_m = \frac{1}{z} U = \frac{U}{z/l} = \frac{U}{\cos \theta} \quad (\because \cos \theta = \frac{z}{l})$$

2. a. is wrong since earth is an accelerated frame and hence cannot be an inertial frame.

b. is correct.

c. is incorrect strictly speaking as Earth is accelerated reference frame [earth is treated as a reference frame for practical examples and Newton's laws are applicable to it only as a limiting case].

d. is correct.

3. a. Since the body is moving in a circular path (Fig. 7.786)

therefore, it needs centripetal force $\left(\frac{Mv^2}{l}\right)$

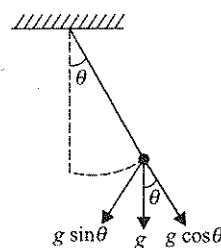


Fig. 7.786

$$\therefore T - mg \cos \theta = \frac{MV^2}{l}$$

Also the tangential acceleration acting on the mass is $g \sin \theta$ Fig. 7.787.

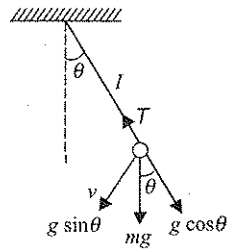


Fig. 7.787

(b) and (c) are correct options.

4. a. At A the horizontal speeds of both the masses is the same (Fig. 7.788). The velocity of Q remains the same in horizontal as no force is acting on the horizontal direction. But in case

of P as shown at any intermediate position, the horizontal velocity first increases (due to $N \sin \theta$), reaches a max value at O and then decreases. Thus it always remains greater than v . Therefore, $t_P < t_Q$.

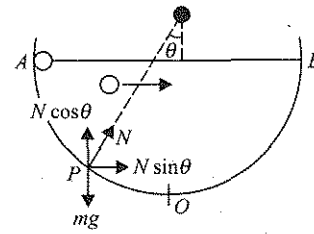


Fig. 7.788

Appendix

Solutions to Concept Application Exercises

Chapter 1

Exercise 1.1

$$1. (1+x)^{-2} = 1 + (-2)x + \frac{(-2)(-2-1)x^2}{2} + \frac{(-2)(-2-1)(-2-2)x^3}{3!} = 1 - 2x + 3x^2 - 4x^3 + \dots$$

$$2. Q\left[\left(1 + \frac{\Delta x}{x}\right)^3 - 1\right] = Q\left[1 + \frac{3\Delta x}{x} - 1\right] = \frac{3Q\Delta x}{x}$$

Exercise 1.2

1. i. $3x + 2y = 0$

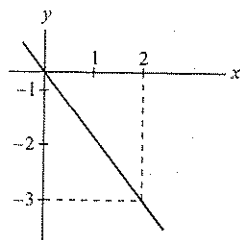


Fig. S-1.1

ii. $x - 3y + 6 = 0 \Rightarrow y = \frac{x}{3} + 2$

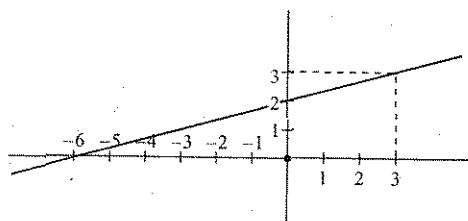


Fig. S-1.2

2. $v = 1 + 2t$

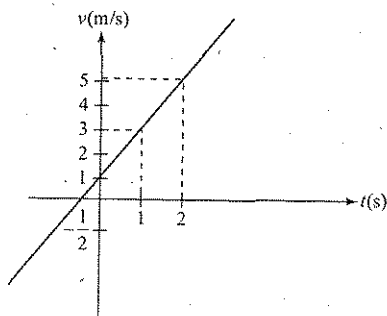


Fig. S-1.3

3. $v = u + at \Rightarrow v = 25 - 2t$

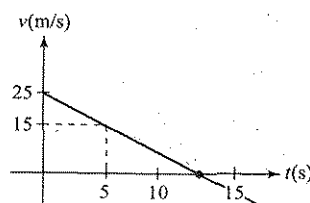


Fig. S-1.4

Exercise 1.3

1. a. $y = x^2 - 8x$, $a = 1$, $b = -8$, $c = 0$

Vertex $x = -\frac{b}{2a} = -\left(\frac{-8}{2 \times 1}\right) = 4$

At $x = 4$, $y = 4^2 - 8 \times 4 = -16$, At $x = 0$, $y = 0$

At $x = 8$, $y = 0$

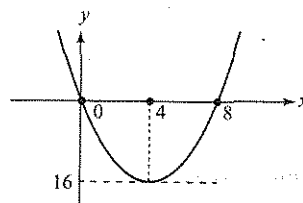


Fig. S-1.5

b. $y = -2x^2 + 3$, $a = -2$, $b = 0$, $c = 3$

Vertex $x = -\frac{b}{2a} = 0$, at $x = 0$, $y = 3$

At $x = 1$, $y = 1$, at $x = -1$, $y = 1$

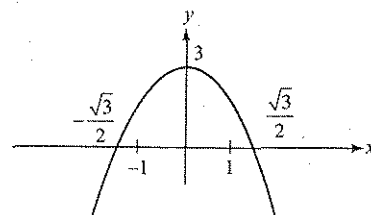


Fig. S-1.6

c. $y = x^2 - 6x + 4$, $a = 1$, $b = -6$, $c = 4$

Vertex: $x = -\frac{b}{2a} = -\frac{(-6)}{2 \times 1} = 3$

At $x = 3, y = -5$; at $x = 0, y = 4$; at $x = 6, y = 4$

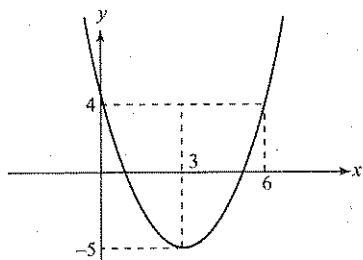


Fig. S-1.7

2. $x = t + t^2, a = 1, b = 1, c = 0$

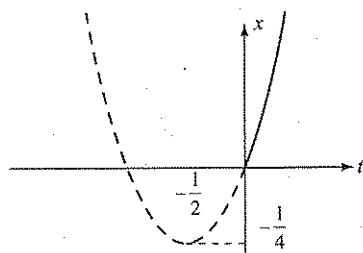


Fig. S-1.8

Vertex: $t = -\frac{b}{2a} = -\frac{1}{2 \times 1} = -\frac{1}{2}$,

at $t = -\frac{1}{2}, x = -\frac{1}{4}$

Exercise 1.4

1. a. $\frac{d}{dx}(9) = 0$, since differentiation of constant is zero.

b. $\frac{d}{dx}(\pi^4) = 0$, since differentiation of constant is zero.

c. $\frac{d}{dx}(2e^3) = 0$, since differentiation of constant is zero.

d. $\frac{d}{dx}(x^2 + 5) = 2x + 0 = 2x$

e. $\frac{d}{dx}[(x+5)^{-1/2}] = -\frac{1}{2}(x+5)^{-3/2}$

f. $\frac{d}{dx}(5x^{3/2}) = 5 \frac{d}{dx}(x^{3/2}) = 5 \times \frac{3}{2} x^{1/2} = \frac{15}{2} x^{1/2}$

g. $\frac{d}{dx}(\sqrt{x+3}) = \frac{d}{dx}[(x+3)^{1/2}] = \frac{1}{2}(x+3)^{-1/2}$

h. $\frac{d}{dx}[(2x^2+9)^3] = 3(2x^2+9)^2(4x) = 12(2x^2+9)^2 x$

2. a. Let $y = (x^2 + 3x)(2x + 7)$

$$\frac{dy}{dx} = (x^2 + 3x) \frac{d(2x+7)}{dx} + \left[\frac{d}{dx}(x^2 + 3x) \right] (2x+7)$$

$$(2x+7) = (x^2 + 3x) 2 + (2x+3)(2x+7) = 6x^2 + 26x + 21$$

b. Let $y = (3x^2 + 2)(4x - 3x^3)$

$$\begin{aligned} \frac{dy}{dx} &= (3x^2 + 2) \frac{d}{dx}(4x - 3x^3) \\ &+ \left[\frac{d}{dx}(3x^2 + 2) \right] (4x - 3x^3) \\ &= (3x^2 + 2)(4 - 9x^2) + 6x(4x - 3x^3) \\ &= -45x^4 + 18x^2 + 8 \end{aligned}$$

c. Let $y = \sqrt{x}(x^3 + x^2 - 3x) = x^{7/2} + x^{5/2} - 3x^{3/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{7}{2} x^{5/2} + \frac{5}{2} x^{3/2} - 3 \times \frac{3}{2} x^{1/2} \\ &= \frac{1}{2\sqrt{x}}(7x^3 + 5x^2 - 9x) \end{aligned}$$

d. Let $y = \sin x \log x$

$$\begin{aligned} \frac{dy}{dx} &= \sin x \frac{d}{dx}(\log x) + \left[\frac{d}{dx}(\sin x) \right] \log x \\ &= \frac{\sin x}{x} + \cos x \log x \end{aligned}$$

3. a. Let $y = \tan^3 x = (\tan x)^3, \frac{dy}{dx} = 3(\tan^2 x)(\sec^2 x)$

b. Let $y = \tan(x^2), \frac{dy}{dx} = \sec^2(x^2) 2x$

c. Let $y = \sin^2(\sqrt{x}) = (\sin \sqrt{x})^2$

$$\begin{aligned} \frac{dy}{dx} &= 2(\sin \sqrt{x}) \cos(\sqrt{x}) \frac{1}{2} x^{-1/2} \\ &= \frac{1}{2\sqrt{x}} \sin(2\sqrt{x}) \end{aligned}$$

4. $x = a(\theta + \sin \theta) \Rightarrow \frac{dx}{d\theta} = a(1 + \cos \theta)$ (1)

$y = a(1 + \sin \theta) \Rightarrow \frac{dy}{d\theta} = a(0 + \cos \theta)$ (2)

Dividing (2) and (1) $\frac{dy/d\theta}{dx/d\theta} = \frac{dy}{dx} = \frac{\sin \theta}{1 + \cos \theta} = \tan(\theta/2)$

5. $\theta = \frac{t^2}{20} + \frac{t}{5}, \omega = \frac{d\theta}{dt} = \frac{2t}{20} + \frac{1}{5} = \frac{t}{10} + \frac{1}{5}$

$$\omega_{t=4s} = \frac{4}{10} + \frac{1}{5} = 0.6 \text{ rad/s}$$

6. $A = 5t^2 + 4t + 8$

Rate of inc. of area:

$$\frac{dA}{dt} = 10t + 4$$

$$\left(\frac{dA}{dt}\right)_{t=3s} = 10 \times 3 + 4 = 34 \text{ m}^2/\text{s}$$

Exercise 1.5

1. a. $\int (x^2 + 2) dx = \int x^2 dx + \int 2 dx = \frac{x^3}{3} + 2x + c$

b. $\int (x^3 - \frac{1}{x} + 3x) dx = \int x^3 dx - \int \frac{1}{x} dx + \int 3x dx$
 $= \frac{x^4}{4} - \log x + \frac{3x^2}{2} + c$

c. $\int (x^{3/2} - x^{1/2} + 5x) dx$
 $= \int x^{3/2} dx - \int x^{1/2} dx + \int 5x dx$
 $= \frac{2x^{5/2}}{5} - \frac{2x^{3/2}}{3} + \frac{5}{2}x^2 + c$

d. $\int (e^{2x} + 5) dx = \int e^{2x} dx + \int 5 dx = \frac{e^{2x}}{2} + 5x + c$

2. a. $\int_1^2 x^3 dx = \left(\frac{x^4}{4}\right)_1^2 = \frac{2^4}{4} - \frac{1^4}{4} = \frac{15}{4}$

b. $\int_u^v Mvdv = \left(M \frac{v^2}{2}\right)_u^v = \frac{M}{2}(v^2 - u^2)$

c. $\int_3^4 \frac{1}{x} dx = (\log_e x)_3^4 = \log_e 4 - \log_e 3 = \log_e (4/3)$

d. $\int_4^9 \sqrt{x} dx = \left(\frac{x^{3/2}}{3/2}\right)_4^9 = \frac{2}{3}(9^{3/2} - 4^{3/2}) = 38/3$

e. $\int_0^{\pi/4} \cos 2x dx = \left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$
 $= \frac{1}{2}[\sin[2[\pi/4]] - \sin[4 \times 0]] = \frac{1}{2}$

Exercise 1.6

1. $y = 6t^2 + 3t + 4 \Rightarrow v = \frac{dy}{dt} = 12t + 3 \text{ m/s}$

2. $v = 12 + 3(t + 7t^2) = 12 + 3t + 21t^2$

$$a = \frac{dv}{dt} = 3 + 42t$$

3. $x = 2 + 5t + 7t^2 \Rightarrow \frac{dx}{dt} = v = 5 + 14t$

a. $v_{t=0} = 5 + 14 \times 0 = 5 \text{ m/s}$

b. $v_{t=4s} = 5 + 14 \times 4 = 61 \text{ m/s}$

c. $a = \frac{dv}{dt} = 14 \text{ m/s}^2$

d. $x_{t=5s} = 2 + 5 \times 5 + 7 \times 5^2 = 202 \text{ m}$

4. $a = t^3 - 3t^2 + 5$

$$v = \int a dt + C_1 = \frac{t^4}{4} - \frac{3t^3}{3} + 5t + C_1$$

At $t = 1 \text{ s}$, $v = 6.25 \text{ m/s}$

$$6.25 = \frac{1^4}{4} - 1^3 + 5 \times 1 + C_1 \Rightarrow C_1 = 2$$

$$\Rightarrow v = \frac{t^4}{4} - t^3 + 5t + 2$$

$$x = \int v dt + C_2 = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + C_2$$

At $t = 1 \text{ s}$, $x = 8.30 \text{ m}$

$$8.30 = \frac{1^5}{20} - \frac{1^4}{4} + \frac{5}{2}(1)^2 + 2 \times 1 + C_2 \Rightarrow C_2 = 4$$

$$x = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5}{2}t^2 + 2t + 4. \text{ now put } t = 2 \text{ s}$$

$$v = \frac{2^4}{4} - 2^3 + 5 \times 2 + 2 = 8 \text{ m/s}$$

$$x = \frac{2^5}{20} - \frac{2^4}{4} + \frac{5}{2}(2)^2 + 2 \times 2 + 4 = 15.6 \text{ m}$$

5. $x = 2t^3 - 3t^2 + 1 \Rightarrow v = \frac{dx}{dt} = 6t^2 - 6t$

a. Put $v = 0 \Rightarrow 0 = 6t^2 - 6t \Rightarrow t = 1 \text{ s}, t = 0 \text{ s}$

b. $x = 0 \Rightarrow 0 = 2t^3 - 3t^2 + 1 \Rightarrow (t-1)^2(2t+1) = 0$
 $\Rightarrow t = 1 \text{ s}, v_{t=1s} = 6 \times 1^2 - 6 \times 1 = 0 \text{ m/s}$

6. $x = t[t-1][t-2] = (t^2-t)(t-2)$
 $= t^3 - 2t^2 - t^2 + 2t = t^3 - 3t^2 + 2t$

$$v = \frac{dx}{dt} = 3t^2 - 6t + 2$$

a. $v_{t=0} = 2 \text{ m/s}$

b. $a = \frac{dv}{dt} = 6t - 6, a_{t=0} = -6 \text{ m/s}^2$

c. $x = 0 \Rightarrow t = 0 \text{ s}, t = 1 \text{ s}, t = 2 \text{ s}$

d. $v = 0 \Rightarrow 3t^2 - 6t + 2 = 0$

$$\Rightarrow t = \frac{6 \pm \sqrt{6^2 - 4 \times 3 \times 2}}{6} = \frac{6 \pm \sqrt{12}}{6}$$

$$= 1 + \frac{1}{\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{3}}$$

$$x = \left(\frac{\sqrt{3}-1}{\sqrt{3}} \right) \left(\frac{\sqrt{3}-1}{\sqrt{3}} - 1 \right) \left(\frac{\sqrt{3}-1}{\sqrt{3}} - 2 \right)$$

$$= \frac{2}{3\sqrt{3}}$$

$$\text{And } x = \left(\frac{\sqrt{3}+1}{\sqrt{3}} \right) \left(\frac{\sqrt{3}+1}{\sqrt{3}} - 1 \right) \left(\frac{\sqrt{3}+1}{\sqrt{3}} - 2 \right)$$

$$= \frac{-2}{3\sqrt{3}}$$

$$\text{e. } a = 6t - 6 = 6 \left(\frac{\sqrt{3}-1}{\sqrt{3}} \right) - 6 = -\frac{6}{\sqrt{3}} = -2\sqrt{3}$$

$$\text{and } a = 6 \left(\frac{\sqrt{3}+1}{\sqrt{3}} \right) - 6 = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

$$7. \text{ a. Distance} = \text{area} = \frac{1}{2} \times 2 \times 10 = 10 \text{ m}$$

$$\text{b. Distance} = \text{area} = 2 \times 10 = 20 \text{ m}$$

$$\text{c. Total distance} = 10 + 20 = 30 \text{ m}$$

$$8. \text{ a. Maximum speed of the car will be at } t = 2 \text{ s, because area is +ve till that time.}$$

$$\text{Change in vel.} = \text{area of } a-t \text{ graph}$$

$$\Rightarrow v_{\max} - 0 = 2 \times 2 \Rightarrow v_{\max} = 4 \text{ m/s}$$

$$\text{b. Since finally the car comes to rest, so both area should be same, } 4 = t_0 \times 4 \Rightarrow t_0 = 1 \text{ s}$$

$$9. \text{ a. } \frac{x_0 - 2}{100} = \frac{4 - x_0}{50} \Rightarrow x_0 = \frac{10}{3} \text{ m}$$

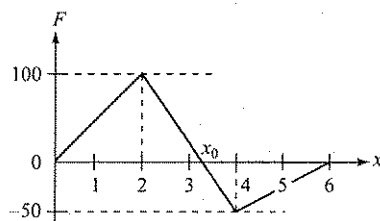


Fig. S-1.9

$$W = \frac{1}{2} x_0 \times 100 - \frac{1}{2} (6 - x_0) 50 = 100 \text{ J}$$

$$\Rightarrow \frac{1}{2} mv^2 = 100 \Rightarrow \frac{1}{2} \times \frac{15}{10} v^2 = 100$$

$$\Rightarrow v = \sqrt{\frac{2000}{15}} = \frac{20}{\sqrt{3}} \text{ m/s}$$

$$\text{b. } W_{\text{rot}_0} = \frac{1}{2} \times \frac{10}{3} \times 100 = \frac{500}{3} \text{ J}$$

$$\Rightarrow \frac{1}{2} mv^2 = \frac{500}{3} \Rightarrow \frac{1}{2} \times \frac{15}{10} v^2 = \frac{500}{3}$$

$$\Rightarrow v = \sqrt{\frac{5000 \times 2}{45}} = \frac{10\sqrt{20}}{3} \text{ m/s}$$

$$= \frac{20}{3} \sqrt{5} \text{ m/s}$$

$$10. x = a + bt + ct^2 \Rightarrow a = \frac{d^2x}{dt^2} = 2c$$

$$11. v = \frac{ds}{dt} = 15 - 0.8t \Rightarrow 7 = 15 - 0.8t \Rightarrow t = 10 \text{ s}$$

$$12. s = t^3 - 6t^2 + 3t + 4 \Rightarrow v = 3t^2 - 12t + 3$$

$$a = 6t - 12 = 0 \Rightarrow t = 2 \text{ s}$$

$$v = 3(2)^2 - 12 \times 2 + 3 = -9 \text{ m/s}$$

$$13. x = ct^2 \Rightarrow v_x = \frac{dx}{dt} = 2ct$$

$$y = bt^2 \Rightarrow v_y = \frac{dy}{dt} = 2bt$$

$$\text{Speed at any time } v = \sqrt{v_x^2 + v_y^2} = 2t\sqrt{c^2 + b^2}$$

$$14. t = \sqrt{x} + 3 \Rightarrow x = (t-3)^2$$

$$v = \frac{dx}{dt} = 2(t-3) = 0 \Rightarrow t = 3 \text{ s}$$

$$x_{t=3\text{s}} = (3-3)^2 = 0$$

Chapter 2

Exercise 2.1

$$1. \text{ Total force } \vec{F} = 2\hat{i} + 3\hat{j} + \hat{k} + \hat{i} + \hat{j} + \hat{k} = 3\hat{i} + 4\hat{j} + 2\hat{k}$$

$$|\vec{F}| \text{ or } F = \sqrt{3^2 + 4^2 + 2^2} = \sqrt{29} \text{ N}$$

$$2. \text{ The required vector is}$$

$$\vec{C} = \frac{|\vec{b}|}{|\vec{a}|} \vec{a} = \frac{\sqrt{7^2 + 24^2}}{\sqrt{3^2 + 4^2}} (3\hat{i} + 4\hat{j})$$

$$= 15\hat{i} + 20\hat{j}$$

$$3. \sin \theta' = \frac{C}{B} \Rightarrow \sin \theta' = \frac{B/2}{B} \Rightarrow \sin \theta' = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

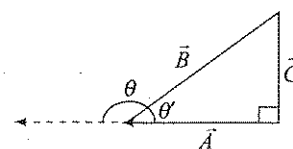


Fig. S-2.1

Now, $\theta + \theta' = 180^\circ \Rightarrow \theta + 30^\circ = 180^\circ \Rightarrow \theta = 150^\circ$

4. $\tan \beta = \frac{4}{3}$ or $\beta = \tan^{-1}\left(\frac{4}{3}\right)$

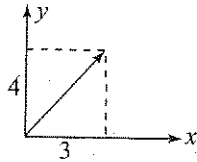


Fig. S-2.2

5. $\vec{A} + \vec{B} + \vec{C} = 0$ or $\vec{C} = -(\vec{A} + \vec{B}) = -(3\hat{i} + 4\hat{k})$

6. $A = 2F, B = \sqrt{2}F, R = \sqrt{10}F$

$$R^2 = A^2 + B^2 = 2AB \cos \theta$$

$$\Rightarrow 10F^2 = 4F^2 + 2F^2 + 2(2F)\sqrt{2}F \cos \theta$$

$$\Rightarrow \cos \theta = 1/\sqrt{2} \Rightarrow \theta = 45^\circ$$

7. Let the two forces be A and B , then given

$$A - B = 10, A^2 + B^2 = 50^2$$

Solve to get $A = 40 \text{ N}, B = 30 \text{ N}$

8. Resultant of two forces, $\sqrt{(F/2)^2 + (F/2)^2} = F/\sqrt{2}$

The third force should be opposite to the resultant and of same magnitude. Hence its magnitude is $F/\sqrt{2}$.

9. Let P be the unknown force

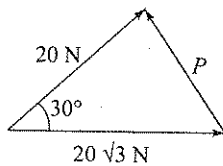


Fig. S-2.3

$$P^2 = (20\sqrt{3})^2 + 20^2 - 2(20)(20\sqrt{3})\cos 30^\circ$$

or $P^2 = 1600 - 1200 = 400$ or $P = 20 \text{ N}$

10. $P + Q = 18$

$$P^2 + Q^2 + 2PQ \cos \theta = 144$$

$$\frac{Q \sin \theta}{P + Q \cos \theta} = \tan 90^\circ = \infty$$

$$\Rightarrow P + Q \cos \theta = 0 \text{ or } Q \cos \theta = -P$$

From equation (ii), $P^2 + Q^2 + 2(P(-P)) = 144$

or $Q^2 - P^2 = 144$ or $(Q - P)(Q + P) = 144$

or $(Q - P)18 = 144$ [From equation (i)]

or $Q - P = 8$

Adding equations (i) and (iii), $2Q = 26$ or $Q = 13$

Now, $P + 13 = 18$

or $P = 5$

Exercise 2.2

1. Area of parallelogram $= |\vec{A} \times \vec{B}|$

2. Their dot product should be zero.

$$(4\hat{i} + \hat{j} - 3\hat{k}) \cdot (2m\hat{i} + 6m\hat{j} + \hat{k}) = 0 \Rightarrow 8m + 6m - 3 = 0$$

$$\Rightarrow m = 3/14$$

3. Given $|\vec{A} \times \vec{B}| = \sqrt{3}\vec{A} \cdot \vec{B} \Rightarrow AB \sin \theta = \sqrt{3} AB \cos \theta$

$$\Rightarrow \tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

4. If θ is the required angle, then

$$\cos \theta = \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot \hat{i}}{\sqrt{1^2 + 1^2 + 1^2} \sqrt{1^2}} = \frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7^\circ \therefore \cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

5.

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= \hat{i}[6 - 8] + \hat{j}[0 + 3] + \hat{k}[4 - 0]$$

$$= -2\hat{i} + 3\hat{j} + 4\hat{k}; |\vec{v}| = v = \sqrt{4 + 9 + 16}$$

$$= \sqrt{29} \text{ units.}$$

6. $\vec{R} = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{A} = 12\hat{i} + 3\hat{j} - 4\hat{k}$

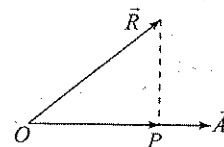


Fig. S-2.4

Component of $\vec{R}$ along $\vec{A}$:

$$OP = \vec{R} \cdot \hat{A} = \frac{\vec{R} \cdot \vec{A}}{A} = \frac{36 - 6 - 4}{\sqrt{12^2 + 3^2 + 4^2}} = 2.$$

Chapter 3

Exercise 3.1

1. No, it is a unit of length.

2. No, the numerical value may change, but not the magnitude.

3. It is not true, constants may have dimensions. For example, G , universal gravitational constant has dimensions.

4. nm stands for nano-meter, a unit of length, mN stands for milli-newton, a unit of force, and Nm stands for newton-meter, a unit of torque.

5. Work, energy and torque all have same dimensions as ML^2T^{-2} .

6. When we add length into length or subtract length from length, we obtain another length.

7. Here Kt is dimensionless. Hence $[K] = [1/t]$

So unit of $K = \text{sec}^{-1} = \text{H}$.

8. Unit of $a = \text{unit of } P \times \text{unit of } V^2$.

$$= \frac{\text{dyne}}{\text{cm}^2} \times \text{cm}^6 = \text{dyne} \times \text{cm}^4$$

9. Planck's constant, $h = \frac{E}{f}$

$$[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

$$10. v = at + \frac{b}{t+c}$$

$$[at] = [v] = [LT^{-1}] \Rightarrow [a] = \frac{[LT^{-1}]}{[T]} \\ = [LT^{-2}]$$

Dimensions of $c = [t] = [T]$ (we can add quantities of same dimensions only).

$$\left[\frac{b}{t+c} \right] = [v] = [LT^{-1}] \Rightarrow [b] = [LT^{-1}][T]$$

$$11. [\phi] = [BA] = \left[\frac{F}{qv} A \right] = \left[\frac{MLT^{-2}L^2}{ATLT^{-1}} \right] = [ML^2T^{-2}A^{-1}]$$

$$12. a[T] = [MLT^{-2}] \Rightarrow a = [MLT^{-3}]$$

$$b[T] = [MLT^{-2}] \text{ or } b = [MLT^{-4}]$$

$$13. 1 \text{ N} = G \frac{1 \text{ kg} \times 1 \text{ kg}}{1 \text{ m}^2} \text{ or } G = 1 \text{ Nm}^2\text{kg}^{-2}$$

$$14. 1 \text{ N} = 6.67 \times 10^{-11} \frac{1 \times 1}{(10)^3} = 6.67 \times 10^{-17} \text{ N}$$

$$15. 1 \text{ J} = 1 \text{ N} \times 1 \text{ km} \\ = 6.67 \times 10^{-17} \text{ N} \times 10^3 \text{ m} = 6.67 \times 10^{-14} \text{ J}$$

Exercise 3.2

1. c. 2.000 cm, because it contains maximum number of significant figures, 4.
2. Not significant. In a number without decimal, the zeros on the right of non-zero digits are not significant.
3. Probable error reduces to $1/5$ as the number of observations is made 5 times.
4. Quantity having higher powers, because errors get multiplied by powers.
5. Here $S = (13.8 \pm 0.2) \text{ cm}$; $t = (4.0 \pm 0.3) \text{ s}$

$$V = \frac{13.8}{4.0} = 3.45 \text{ ms}^{-1}$$

$$\frac{\Delta V}{V} = \pm \left(\frac{\Delta S}{S} + \frac{\Delta t}{t} \right) = \pm \left(\frac{0.2}{13.8} + \frac{0.3}{4.0} \right) \\ = \pm 0.0895$$

$$\Delta V = \pm 0.0895 \times 3.45 = \pm 0.3$$

(rounded off to one place of decimal)

$$V = (3.45 \pm 0.3) \text{ ms}^{-1}$$

$$\text{Percentage error} = \frac{\Delta V}{V} \times 100 \\ = 0.0895 \times 100 = 8.95\%$$

$$6. \text{ Volume } V = \frac{4}{3} \pi r^3 \Rightarrow \Delta V = \frac{4}{3} \pi 3r^2 \Delta r$$

$$\frac{\Delta V}{V} \times 100 = 3 \frac{\Delta r}{r} \times 100 = 3 \times 1\% = 3\%$$

$$7. \text{ In parallel, } R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{5.0 \times 10.0}{5.0 + 10.0} = \frac{50}{15} = 3.3 \Omega$$

$$\text{Also } \frac{\Delta R_p}{R_p} \times 100 = \frac{\Delta R_1}{R_1} \times 100 + \frac{\Delta R_2}{R_2} \times 100 \\ + \frac{\Delta(R_1 + R_2)}{R_1 + R_2} \times 100 \\ = \frac{0.2}{5.0} \times 100 + \frac{0.1}{10.0} \times 100 + \frac{0.3}{15} \times 100 \\ = 7\%$$

$$\therefore R_p = 3.3 \Omega \pm 7\%$$

8. The final result should contain three significant figures.

9. Length $l = 2.53 + 1.27 = 3.80 \text{ cm}$

$$\Delta l = 0.01 + 0.01 = 0.02$$

(Most probable errors of both the rods are added)

Hence true value $= (3.80 \pm 0.02) \text{ cm}$

$$10. \text{ Since } \rho = \frac{m}{\pi r^2 l}$$

$$\left(\frac{\Delta \rho}{\rho} \right) \times 100 = \left(\frac{\Delta m}{m} \times \frac{2\Delta r}{r} + \frac{\Delta L}{L} \right) \times 100 \\ = \left(\frac{0.003}{0.3} + 2 \times \frac{0.005}{0.5} + \frac{0.06}{6} \right) \times 100 \\ = (0.01 + 0.02 + 0.01) \times 100 = 4$$

11. Maximum percentage error in $P = 4\% + 2 \times 2\% = 8\%$

12. Maximum error in density $= 3 + 3 \times 2 = 9\%$.

Exercise 3.3

1. a. Main scale division $= x = 1 \text{ mm}$

$$\text{Let } y \text{ be the vernier scale division, then } y = \frac{90}{100} = 0.9 \text{ mm}$$

$$\text{Vernier constant: } x - y = 1 - 0.9 = 0.1 \text{ mm}$$

2. c. $19x = 20y$ and $x - y = 0.1$

$$\text{Solve to get } x = 2 \text{ mm}$$

3. a. Maximum error is taken to be half of least count.

$$4. \text{ a. Least count} = \frac{\text{Pitch}}{\text{No. of division}} = \frac{1 \text{ mm}}{200} = 5 \times 10^{-3} \text{ mm}$$

Chapter 4

Exercise 4.1

- True, because a revolving object is under acceleration.
 - True, because for a constant velocity, acceleration is zero.
 - True, when a plane takes off it has varying velocity, hence under acceleration.
 - False, a person in circular motion is under acceleration.
 - False, here velocity is varying so the guard is under acceleration.
- No, if velocity of a body is zero, there may be acceleration in the body. For example, a body at its highest point during vertical motion.
 - No, if acceleration of a body is zero, its velocity may be constant or zero.
 - Yes, change in velocity = acceleration $\times$ time, and average acceleration = change in velocity/time
 - True, here $u = 0$, $s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2 \Rightarrow s \propto t^2$
 - True, $D_n = u + \frac{a}{2}(2n-1) \Rightarrow D_n \propto (2n-1)$
- Yes, if its velocity is slowing down towards north.
 - Yes, if velocity and acceleration are in opposite direction.
 - Yes, if an accelerating object starts decelerating.
 - No, because average speed can also be greater than average velocity.
 - No, change in velocity takes place in the direction of acceleration.
- Yes, for example, a freely falling particle at its highest point has zero velocity but accelerating downward.
 - No, if distance is zero then displacement is also zero.
 - Yes, if velocity is also in negative direction.
 - Yes, if motion takes place continuously in one direction.
- Distance = $5 = \pi r \Rightarrow r = 5/\pi$ m
Displacement = $2r = 10/\pi$ m

$$\frac{\text{Distance}}{\text{Displacement}} = \frac{5}{10/\pi} = \frac{11}{7}$$

$$6. \widehat{AB} = R\theta \Rightarrow 10 = \frac{15}{\pi}\theta \Rightarrow \theta = \frac{2\pi}{3}$$

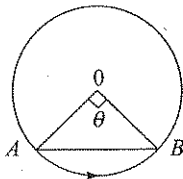


Fig. S-4.1

$$AB = 2(OB) \sin 60^\circ = 2 \times \frac{15}{\pi} \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{\pi} \text{ cm}$$

$$7. v_{av} = \frac{s}{t} = \frac{100 \times 10 + 200 \times 20}{10 + 20} = \frac{500}{3} \text{ m/s}$$

$$8. AB = 10 \times 10 = 100 \text{ m}, BC = 20 \times 10 = 200 \text{ m}$$

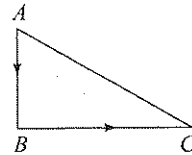


Fig. S-4.2

$$\text{Displacement, } AC = \sqrt{AB^2 + BC^2} = 100\sqrt{5} \text{ m}$$

$$\text{Av. vel.} = \frac{AC}{t} = \frac{100\sqrt{5}}{20} = 5\sqrt{5} \text{ m/s}$$

$$\text{Av. speed} = \frac{AB + BC}{t} = \frac{100 + 200}{20} = 15 \text{ m/s}$$

- $\vec{r}_1 = 3\hat{i} + 2\hat{j}$, $\vec{r}_2 = 7\hat{i} + 6\hat{j}$, $\vec{d} = \vec{r}_2 - \vec{r}_1 = 4\hat{i} + 4\hat{j}$
- $u = 108 \text{ km/h} = 30 \text{ m/s}$, $v = 36 \text{ km/h} = 10 \text{ m/s}$
 $v^2 = u^2 + 2as \Rightarrow 10^2 = 30^2 + 2a \times 200 \Rightarrow a = -2 \text{ m/s}^2$
 $v = u + at \Rightarrow 10 = 30 - 2t \Rightarrow t = 10 \text{ s}$

$$11. s \propto t^2 \Rightarrow s = kt^2 \Rightarrow v = \frac{ds}{dt} = 2kt$$

$$a = \frac{dv}{dt} = 2k \rightarrow (\text{constant})$$

- $8 = 0 + a_1 \times 10 \Rightarrow a_1 = 0.8 \text{ m/s}^2$
 $0^2 = 8^2 + 2a_2 \times 64 \Rightarrow a_2 = -0.5 \text{ m/s}^2$
 $0 = 8 + a_2 t_3 \Rightarrow t_3 = 16 \text{ s}$

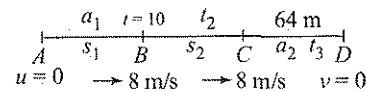


Fig. S-4.3

$$S_1 = \frac{1}{2}a_1 t^2 = \frac{1}{2}(0.8)10^2 = 40 \text{ m}$$

$$S_2 = 584 - 40 - 64 = 480 \text{ m}, t_2 = \frac{s_2}{v} = \frac{480}{8} = 60 \text{ s}$$

$$\text{Total time taken} = 10 + 60 + 16 = 86 \text{ s}$$

$$13. 10 = u + \frac{a}{2}(2 \times 2 - 1) \text{ and } 25 = u + \frac{a}{2}(2 \times 5 - 1)$$

$$\text{Solving, we get } u = \frac{5}{2} \text{ m/s}, a = 5 \text{ m/s}^2$$

$$D_7 = \frac{5}{2} + \frac{5}{2}[2 \times 7 - 1] = 35 \text{ m}$$

$$14. 25 = u + \frac{a}{2}(2 \times 5 - 1) \text{ and } 33 = u + \frac{a}{2}(2 \times 7 - 1)$$

$$\text{Solving, we get } u = 7 \text{ m/s}, a = 4 \text{ m/s}^2$$

$$15. V_{rel} = 50 + 40 = 90 \text{ km/h} = 25 \text{ m/s}$$

$$a_{rel} = 30 + 20 = 50 \text{ cm/s}^2 = 0.5 \text{ m/s}^2$$

$$s_{rel} = u_{rel} t + \frac{1}{2} a_{rel} t^2 \Rightarrow 100 + 100 = 25t + \frac{1}{2} 0.5 t^2$$

$$\text{Solving, we get } t = 10\sqrt{33} - 55 \text{ s}$$

Exercise 4.2

$$1. x = t^3 + 3t^2 + 2t \Rightarrow v = \frac{dx}{dt} = 3t^2 + 6t + 2$$

$$\Rightarrow a = \frac{dv}{dt} = 6t + 6$$

$$2. x = 3 + 8t + 7t^2, v = \frac{dx}{dt} = 8 + 14t$$

$$v_{t=2s} = 8 + 14 \times 2 = 36 \text{ m/s}$$

$$a = \frac{dv}{dt} = 14 \text{ m/s}^2$$

$$3. \int_2^v dv = \int_0^2 a dt$$

$$v - 2 = \left[\frac{3t^3}{3} + \frac{2t^3}{2} + 2t \right]_0^2 \Rightarrow v = 18 \text{ m/s}$$

$$4. x = \frac{a_1}{2}t + \frac{a_2}{3}t^2 \Rightarrow a = \frac{d^2x}{dt^2} = \frac{2a_2}{3}$$

$$5. s = t^3 - 6t^2 + 3t + 4, v = \frac{ds}{dt} = 3t^2 - 12t + 3$$

$$a = \frac{dv}{dt} = 6t - 12$$

$$\text{Put } a = 0 \Rightarrow 6t - 12 = 0 \Rightarrow t = 2 \text{ s}$$

$$v_{t=2s} = 3(2)^2 - 12 \times 2 + 3 = -9 \text{ m/s}$$

Exercise 4.3

1. i. a. False, time of ascent = time of descent.

b. True, $D_n = u + \frac{a}{2}(2n-1) = 0 + \frac{10}{2}(2 \times 3 - 1) = 25 \text{ m}$

c. True, because at the time of dropping the velocity of the packet is upward and same as that of balloon.

d. True, acceleration due to gravity is same for all bodies.

ii. a. only velocity is zero

b. time of descent

c. second half

2. a. $H = 20 = \frac{u^2}{2g} \Rightarrow u = \sqrt{2g \times 20} = 20 \text{ m/s}$

b. $t_a = \frac{u}{g} = \frac{20}{10} = 2 \text{ s}$

c. On hitting the ground, the velocity will be equal to the initial velocity but in opposite direction. Hence, answer is $-u = 20 \text{ m/s}$

d. $S_1 = 20 \times 0.5 - \frac{1}{2} 10 (0.5)^2$,

$$S_2 = 20 \times 2.5 - \frac{1}{2} 10 (3.5)^2$$

$$\text{Required displacement} = S_2 - S_1 = 10 \text{ m}$$

e. $15 = 20t - \frac{1}{2} 10t^2 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1 \text{ s}, 3 \text{ s}$

3. $S = ut + \frac{1}{2}at^2 \Rightarrow -40 = 10t - \frac{1}{2}10t^2 \Rightarrow t^2 - 2t - 8 = 0$
 $\Rightarrow t = 4 \text{ s}, -2 \text{ s}$

Ignoring -ve value, we get $t = 4 \text{ s}$

4. $v = u + at \Rightarrow 0 = 20 - g \sin \theta \times 4$

$$\Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

$$v^2 = u^2 + 2as$$

$$0^2 = (20)^2 + 2(-g \sin \theta)(h/\sin \theta) \Rightarrow h = 20 \text{ m}$$

5. a. $s = ut + \frac{1}{2}at^2 \Rightarrow -200 = 0 \times t + \frac{1}{2}(-10)t^2$
 $\Rightarrow t = \sqrt{40} \text{ s}$

b. $-200 = 10t - \frac{1}{2}10t^2 \Rightarrow t^2 - 2t - 40 = 0$

$$\Rightarrow t = 1 + \sqrt{41} \text{ s}$$

c. $-200 = -10t - \frac{1}{2}10t^2 \Rightarrow t^2 + 2t - 40 = 0$
 $\Rightarrow t = -1 + \sqrt{41} \text{ s}$

6. $H = \frac{u^2}{2g}, u = \frac{1}{2}g(t_1 + t_2), h = \frac{1}{2}gt_1t_2$

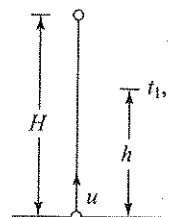


Fig. S-4.4

Given $\frac{t_1}{t_2} = \frac{1}{3}$; solving, we get $h = \frac{3}{4}H$

7. $u = \sqrt{2 \times \frac{g}{8}H} \Rightarrow u = \frac{\sqrt{gH}}{2}$

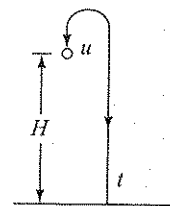


Fig. S-4.5

$$-H = ut - \frac{1}{2}gt^2 \Rightarrow gt^2 - 2ut - 2H = 0$$

$$gt^2 - \sqrt{gH}t - 2H = 0$$

$$\Rightarrow gt^2 - 2\sqrt{gH}t + \sqrt{gH}t - 2H = 0$$

$$\sqrt{g}t[\sqrt{g}t - 2\sqrt{H}] + \sqrt{H}[\sqrt{g}t - 2\sqrt{H}] = 0$$

$$\Rightarrow (\sqrt{g}t + \sqrt{H})(\sqrt{g}t - 2\sqrt{H}) = 0 \Rightarrow t = 2\sqrt{\frac{H}{g}}$$

8. Velocity after 50 m fall, $u_1 = \sqrt{2gh} = \sqrt{100g}$

$$3^2 = u_1^2 + 2 \times (2)h \Rightarrow 9 = 100g - 4h$$

$$h = \frac{100 \times 9.8 - 9}{4} = 242.75 \approx 243 \text{ m}$$

$$\text{Net height} = -243 + 50 = 293 \text{ m}$$

9. $h = \frac{1}{2}gn^2$, $\frac{h}{2} = \frac{1}{2}g(n-1)^2$, where n is the total time taken. Solving, we get

$$n = \frac{\sqrt{2}}{\sqrt{2}-1} \Rightarrow n = \left(\frac{\sqrt{2}}{\sqrt{2}-1} \right) \left(\frac{\sqrt{2}+1}{\sqrt{2}+1} \right) = 2 + \sqrt{2} \text{ s}$$

10. Every ball will travel a distance of 5 m in the last second of going up to its highest point, provided its velocity is $> 10 \text{ m/s}$ initially (It at least should travel a maximum height of 5 m).

Exercise 4.4

1. a. We know that the slope of position-time graph is equal to velocity. So

- as the slope is zero, it is zero velocity
- as the slope is positive and constant, it is constant positive velocity
- as the slope is infinite it is infinite velocity
- as the slope is negative and constant, it is constant negative velocity
- as the slope is positive and increasing, it is positive increasing velocity
- as the slope is positive and decreasing, it is positive decreasing velocity
- as the slope is positive and increasing, it is positive increasing velocity
- as the slope is positive and decreasing, it is positive decreasing velocity

- b. True, as $dv/dt = \text{acceleration}$

- c. Change in velocity

- d. No, because it will indicate infinite acceleration which is not possible

- e. Zero, because acceleration will be zero in uniform motion.

2.

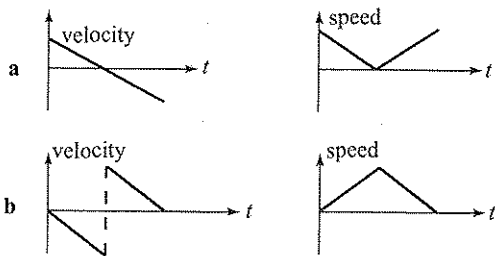


Fig. S-4.6

3. a. $a = 2 \text{ m/s}^2$, $u = 8 \text{ m/s}$

$$v = u + at = 8 + 2t, S = ut + \frac{1}{2}at^2 = 8t + t^2$$

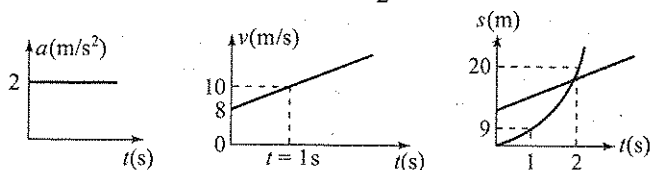


Fig. S-4.7

- b. $a = -2 \text{ m/s}^2$, $u = 8 \text{ m/s}$

$$v = u + at = 8 - 2t, S = ut + \frac{1}{2}at^2 = 8t - t^2$$

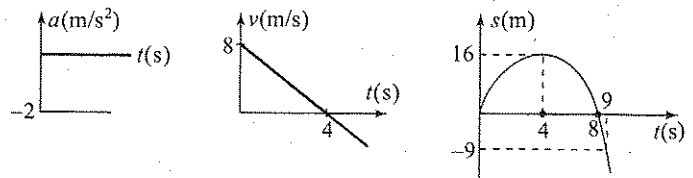


Fig. S-4.8

- c. $a = 2 \text{ m/s}^2$, $u = -8 \text{ m/s}$

$$v = u + at = -8 + 2t, S = ut + \frac{1}{2}at^2 = -8t + t^2$$

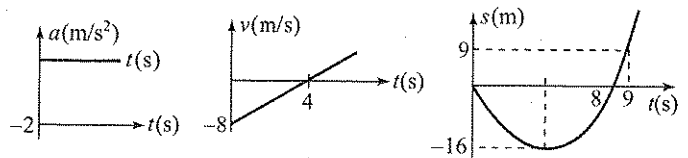


Fig. S-4.9

- d. $a = -2 \text{ m/s}^2$, $u = -8 \text{ m/s}$

$$v = u + at = -8 - 2t, S = ut + \frac{1}{2}at^2 = -8t - t^2$$

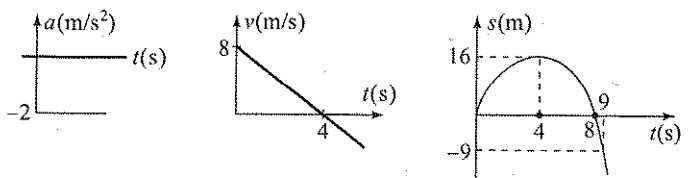


Fig. S-4.10

4. Let us first find the change in velocity. Change in velocity is the area under acceleration-time graph. For first 20 s,

$$\text{Area} = \Delta v = \frac{1}{2} \times 10 \times 20 + 10 \times 20 = 300 \text{ m/s}$$

$$\text{Av. acceleration} = \frac{\Delta v}{\Delta t} = \frac{300}{20} = 15 \text{ m/s}^2$$

5. From 0 to 4 s, at $t = 0$, $a = 5 \text{ m/s}^2$

$$v_1 = u + at = 0 + 5 \times 4 = 20 \text{ m/s}$$

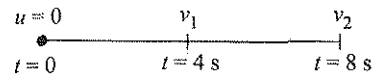


Fig. S-4.11

From 4 to 8 s, $a = -5 \text{ m/s}^2$

$$v_2 = v_1 + at \Rightarrow v_2 = 20 - 5 \times 4 = 0 \text{ m/s}$$

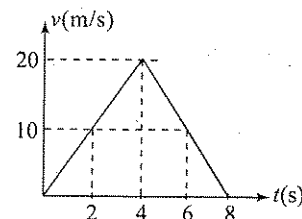


Fig. S-4.12

Maximum velocity is 20 m/s at $t = 4$ s.

Displacement from 2 to 6 s = area from 2 to 6 s

$$= 10 \times 4 + (1/2) \times 4 \times 10 = 60 \text{ m}$$

Since the motion takes place along the same direction only, so distance = displacement = 60 m

6. No, As shown, at a given instant of time, the body is at two different positions which is not possible.



Fig. S-4.13

Chapter 5

Exercise 5.1

1. a. True, as the ball gains horizontal velocity due to the motion of the train and also it moves downward with constant acceleration resulting the net path to be parabolic w.r.t. an observer on the ground. However w.r.t. an observer in train the path of the ball will be a straight line vertically downward.
- b. False, the path is a straight line vertically downward.
- c. True, let the boat moves at an angle θ as shown in Fig. S-5.1

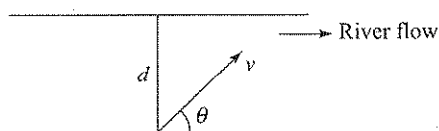


Fig. S-5.1

$$\text{Then time taken to cross the river, } t = \frac{d}{V \sin \theta}$$

$$\text{For } t_{\min}, \sin \theta = 1 \Rightarrow \theta = 90^\circ$$

Hence, boat needs to move perpendicularly to the direction of flow of river.

- d. False, θ should be $> 90^\circ$

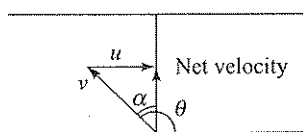


Fig. S-5.2

$$\text{e. True, } \vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$$

Let rain be falling vertically

$$\tan \theta = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1} \left(\frac{v_m}{v_r} \right)$$

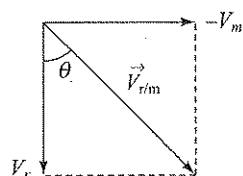


Fig. S-5.3

$$\text{f. True, } \sin \alpha = \frac{u}{v} = \frac{0.25}{0.5} = \frac{1}{2} \Rightarrow \alpha = 30^\circ$$

$$\theta = 90^\circ + \alpha = 120^\circ$$

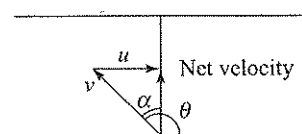


Fig. S-5.4

2. Distance to be covered = 1000 + 200 = 1200 m
Velocity = 72 km/h = 20 m/s

$$t = \frac{\text{Distance}}{\text{Velocity}} = \frac{1200}{20} = 60 \text{ s}$$

3. Required separation = $(v_2 - v_1)t$

$$= (72 - 36) \frac{20}{60} = 12 \text{ km}$$

$$4. t = \frac{\text{Total length}}{\text{Relative velocity}} = \frac{110 + 90}{(36 + 54) \times (5/18)} = 8 \text{ s}$$

5. Required separation = $(v_2 - v_1)t$

$$= (10 - 6)3 = 12 \text{ km}$$

6. $v_A = 54$, $v_B = -90$



Fig. S-5.5

Relative velocity of B w.r.t. A, $v_{B/A} = v_B - v_A$

$$= -90 - 54 = -144 \text{ km/h} = -40 \text{ m/s} = 40 \text{ m/s, west}$$

Relative velocity of ground w.r.t. B

$$v_{G/B} = v_G - v_B = 0 - (-90) = 90 \text{ km/h} = 25 \text{ m/s, east}$$

7. 6 m/s due south. Because relative velocity of B w.r.t. A will be equal and opposite to the relative velocity of A w.r.t. B.

$$8. \text{ a. } \vec{v}_A = 5\hat{i}, \vec{v}_B = 7\hat{j}$$

Relative velocity of B w.r.t. A

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 7\hat{j} - 5\hat{i} = -5\hat{i} + 7\hat{j}$$

$$v_{BA} = \sqrt{5^2 + 7^2} = \sqrt{74} \text{ m/s}$$

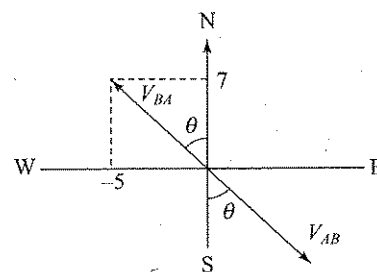


Fig. S-5.6

$$\tan \theta = \frac{5}{7} \Rightarrow \theta = \tan^{-1} \left(\frac{5}{7} \right) \text{ W of N}$$

b. $v_{AB} = \sqrt{74} \text{ m/s}$, at $\tan^{-1}\left(\frac{5}{7}\right)$ E of S

9. $\vec{v}_c = -15\hat{j}$

$$\vec{v}_b = -10\sqrt{2} \cos 45^\circ \hat{i} + 10\sqrt{2} \sin 45^\circ \hat{j} = -10\hat{i} + 10\hat{j}$$

$$\vec{v}_{c/b} = \vec{v}_c - \vec{v}_b = -10\hat{i} - 25\hat{j}$$

$$v_{c/b} = \sqrt{10^2 + 25^2} = \sqrt{725} = 5\sqrt{29} \text{ m/s}$$

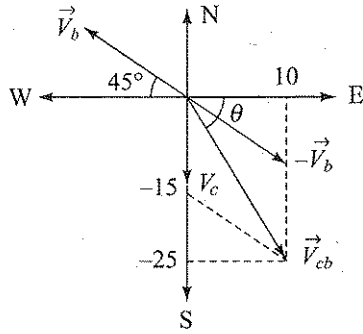


Fig. S-5.7

$$\tan \theta = \frac{25}{10} = 2.5, \quad \theta = \tan^{-1}(2.5) \text{ S of E}$$

10. i. When she walks in same direction, relative velocity of woman w.r.t. ground:

$$v_1 = 1 + 1.5 = 2.5 \text{ m/s}, \text{ time taken} = \frac{s}{v_1} = \frac{35}{2.5} = 14 \text{ s}$$

- ii. When she walks in opposite direction, relative velocity of woman w.r.t. ground:

$$v_2 = 1.5 - 1 = 0.5 \text{ m/s}, \text{ time taken} = \frac{s}{v_2} = \frac{35}{0.5} = 70 \text{ s}$$

11. a. $t = \frac{d}{v \sin \theta} = \frac{800}{2.5 \times \frac{5}{18} \times \sin 120^\circ} = 1330 \text{ s}$

b. $x = (u + v \cos \theta) t = (1.5 + 2.5 \cos 120^\circ) \times \frac{5}{18} \times 1330 = 92.4 \text{ m}$

12. $\vec{v}_c = 40 \times \frac{5}{18} = \frac{100}{9} \text{ km/h}$, $\vec{v}_r = 20 \text{ m/s}$

$$\theta = \tan^{-1}\left(\frac{v_c}{c_r}\right) = \tan^{-1}\left(\frac{5}{9}\right)$$

Exercise 5.2

- No, they do not depend upon mass.
- All quantities will increase, because g is less on moon in comparison to that on earth.
- (a) Highest point.
(b) Point of projection and point of return.
- Let the other angle is θ , then $\theta + 60^\circ = 90^\circ \Rightarrow \theta = 30^\circ$

$$T = \frac{2u \sin \theta}{g}, \text{ as } \theta < 60^\circ, \text{ so it will take less time.}$$

- The gun is lined slightly above the target. This is to accommodate the effect of gravity, as due to gravity the bullet will descend some height by the time it reaches the target.
- True, acceleration during projectile motion is same at all points and is equal to g , the acceleration due to gravity. It remains constant both in magnitude and direction.
- At highest point, velocity is in horizontal direction and acceleration (due to gravity, g) is in vertical direction. So angle between them is 90° .

8. $\frac{H_1}{H_2} = \frac{\sin^2 \alpha}{\sin^2 (90^\circ - \alpha)} \Rightarrow \frac{H_1}{H_2} = \tan^2 \alpha : \text{ as } H \propto \sin^2 \alpha$

- (i) Highest point, height is maximum.
(ii) Point of projection, velocity is maximum, hence K.E. is maximum.
(iii) Same at all points, sum of K.E. + P.E. = total mechanical energy, it always remains same.
- Since their time of flight is same, so maximum height attained by them will also be same. It is because both depend only upon vertical component of velocity.

11. $T = \frac{2u \sin \theta}{g} = \frac{2 \times 25 \times \sin 60^\circ}{10} = \frac{5\sqrt{3}}{2} \text{ s}$

$$R = u \cos \theta T = 25 \cos 60^\circ \times \frac{5\sqrt{3}}{2} = \frac{125\sqrt{3}}{4} \text{ m}$$

12. a. $\vec{v} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}$
 $= 100 \cos 30^\circ \hat{i} + (100 \sin 30^\circ - 10 \times 2) \hat{j} = 50\sqrt{3} \hat{i} + 30 \hat{j}$

b. $\vec{u} = u \cos \theta \hat{i} + u \sin \theta \hat{j} = 100 \cos 30^\circ \hat{i} + 100 \sin 30^\circ \hat{j}$
 $= 5\sqrt{3} \hat{i} + 50 \hat{j}$

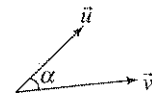


Fig. S-5.8

Angle between $\vec{u}$ and $\vec{v}$, $\cos \alpha = \frac{\vec{u} \cdot \vec{v}}{uv}$

$$= \frac{50\sqrt{3} \times 50\sqrt{3} + 30 \times 50}{\sqrt{(50\sqrt{3})^2 + 30^2} \sqrt{(50\sqrt{3})^2 + (50)^2}} = \frac{9}{2\sqrt{21}}$$

$$\Rightarrow \alpha = \cos^{-1}\left[\frac{9}{2\sqrt{21}}\right]$$

c. $H = \frac{u_y^2}{2g} = \frac{50^2}{2 \times 10} = 125 \text{ m}$

13. a. $-15 = 20 \sin 30^\circ t - \frac{1}{2} 10 t^2 \Rightarrow t^2 - 2t - 3 = 0$
 $\Rightarrow (t - 3)(t + 1) = 0 \Rightarrow t = 3 \text{ s}$

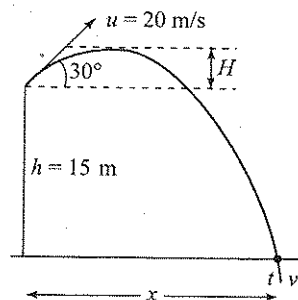


Fig. S-5.9

$$\text{b. } x = 20 \cos 30^\circ \times t = 20 \frac{\sqrt{3}}{2} \times 3 = 30\sqrt{3} \text{ m}$$

$$\text{c. } v^2 = u^2 + 2gh = 20^2 + 2 \times 10 \times 15 = 700$$

$$\Rightarrow v = 10\sqrt{7} \text{ m/s}$$

Let the angle made by v with the horizontal is α

$$\text{Then } v \cos \alpha = u \cos \theta \Rightarrow 10\sqrt{7} \cos \alpha = 20 \cos 30^\circ$$

$$\Rightarrow \cos \alpha = \sqrt{\frac{3}{7}}$$

$$\text{d. } H = \frac{u^2 \sin^2 \theta}{2g} = \frac{20^2 \sin^2 30^\circ}{2 \times 10} = 5 \text{ m}$$

Maximum height attained above ground

$$= h + H = 15 + 5 = 20 \text{ m}$$

14. Both will reach at the same time, because initial vertical velocity of both is zero and the time of flight depends upon the vertical component of velocity only.

15. On reaching the ground:

$$V_A = \sqrt{u^2 + 2gh}, V_B = \sqrt{2gh}$$

$$\text{Clearly, } v_A > v_B$$

Exercise 5.3

$$1. \text{ a. } \omega = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/s}$$

$$\text{b. } \omega = \frac{2\pi}{3600} = \frac{\pi}{1800} \text{ rad/s}$$

$$\text{c. } \omega = \frac{2\pi}{12 \times 3600} = \frac{\pi}{21600} \text{ rad/s}$$

$$2. a_c = \omega^2 r = \left(\frac{10 \times 2\pi}{20} \right)^2 \times 2 = 19.74 \text{ m/s}^2$$

$$3. a = \sqrt{\left(\frac{30^2}{500} \right)^2 + 2^2} = 2.7 \text{ m/s}^2$$

$$a = \sqrt{\left(\frac{(15/2)^2}{80} \right)^2 + (0.5)^2} = 0.86 \text{ m/s}^2$$

$$\tan \alpha = \frac{a_t}{a_c} = \frac{0.5 \times 4 \times 80}{15 \times 15} = \frac{32}{45} \Rightarrow \alpha = \tan^{-1} \left(\frac{32}{45} \right)$$

$$4. a_c = \left(\frac{2\pi}{2} \right)^2 \frac{100}{100} = 9.87 \text{ m/s}^2$$

$$5. a_c = \omega^2 R = \left(\frac{2\pi}{T} \right)^2 R = \frac{4\pi^2 R}{T^2}$$

Chapter 7

Exercise 7.1

1. a. The horse cart will not have any reaction on it in empty space.
- b. Because of inertia.
- c. As $F = \frac{\Delta P}{\Delta t}$, so if the player moves his hands backward, Δt will increase and hence F will decrease.
2. a. Force on a body acts in a direction of change in its momentum. In both cases change in momentum of ball is along XO . Hence, in both the cases force will be along x -direction.

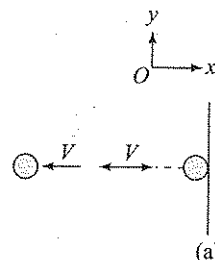


Fig. S-7.1

- b. Case (a)

Momentum of the ball before reflection = mv (along OX)

Momentum after reflection = mg (along XO)

$$= -mv \text{ (along } XO)$$

Now, I = change in momentum

$$= -mv - mv = -2mv \text{ (along } OX)$$

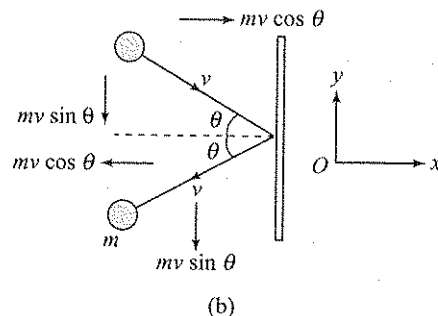


Fig. S-7.2

- Case (b)

$$I' = -mv \cos \theta - mv \cos \theta = -2mv \cos \theta$$

$$\text{Ratio} = \frac{I}{I'} = \frac{-2mv}{-2mv \cos \theta}; \frac{I}{I'} = \frac{1}{\cos \theta}$$

3. Just before $t = 2$ s, the velocity of the particle is

$$u = \frac{2-0}{2-0} = 1 \text{ cm/s} = 0.01 \text{ m/s}$$

Just after $t = 2$ s, the velocity of the particle is

$$v = \frac{2-0}{2-0} = -1 \text{ cm/s} = -0.01 \text{ m/s}$$

The magnitude of impulse $J = |m(v - u)|$

$$= 10.04 (-0.01 - 0.01) = 8 \times 10^{-4} \text{ Ns}$$

The given $x-t$ graph (see Fig. 7.11, Chapter 7) may represent the repeated rebounding of a particle between two elastic walls at $x = 0$ and $x = 2$ cm. The particle will get an impulse of 8×10^{-4} Ns after every 2 s.

4. Velocity of the ball just before collision: $v^2 = 0 + 2gh$

$$v = v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}$$

$\therefore$ Momentum of the ball before collision, $\vec{P}_i = m\vec{v}_i$

$$= 0.050 \times (-10) \hat{j} \text{ Ns} = -0.50 \hat{j} \text{ Ns}$$

Velocity of the ball just after collision, $v_f = \sqrt{2gh}$

$$= \sqrt{2 \times 10 \times 1.25} = 5.0 \text{ m/s}$$

$\therefore$ Momentum of the ball just after collision,

$$\vec{P}_f = m\vec{v}_f = 0.050 \times (5.0 \hat{j}) = 0.25 \hat{j} \text{ Ns}$$

Now impulse imparted by the ground on the ball

$$\vec{I} = \Delta \vec{P} = \vec{P}_f - \vec{P}_i = 0.25 \hat{j} - (-0.50 \hat{j}) = 0.75 \hat{j} \text{ Ns}$$

$$\text{Required force, } F = \frac{\Delta P}{\Delta t} = \frac{0.75}{0.1} = 7.5 \text{ N}$$

5. After 3 s of pouring, the bucket contains $(3 \text{ s})(0.25 \text{ L/s}) = 0.75 \text{ L}$ of water, with mass $0.75 \text{ L} \times (1 \text{ kg/L}) = 0.75 \text{ kg}$, and feeling gravitational force $0.75 \text{ kg} (9.8 \text{ m/s}^2) = 7.35 \text{ N}$. The scale through the bucket must exert 7.35 N upward on this stationary water to support its weight. The scale must exert another 7.35 N to support the 0.75 kg bucket itself.

Water is entering the bucket with the speed given by

$$mgy_{\text{top}} = (1/2)mv_{\text{impact}}^2$$

$$v_{\text{impact}} = (2gy_{\text{top}})^{1/2} = [2(9.8 \text{ m/s}^2)(2.6 \text{ m})]^{1/2} = 7.14 \text{ m/s}$$

downward.

The scale exerts an extra upward force to stop the downward motion of this additional water, as described by

$$m v_{\text{impact}} + F_{\text{extra}} t = m v_f$$

The rate of change of momentum is the force itself

$$(dm/dt) v_{\text{impact}} + F_{\text{extra}} = 0$$

$$F_{\text{extra}} = -(dm/dt) v_{\text{impact}} = -(0.25 \text{ kg/s})(-7.14 \text{ m/s}) = +1.78 \text{ N}$$

Altogether the scale must exert

$$7.35 \text{ N} + 7.35 \text{ N} + 1.78 \text{ N} = 16.5 \text{ N}$$

6. Consider a mass Δm of liquid flowing across the corner in time Δt . We will apply Newton's second law of motion to the mass Δm .

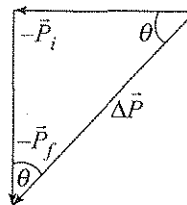


Fig. S-7.3

Magnitude of initial momentum $= P_i = (\Delta m)V$

Magnitude of final momentum $= P_f = (\Delta m)V$

Change in momentum $= \Delta \vec{P} = \vec{P}_f - \vec{P}_i$

$\Delta \vec{P}$ can be calculated by the vector subtraction, geometrically. As $P_i = P_f \Rightarrow \theta = 45^\circ$

$$\Delta P = \sqrt{(P_i^2 + P_f^2)} = \sqrt{2(\Delta m)^2 V^2} = \sqrt{2} \Delta m V$$

$$\text{Force exerted on the liquid} = \frac{\Delta P}{\Delta t} = \frac{\sqrt{2} \Delta m V}{\Delta t} = \sqrt{2} V \rho A V.$$

Hence, the pipe must be pushed at the corners with force $\sqrt{2} \rho A V^2$ at an angle of 45° .

7. Impulse, $I = 2 mu \cos 30^\circ = 2 \times 3 \times 10 \times \frac{\sqrt{3}}{2} = 30\sqrt{3} \text{ Ns}$

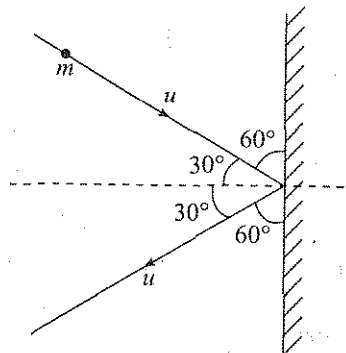


Fig. S-7.4

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{30\sqrt{3}}{0.2} = 150\sqrt{3} \text{ N}$$

8. a. The impulse is to the right and equal to the area under the $F-t$ graph (see Fig. 7.15, Chapter 7): $I = \{[5 + 1]/2\} \times 4 = 12 \text{ Ns}$
- b. $m\vec{v}_i + \vec{F}t = m\vec{v}_f$
 $\Rightarrow 2.5 \times 0 + 12 = (2.5)v \Rightarrow v = 4.8 \text{ m/s}$
- c. From the same equation,
 $\Rightarrow 2.5 \times (-2) + 12 = (2.5)v \Rightarrow v = 2.8 \text{ m/s}$
- d. $F_{\text{avg}} \Delta t = 12.0 \Rightarrow F_{\text{avg}} = 2.40 \text{ N}$

Exercise 7.2

- False. F is not acting on M .
- No force is acting in horizontal direction on m .

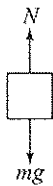


Fig. S-7.5

No relative motion of m . Acceleration of m is zero.

3. Weight shown by machine is $N = Mg \cos \theta$. So the mass shown by the machine will be $N/g = M \cos \theta$

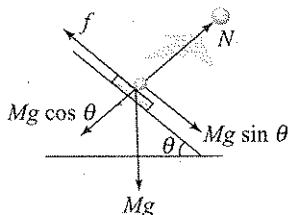


Fig. S-7.6

4. $10 + 10 = mg \Rightarrow m = 2 \text{ kg}$

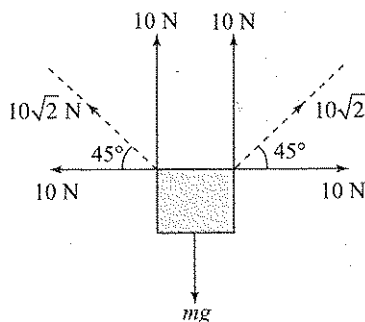


Fig. S-7.7

5. Let the man exerts force F on string.

$$2F = F + N + mg \quad (1)$$

$$F + N = Mg \quad (2)$$

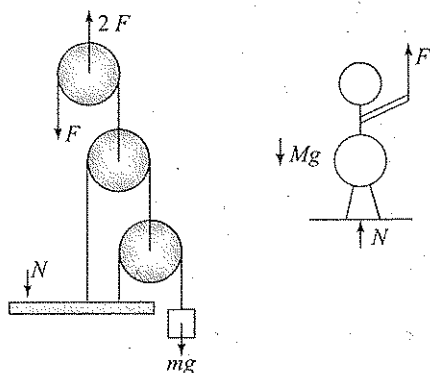


Fig. S-7.8

From equations (1) and (2): $F = \frac{(M+m)g}{2}$ and $N = \frac{(M-m)g}{2}$

6. From the FBD of the block it is clear that tension in lower cord is 400 N. Figure S-7.9 shows the junction where the three cords meet.

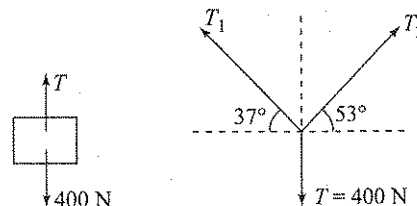


Fig. S-7.9

As the system is in equilibrium, net sum of all the forces at the junction must be zero. For this we resolve the tensions in the horizontal and the perpendicular direction. In horizontal direction, we get

$$\cos 53^\circ T_2 - \cos 37^\circ T_1 = 0 \quad (i)$$

In vertical direction, we get

$$\sin 37^\circ T_1 + \sin 53^\circ T_2 - 400 = 0 \quad (ii)$$

On solving the above equations, we get

$$T_1 = 240 \text{ N and } T_2 = 320 \text{ N}$$

7. A horizontal string cannot balance a vertical weight.

$$\sum F_y = 0 \Rightarrow T_1 = 100 \text{ N, } \sum F_x = 0 \Rightarrow T_2 = 0$$

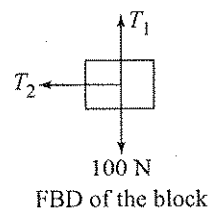


Fig. S-7.10

8. The free body diagram is drawn so that only the string T_4 is cut, as shown in Fig. S-7.11

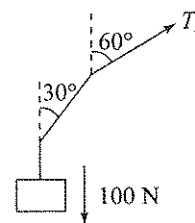


Fig. S-7.11

$$\sum F_y = 0 \Rightarrow T_4 \cos 60^\circ = 100 \Rightarrow T_4 = 200 \text{ N}$$

9. Pseudo force $= ma$ in backward direction.

10. In all the three cases the spring balance reads 10 kg.

Let us cut a section inside the spring as shown in Fig. S-7.12

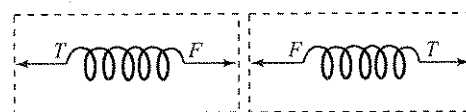


Fig. S-7.12

As each part of the spring is at rest, so $F = T$. As the block is stationary, so $T = F = 100 \text{ N}$.

11. Let T be the reading of the spring balance. Then

for 20 kg block: $20g - T = 20a$

(i)

for 10 kg block: $T - 10g = 10a$

(ii)

Solving equations (i) and (ii), we get $a = \frac{g}{3}$, $T = \frac{40g}{3}$.

So the spring balance reading is $\frac{40}{3} \text{ kg}$.

12. The FBD for the two cases is shown in Fig. 7.13.

In first case, let the force exerted by the man on the floor is N_1 . Consider the forces inside the dotted box,

We have, $N_1 = T + 50g$

Block is to be raised without acceleration, so $T = 25g$

$\therefore N_1 = 25g + 50g = 75g = 75 \times 10 = 750 \text{ N}$

In second case, let the force exerted by the man on the floor is N_2 . Consider the forces inside the dotted box,

we have, $N_2 = 50g - T$, and $T = 25g$

$\therefore N_2 = 50g - 25g = 25g = 25 \times 10 = 250 \text{ N}$

As the floor yields to a downward force of 700 N, so the man should adopt the second case.

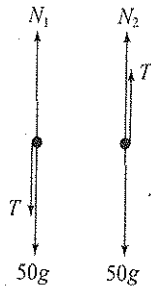


Fig. S-7.13

13. Since both the blocks are in contact, therefore they will move together with an acceleration

$$a_x = \frac{F_{\text{net}}}{M_{\text{total}}} = \frac{3}{2+1} = 1 \text{ m/s}^2 \quad (\text{i})$$

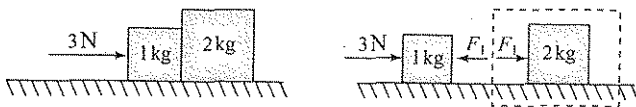


Fig. S-7.14

Let the force of interaction between them is F_1 .

By Newton's second law for 2 kg block, we have

$$F_1 = 2a = 2 \times 1 = 2 \text{ N}$$

The same result can also be obtained from 1 kg block

$$3 - F_1 = 1a \Rightarrow F_1 = 2 \text{ N}$$

Let the force of interaction between them is F_2

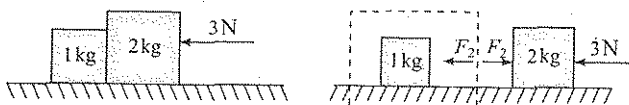


Fig. S-7.15

By Newton's second law for 1 kg block,

we have $F_2 = 1a = 1 \times 1 = 1 \text{ N}$

14. By Newton's second law, we have

$$1g - T_1 = 1a \quad (\text{i})$$

$$(T_1 + 2g) - T_2 = 2a \quad (\text{ii})$$

$$\text{and } T_2 - 2g = 2a \quad (\text{iii})$$

Solving equations (i), (ii), and (iii), we get

$$a = 2 \text{ m/s}^2, T_1 = 8 \text{ N},$$

$$\text{and } T_2 = 24 \text{ N}$$

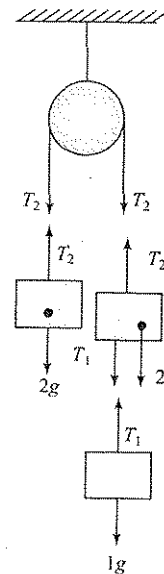


Fig. S-7.16

By shortcut method:

$$a = \frac{[(2+1)-1]g}{(1+2+2)} = 2 \text{ m/s}^2$$

$$T_1 = m(g - a) = 1(10 - 2) = 8 \text{ N}$$

- 15.

Note: As the motion of the block is on smooth horizontal surface, so no need to mark the forces along vertical direction.

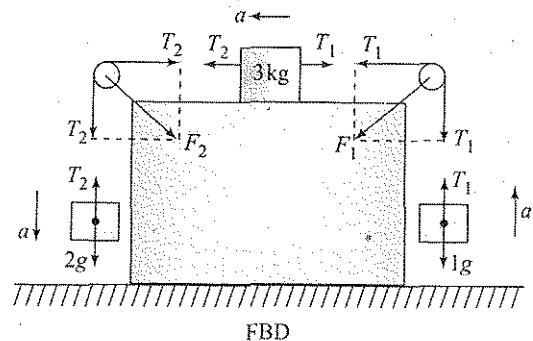


Fig. S-7.17

By Newton's second law, we have

$$T_1 - 1g = 1a \quad (i)$$

$$T_2 - T_1 = 3a \quad (ii)$$

$$\text{and } 2g - T_2 = 2a \quad (iii)$$

Solving above equations, we get

$$a = \frac{g}{6} \text{ m/s}^2, \quad T_1 = \frac{7g}{6} \text{ N}, \quad \text{and } T_2 = \frac{5g}{3} \text{ N}$$

$$\text{Force on pulley } P_1, F_1 = \sqrt{T_1^2 + T_1^2} = \sqrt{2} T_1$$

$$\text{Force on pulley } P_2, F_2 = \sqrt{T_2^2 + T_2^2} = \sqrt{2} T_2$$

By shortcut method:

$$a = \frac{\text{Unbalanced load}}{\text{Total mass}} = \frac{(2-1)g}{(1+3+2)}$$

$$= \frac{g}{6} \text{ m/s}^2$$

$$T_1 = m_{\text{up}}(g+a) = 1 \left(g + \frac{g}{6} \right) = \frac{7g}{6} \text{ N}$$

$$T_2 = m_{\text{down}}(g-a) = 2 \left(g - \frac{g}{6} \right) = \frac{5g}{3} \text{ N}$$

16. By Newton's second law

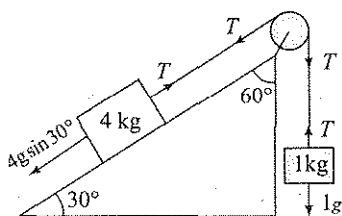


Fig. S-7.18

$$4g \sin 30^\circ - T = 4a \quad (i)$$

$$\text{and } T - 1g = 1a \quad (ii)$$

Solving equations (i) and (ii), we get

$$a = 2 \text{ m/s}^2 \quad \text{and } T = 12 \text{ N}$$

By shortcut method: $a = \frac{\text{Unbalanced load}}{\text{Total mass}}$

$$= \frac{4g \sin 30^\circ - 1g}{4+1} = 2 \text{ m/s}^2$$

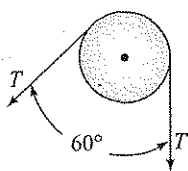


Fig. S-7.19

$$T = M_{\text{up}}(g+a) = 1(10+2) = 12 \text{ N}$$

$$\text{Force on the pulley, } F = \sqrt{T^2 + T^2 + 2TT \cos 60^\circ} = \sqrt{3} T$$

17. Let tension in the rope is T . By Newton's second law, we have

$$T - (10g + 8g) = 10 \times 2 + 8 \times 0$$

$$\text{or } T = 18g + 20 = 200 \text{ N}$$

18. When the student takes a step on the platform of the balance, there is a sudden downward change in motion in the beginning, so there is downward acceleration. Also, there is a sudden upward change at the end of the step, so the acceleration is in upward direction.

19. Suppose $F_2 > F_1$. Then the motion of the rod will take place in the direction of the force F_2 with acceleration, say, a . Considering the motion of the entire rod, we have $F_2 - F_1 = \text{net force on the rod} = mL \times a$ where m is the mass of the rod per unit length of it.

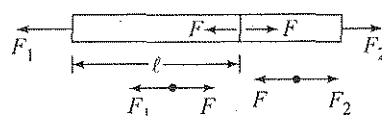


Fig. S-7.20

Considering the motion of the length ℓ from the first end $F - F_1 = m\ell a$. Dividing, we get

$$\frac{F - F_1}{F_2 - F_1} = \frac{\ell}{L} \quad \text{or } F = \frac{(F_2 - F_1)\ell}{L} + F_1$$

20. Let acceleration of mass m relative to wedge down the plane is a_r . Its absolute acceleration in horizontal direction is $a_r \cos 60^\circ - a$ (towards right). Hence, let N be the normal reaction between the mass and the wedge. Then, $N \sin \theta = Ma = m(a_r \cos 60^\circ - a)$

$$\text{or, } a_r = \frac{(M+m)a}{m \cos 60^\circ} = \frac{(4+1)(2)}{(1)(1/2)} = 20 \text{ m/s}^2$$

21. The point to this problem is that the monkey and the bananas have the same weight, and the tension in the string is the same at the point where the bananas are suspended and where the monkey is pulling. In all the cases, the monkey and the bananas will have the same net force and hence the same acceleration, direction, and magnitude.

a. The bananas move up.

b. The monkey and bananas always move at the same velocity, so the distance between them stays the same.

c. Both the monkey and the bananas are in free fall, and as they have the same initial velocity, the distance between them does not change.

d. The bananas will slow down at the same rate as the monkey; if the monkey comes to a stop, so will the bananas.

22. From $t = 0$ to 2 s, the lift has a uniform acceleration

$$a_1 = \frac{\Delta v}{\Delta t} = \frac{3.6-0}{2-0} = 1.8 \text{ m/s}^2$$

From $t = 2$ to 10 s, the lift has a constant speed and hence acceleration a_2 during the period is 0 m/s^2 .

From $t = 10$ s to 12 s, the lift has a uniform acceleration. If a_3 is the acceleration during the period, then

$$a_3 = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{0-3.6}{12-10} = -1.8 \text{ m/s}^2$$

- a. here $m = 1500 \text{ kg}$, $g = 9.8 \text{ m/s}^2$
- At $t = 1 \text{ s}$, $a = a_1 = 1.8 \text{ m/s}^2$
 $\therefore$ tension $T_1 = m(g + a) = 1500(10 + 1.8) = 17,700 \text{ N}$
 - At $t = 6 \text{ s}$, $a = a_2 = 0$
 $\therefore$ tension $T_2 = m(g + a_2) = mg = 1500 \times 10 = 15,000 \text{ N}$
 - At $t = 11 \text{ s}$ $a = a_3 = -1.8 \text{ m/s}^2$
 $\therefore$ tension $T_3 = m(g + a_3) = 1500(10 - 1.8) = 12,300 \text{ N}$
- b. Height reached by the lift
 $=$ Area of enclosed by $v-t$ curve and t -axis (see Fig. 7.86, Chapter 7)
 $=$ Area of quadrilateral $OABC = 36 \text{ m}$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{26}{12} = 3 \text{ m/s}$$

$$\begin{aligned} \text{Average acceleration} &= \frac{\text{Net change in velocity}}{\text{Total time}} \\ &= \frac{0}{12} = 0 \text{ m/s}^2 \end{aligned}$$

23. a. When the lift moves upward with an acceleration, then the apparent weight is

$$\begin{aligned} W' &= W + ma \text{ or } W' = W + \frac{W}{g}a \\ &= W \left(1 + \frac{a}{g} \right) \end{aligned}$$

here $W' = 50 \text{ N}$, $a = 2.45 \text{ m/s}^2$, $g = 9.8 \text{ m/s}^2$

$$50 = \left(1 + \frac{2.45}{9.8} \right) W \times \frac{5}{4} \Rightarrow W = \frac{4}{5} \times 50 = 40 \text{ N}$$

- b. Again if a is upward acceleration, then $W' = W$

$$\left(1 + \frac{a}{g} \right) \text{ gives}$$

$$30 = 40 \left(1 + \frac{a}{g} \right) \Rightarrow \frac{a}{g} = \frac{30}{40} - 1 = -\frac{1}{4}$$

$$a = -\frac{g}{4} \Rightarrow \frac{a}{g} = \frac{30}{40} - 1 = -\frac{1}{4} \Rightarrow a = -\frac{g}{4}$$

(Negative sign shown that lift descends)

i.e., lift moves downward with acceleration $\frac{g}{4}$.

- c. When the elevator cable breaks, the lift falls freely with acceleration g .

$$\therefore \text{apparent weight } W' = W \left(1 - \frac{g}{g} \right) = 0 \text{ N}$$

24. a. $\sum F_x = Ma$

$$a = \frac{T}{M} = \frac{18.0 \text{ N}}{5.00 \text{ kg}} = 3.60 \text{ m/s}^2 \text{ to the right.}$$

- b. If $v = \text{constant}$, $a = 0$, so $T = 0$

(This is also known as an equilibrium situation).

- c. Someone in the car (non-inertial observer) claims that the

forces on the mass along x are T and a fictitious force $(-Ma)$. Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x -direction.

Exercise 7.3

1. Let in a given time displacement A is x_1 , B is x_2 , and that of Y is x_3 in downward direction.

As length of string will remain constant, so we can write

$$\begin{aligned} x_1 + (x_1 - x_3) + x_3 + (x_2 - x_3) + x_2 &= 0 \\ \Rightarrow 2x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

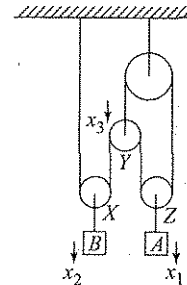


Fig. S-7.21

- a. Differentiating, we get $2v_1 + 2v_2 - v_3 = 0$
 $\Rightarrow 2v_1 + 2(-1) - (-2) = 0 \Rightarrow v_1 = 0$
- b. Again differentiating, we get $2a_1 + 2a_2 - a_3 = 0$
 $\Rightarrow 2a_1 + 2(-3) - 1 = 0 \Rightarrow a_1 = 7/2 \text{ m/s}^2$
2. a. $v_p = \frac{v_A + v_B}{2} \Rightarrow 5 = \frac{v_A + 10}{2} \Rightarrow v_A = 0$
- b. $5 = \frac{v_A - 20}{2} \Rightarrow v_A = 30 \text{ m/s}$
3. a. Length of the string between A and D will remain the same. For this, $(a_1 - a_4) + (a_2 - a_4) + a_2 = 0$
 $\Rightarrow a_1 + 2a_2 - 2a_4 = 0$ (i)

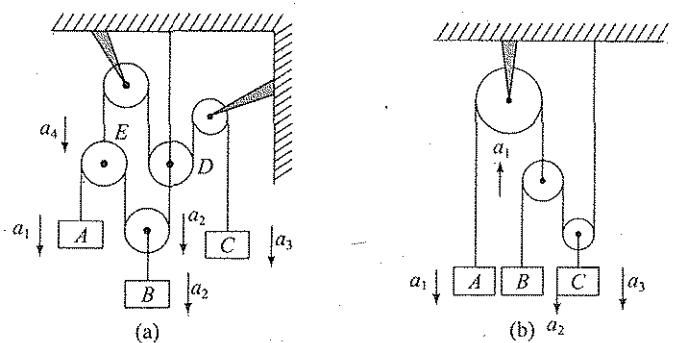


Fig. S-7.22

Length of the string from E to C will remain the same.

$$a_4 + a_3 = 0 \Rightarrow a_4 = -a_3 \quad \text{(ii)}$$

From equations (i) and (ii) $a_1 + 2a_2 + 2a_3 = 0$

b. $(a_1 + a_2) + (a_1 + a_3) + a_3 = 0$
 $\Rightarrow 2a_1 + a_2 + 2a_3 = 0$

4. Components of velocities along the string should be the same.
So $v = u \cos \theta$

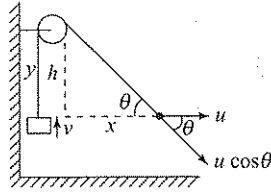


Fig. S-7.23

Alternatively:

Length of string, $\ell = y + \sqrt{h^2 + x^2}$

Differentiating we get $0 = \frac{dy}{dt} + \frac{2x}{2\sqrt{h^2 + x^2}} \frac{dx}{dt}$

$$\Rightarrow 0 = -v + (\cos \theta) u \Rightarrow v = u \cos \theta$$

5. let the speed of the ring is u , then $u \cos \theta = v$

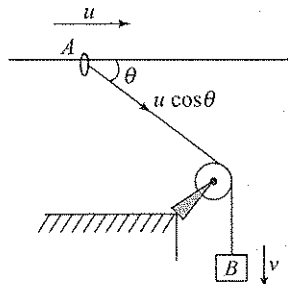


Fig. S-7.24

$$\Rightarrow u = v / \cos \theta = v \sec \theta$$

6. a. Length of (1) and (3) will increase at the cost of part 4 (Fig. S-7.25). Hence decrease in length of (4) will be twice of increase in lengths of either 1 or 3.

So $a_A = 2a$

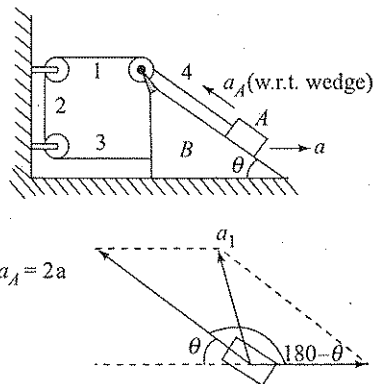


Fig. S-7.25

$$b. a_1 = \sqrt{a^2 + a_A^2 + 2a_A \cos(180 - \theta)} = a\sqrt{5 - 4\cos \theta}$$

7. a. i. Length of (5) will decrease at the cost of increase in length of (1) and (3).

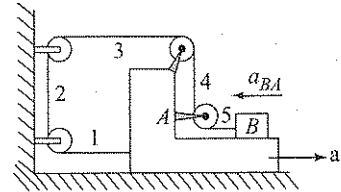


Fig. S-7.26

Hence, $a_{BA} = 2a$

$$ii. a_{BG} = a - a_{BA} = a - 2a = -a$$

- b. i. $a_{BA} = 3a$

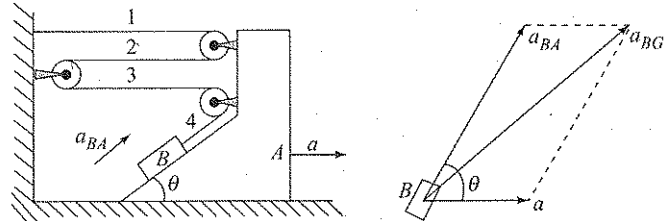


Fig. S-7.27

$$ii. a_{BG} = \sqrt{a^2 + a_{BA}^2 + 2aa_{BA} \cos \theta} = a\sqrt{10 + 6\cos \theta}$$

8. $ma \rightarrow$ pseudo force; $ma \cos \theta = mg \sin \theta$

$$\Rightarrow a = g \tan \theta$$

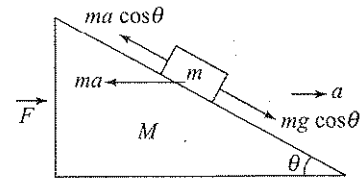


Fig. S-7.28

$$F = (M + m)a = (M + m)g \tan \theta$$

9. i. Length of the string: $\ell = x + \sqrt{h^2 + y^2}$

Differentiating, we get $\frac{d\ell}{dt} = \frac{dx}{dt} + \frac{2y}{2\sqrt{h^2 + y^2}} \frac{dy}{dt}$

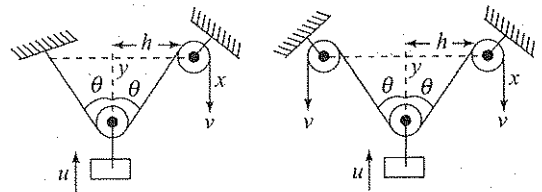


Fig. S-7.29

$$\Rightarrow 0 = v + \frac{2y}{\sqrt{h^2 + y^2}} (-u)$$

$$\Rightarrow 0 = v - 2u \cos \theta \Rightarrow u = v/2 \cos \theta$$

ii. Length of string, $\ell = 2x + \sqrt{h^2 + y^2}$

Again differentiate to get $u = v/\cos\theta$

10. i. For block $2M$, $2T = 2Ma$ (i)

For block M , $Mg - T = M2a$ (ii)

Solving equations (i) and (ii), we get $a = g/3$.

ii. For block $2M$,
 $2Mg \sin\theta - T' = 2Ma$ (i)

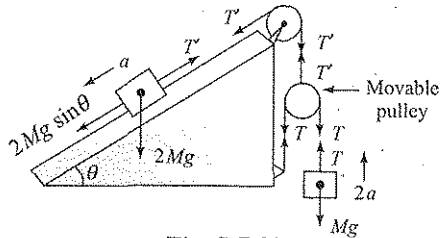


Fig. S-7.30

For movable pulley,

$$T' - 2T = 0 \times a \Rightarrow T' = 2T \quad (\text{ii})$$

For block M , $T - Mg = M2a$ (iii)

Solving equations (i), (ii), and (iii), we get $a = \frac{g[\sin\theta - 1]}{3}$

This is negative, hence directions of accelerations will be opposite to those shown in Fig. S-7.30.

Note: Here we assume that the block $2M$ is moving down the plane. You may assume that it is moving up the plane. The magnitude of the acceleration found will remain the same.

iii. For block $2M$,
 $2Mg \sin\theta - T = 2M(2a)$ (i)

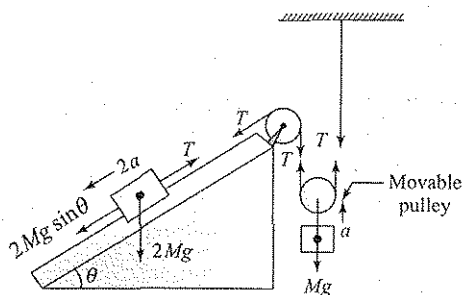


Fig. S-7.31

For block M , $2T - Mg = Ma$ (ii)

Solving equations (i) and (ii), we get $a = \frac{g[4\sin\theta - 1]}{9}$

11. Components of acceleration of M and m perpendicular to the incline should be same as both always remain in contact.

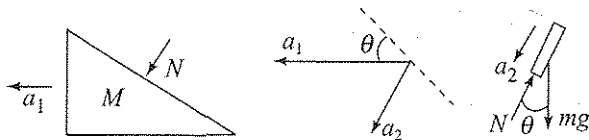


Fig. S-7.32

For this $a_2 = a_1 \sin\theta$ (i)

$N \sin\theta = Ma_1$ (ii)

$mg \cos\theta - N = ma_2$ (iii)

Solving equations (i), (ii), and (iii), we get

$$a_1 = \frac{mg \sin\theta \cos\theta}{M + m \sin^2\theta}, \quad a_2 = \frac{mg \sin^2\theta \cos\theta}{M + m \sin^2\theta}$$

12. $7F - N - Mg = Ma$ (i)

$F + N - mg = ma$ (ii)

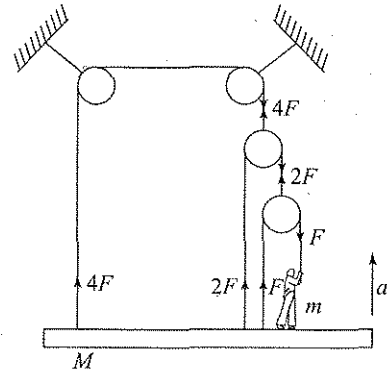


Fig. S-7.33

From equations (i) and (ii), we get $N = F \left[\frac{7m - M}{M + m} \right]$

Now for both to move together, $N \geq 0 \Rightarrow m/M \geq 1/7$

13. Acc. of wedge should be $a_1 = g \tan\theta$

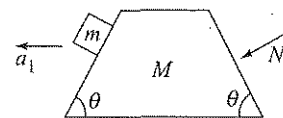


Fig. S-7.34

$$N \sin\theta = 2Ma_1 = 2Mg \tan\theta \Rightarrow N = \frac{2Mg}{\cos\theta}$$

$mg - N \cos\theta = ma_2$ where $a_2 = a_1 \tan\theta$

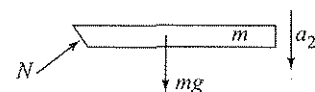


Fig. 7.35

Solving, we get $m = \frac{2M}{1 - \tan^2\theta}$

14. $v_1 = 1 \text{ m/s}$, $v_2 = 3 \text{ m/s}$

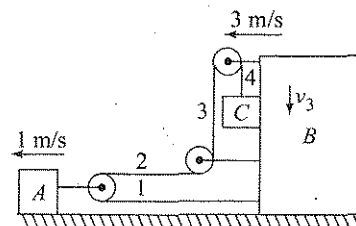


Fig. S-7.36

Let the velocity of c is v_3 downward w.r.t. B . Then

$$(v_1 - v_2) + (v_1 - v_2) + v_3 = 0$$

$$\Rightarrow v_3 = 2(v_2 - v_1) = 2(3 - 1) = 4 \text{ m/s}$$

$$\text{Net velocity of } C = \sqrt{v_2^2 + v_3^2} = \sqrt{3^2 + 4^2} = 5 \text{ m/s}$$

15. Case I: $T - N = 40a$, $20g - T = 20a$

$$N = 20a \Rightarrow a = g/4$$

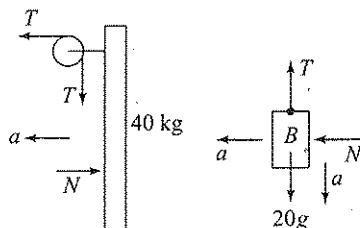


Fig. S-7.37

$$\text{Acceleration of } B = \sqrt{2}a = g/2\sqrt{2}$$

Case II: $T = 40a$, $20g - T = 20a \Rightarrow a = g/3$

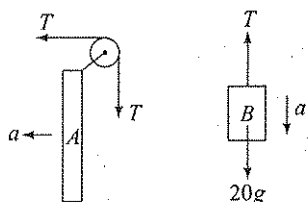


Fig. S-7.38

$$\text{Ratio: } \frac{g}{2\sqrt{2}} \times \frac{3}{g} = \frac{3n}{2\sqrt{2}} \Rightarrow n = 1$$

Exercise 7.4

1. a. When $F_{\text{ext}} = 0$, $F_{\text{friction}} = 0$

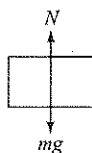


Fig. S-7.39

b. First, we calculate maximum friction force.

$$F_{\text{max}} = \mu_s N = (0.40 \times 20) = 8 \text{ N}$$

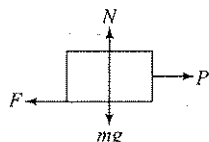


Fig. S-7.40

Since in this case, $P < F_{\text{max}}$, the block is in static equilibrium, i.e.,

$$F = P = 5 \text{ N}$$

$$\text{c. } P_{\text{min}} = F_{\text{max}} = \mu_s N = 8 \text{ N}$$

d. When the block is in motion, $F = \mu_k N = 0.2 \times 20 = 4 \text{ N}$

$$P_{\text{min}} = F_{\text{kinetic}} = \mu_s N = (0.20)(20 \text{ N}) = 4 \text{ N}$$

e. Since in this case $P > \mu_s N$, the block accelerates and the friction force will be kinetic.

2. a.

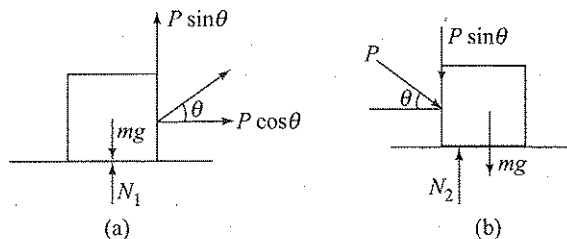


Fig. S-7.41

In first case: $N_1 = mg - P \sin \theta$

In second case: $N_2 = mg + P \sin \theta$

Since normal force in second case is greater so the friction will be greater in second case when the block impends the motion. Hence in first case, less effort is required.

$$\text{b. } P \cos \theta = \mu N_1 = \mu (mg - P \sin \theta)$$

$$\Rightarrow P = \frac{\mu mg}{\cos \theta + \mu \sin \theta}$$

c. For pushing, $P \cos \theta > \mu N_2$

$$\Rightarrow P \cos \theta > \mu (mg + P \sin \theta)$$

$$\Rightarrow P > \frac{\mu mg}{\cos \theta - \mu \sin \theta}$$

As θ increases, denominator $\cos \theta - \mu \sin \theta$ decreases and hence P increases. Let at $\theta = \theta_0$, denominator becomes zero. So $\cos \theta_0 - \mu \sin \theta_0 = 0 \Rightarrow \theta_0 = \cot^{-1}(\mu)$.

At $\theta = \theta_0$, $P > \infty$ which is not possible.

$$3. f_\ell = \mu_s N = \mu_s mg = 0.3 \times 1 \times 10 = 3 \text{ N}$$

$$f_\ell = \mu_s N = \mu_s mg = 0.25 \times 1 \times 10 = 2.5 \text{ N}$$

a. $F = 1 \text{ N}$, $F < f_\ell$, so the motion will not start and the friction will be static. So friction $f = F = 1 \text{ N}$.

b. $F = 2 \text{ N}$, $F < f_\ell$, so the motion will not start and the friction will be static. So friction $f = F = 2 \text{ N}$.

c. $f = 3 \text{ N}$, $F = f_\ell$. Here the motion is just about to start. Hence the friction force will be limiting. So friction $f = f_\ell = 3 \text{ N}$.

d. $F = 4 \text{ N}$, $F > f_\ell$, so the motion will start and friction will be kinetic. Hence friction will be $f = f_k = 2.5 \text{ N}$.

e. $F = 20 \text{ N}$, $F > f_\ell$, so motion will start and friction will be kinetic. Hence friction is $f = f_k = 2.5 \text{ N}$.

$$4. \text{ Here } f_\ell = 10 \text{ N} \Rightarrow \mu_s mg = 10$$

$$\Rightarrow \mu_s = \frac{10}{mg} = \frac{10}{5 \times 10} = 0.2$$

$$f_k = 8 \text{ N} \Rightarrow \mu_k mg = 8$$

$$\Rightarrow \mu_k = \frac{8}{mg} = \frac{8}{5 \times 10} = 0.16$$

5. Since the body is not moving.

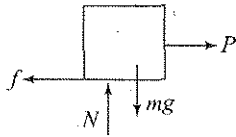


Fig. S-7.42

So $f = P$ and $N = mg$

Net force applied by surface on the body:

$$F = \sqrt{f^2 + N^2} = \sqrt{P^2 + m^2 g^2}$$

$$F = \sqrt{f^2 + N^2} = \sqrt{P^2 + m^2 g^2}$$

6. i. $f_{\max} = \mu N = (0.2)(100) = 20 \text{ N}$

Since $mg > f_{\max}$, therefore, friction force is equal to $f_{\max}$.
i.e., $f = f_{\max} = 20 \text{ N}$

- ii. $f_{\max} = \mu N = 0.2 \times 500 = 100 \text{ N}$

Since $mg > f_{\max}$, therefore, friction force is equal to mg .
This means that $f = mg = 50 \text{ N}$

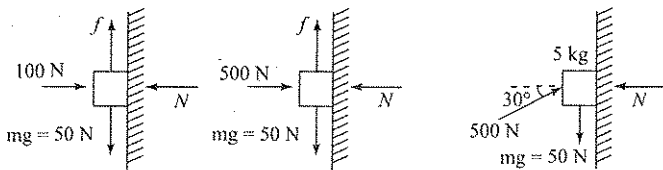


Fig. S-7.43

- iii. $f_{\max} = \mu N = \mu(100 \cos 30^\circ) = (0.2)100 \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ N}$

Since the vertical component of the external force can balance the weight of the block, therefore $f = 0$.

7.

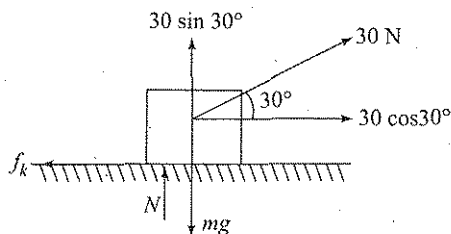


Fig. S-7.44

$$N = mg - 30 \sin 30^\circ$$

As the speed is constant, so the acceleration is zero.

$$30 \cos 30^\circ = f_k = \mu_k N$$

$$\Rightarrow 30 \cos 30^\circ = \mu_k (mg - 30 \sin 30^\circ)$$

Solving, we get $\mu_k = 3\sqrt{3}/7$

8.

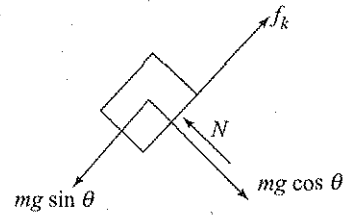


Fig. S-7.45

$$N = mg \cos \theta$$

$$f_k = \mu_k N = \mu_k mg \cos \theta$$

Hence, required force is $\sqrt{f_k^2 + N^2} = 6\sqrt{13} \text{ N}$

9. Apply $a = g \sin 60^\circ - \mu_k g \cos 60^\circ$

and get $\mu_k = \sqrt{3} - 1$

10. From FBD friction, $f = mg \sin \alpha$

$$N = mg \cos \alpha, \quad \frac{f}{N} = \tan \alpha$$

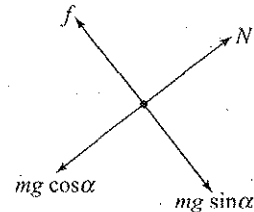


Fig. S-7.46

For the maximum possible value of α , the friction becomes limiting friction, so $\mu = \tan \alpha \Rightarrow \cot \alpha = 3$

11.

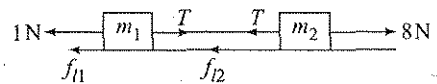


Fig. S-7.47

$$f_{11} = \mu_1 m_1 g = 0.1 \times 2 \times 10 = 2 \text{ N}$$

$$f_{12} = \mu_2 m_2 g = 0.2 \times 3 \times 10 = 6 \text{ N}$$

$$T + f_{12} = 8 \Rightarrow T = 2 \text{ N}$$

$$f_1 + 1 = T \Rightarrow f_1 = T - 1 = 2 - 1 = 1 \text{ N}$$

12. $f_t = \mu N = 0.4 \times 2 \text{ g} = 8 \text{ N}$

$$F = 2.5 \text{ N}$$

Since $F < f_t$. So the block will not move and friction will be static and equal to 2.5 N.

13. Here the weights of the blocks will balance each other so that no friction force is required.

14. Push is a force directed inward (towards the centre) and pull is a force directed away from the centre. Let P be the force required to move the roller by pulling and Q be the force required to move it by pushing applied at the same inclination θ to the horizontal. Considering free body diagrams in the two cases.

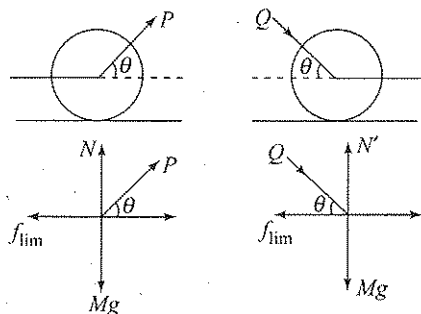


Fig. S-7.48

$N + P \sin \theta = mg$ $P \cos \theta = f_{\text{lim}} = \mu_s N$ Hence $P = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta}$ $\therefore P < Q$	$N - Q \sin \theta = mg$ $Q \cos \theta = f'_{\text{lim}} = \mu_s N'$ Hence $Q = \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta}$
--	--

This is why it is easier to pull than to push a lawn roller.

15. No. Let P be the pull of the engine. Considering the first four compartments

$$P - 4f - T = 4ma$$

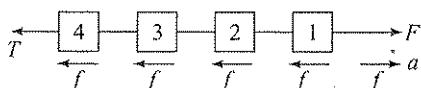


Fig. S-7.50

where f = frictional force on each compartment, m = mass of each compartment, a = acceleration of the train, T = tension. Considering the free body diagrams of the 21st compartment

$$P - 21f - T' = 21ma$$

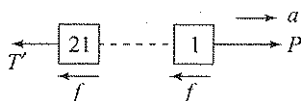


Fig. S-7.50

Hence $T = P - 4(f + ma)$ and

$$T' = P - 21(f + ma)$$

$$\therefore T' < T$$

16. a. If no horizontal force is applied, no friction force is needed to keep the box in equilibrium.
 b. The maximum static friction force is,
 $\mu_s N = \mu_s W = (0.40)(40.0) = 16.0 \text{ N}$
 so the box will not move and the friction force balances the applied force of 6.0 N.
 c. The maximum friction force found in part (b), 16.0 N.
 d. From $f_k = \mu_k N = (0.20)(40.0 \text{ N}) = 8.0 \text{ N}$.
 e. The applied force is enough either to start the box moving or to keep it moving. The answer to part (d) is independent of speed (as long as the box is moving), so the friction force is 8.0 N. The acceleration is $(F - f_k)/m = 2.45 \text{ m/s}^2$

17. $T = Mg$ (T = tension in the rope)

$N = 60g - T \sin 60^\circ$ (N is normal reaction between the man and the ground) and $T \cos 60^\circ = \mu N$

Solving these three equations with proper substitutions, we get $M = 32.15 \text{ kg}$

18. For an angle of 45.0° , the tensions in the horizontal and vertical wires will be the same.

a. The tension in the vertical wire will be equal to the weight $w = 12.0 \text{ N}$. This must be the tension in the horizontal wire, and hence the friction force on block A is also 12.0 N.

b. The maximum frictional force is

$\mu_s w_A = (0.25)(60.0 \text{ N}) = 15 \text{ N}$, this will be the tension in both the horizontal and vertical parts of the wire, so the maximum weight is 15 N.

19. The block has a tendency to slide down. A block placed on an inclined plane has a tendency to slide down the inclined plane due to its weight if the inclination angle of plane is greater than the angle of repose.

$$\text{Angle of repose, } \phi = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ$$

Here $\theta > \phi$, hence the block has a tendency to slide down

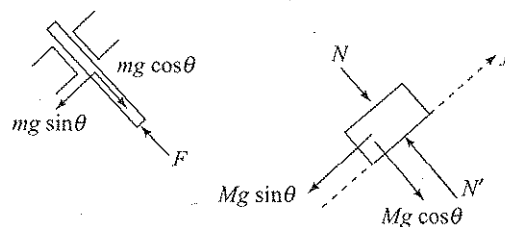


Fig. S-7.51

$$Mg \cos \theta + N = N'$$

$$N' = Mg \cos \theta + mg \cos \theta \quad (i)$$

For the block to remain stationary

$$f = Mg \sin \theta \quad (ii)$$

If f is static in nature; $f < f_{\text{max}}$

$$Mg \sin \theta < \mu N'$$

$$Mg \sin \theta < \mu (Mg \cos \theta + mg \cos \theta)$$

$$Mg \sin \theta - \mu Mg \cos \theta < \mu mg \cos \theta$$

$$m > M \left(\frac{\tan \theta}{\mu} - 1 \right) \Rightarrow m > 10 \left(\frac{\sqrt{3}}{1/\sqrt{3}} - 1 \right)$$

$$\Rightarrow m > 20 \text{ kg}$$

20. Here also angle of inclination of the plane $>$ angle of repose

$$\left[\tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ \right]$$

Hence, the block has a tendency to slide down the plane.

Free body diagram of the block as seen from the frame of reference of the wedge.

Considering the equilibrium of M in the plane of wedge.

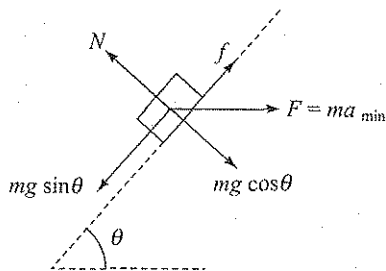


Fig. S-7.52

$$mg \sin \theta = f + ma_{\min} \cos \theta$$

$$f = mg \sin \theta - ma_{\min} \cos \theta \quad (i)$$

For friction to be static nature

$$f < f_{\max} \Rightarrow f < \mu N$$

$$mg \sin \theta - ma_{\min} \cos \theta < (ma_{\min} \sin \theta + mg \cos \theta)$$

$$a_{\min} > g \left[\frac{\sin \theta - \mu \cos \theta}{\mu \sin \theta + \cos \theta} \right] \Rightarrow a_{\min} > g \left[\frac{\tan \theta - \mu}{\mu \tan \theta + 1} \right]$$

$$\Rightarrow a_{\min} > 10 \left[\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{\sqrt{3} \frac{1}{\sqrt{3}} + 1} \right] \Rightarrow a_{\min} > \frac{10}{\sqrt{3}} \text{ m/s}^2$$

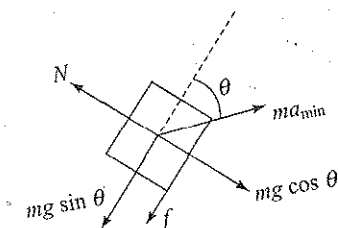


Fig. S-7.53

$$ma_{\max} \cos \theta = f_t + mg \sin \theta$$

$$N = ma_{\max} \sin \theta$$

$$= \mu(ma_{\max} \sin \theta + mg \cos \theta) + mg \sin \theta$$

$$\Rightarrow a_{\max} = \frac{mg \sin \theta + \mu mg \cos \theta}{m \cos \theta - \mu m \sin \theta}$$

$$= \frac{g(\tan \theta + \mu)}{(-1\mu \tan \theta)}$$

$$a_{\max} = \frac{10 \left(\sqrt{3} + \frac{1}{\sqrt{3}} \right)}{1 - \frac{1}{\sqrt{3}} \sqrt{3}} = \infty$$

So there is no limit on maximum acceleration.

Exercise 7.5

1. Considering the string over the left nail, the vertical components of the forces of tension T acting on the weights are mg if the string is secured on the nail. From Newton's third law the knot (point O) is acted upon by the same forces T , whose resultant is $2mg$.

For the string on the right, the vertical component of the strings tension T' is equal to $2mg$ (if the mass does not move down). The tension of the string itself, however, is $T' > 2mg$. Therefore, the system is not in equilibrium. The right hand mass will have a greater pull.

2. No, force cannot determine the direction of motion. Yes, force determines the direction of acceleration. For example, in circular motion force is directed towards the centre but the motion is along the tangent, but acceleration is definitely along the normal which is the direction of the force.
3. a. The normal force is always perpendicular to the surface that applies the force. Because your car maintains its orientation at all points on the ride, the normal force is always upward.
b. Your centripetal acceleration is downward toward the center of the circle, so the net force on you must be downward.
4. (a) Because the speed is constant, the only direction the force can have is that of the centripetal acceleration. The force is larger at (C) than at (A) because the radius at (C) is smaller. There is no force at (B) because the wire is straight. (b) In addition to the forces in the centripetal direction in (a), there are now tangential forces to provide the tangential acceleration. The tangential force is the same at all the three points because the tangential acceleration is constant.

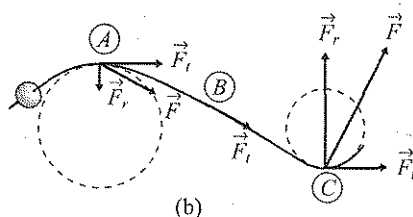
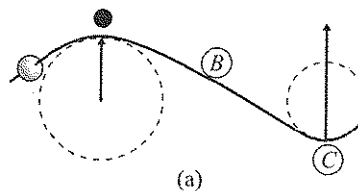


Fig. S-7.54

5. Outer wall will exert non-zero normal force. It is because the block requires inward force on it which will provide the centripetal force to the block. Inward force can be given by outer wall only.

$$6. \text{ a. } N = \frac{mv^2}{R}, f - mg = 0, f = \mu_s N, v = \frac{2\pi R}{T}$$

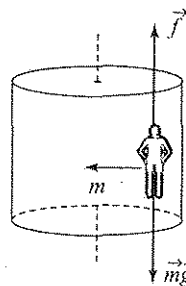


Fig. S-7.55

Solving, we get $T = \sqrt{\frac{4\pi^2 R \mu_s}{g}}$

b. $T = 2.54 \text{ s}$

$$\frac{\text{rev}}{\text{min}} = \frac{1 \text{ rev}}{2.54 \text{ s}} \left(\frac{60 \text{ s}}{\text{min}} \right) = 23.6 \frac{\text{rev}}{\text{min}}$$

c. The gravitational and the frictional forces remain constant. The normal force increases. The person remains in motion with the wall.

d. The gravitational force remains constant. The normal and the frictional forces decrease. The person slides relative to the wall and downward into the pit.

7. Let the tension at the lowest point be T .

$$\sum F = ma; T - mg = ma_c = \frac{mv^2}{r}$$

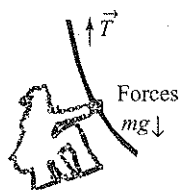


Fig. S-7.56

$$T = m \left(g + \frac{v^2}{r} \right) T = (90.0 \text{ kg}) \times \left[1.0 \text{ m/s}^2 + \frac{(8.00 \text{ m/s}^2)}{16.0 \text{ m}} \right] = 1.26 \text{ kN} > 1000 \text{ N}$$

He does not make it across the river because the vine breaks.

8. a. $v = 20.0 \text{ m/s}$, N = force of track on roller coaster, and $R = 1.00 \text{ m}$.

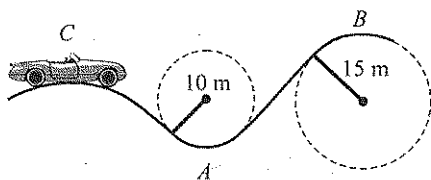


Fig. S-7.57

$$\sum F = \frac{Mv^2}{R} = N - Mg$$

From this, we find

$$n = Mg + \frac{Mv^2}{R} = (500 \text{ kg})(10 \text{ m/s}^2) + \frac{(500 \text{ kg})(2.0 \text{ m/s}^2)}{10.0 \text{ m}}$$

$$N = 5000 \text{ N} + 20,000 \text{ N} = 2.50 \times 10^4 \text{ N}$$

b. At B, $N - Mg = -\frac{Mv^2}{R}$

The maximum speed at B corresponds to $N = 0$

$$-Mg = -\frac{mv_{\text{max}}^2}{R}$$

$$v_{\text{max}} = \sqrt{Rg} = \sqrt{15.0(10.0)} = 5\sqrt{6} \text{ m/s}$$

9. a. Since the object of mass m_2 is in equilibrium,

$$\sum F_y = T - m_2g = 0 \text{ Or } T = m_2g$$

b. The tension in the string provides the required centripetal acceleration of the puck.

$$\text{Thus, } F_c = T = m_2g.$$

c. From $F_c = \frac{m_1 v^2}{R}$, we have $v = \sqrt{\frac{RF_c}{m_1}} = \sqrt{\left(\frac{m_2}{m_1}\right)gR}$

d. The puck will spiral inward, gaining speed as it does so. It gains speed because the extra-large string tension produces forward tangential acceleration as well as inward radial acceleration of the puck, pulling at an angle of less than 90° to the direction of the inward spiraling velocity.

e. The puck will spiral outward, slowing down as it does so.

10. $r = R \sin \theta$, $N \cos \theta = mg$ (i)

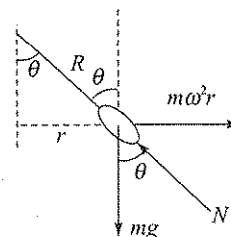


Fig. S-7.58

$$N \sin \theta = m\omega^2 r$$

$$\Rightarrow N = m\omega^2 R \quad \text{(ii)}$$

From equations (i) and (ii), we get $\cos \theta = \frac{g}{\omega^2 R} \Rightarrow \theta =$

$$\cos^{-1} \left(\frac{g}{\omega^2 R} \right)$$

11.

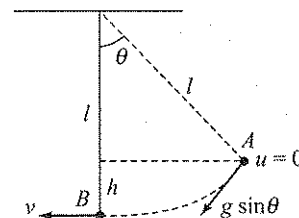


Fig. S-7.59

Acceleration at point A, $a_1 = g \sin \theta$, at point B: $a_2 = \frac{v^2}{\ell}$

where $v = \sqrt{2gh} = \sqrt{2g(\ell - \ell \cos \theta)}$

$\Rightarrow a_2 = 2g(1 - \cos \theta)$

Given $a_1 = a_2 \Rightarrow g \sin \theta = 2g(1 - \cos \theta)$

Squaring both sides and solving, we get

$$\cos \theta = \frac{3}{5} \Rightarrow \theta = 53^\circ$$

12. $v = \sqrt{2gh} = \sqrt{2g\ell \cos \theta}$, $a_t = g \tan \theta$

$$a_c = \frac{v^2}{\ell} = 2g \cos \theta$$

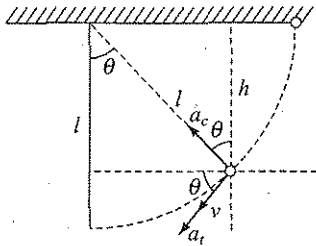


Fig. S-7.60

Net acceleration will be horizontal if the vertical components of a_c and a_t cancel each other.

For this, $a_c \cos \theta = a_t \Rightarrow 2g \cos^2 \theta = g \sin 2\theta$

$\Rightarrow \tan \theta = \sqrt{2}$

13. i. Force on the particle when the fan is at full speed

$$\begin{aligned} F_n &= m\omega^2 R = \left(\frac{1}{1000}\right) \left(\frac{2\pi f}{60}\right)^2 R \\ &= \left(\frac{1}{1000}\right) \left(\frac{2\pi \times 1500}{60}\right)^2 \left(\frac{60}{100}\right) \\ &= \frac{3}{2} \pi^2 \text{ N} \end{aligned}$$

ii. The friction force exerts this force.

iii. $F_{\text{particle}} = F_n = \frac{3}{2} \pi^2$

14. a. If the block does not slip $f \leq \mu N$

$$m\omega^2 L \leq \mu mg \Rightarrow \omega \leq \left(\frac{\mu g}{L}\right)^{1/2}$$

b. It is the case of non-uniform circular motion.

$$F_{\text{tangential}} = m \frac{dv}{dt} = m \frac{d(\omega L)}{dt} = mL \frac{d\omega}{dt} = m\alpha L$$

$$F_{\text{tangential}} = m\alpha L \quad (i)$$

$$F_{\text{radial}} = m\omega^2 L \quad (ii)$$

Net force $F_{\text{net}} = \sqrt{(m\alpha L)^2 + (m\omega^2 L)^2}$

Friction force $f = F_{\text{net}}$

$$f = \sqrt{(m\alpha L)^2 + (m\omega^2 L)^2}$$

$$f \leq \mu N \Rightarrow \sqrt{(m\alpha L)^2 + (m\omega^2 L)^2} \leq \mu mg$$

$$m^2 \alpha^2 L^2 + m^2 \omega^4 L^2 \leq \mu^2 m^2 g^2$$

$$\omega^4 \leq \mu^2 g^2 - \alpha^2 \Rightarrow \omega \leq \left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2 \right]^{1/4}$$

15. a. rim
b.

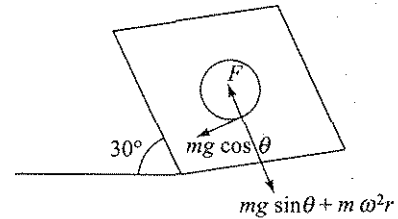


Fig. S-7.61

$$f \geq mg \sin \theta + m \omega^2 r$$

$$\mu mg \cos \theta \geq mg \sin \theta + m \omega^2 r$$

$$\begin{aligned} \mu \geq \tan \theta + \frac{\omega^2 r}{g \cos \theta} &= \frac{1}{\sqrt{3}} + \frac{4\pi^2 \times 1089 \times 30}{36 \times 1000 \times \sqrt{3}} \\ &= \frac{1}{\sqrt{3}} + \frac{4\pi^2 \times 363}{4 \sqrt{3} \times 1000} \\ &= \frac{1}{\sqrt{3}} (1 + \pi^2 \times .363) \end{aligned}$$

$$\frac{1}{\sqrt{3}} + (1 + 3.5) \Rightarrow \frac{1}{\sqrt{3}} (4.5) = 2.6$$

$$\mu_{\text{least}} = 2.6$$

16.

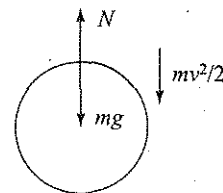


Fig. S-7.62

a.
$$N - mg = -\frac{mv^2}{r}$$

To feel weightless at top, $N = 0$

$$mg = \frac{mv^2}{r} \Rightarrow v = \sqrt{rg} = \sqrt{200} = 10\sqrt{2} \text{ m/s}$$

b.

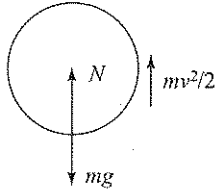


Fig. S-7.63

$$N - mg = \frac{mv^2}{r}$$

$$N = m \left(g + \frac{v^2}{r} \right) = 1200 \text{ N}$$

17. At the time of slipping, maximum friction acts on the body.

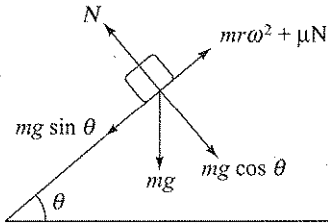


Fig. S-7.64

In the frame of rod,

$$mg \sin \theta = \mu N + mr\omega^2$$

Also,
$$N = mg \cos \theta$$

$$mg \sin \theta = \mu mg \cos \theta + mr\omega^2$$

Here
$$r = 0.45 \text{ m}, \theta = 50^\circ$$

$$\sin 50^\circ = 0.766 \text{ and } \cos 50^\circ = 0.64$$

$$\mu = \frac{mg \sin \theta - mr\omega^2}{mg \cos \theta} = \frac{g \sin \theta - r\omega^2}{g \cos \theta} = 0.55$$

18. If v be the speed of the particle when radius vector makes an angle θ with vertical, then

$$(\cos \theta)mg - N = \frac{mv^2}{r} = \frac{m}{r} [u^2 + 2gr(1 - \cos \theta)]$$

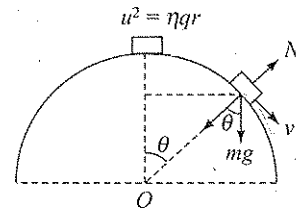


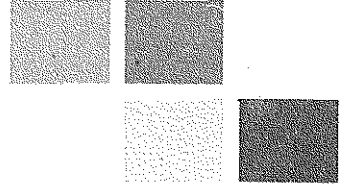
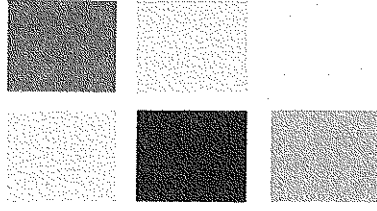
Fig. S-7.65

$$mg \cos \theta = \frac{m}{r} [(\eta + 2)gr - 2gr \cos \theta]$$

when the particle loses contact $N = 0$

$$\cos \theta = (\eta + 2) - 2 \cos \theta \Rightarrow \cos \theta = \left(\frac{\eta + 2}{3} \right)$$

Appendix



IIT-JEE 2010 Solved Paper

Physics

Paper 1

Objective Type

Multiple choice questions with one correct answer

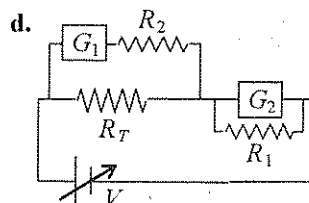
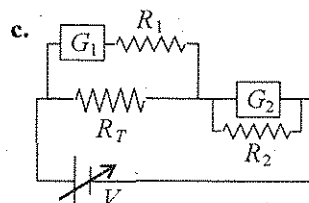
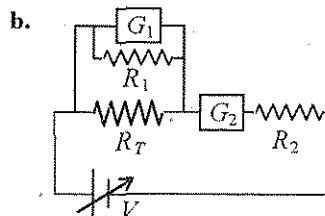
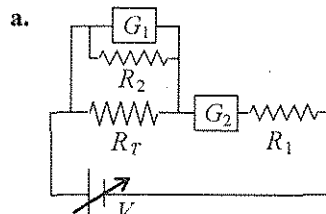
1. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100} , R_{60} and R_{40} , respectively, the relation between these resistances is

- a. $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ b. $R_{100} = R_{40} + R_{60}$
 c. $R_{100} > R_{60} > R_{40}$ d. $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

Sol. d.

Power $\propto 1/R$

2. To verify Ohm's law, a student is provided with a test resistor R_T , a high resistance R_1 , a small resistance R_2 , two identical galvanometers G_1 and G_2 , and a variable voltage source V . The correct circuit to carry out the experiment is



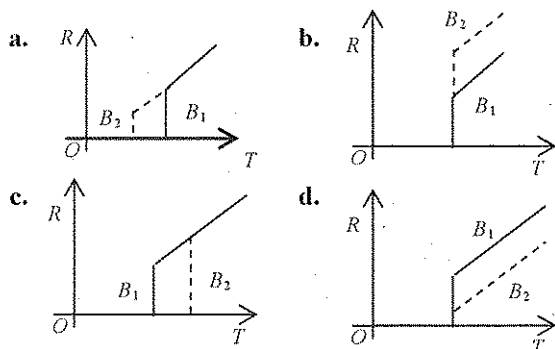
Sol. c.

G_1 is acting as voltmeter and G_2 is acting as ammeter.

3. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

the critical temperature $T_c(B)$ is a function of the magnetic field strength B . The dependence of $T_c(B)$ on B is shown in the figure.

14. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1 which of the following graphs shows the correct variation of R with T in these fields?



Sol. a.

Larger the magnetic field, smaller the critical temperature.

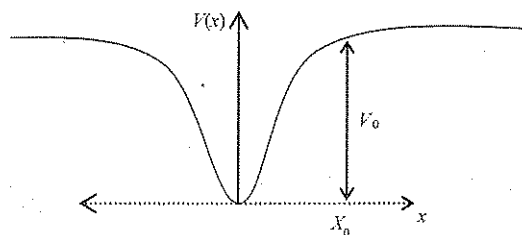
15. A superconductor has $T_c(0) = 100$ K. When a magnetic field of 7.5 T is applied, its T_c decreases to 75 K. For this material, one can definitely say that when

- $B = 5$ T, $T_c(B) = 80$ K
- $B = 5$ T, 75 K $< T_c(B) < 100$ K
- $B = 10$ T, 75 K $< T_c < 100$ K
- $B = 10$ T, $T_c = 70$ K

Sol. b. Self-explanatory

Problems 16–18:

When a particle of mass m moves on the x -axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{m/k}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x -axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| > X_0$ (see figure).



16. If the total energy of the particle is E , it will perform periodic motion only if

- $E < 0$
- $E > 0$
- $V_0 > E > 0$
- $E > V_0$

Sol. c.

Energy must be less than V_0 .

17. For periodic motion of small amplitude A , the time period T of this particle is proportional to

- $A\sqrt{\frac{m}{\alpha}}$
- $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
- $A\sqrt{\frac{\alpha}{m}}$
- $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

Sol. b.

$$[\alpha] = ML^{-2}T^{-2}$$

Only option (b) has dimension of time. Alternatively,

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + kx^4 = kA^4$$

$$\left(\frac{dx}{dt}\right)^2 = \frac{2k}{m}(A^4 - x^4)$$

$$4\sqrt{\frac{m}{2k}} \int_0^A \frac{dx}{\sqrt{A^4 - x^4}} = \int dt = T$$

$$4\sqrt{\frac{m}{2k}} \frac{1}{A} \int_0^1 \frac{du}{\sqrt{1 - u^4}} = T$$

Substituting, we get $x = Au$.

18. The acceleration of this particle for $|x| > X_0$ is

- proportional to V_0
- proportional to V_0/mX_0
- proportional to $\sqrt{V_0/mX_0}$
- zero

Sol. d.

As potential energy is constant for $|x| > X_0$, the force on the particle is zero. Hence, acceleration is zero.

Integer type

This section contains ten questions. The answer to each question is a single-digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

19. Gravitational acceleration on the surface of a planet is $\sqrt{6}/11g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $2/3$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1} , the escape speed on the surface of the planet in kms^{-1} will be _____.

Sol. (3)

$$\frac{g'}{g} = \frac{\sqrt{6}}{11}, \frac{\rho'}{\rho} = \frac{2}{3}$$

$$\text{Hence, } \frac{R'}{R} = \frac{3\sqrt{6}}{22}$$

$$\frac{v'_{\text{esc}}}{v_{\text{esc}}} \propto \sqrt{\frac{R'^2 \rho'}{R^2 \rho}} = \frac{3}{11}$$

$$v'_{\text{esc}} = 3 \text{ km/s}$$

20. A piece of ice (heat capacity = $2100 \text{ J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ and latent heat = $3.36 \times 10^5 \text{ J kg}^{-1}$) of mass m grams is at -5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally, when the ice-water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of m is _____.

Sol. (8)

$$420 = (m \times 2100 \times 5 + 1 \times 3.36 \times 10^5) \times 10^{-3}$$

where m is in gm.

21. A stationary source is emitting sound at a fixed frequency f_0 , which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0 . What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1} .

Sol. (7)

$$f_{\text{app}} = f_0 \frac{c+v}{c-v}$$

$$df = \frac{2f_0 c}{(c-v)^2} dv$$

where c is speed of sound

$$df = \frac{1.2}{100} f_0$$

$$\text{Hence, } dv \approx 7 \text{ km/hr.}$$

22. The focal length of a thin biconvex lens is 20 cm . When an object is moved from a distance of 25 cm in front of it to 50 cm , the magnification of its image changes from m_{25} to m_{50} . The ratio m_{25}/m_{50} is _____.

Sol. (6)

$$m = \frac{f}{f+u}$$

23. An α -particle and a proton are accelerated from rest by a potential difference of 100 V . After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio λ_p/λ_α to the nearest integer, is _____.

Sol. (3)

$$\frac{1}{2}mv^2 = qV$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \sqrt{8} \approx 3$$

24. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R , the rate of heat produced in R is J_1 . When the same batteries are connected in parallel across R , the rate is J_2 . If $J_1 = 2.25 J_2$, then the value of R in Ω is _____.

Sol. (4)

$$J_1 = \left(\frac{2E}{R+2} \right)^2 R$$

$$J_2 = \left(\frac{E}{R+1/2} \right)^2 R \text{ since } J_1/J_2 = 2.25$$

$$R = 4 \Omega$$

25. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2 , respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm . Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B ?

Sol. (9)

$$\lambda_m T = \text{constant}$$

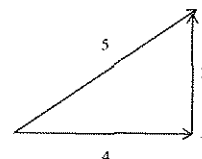
$$\lambda_A T_A = \lambda_B T_B$$

$$\text{Rate of total energy radiated} \propto AT^4$$

26. When two progressive waves $y_1 = 4 \sin(2x - 6t)$ and $y_2 = 3 \sin(2x - 6t - \frac{\pi}{2})$ are superimposed, the amplitude of the resultant wave is _____.

Sol. (5)

Two waves have phase difference $\pi/2$.



27. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is $4.9 \times 10^{-7} \text{ m}^2$. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1} . If the Young's modulus of the material of the wire is $n \times 10^9 \text{ Nm}^{-2}$, the value of n is _____.

Sol. (4)

$$\omega = \sqrt{\frac{YA}{mL}}$$

28. A binary star consists of two stars A (mass $2.2M_s$) and B (mass $11M_s$), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is _____.

Sol. (6)

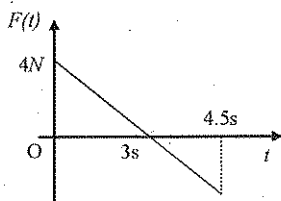
$$\frac{L_{\text{total}}}{L_B} = \frac{m_1 r_1^2}{m_2 r_2^2} + 1$$

Paper 2

Objective Type

Multiple choice questions with one correct answer

1. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



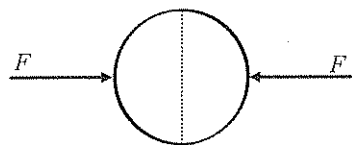
- a. 4.50 J b. 7.50 J
c. 5.06 J d. 14.06 J

Sol. c.

Area under $F-t$ curve = 4.5 kg-m/sec

$$\text{K.E.} = \frac{1}{2}(2) \left(\frac{4.5}{2} \right)^2 = 5.06 \text{ J}$$

2. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F . F is proportional to



- a. $\frac{1}{\epsilon_0} \sigma^2 R^2$ b. $\frac{1}{\epsilon_0} \sigma^2 R$
c. $\frac{1}{\epsilon_0} \frac{\sigma^2}{R}$ d. $\frac{1}{\epsilon_0} \frac{\sigma^2}{R^2}$

Sol. a.

$$\text{Pressure} = \frac{\sigma^2}{2\epsilon_0} \text{ and force} = \frac{\sigma^2}{2\epsilon_0} \times \pi R^2$$

3. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $(81\pi/7) \times 10^5 \text{ Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3} \text{ ms}^{-1}$. Given $g = 9.8 \text{ ms}^{-2}$, viscosity of the air $= 1.8 \times 10^{-5} \text{ N s m}^{-2}$ and the density of oil $= 900 \text{ kg m}^{-3}$, the magnitude of q is
- a. $1.6 \times 10^{-19} \text{ C}$ b. $3.2 \times 10^{-19} \text{ C}$
c. $4.8 \times 10^{-19} \text{ C}$ d. $8.0 \times 10^{-19} \text{ C}$

Sol. d.

$$\frac{4}{3}\pi R^3 \rho g = qE = 6\pi\eta R v_T$$

$$\therefore q = 8.0 \times 10^{-19} \text{ C}$$

4. A vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier callipers, the least count is
- a. 0.02 mm b. 0.05 mm
c. 0.1 mm d. 0.2 mm

Sol. d.

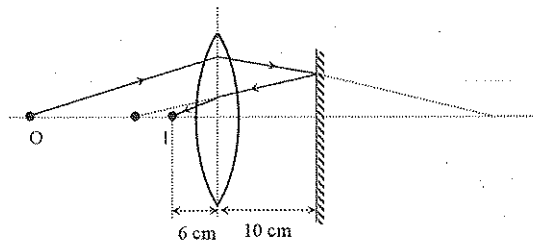
$$\text{L.C.} = 1 \text{ M.S.D} - 1 \text{ V.S.D}$$

$$\left(1 - \frac{16}{20} \right) \text{ M.S.D}$$

$$\left(1 - \frac{4}{5} \right) (1 \text{ mm}) = 0.2 \text{ mm}$$

5. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is
- a. virtual and at a distance of 16 cm from the mirror
b. real and at a distance of 16 cm from the mirror
c. virtual and at a distance of 20 cm from the mirror
d. real and at a distance of 20 cm from the mirror

Sol. b.



6. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1} , the mass of the string is
- a. 5 g b. 10 g
c. 20 g d. 40 g

Sol. b.

$$\frac{V_s}{4L_p} = \frac{2\sqrt{\frac{T}{\mu}}}{2ls}$$

Integer type

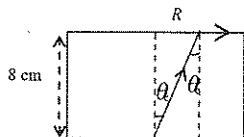
This section contains five questions. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

7. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R ?

Sol. (6)

$$\sin \theta_c = 3/5$$

$$\therefore R = 6 \text{ cm}$$



8. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 seconds. What is the speed of the object in km per hour?

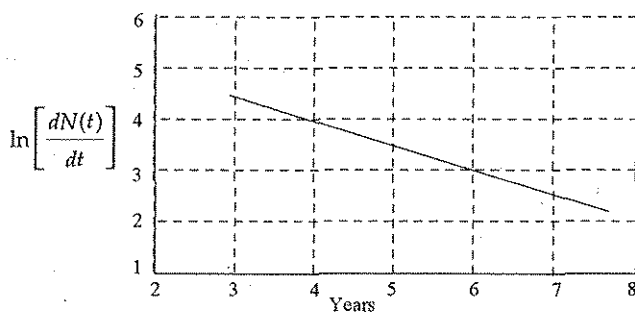
Sol. (3)

$$\text{For } v_1 = \frac{50}{7} \text{ m, } u_1 = -25 \text{ m}$$

$$v_2 = \frac{25}{3} \text{ m, } u_2 = -50 \text{ m}$$

$$\text{Speed of object} = \frac{25}{30} \times \frac{18}{5} = 3 \text{ kmph.}$$

9. To determine the half-life of a radioactive element, a student plots a graph of $\ln \left[\frac{dN(t)}{dt} \right]$ versus t . Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t . If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is _____.



Sol. (8)

$$N = N_0 e^{-\lambda t}$$

$$\ln[dN/dt] = \ln(N_0 \lambda) - \lambda t$$

$$\text{From graph, } \lambda = \frac{1}{2} \text{ per year}$$

$$t_{1/2} = \frac{0.693}{1/2} = 1.386 \text{ year}$$

$$4.16 \text{ yrs} = 3t_{1/2}$$

$$\therefore p = 8$$

10. A diatomic ideal gas is compressed adiabatically to $1/32$ of its initial volume. If the initial temperature of the gas is T_i (in kelvin) and the final temperature is aT_i , the value of a is _____.

Sol. (8)

$$TV^{\gamma-1} = \text{constant}$$

$$TV^{7/5-1} = aT_i \left(\frac{V}{32} \right)^{7/5-1}$$

$$\therefore a = 4.$$

11. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them becomes 4 volt? [take $\ln 5 = 1.6$, $\ln 3 = 1.1$]

Sol. (2)

$$4 = 10(1 - e^{-t/4})$$

$$\therefore t = 2 \text{ sec}$$

Linked comprehension type**Problems 12–14:**

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R . When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

12. If the radius of the opening of the dropper is r , the vertical force due to the surface tension on the drop of radius R (assuming $r \ll R$) is _____.

a. $2\pi rT$

b. $2\pi RT$

c. $\frac{2\pi r^2 T}{R}$

d. $\frac{2\pi R^2 T}{r}$

Sol. c.

$$\text{Surface tension force} = 2\pi rT \frac{r}{R} = \frac{2\pi r^2 T}{R}$$

13. If $r = 5 \times 10^{-4} \text{ m}$, $\rho = 10^3 \text{ kg m}^{-3}$, $g = 10 \text{ m/s}^2$, $T = 0.11 \text{ Nm}^{-1}$, the radius of the drop when it detaches from the dropper is approximately _____.

a. $1.4 \times 10^{-3} \text{ m}$

b. $3.3 \times 10^{-3} \text{ m}$

c. $2.0 \times 10^{-3} \text{ m}$

d. $4.1 \times 10^{-3} \text{ m}$

Sol. a.

$$\frac{2\pi rT}{R} = mg = \frac{4}{3}\pi R^3 \rho g$$

14. After the drop detaches, its surface energy is _____.

- a. $1.4 \times 10^{-6} \text{ J}$ b. $2.7 \times 10^{-6} \text{ J}$
c. $5.4 \times 10^{-6} \text{ J}$ d. $8.1 \times 10^{-6} \text{ J}$

Sol. b.

$$\text{Surface energy} = T(4\pi R^2) = 2.7 \times 10^{-6} \text{ J}$$

Problems 15–17:

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

15. A diatomic molecule has moment of inertia I . By Bohr's quantization condition its rotational energy in the n^{th} level ($n = 0$ is not allowed) is _____.

- a. $\frac{1}{n^2} \left(\frac{h^2}{8\pi^2 I} \right)$ b. $\frac{1}{n} \left(\frac{h^2}{8\pi^2 I} \right)$
c. $n \left(\frac{h^2}{8\pi^2 I} \right)$ d. $n^2 \left(\frac{h^2}{8\pi^2 I} \right)$

Sol. d.

$$L = \frac{nh}{2\pi}$$

$$\text{K.E.} = \frac{L^2}{2I} = \left(\frac{nh}{2\pi} \right)^2 \frac{1}{2I}$$

16. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $4/\pi \times 10^{11} \text{ Hz}$. Then the moment of inertia of CO molecule about its centre of mass is close to _____. (Take $h = 2\pi \times 10^{-34} \text{ Js}$)

- a. $2.76 \times 10^{-46} \text{ kg m}^2$
b. $1.87 \times 10^{-46} \text{ kg m}^2$
c. $4.67 \times 10^{-47} \text{ kg m}^2$
d. $1.17 \times 10^{-47} \text{ kg m}^2$

Sol. b.

$$h\nu = kE_{n=2} - kE_{n=1}$$

$$I = 1.87 \times 10^{-46} \text{ kg m}^2$$

17. In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u. = $\frac{5}{3} \times 10^{-27} \text{ kg}$, is close to _____.

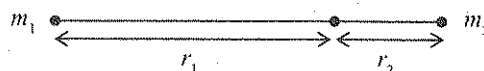
- a. $2.4 \times 10^{-10} \text{ m}$ b. $1.9 \times 10^{-10} \text{ m}$
c. $1.3 \times 10^{-10} \text{ m}$ d. $4.4 \times 10^{-11} \text{ m}$

Sol. c.

$$r_1 = \frac{m_2 d}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 d}{m_1 + m_2}$$

$$I = m_1 r_1^2 + m_2 r_2^2$$

$$\therefore d = 1.3 \times 10^{-10} \text{ m}$$



Match the column type

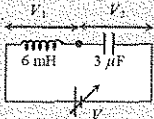
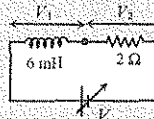
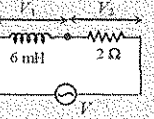
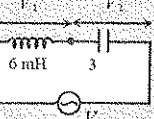
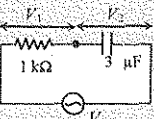
Match the entries in Column I with appropriate options in Column II.

18. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1 , μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II.

Column I	Column II
a. $\mu_1 < \mu_3$	p.
b. $\mu_1 < \mu_2$	q.
c. $\mu_2 < \mu_3$	r.
d. $\mu_2 < \mu_1$	s.
	t.

Sol. a. $\rightarrow$ p., r.; b. $\rightarrow$ q., s., t.; c. $\rightarrow$ p., r., t.; d. $\rightarrow$ q., s.

19. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in Column I. Match the two

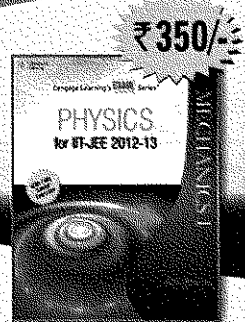
Column I	Column II
a. $I \neq 0$, V_1 is proportional to I	p. 
b. $I \neq 0$, $V_2 > V_1$	q. 
c. $V_1 = 0$, $V_2 = V$	r. 
d. $I \neq 0$, V_2 is proportional to I	s. 
	t. 

Sol. a. $\rightarrow$ r, s, t.; b. $\rightarrow$ q, r, s, t.; c. $\rightarrow$ p, q.; d. $\rightarrow$ q, r, s, t.

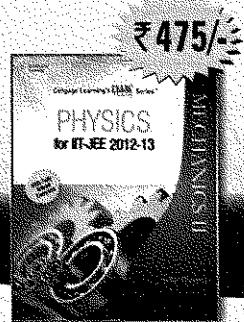
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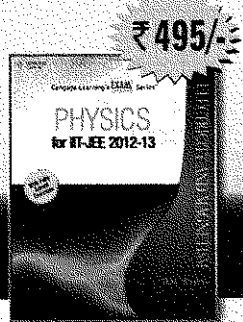
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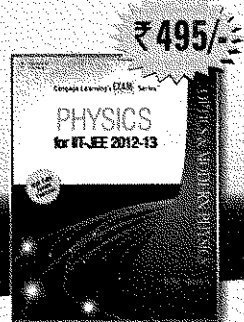
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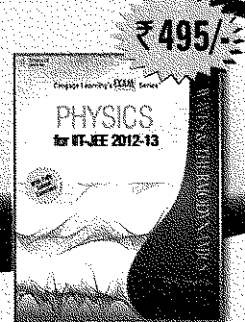
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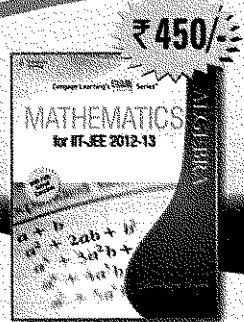


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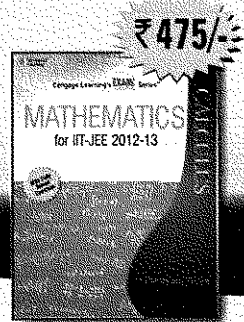


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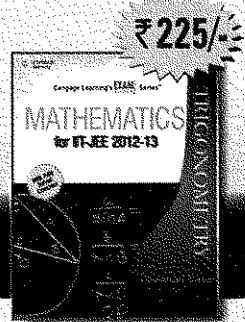
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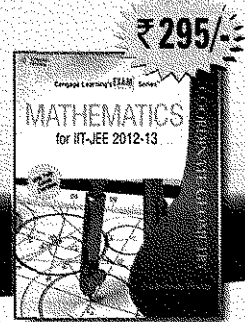
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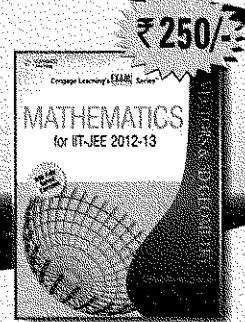
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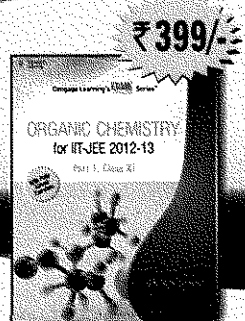


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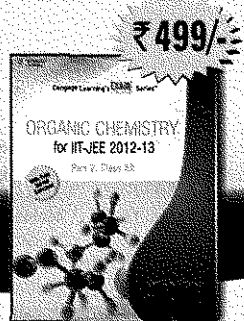


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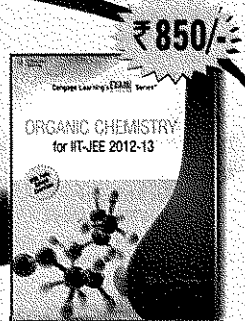
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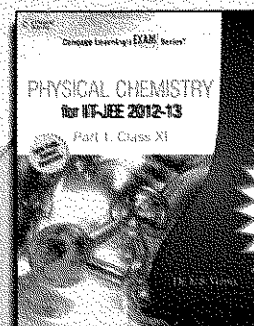
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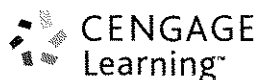
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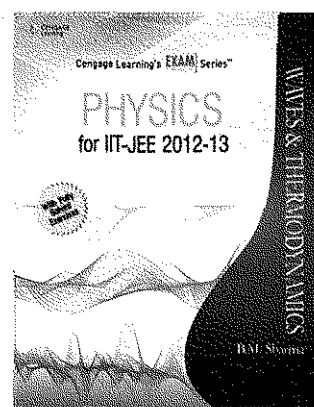
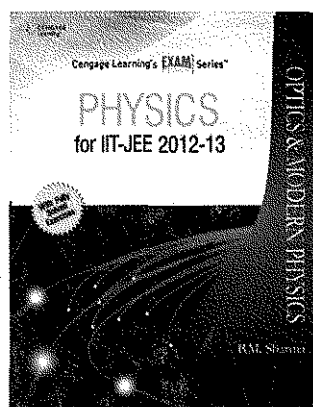
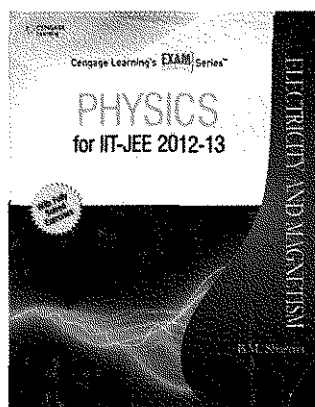
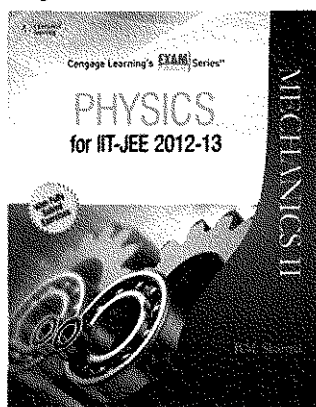
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